



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

University of Wisconsin

LIBRARY

Class **SQB**

Book **C19**

11/20/1914
No. _____
11/20/1914

·
A TREATISE

ON

PLANE SURVEYING.

·

BY

DANIEL CARHART, C.E.,
PROFESSOR OF CIVIL ENGINEERING IN THE WESTERN
UNIVERSITY OF PENNSYLVANIA.



BOSTON:
GINN & COMPANY, PUBLISHERS.
1888.

Entered, according to Act of Congress, in the year 1887, by
DANIEL CARHART,
in the Office of the Librarian of Congress, at Washington.

TYPOGRAPHY BY J. S. CUSHING & Co., BOSTON.
PRESSWORK BY GINN & Co., BOSTON.

21680

627.123

S2E

.C19

PREFACE.

THIS work, as its name indicates, extends over the field of plane surveying. It illustrates and describes the instruments employed, their adjustments and uses; it exemplifies the best methods of solving the common problems occurring in practice, and furnishes solutions for many special cases which not unfrequently present themselves. An experience of twenty years in the field and in technical schools confirms the opinion that a work of this kind should be eminently practical; that the student who desires to become a reliable surveyor needs frequently to manipulate the various surveying instruments in the field, to solve many examples in the class-room, and to exercise good judgment in all these operations. With this in view, therefore, the different methods of surveying are treated, directions for using the instruments are given, and these are supplemented by numerous examples to be solved, by various field exercises to be performed, and by many queries to be answered.

Chapter I. is devoted to Chain Surveying, in which directions are given for measuring and ranging out lines, and methods of overcoming obstacles, recording field notes, obtaining areas, and plotting a chain survey.

Chapter II. treats of Compass and Transit Surveying, or when, in addition to the chain, an instrument for measuring angles is employed. In this chapter the compass and transit, the solar attachment, the adjustments of these, and auxiliaries of the transit, such as the stadia wires, gradienter, etc., are fully illustrated and described, and their uses shown. Here the various methods of obtaining the data requisite to deter-

mine the area, as well as the different methods employed in calculating the contents of land, are exhibited. Tests of the accuracy of a survey are indicated, numerous methods of overcoming obstacles, supplying omissions, of ascertaining heights and distances, of keeping the field notes, and of plotting a survey are given, while the uses of the solar attachment in determining the latitude of a station and its geographic meridian are exemplified.

The student now having been taught how to survey land, using a needle instrument, should become acquainted with the declination of the magnetic needle, or variation of the compass, as it is frequently called. This subject is accordingly discussed in Chapter III. Some of the tables and much of the matter is taken from the Reports of the United States Coast and Geodetic Survey. The student will do well to give this chapter a careful inspection, examining the tables and formulas and the directions for determining the true meridian, thus being prepared with facts, figures, and methods, which will enable him intelligently to undertake the retracing of old lines, as well as to establish with considerable precision his geographic meridian, and thereby obtain the declination of the needle.

Chapter IV. is devoted to Laying Out and Dividing Up Land. This subject is of more importance than some suppose, especially to practitioners in the older States of the Union, and is here treated very fully. The principal cases are exemplified, and general directions and suggestions given, so that, it is believed, with a thorough knowledge of this chapter, the student will be enabled, without embarrassment, to meet the requirements of an extensive practice.

The description, adjustment, and use of the Plane Table form the subject of Chapter V. This instrument is being employed more frequently than formerly in park surveys, in determining positions in harbors, along the lines of proposed highways, in "filling in" large surveys, and generally in locating points where extreme accuracy is not required.

In Chapter VI. the system employed by the government in the Survey of the Public Lands is set forth. The description and adjustment of the Solar Compass, which is used quite extensively in these surveys, precede an account of the origin of the system, and the leading points in the "Instructions to Surveyors-General" from the commissioner of the land office. A form of recording the notes extracted from the "Instructions" is also given, the chapter closing with formulas and a table for determining the inclination of meridians and deviation of parallels.

Chapter VII., on City Surveying, is from the pen of my friend and former colleague, Frederic H. Robinson, C.E., City Engineer of Wilmington, Del. This subject has received but little notice from writers on surveying, although the need of some systematic and practical treatment of it has long been recognized. It therefore affords me much pleasure to acknowledge my indebtedness to Professor Robinson for supplying this want, and so enhancing the value of this publication as a textbook. Experience in teaching, and ten years' practice in city surveys and improvements, eminently qualify him to speak on this important subject with authority and in a manner readily understood by students.

The special instruments needed in this branch of surveying are illustrated and described; the adjustment of the Y-level and directions how to level and to record the notes are given; more refined means of measuring lines are discussed; temperature, pull, sag, wind, etc., are considered, and corrections indicated; best directions and width of streets, together with the subject of grades, sewers, the establishment of permanent reference points, and adjusting property lines, are fully set forth.

To my college classmate and esteemed friend, F. Z. Schellenberg, C.E., Superintendent of Westmoreland Coal Co., Irwin, Pennsylvania, I am indebted for Chapter VIII., on Mine Surveying. This chapter, though in general explanatory of what is applicable and peculiar to this branch of surveying,

includes directions for running contours and sketching topography. It is replete with suggestions that will be valued when, by the aid of the study of mine workings themselves and their ground, illustrations will be afforded which otherwise, as drawings alone, cannot readily be understood.

The Judicial Functions of Surveyors, as given by Chief Justice Cooley, are set forth in an Appendix.

. Those who are familiar with the elegant tables of logarithms of numbers and of trigonometrical functions prepared by Professor Wentworth, will likely recognize the use of his electroplates, from which I have been permitted to print Tables I., III., IV., and VII. To him my personal acknowledgments are due. The plates from which Tables II., V., VI., VIII., and IX. are printed were prepared expressly for this work. It is thought that the *four-place* tables of the natural trigonometrical functions will be found very useful in connection with surveying and engineering operations. They are believed to be correct, having been very carefully compared with others whose accuracy is unquestioned.

In addition to acknowledgments made elsewhere, I take pleasure in expressing here my thanks to Messrs. W. and L. E. Gurley, of Troy, New York, for the use which I have been permitted to make of their valuable catalogue, in the description of certain instruments, and for the loan of several plates for the engraving of instruments; also to Messrs. Fauth and Co., Washington, D.C., and to Messrs. Heller and Brightly, and Messrs. Young and Sons, Philadelphia, Pa., for plates which they kindly furnished for the illustration of the subject.

D. C.

WESTERN UNIVERSITY OF PENNSYLVANIA,
DECEMBER, 1887.

CONTENTS.



| | PAGE |
|---|------|
| Definitions, and division of the subject..... | 1 |

CHAPTER I.

CHAIN SURVEYING.

SECTION I.

INSTRUMENTS.

| | |
|------------------------------|---|
| Gunter's chain..... | 3 |
| Two-pole or half chain | 3 |
| The engineer's chain | 4 |
| The tape measure..... | 4 |
| Marking-pins..... | 4 |
| Straight poles | 4 |

SECTION II.

CHAINING.

| | |
|--|--------|
| How to chain | 4 |
| Tallying | 5 |
| Error in chaining | 6 |
| Chaining on sloping ground..... | 8 |
| Field exercises..... | 10 |
| Ranging out lines..... | 10 |
| Over a hill ; across a valley..... | 11 |
| Through a wood | 12 |
| Field exercises..... | 13 |
| To set off a perpendicular from a line | 13, 14 |
| To let drop a perpendicular from a point to a line | 14, 15 |
| Through a point to run a line parallel to a given line | 16, 17 |
| Obstacles to alignment | 17, 18 |

| | PAGE |
|------------------------------------|-------|
| Obstacles to measurement | 18-20 |
| Measurement of heights | 20 |
| Examples and field exercises | 21 |

SECTION III.

RECORDING THE FIELD NOTES.

| | |
|------------------|-------|
| By sketch | 22 |
| In columns | 23-25 |

SECTION IV.

MAPPING AND PLOTTING.

| | |
|--|--------|
| Instruments useful for plotting a chain survey | 26 |
| Drawing-board, T-square, triangles, etc. | 26, 27 |
| Scales | 28 |
| Drawing to scale | 29 |
| To ascertain unknown scale | 30 |

SECTION V.

ON AREAS, AND ILLUSTRATIVE EXAMPLES.

| | |
|--|----|
| Areas : The area of a triangle | 31 |
| " " rectangle | 31 |
| " " parallelogram | 31 |
| " " trapezoid | 31 |
| " " regular hexagon | 31 |
| " " " octagon | 31 |
| " " " polygon | 32 |
| Table of areas of regular polygons | 33 |
| The area of a circle | 33 |
| " " quadrant | 34 |
| " " sextant | 34 |
| " " circular ring | 34 |
| " " segment | 34 |
| " " ellipse | 35 |

EXAMPLES INDICATING HOW TO SURVEY LAND, TO PLOT THE SURVEY,
AND TO COMPUTE THE AREA.

| | |
|----------------------------|--------|
| To survey a triangle | 35, 36 |
| " " rectangle | 37 |
| Examples | 37 |

CONTENTS.

ix

| | PAGE |
|---|--------|
| To survey a parallelogram | 38 |
| " " trapezoid | 38 |
| Examples | 38, 39 |
| To survey, plot, and compute the area of a trapezium | 39 |
| Examples | 40 |
| To survey, plot, and compute the area of a polygon, regular or irreg- ular | 40, 41 |
| Examples | 41, 42 |
| To survey, plot, and compute the area of a circle and a circular ring. | 43 |
| " " " " " " sector and a segment | 43 |
| Examples | 43 |

SECTION VI.

OFFSETS AND TIE-LINES.

| | |
|---|--------|
| Remarks and illustrations | 44 |
| Rectangular co-ordinates and their application to the computation of areas | 45 |
| Formula and rule | 46 |
| Examples | 47 |
| Additional formulas and rule | 48 |
| Examples | 48-50 |
| Tie-lines used to survey a lake or pond | 51 |
| Miscellaneous examples | 51, 52 |
| Field exercises | 53 |

CHAPTER II.

COMPASS AND TRANSIT SURVEYING.

SECTION I.

DEFINITIONS AND DESCRIPTION OF INSTRUMENTS.

| | |
|--|--------|
| A meridian plane; meridian line; the magnetic needle; the magnetic meridian | 54 |
| The azimuth, bearing, and meridian distance of a line | 55 |
| Horizontal angle; vertical angle; angle of elevation; angle of depres- sion | 55 |
| The surveyor's compass | 55, 56 |
| To adjust the compass | 56 |
| Engraving of compass | 57 |
| Caution when handling the compass | 59 |

| | PAGE |
|---|-------------|
| To re-magnetize the needle | 60 |
| Weight of compass | 60 |
| The vernier | 61 |
| Formulas for determining the least count..... | 61 |
| How to space a vernier for a given least count..... | 62 |
| To read an instrument having a vernier..... | 62 |
| Exercises on designing verniers | 62 |
| Engraving of surveyor's transit | 64 |
| Description of transit, and section of telescope and cross-wires | 65, 66 |
| Sectional view through the spindle of transit | 67 |
| The needle, tangent movements, levels, and verniers..... | 68, 69 |
| The sockets, levelling-plate, and tripod..... | 70, 71 |
| To adjust the transit..... | 71-75 |
| To use the transit..... | 76 |
| Transit attachments and their adjustments | 77, 78 |
| The solar attachment | 79, 80 |
| Engraving of solar attachment (Gurley's)..... | 81 |
| The adjustment of the solar attachment..... | 83-85 |
| To find the latitude by means of the solar attachment | 85 |
| To run lines with the solar attachment..... | 87 |
| Saegmuller's solar attachment and its adjustments | 88 |
| Engraving of Saegmuller's attachment..... | 89 |
| Latitude by means of Saegmuller's attachment | 91 |
| Latitude by a circumpolar star..... | 91 |
| Table of refraction..... | 92 |
| To find the meridian and declination of the needle, using the attach- ment | 93 |
| Time and azimuth with the solar attachment | 94 |
| Error in declination or latitude causing an error in azimuth | 94 |
| Table of errors in azimuth for one-minute error in latitude or declina- tion | 95 |
| The stadia or micrometer..... | 95, 96 |
| Formula when stadia is perpendicular to level line of sight | 97 |
| Formula when line of sight is not level..... | 99 |
| Examples | 100 |
| Gradiometer | 100, 101 |

SECTION II.

BEARINGS WITH COMPASS, AND ANGLES WITH TRANSIT.

| | |
|--|-----|
| To obtain the bearing of a line | 102 |
| Reverse bearing; local attraction..... | 103 |

CONTENTS.

xi

| | PAGE |
|---|---------|
| Proof-bearings and tests of accuracy | 104 |
| Suggestions and field exercises | 105 |
| To measure angles with the transit, by repetition | 106 |
| " " " " by series; remark | 107 |
| Deflection angle; traversing, or surveying by the back angle. | 108-110 |
| To traverse a road or a small stream | 111 |
| Problems on bearings and angles | 112-115 |
| To change the bearings of the sides of a survey | 116 |
| Examples and field exercises | 117-119 |

SECTION III.

PROBLEMS ON PERPENDICULARS AND PARALLELS.

| | |
|--|----------|
| To let fall a perpendicular from a given point to a line | 120, 121 |
| To prolong a line past an obstacle (various methods) | 121-124 |
| To measure a line past an obstacle (various methods) | 124-127 |
| Examples and field exercises | 127, 128 |

SECTION IV.

HEIGHTS AND DISTANCES.

| | |
|--|----------|
| Measurement of accessible heights | 129 |
| Examples | 130 |
| Measurement of inaccessible heights | 130-132 |
| Examples | 132, 133 |
| Inaccessible distances | 135 |
| The three-point problem | 136-138 |
| Miscellaneous problems and field exercises | 140, 141 |

SECTION V.

RECORDING THE FIELD NOTES.

| | |
|------------------------------|---------|
| Tabular form | 142 |
| By sketch | 142-144 |
| In columns | 147-149 |
| Large plot of a survey | 153 |

SECTION VI.

LATITUDES AND DEPARTURES.

| | |
|---|----------|
| Definition of latitude and departure | 154 |
| Formulas and examples on latitudes and departures | 155 |
| The traverse table and explanation of the same | 156, 157 |

| | PAGE |
|--|----------|
| The use of the traverse table exemplified..... | 157, 158 |
| Table of sines and cosines employed; examples..... | 159 |
| Testing a survey..... | 160 |
| Correcting latitudes and departures..... | 161 |
| “ “ “ “ sides weighted..... | 165 |

SECTION VII.

SUPPLYING OMISSIONS.

| | |
|--|-----|
| General observations on supplying omissions | 166 |
| Determination of one side | 167 |
| “ “ the length of two sides..... | 169 |
| “ “ bearing of two sides | 172 |
| “ “ bearing of one side and length of another..... | 174 |

SECTION VIII.

PLOTING A COMPASS OR TRANSIT SURVEY.

| | |
|-----------------------------------|----------|
| Using the protractor | 178, 179 |
| By latitudes and departures | 180, 181 |
| Using cross-section paper | 182 |

SECTION IX.

ON DETERMINING AREAS.

| | |
|--|----------|
| Determining the area of triangles and parallelograms | 183 |
| “ “ “ trapezoids and trapeziums | 184 |
| Examples | 185 |
| Determining the area of polygons | 186 |
| General method for determining the area of any rectilinear figure..... | 187-190 |
| Examples | 191-193 |
| Method by total latitudes and departures..... | 193, 194 |
| Examples | 194-196 |
| Determining areas when offsets are taken | 196, 197 |
| Field exercises | 197, 198 |

CHAPTER III.

DECLINATION OF THE NEEDLE, OR VARIATION OF THE COMPASS.

| | |
|----------------------------|-----|
| Irregular changes..... | 199 |
| The diurnal variation..... | 200 |

CONTENTS.

xiii

| | PAGE |
|---|----------|
| Table for reducing observed declinations | 201 |
| The secular variation | 201 |
| Line of no declination, or agonic line | 201, 202 |
| Formulas expressing the magnetic declination | 203 |
| Tables exhibiting annual variation, etc. | 206 |
| Effects of the secular change | 208 |
| Rule for obtaining bearings when variation is known | 209 |
| Change determined by old lines | 210 |
| Examples | 211, 212 |
| To obtain the true bearing of a line | 212 |
| To determine the declination, by Polaris | 214 |
| Tables of elongation and azimuth of Polaris | 216, 217 |
| To establish a true meridian with a transit | 218 |
| To obtain approximately the meridian | 219 |

CHAPTER IV.

LAYING OUT AND DIVIDING LAND.

SECTION I.

LAYING OUT LAND.

| | |
|--------------------------------------|----------|
| Triangles; examples | 221-223 |
| Squares | 223 |
| Rectangles; examples | 224, 225 |
| Parallelograms; examples | 225, 226 |
| Polygons; examples | 227, 228 |
| Circles and ellipses; examples | 228-231 |
| Additional problems | 232 |

SECTION II.

DIVIDING LAND.

| | |
|---|---------|
| Triangles, various cases; numerous examples | 232-240 |
| Trapezoids, " " | 241-246 |
| Trapeziums, " " | 246-252 |
| Polygons, " " | 253-258 |
| Irregular boundary | 258 |
| Straightening boundary lines | 258 |
| Miscellaneous examples | 259-261 |

CHAPTER V.

PLANE TABLE SURVEYING.

| | PAGE |
|--|----------|
| The plane table | 262 |
| Engraving of plane table | 263 |
| The alidade and declinator | 265 |
| Adjustments | 266 |
| Surveying by radiation, progression | 267, 268 |
| " " intersection, resection | 269 |
| The three-point problem | 270 |
| Bessel's method by inscribed quadrilateral | 270 |
| The two-point problem | 271 |
| Practical suggestions | 273 |
| Field exercises | 274 |

CHAPTER VI.

THE SURVEY OF THE PUBLIC LANDS.

| | |
|---|----------|
| The solar compass | 275 |
| The latitude and declination arcs | 276 |
| Engraving of solar compass | 277 |
| The hour arc and polar axis | 279 |
| Principles of the solar compass | 280 |
| Allowance for declination | 281 |
| To adjust the solar compass | 283-285 |
| To use the solar compass | 285 |
| To set off the declination | 285, 286 |
| Allowance for refraction | 287 |
| Table of refractions in declination | 288, 289 |
| Explanation of the table of refractions | 290 |
| Problems in declination and refraction | 290, 291 |
| To set off the latitude | 291 |
| To run lines with the solar compass | 292 |
| Allowance for the earth's curvature | 293 |
| Time of day by the sun | 294 |
| Caution as to the false image | 294 |
| Approximate bearings, and time for using solar compass | 295 |
| Origin of the system for the survey of the public lands | 295 |
| Plan of a township, showing sections | 296 |

CONTENTS.

XV

| | PAGE |
|---|----------|
| System of rectangular surveying | 297 |
| Conformity to the meridian required..... | 298 |
| How to survey section lines ; plan of township..... | 299 |
| An instrument operating independently of needle required..... | 300 |
| The four-pole chain to be adjusted to 66.06 feet..... | 301 |
| Process of chaining | 302 |
| Marking lines | 303 |
| Obstacles in line ; witness points | 304 |
| Establishing corners | 304 |
| Miscellaneous directions..... | 305, 306 |
| Quarter-section corners ; witness corner | 307 |
| Meandering | 307 |
| Base line | 308 |
| Principal meridian ; standard parallels | 309 |
| Auxiliary meridians ; township lines..... | 310 |
| Method of subdividing townships..... | 311, 313 |
| Subdivision of sections | 314-316 |
| Re-establishment of lost corners..... | 316 |
| Objects required to be noted | 316, 317 |
| Specimen field notes | 318-321 |
| Inclination of the meridian | 322, 323 |
| Convergency of meridians ; deflection of range-lines... | 324 |
| Table of inclination and convergency of meridians | 325 |

CHAPTER VII.

CITY SURVEYING.

| | |
|---------------------|----------|
| Introduction | 326, 327 |
|---------------------|----------|

SECTION I.

FIELD INSTRUMENTS, THEIR ADJUSTMENTS AND GENERAL USE.

| | |
|---|----------|
| Transit, engraving of, showing gradienter, etc..... | 328, 329 |
| Rods, steel tapes, etc. | 331, 332 |
| Locke's hand-level | 333 |
| Effects of temperature, sag, and wind considered..... | 334 |
| Corrections for these ; illustrations | 335, 336 |
| Suggestions on measurement..... | 336 |
| The Y-level | 337 |
| Engravings of Y-level..... | 338, 339 |
| Adjustments of the level | 340-343 |

| | PAGE |
|--|----------|
| To use the level | 344 |
| Engravings of New York and Philadelphia levelling-rods | 345 |
| Description of " " " " " " | 347, 348 |
| The rod-level; engraving of same | 349 |
| Levelling defined | 349, 350 |
| Curvature and refraction considered | 350, 351 |
| Exercises; levelling by fore and back sights | 352, 353 |
| Levelling, general method | 354, 355 |
| Form of record; observations on bench marks | 356 |
| General method of running a grade line | 357 |
| Office instruments | 358 |

SECTION II.

A. FIELD WORK.

| | |
|---|----------|
| Public work; street lines; direction and width of streets | 358, 359 |
| The alignment, how made | 360 |
| Preservation of points in line | 361 |
| Street grades; the profile | 362, 363 |
| Record of levels for profile | 365 |
| Marking of lines and grades | 366 |
| Private work | 366 |
| Diagram of lot and block | 367 |
| Locating lot corners from descriptions in deed | 369 |
| How to proceed when obstacles intervene | 370 |

B. OFFICE WORK.

| | |
|--|----------|
| Public work; plans, profiles; exercise | 372, 373 |
| Determination of grades | 374 |
| Diagram | 375 |
| Drainage | 377 |
| Private work | 378 |
| Conclusion; books | 378, 379 |

CHAPTER VIII.

MINE SURVEYING.

| | |
|--|----------|
| Its purpose | 380 |
| Instruments employed in making the alignment | 381, 382 |
| Stations, where made, and how designated | 382 |
| Angles; initial course; reduced courses | 382 |

CONTENTS.

xvii

| | PAGE |
|---|----------|
| Curves, how laid out | 383, 384 |
| The record | 385 |
| Mapping and plotting ; inside and outside work | 385, 386 |
| The use of the level and rod | 387, 388 |
| The use of the vertical circle | 388, 389 |
| Transference of points from surface to bottom of shaft | 389 |
| Special appliances for taking courses on pitches at high angles | 390 |
| The hanging compass and hanging clinometer | 390, 391 |
| Topography, its use, how taken | 391, 393 |
| Employment of the plane table | 393 |
| Contour lines | 393 |
| Angular cross-sectioning | 394 |
| Transit as first made in 1831 | 396 |

APPENDIX.

| | |
|--|---------|
| The judicial functions of surveyors, | 397-411 |
|--|---------|

TABLES.

| | |
|--|--------|
| Logarithms of numbers | 1-19 |
| Approximate equation of time | 20 |
| Logarithms of trigonometric functions | 21-49 |
| For determining with greater accuracy than by the preceding | 50, 51 |
| Lengths of degrees of latitude and longitude | 52 |
| Miscellaneous formulas, and equivalents of metres, chains, and feet, | 53 |
| Traverse | 54-61 |
| Natural sines and cosines | 62-70 |
| Natural tangents and cotangents | 71-79 |

For tables in the body of the book, see "Tables" in Index.

SURVEYING.

DEFINITIONS, AND DIVISION OF THE SUBJECT.

1. Surveying is the art of determining and delineating the relative position of points upon the surface of the earth. It consists principally in measuring, laying out, and dividing land ; in establishing lost positions ; in the measurement of heights and distances ; and in the graphical representation of the peculiarities of any part of the earth's surface.

2. It may be divided into two parts : **PLANE SURVEYING** and **GEODETIC SURVEYING**.

In **Plane Surveying** the spherical form of the earth is neglected ; in other words, the portion of the earth included in the survey is regarded as a horizontal plane. This may be done without sensible error where, as in ordinary land surveying, the operations are limited to surfaces of small extent.

In **Geodetic Surveying** the shape of the earth is regarded, since the surfaces under consideration are so extensive, as in the United States Coast and Geodetic Surveys, sensible errors would otherwise arise.

REMARK. The spherical excess of a spherical triangle, each of whose sides is one mile, is less than six-thousandths of a second. The excess amounts to only one second for an area of 75.5 square miles, each side of the equilateral triangle being then about 13 miles.

3. In the following pages Plane Surveying only will be considered, and the subject treated under the following heads :

CHAIN SURVEYING.

COMPASS AND TRANSIT SURVEYING.

PLANE TABLE SURVEYING.

GOVERNMENT SURVEYING.

CITY SURVEYING.

MINE SURVEYING.

In Plane Surveying there are usually three operations :

- 1. *The Field Work.***
- 2. *The Graphical Representation, or Plot.***
- 3. *The Computation.***

CHAPTER I.

CHAIN SURVEYING.

SECTION I.

INSTRUMENTS.

4. Chain Surveying has chiefly for its object the determination of areas from data obtained by direct measurement of distances between points. The instruments needed are therefore simply those for measuring *lines*.

5. Gunter's Chain, so called from its inventor, is generally used for this purpose. It is made of iron or steel wire, is 66 feet in length, and divided into 100 links, so that each link, with half the rings connecting it with the adjoining links, is seven and ninety-two hundredths inches (7.92), or one-hundredth of a chain. Swivels are inserted to keep it from twisting, and every tenth link has a metallic mark attached, so that the number of tens from either end is readily ascertained. Its advantages in surveying farms or fields are apparent: there being 4840 square yards in an acre, and the chain 22 yards long, a square chain will contain one-tenth of an acre; or, there being 10,000 square links in a square chain, which is one-tenth of an acre, 100,000 square links are equivalent to an acre. Hence, if the area of a field is calculated in links, the area is at once shown in acres, by cutting off the last five figures. If the area is found in chains, then since there are ten square chains in an acre, the area is given in acres by cutting off the last figure.

6. A Two-Pole, or Half-Chain is sometimes used instead of Gunter's Chain. It is quite convenient for measuring lines where the ground is rough and hilly.

7. The Engineer's Chain is used in surveying railroads and canals, and generally where extensive line surveys are being conducted; hence not unfrequently it is employed in connection with these surveys, as well as otherwise, in determining areas. It is 100 feet in length, and is divided into 100 links, every tenth link being marked by a piece of brass, as in the four-pole chain.

8. The Tape Measure is very convenient for taking offsets in a survey, for measuring the boundaries of city lots, cross-sectioning in railroad work, etc. Tapes are "metallic," or steel, and made of various lengths,* — 50 feet or 100 feet are commonly used, — and divided into feet and inches, or feet and tenths of a foot. The latter graduation is preferable for the railroad engineer, and the former for the city engineer.

9. Eleven Marking-Pins, 12 or 14 inches long, one of which is made of brass, the others of No. 4 iron wire or No. 6 steel, all pointed at one end and formed into a ring at the other, are used in chaining.

10. Straight Poles about 8 feet long, shod at the bottom with a conical shoe, point down, and painted alternately red and white in foot-width bands, are used to indicate the direction of the line which is being measured, or the position of points to be located.†

SECTION II.

A. CHAINING.

11. Two men are required, a "leader" and a "follower," or *head* and *hind* chainman. The chain is first thrown out in the general direction of the line which it is desired to measure, and

* Steel tapes 1000 feet in length have been frequently used for special purposes. See *Mine Surveying*, p. 380.

† See Article 383.

examined carefully to see if there are any kinks in it, or bends in the links; the leader having the marking-pins in one hand takes hold of the forward end of the chain with the other, and moves on as nearly as he may judge in the direction of the line; the follower places the rear end of the chain at the station whence the line is to be measured, directs the leader by signals as he approaches the chain's length to get in *line*, and then calls, "halt"; then the chain must be drawn taut and straight, and the follower having his end of the chain precisely at the starting-point, calls out, "down"; the leader then thrusts one of the iron marking-pins into the ground exactly at the end of the chain and calls out, "down," which is the signal to the follower to advance: proceeding as before until the second length of chain is measured, which is indicated by the follower coming to the pin set in the ground by the leader, when the follower cries, "halt," and after placing his end of the chain at the pin, the chain having been drawn taut and straight as before, calls, "down"; the leader, as before, leaving a pin to mark the end of the chain, repeats, "down"; the follower then takes up the pin first placed by the leader, and moves on; thus the party proceeds until the end of the line is reached, the leader placing the pins at his end of the chain, and the follower picking them up at his end.

If the line ends with less than the length of the chain, the leader places his end at the point which marks the extremity of the line, calls out, "down"; the follower then reads off the number of links between the last pin and the end of the line. The number of whole chain's length of the line is shown by the pins in the hands of the follower, and the number of links counted off added thereto will give the total length in chains and links.

12. Tally. If the line exceeds eleven chains in length, a transfer of pins from the hind chainman to the head chainman is necessary; this is called *tallying*, and is performed in the following manner: At the end of the eleventh chain, the *brass*

pin — the last pin left in the hands of the leader — is placed, when he call out “tally”; at this signal the follower drops his end of the chain, advances to the leader, counts over with him the ten iron pins which he has gathered up, and transfers them to the leader, who then withdraws the brass pin, sets an iron one in its place, and the measuring is continued as before.* Each tally should be recorded, especially when chaining very long distances, to avoid error in the final count.† It is obvious that the total length of the line will be equal to the chains and links as indicated above, plus the number of tens shown by the tallies.

13. The surveyor should guard against error in chaining, by frequently testing his chain, to see that it is of the proper length, — if it has been stretched, make a file mark showing its true length, — and when in use, see that it is drawn straight, that the forward chainman sticks the pin in line *exactly* at the end of the chain, or at the mark indicating its true length, and as nearly vertical as possible; ‡ and when obtaining the number of links at the end of the line, see that they are not counted

* Some surveyors use only ten marking-pins, and tally by marking the end of the eleventh chain with a pencil, the finger, or a scratch on the ground, and when the ten pins are transferred to the leader, one of them is thrust in the place thus indicated, and the work is continued as before.

† In chaining long distances where there are several tallies, the leader and follower may, at each tally, change places, and thereby lessen the liability to error in the final count. See Articles 352, 353.

‡ “It has been found by many trials with as good men as can generally be obtained, that with two sets of chainmen instructed alike in the proper manner of keeping their chain level and straight on the line, and of setting the tally pins plumb, as well as holding the ends of the chain to them, a difference has sometimes been made of 36 links, and an average difference of 15 or 16 links to a mile in common timbered land.” — *Burt*, “Government Surveying,” p. 35.

The surveyor should have laid down by means of a standard steel tape or otherwise, in a convenient place, and between permanent marks in the ground or on the floor of a large hall, the exact length of a standard chain by which he could test his chain from time to time.

from the wrong end of the chain, nor the wrong way from the brass mark.

The pull on the chain, when in use, has a tendency to increase its length; and moreover, since there are a great number of wearing surfaces, if each of these be worn by an extremely small amount, the chain will be considerably elongated.

In either the surveyor's or engineer's chain there are two small links which connect with the two pieces of wire which form the principal part of what is called the *link* of the chain, thus giving six wearing surfaces to every link; therefore, if each of these surfaces wears only .005 of an inch, the chain will be increased in length *three inches*, so that in measuring only a quarter of a mile with a four-pole chain, the error from this cause alone would be *five feet*,* making an error in area of about 4.9 acres in a tract one mile square. This stretching of the chain is partially compensated by the difficulty, and often impracticability, of drawing the chain *precisely straight*; and so long as the chain is not elongated beyond one-tenth or one-twelfth of one per cent of its length, it may be relied on for accurate work.†

The true length of a line which has been measured by a chain stretched beyond the standard length may be found from the proportion:

The length of standard chain : the length of chain used
:: the distance measured : the true distance.

* This error, it is perceived, increases directly with the number of applications of the chain: it is called *cumulative*. The error arising from erroneous setting of the pin is termed *compensative*, that is, it is as likely to be additive as subtractive, and it is shown by the Method of Least Squares, that for this class of errors the square root of the number of errors are probably not compensated. If the error in setting is one inch, in chaining a mile with a Gunter's chain, the probable error would be $\sqrt{80}$ = about 9 inches.

† To remove the difficulty of drawing the chain perfectly straight, the instructions issued from the United States Land Office, 1880, to Government Surveyors-General, states that the 66 feet chain must be 66.06 feet. See p. 301.

For example, if, with a chain stretched one link over the standard, a line be measured for 2000 feet, we should have

$$100 : 101 = 2000 : 2020, \text{ the true distance.}$$

In like manner, for the area of a tract measured with a stretched chain :

The square of the length of the standard chain
 : the square of the length of the chain used
 :: the computed area
 : the true area.

If the chain was stretched one link, as in the above example, and the area computed therefrom 20 acres, we should have

$$100^2 : 101^2 = 200 \text{ sq. chs.} : 204.02 \text{ sq. chs. for the true area} \\ = \frac{100}{101} \text{ of the computed area, nearly.}$$

In general, if A = true area, A_1 = computed area, L = length of chain, and dL = error in its length (always small). Then $A : A_1 = (L \pm dL)^2 : L^2$.

Reducing and rejecting d^2 as inconsiderable, there results $A = (1 \pm 2d)A_1$; or, the correction to be applied to obtain the true area $= 2dA_1$.

This correction is additive when the chain is too long, which is the usual case, and subtractive when the chain is too short.

14. The surfaces to be measured are in general uneven and broken, not plane; but however great the inequalities, the area of a tract is considered to be that part of the horizontal plane which is intercepted by vertical planes through its boundaries.* The horizontal distance is therefore required; hence, when the

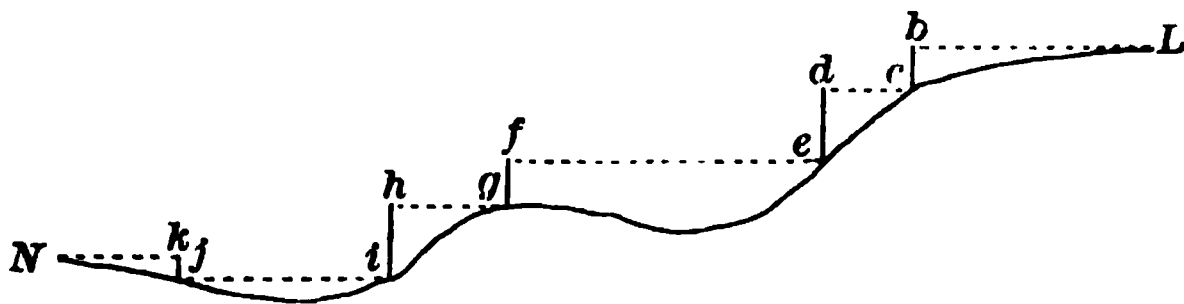
* A vertical line is a line directed to the centre of the earth, or it is a line having a plummet freely suspended to it, and at a state of rest; a *plumb line*.

A vertical plane is a plane embracing a vertical line.

A horizontal line is a line perpendicular to a vertical line.

A horizontal plane is a plane perpendicular to a vertical line.

ground slopes, it is necessary to raise the down-hill end of the chain. If the slope is considerable, only a part of the chain should be used. For example, to measure from L down to N , the follower holds one end of the chain at L , while the leader, stretching the other towards N , takes as much of it as he can raise to a horizontal position b , and, holding a plummet there, fixes the point c ; the follower, who is now signalled to come forward, places at c that point in the chain whence the plummet was suspended to fix c , while the leader advances and, using as much of the chain as possible, locates e , and so on: when the end of the chain is reached, a pin should be transferred



from the leader to the follower. Where great accuracy is not required, a marking-pin or pebble may be dropped to indicate the points c , e , etc.* To measure up hill from N to L is less accurate, on account of the difficulty experienced by the follower in holding his end of the chain at the points h , f , d , etc., over their counterparts, i , g , e , etc.

When chaining steep hills, especially if through a wood or over rough, rocky ground, the work may be greatly facilitated by an extra chainman. He may assist in getting line, straightening the chain, noting the points c , e , etc., marked by the plumb bob, and other duties.†

* If in connection with the chain a survey is being made with an instrument for measuring angles, — vertical and horizontal, — the inclination of a slope may be observed, and the length of it measured; then the horizontal distance required will be equal to the measured distance multiplied by the natural cosine of the angle of inclination.

† For extreme accuracy in measuring lines, see Chapter VII. Article 389.

EXERCISES.

1. Set two marks on gently undulating ground and about 1000 feet apart, and measure forward and back between these points several times; the same party once at least each way.

2. The same between points on hilly and, if possible, bush land.

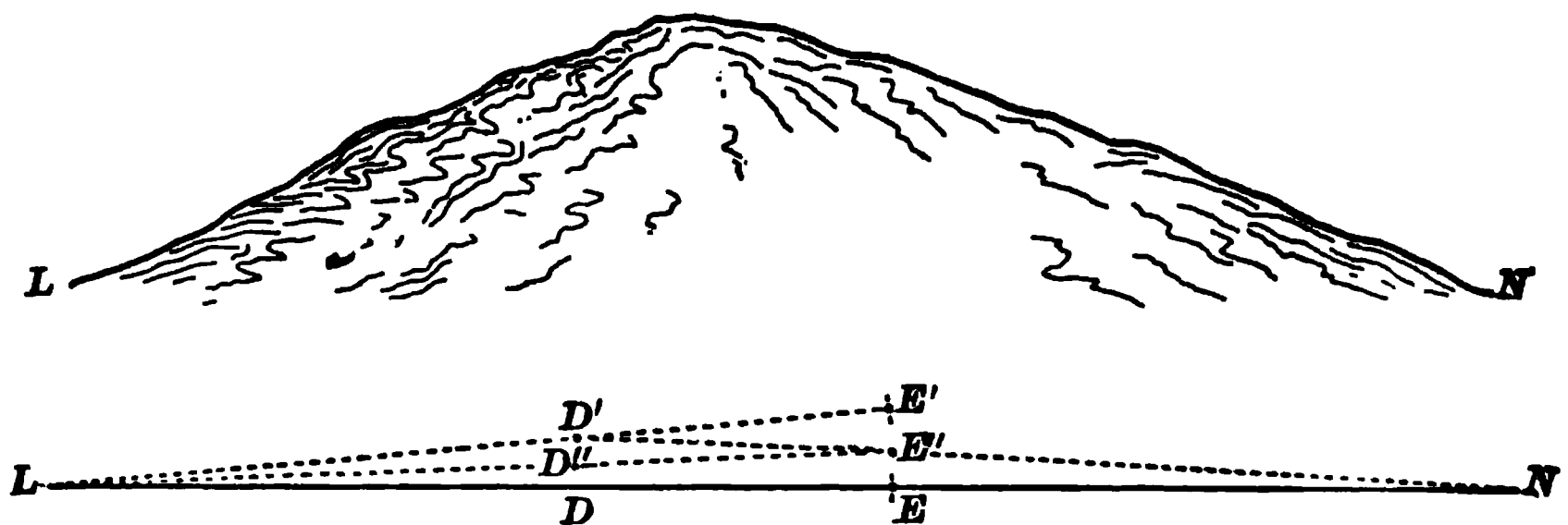
3. Chain down a steep hill, and chain up between the same points.

B. RANGING OUT LINES.

15. If in chaining any line, as LN , from L toward N , a rod at N can be constantly seen by the rear chainman, he can keep the leader in line by ranging him with L ————— N the flagstaff at N . If, however, a hill intervenes, a valley, or brush or woodland interfering with the alignment, then the line must be first *ranged out* or points determined in it before the chaining can be performed.

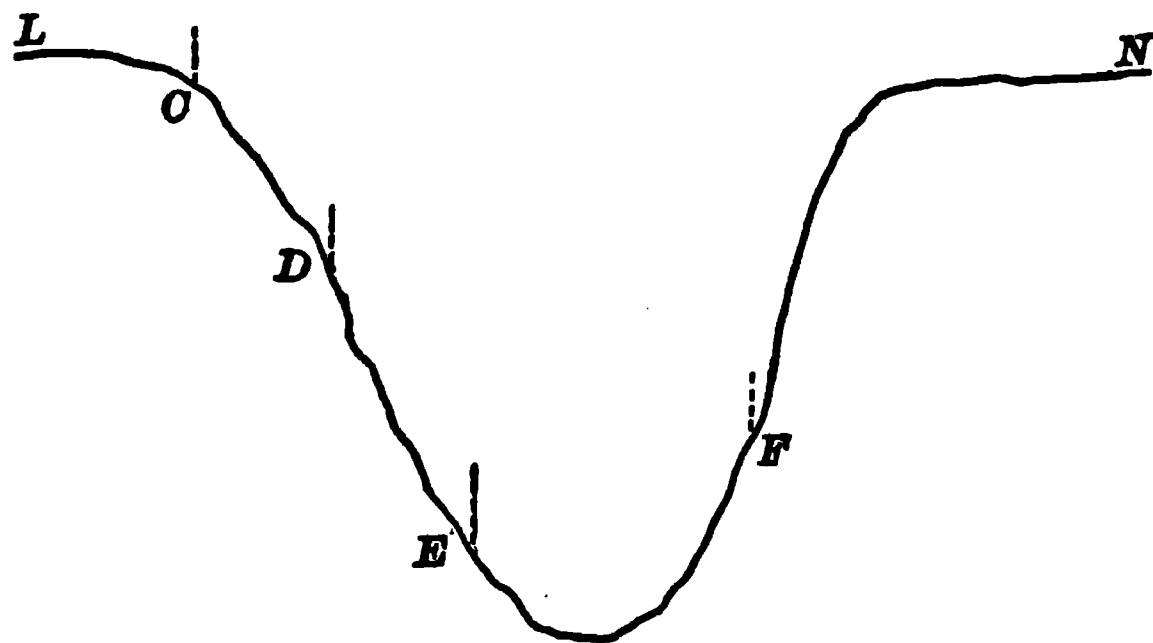
16. **Ranging out a Line.** To range out a line requires three persons, each having a rod eight or ten feet long, and a plummet to indicate when his rod is vertical. Calling these men A , B , and C , and supposing A and B in the line, C goes forward, and sighting back to A and B , puts his rod in line; A then advances beyond C and sets his rod in line with C and B ; next B advances and places his rod in line with C and A , and so on the line may be extended any desired length. If, as frequently is the case, one of the party has had more experience or is naturally better qualified for *sighting a line*, the best results would be obtained by such an one setting all the rods; for example, C would place his rod in line, then call up A , to whom he would turn over the rod just set, and go forward to line the next; after which call up B , exchange rods with him, and so on.

17. Over a Hill. To fix points in a line over a hill, both ends of which are visible from points near the summit, proceed as follows :



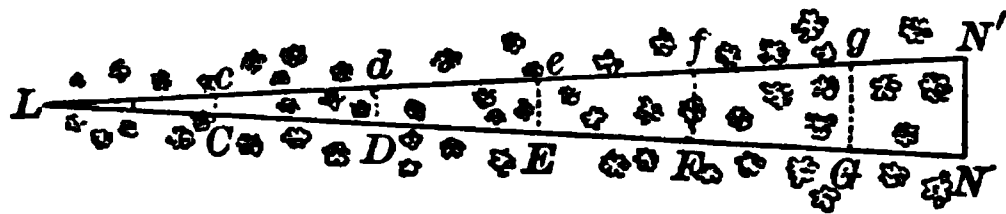
Place a flagstaff at L , another at N . A man at E' signals one at D' in line with L ; D' then directs E' to E'' in line with N ; and so on alternately, until the men are at D and E in the line LN .

18. Across a Valley. To locate points in a line, the ends of which may be seen from each other, but which are separated by a wide, deep valley.



Fix a point C in line with LN ; then a man holding a plumb line at C , and sighting N can direct the setting of the stakes D , E , F , and others.

19. Through a Wood. In chaining through a wood or thick brush land, where the ends cannot be seen from each other, a line* is measured as nearly as may be in the direction of the desired line, and stakes driven every two or three chains, or oftener if necessary. When the end of the line is reached, the distance to the corner is measured, and, by proportion, the amount to move each stake to bring it into line is determined.



For example, let LN be the true line, and LN' the measured line; c, d, e , etc., points three chains apart. Now, if the length LN' equals 17.40 chains, and NN' measured at right angles to $LN' = 35$ links, LN^\dagger will equal

$$\sqrt{LN'^2 + NN'^2},$$

and

$$LN'(1740 \text{ links}) : NN'(35 \text{ links}) \\ = Lg (1500 \text{ links}) : gG (30 \text{ links});$$

or 30 links from g at right angles to LN' will indicate the position of G , a point in the true line LN .

$$1740 : 35 = 1200 : 24, \text{ the distance } fF,$$

$$1740 : 35 = 900 : 18, \text{ the distance } eE;$$

and so on.

Or, after finding the first distance to set off, either gG or cC , the others are readily obtained by taking a proportional part of this distance, shown by the several divisions of the line thus: gG represents the fifth division, fF the fourth, eE the third, and so on; hence, if gG is 30 links, fF will be $\frac{4}{5}$ of 30, or 24,

* Called a *random line* or *trial line*.

† If the distance NN' is a small per cent of the total length of the line, the shortest distance between the ends of the lines may be taken for NN' , and the length of the measured line for that of the true line. See Article 177.

links; eE , $\frac{2}{3}$ of 30, or 18; dD , $\frac{2}{3}$ of 30, or 12; and cC , $\frac{1}{3}$ of 30, or 6 links.

EXERCISES.

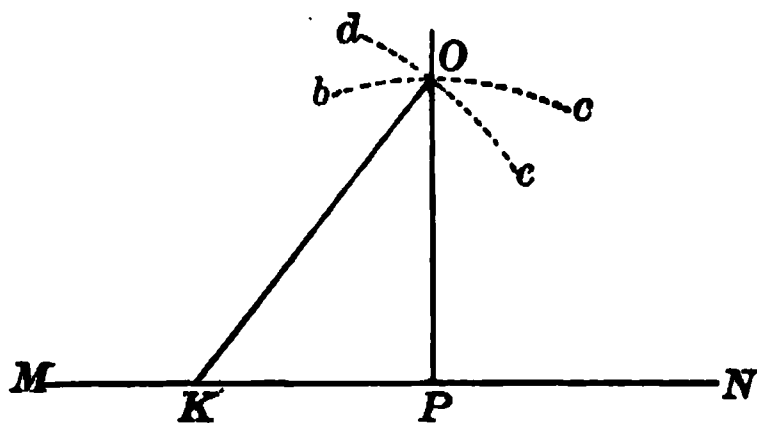
1. Let each student range out a line of several hundred feet, setting all the poles forward, and back again to the starting-point, and on different kinds of ground, undulating, hilly, and bushy.

2. Measure a line through a wood or where the ends are not visible from each other. Set stakes, as indicated in Article 19, in the true line 200 feet apart. See how near these stakes are placed in line by ranging.

C. SETTING OFF PERPENDICULARS.

20. *To erect a perpendicular at a given point in a line.*

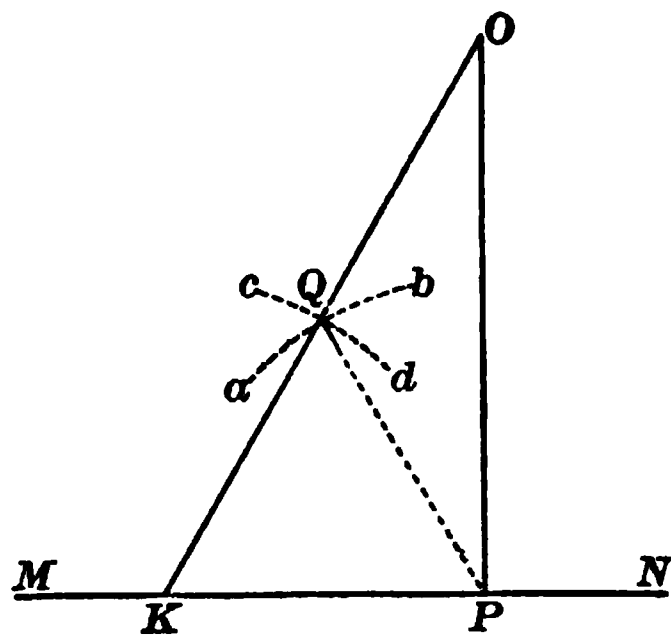
Let MN be the given line, and P the point at which it is desired to erect a perpendicular. Since a triangle formed of the sides 3, 4, and 5, or any multiple of these, will contain a right angle, we may take parts of a chain representing these distances M K P or multiples, having the angle made by the shorter sides at P , and set off a perpendicular to a given line, thus: Fasten one end of the chain at K , 30 links from P , the end of the ninetieth link at P ; then when both parts of the chain are drawn straight by a pull at the fiftieth link, the end of that link will indicate the point O which if connected with P will give the perpendicular required.



21. *If the perpendicular is to be of considerable length, then a greater length than $PO = 40$ links should be used, and the following method would be better: Fasten one end of the chain at P , and with the eightieth link describe an arc bc ; measure*

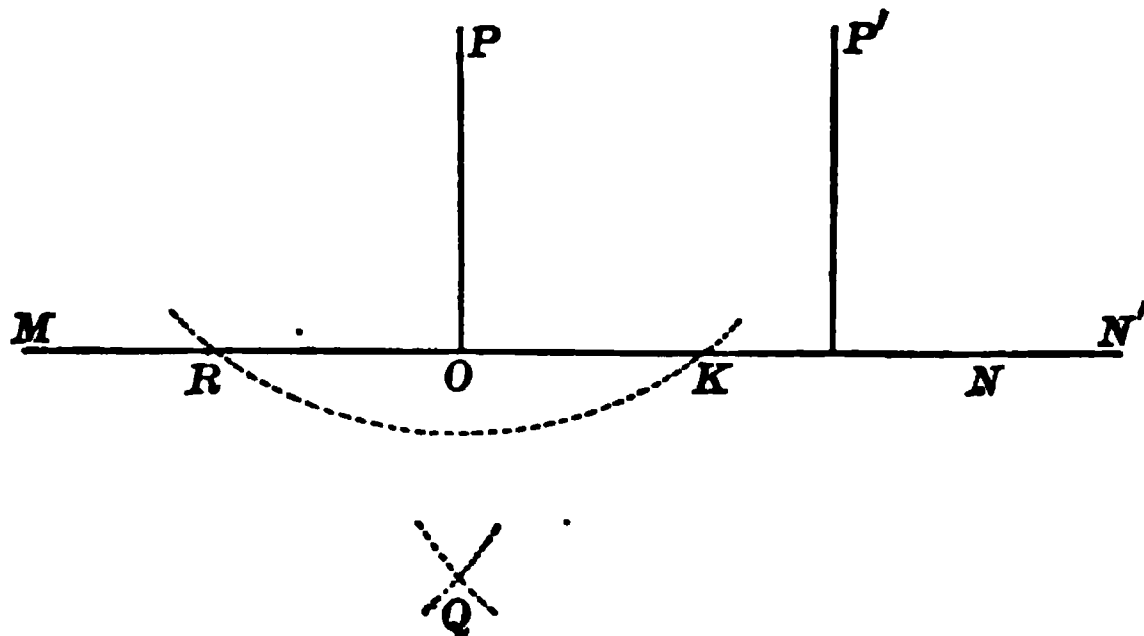
$PK = 60$ links, and with K as a centre, and with a radius $= 100$ links, the whole length of the chain, describe another arc de ; the intersection of these arcs will give the point O required.

22. Another Method. With the whole length of the chain as a radius, and P as a centre, describe an arc ab ; locate K a chain from P , and with the same radius, and with a centre K ,



describe an arc cd cutting ab in Q ; extend KQ to O , so that $OQ = QK$, then will OP be the perpendicular to the line MN at the point P . Why?

23. To let drop a perpendicular on a line from a given point without the line.



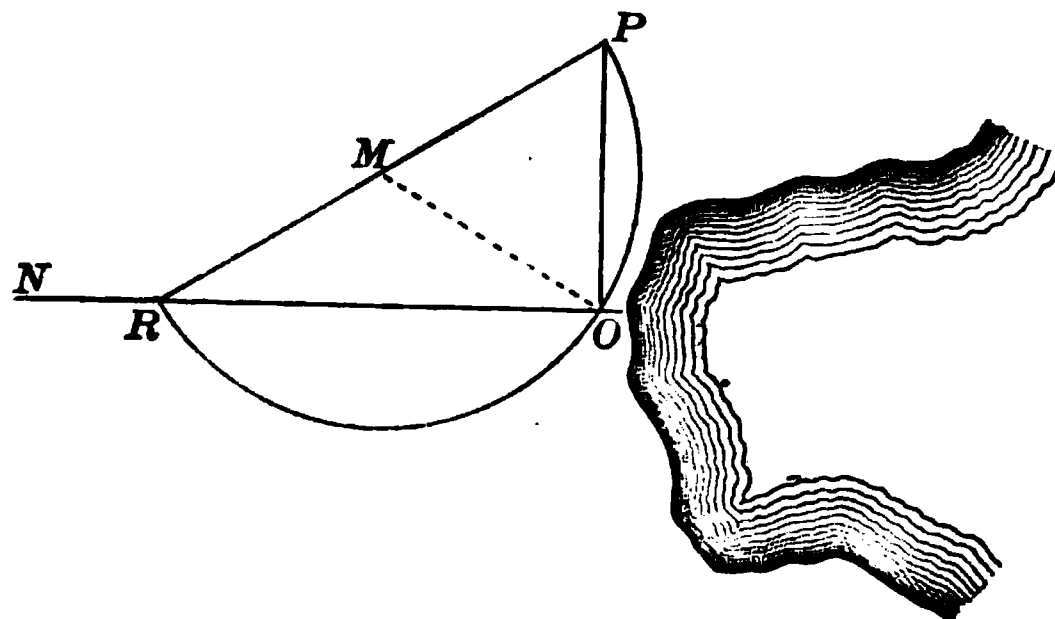
First, *When the point is accessible.*

Let MN represent the line, and P the point. With a length

of chain somewhat greater than PO , describe an arc cutting MN in the points R and K . With centres R and K , and any radius greater than the half of RK , describe arcs intersecting in Q . A line drawn from P to O in the direction of Q will be the perpendicular required.

If the point is at P' at or nearly opposite one end of the line, extend the line if it be possible to N' until a sufficient distance is obtained to describe the arcs required.

24. Or if it is impracticable to prolong the line, as in the figure, where a pond of water prevents, proceed as follows :



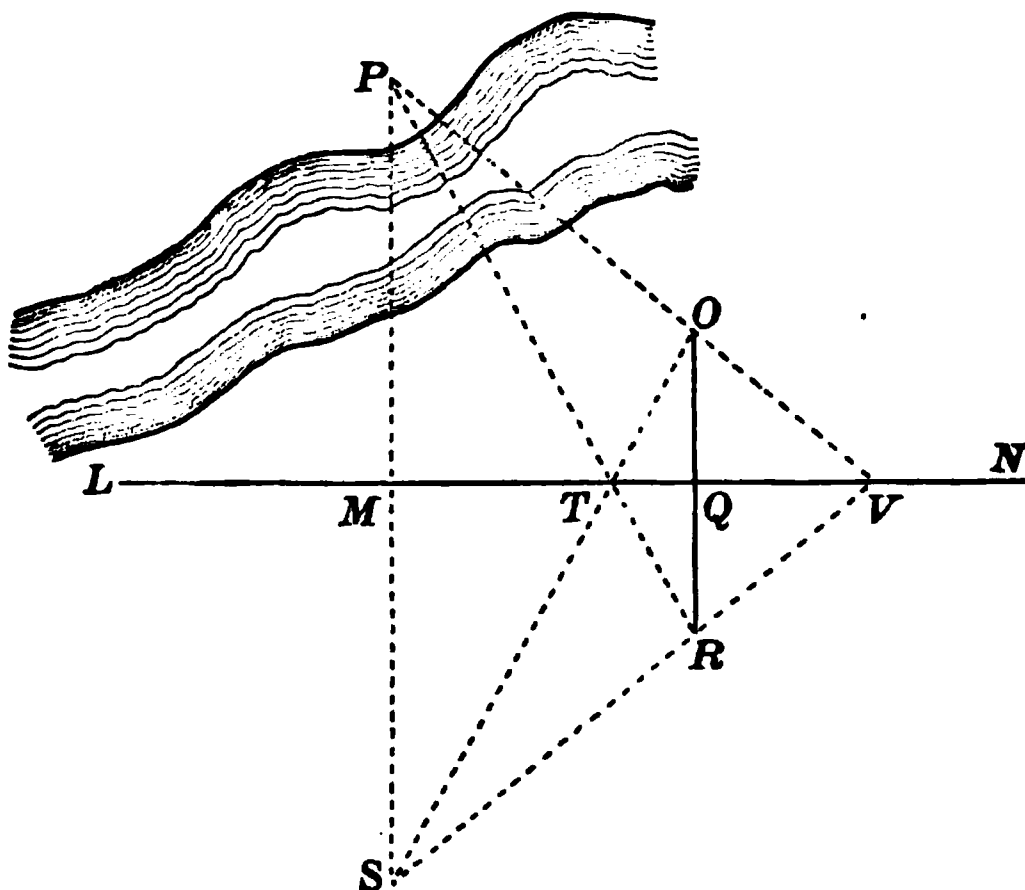
Extend the chain or any convenient portion of it from P to any point R in the line NO . Fix the middle point of RP , as M , and with this as a centre, and a radius MP , or its equal MR , describe an arc cutting the given line in O . Join PO for the perpendicular required.*

25. Second, *When the point is inaccessible.*

Let P be the given point, and LN the line. At any convenient point Q in the line LN erect the perpendiculars QO and QR of equal length. Locate V in the line PO and T in the line RP ; then if a point S be found at the intersection of the

* The angle ROP is measured by one-half a semi-circumference, and is therefore a right angle.

prolongation VR and OT , and a point M be located in LN and SP , a line joining M and P will be the perpendicular sought. Why?



26. Optical Square. To set off perpendiculars from a line, an instrument called the *optical square* may be used. It is a small cylindrical box containing a mirror, from the upper half of which the silvering is removed. The glass is placed so as to make half a right angle with the line of sight, hence two objects seen in it, the one by direct vision, and the other by reflection, subtend at the point of observation a right angle.

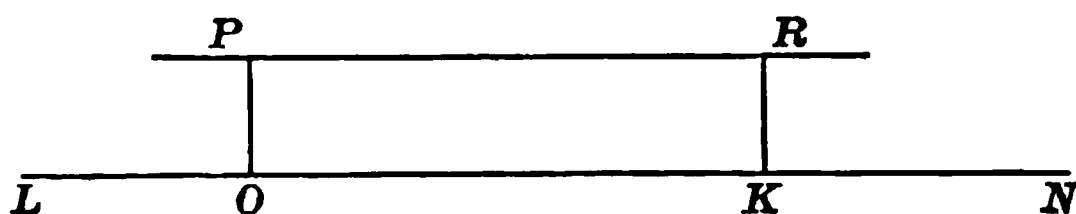
Or the *surveyor's cross*, which is simply two pairs of sights set at right angles to each other, and supported upon a staff.*

D. RUNNING PARALLELS.

27. *Through a given point to run a parallel to a given line, the point and line both being accessible.*

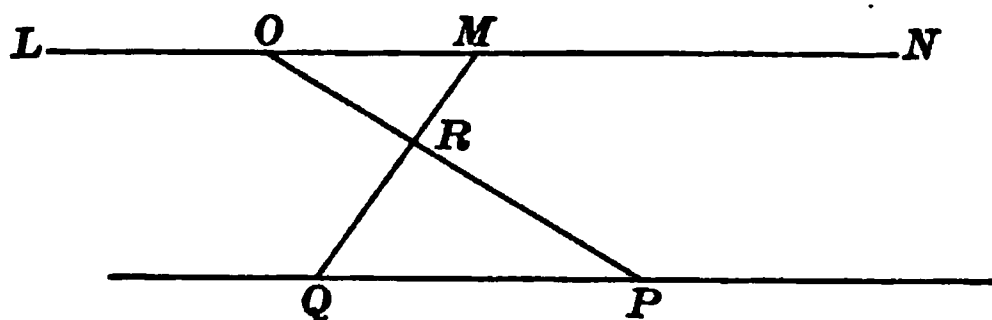
* While these instruments may be employed in chain surveying, neither of them is used in the ordinary practice of a surveyor, as perpendiculars are expeditiously set off by means of the compass or transit.

Let LN represent the line, and P the point. Let drop a perpendicular PO , and at some other point K ; erect a perpen-



dicular $KR = PO$. A line drawn through P and R will be the parallel required.

28. Otherwise. From any point O in LN run an oblique line to the point P . Through any point R in PO measure a

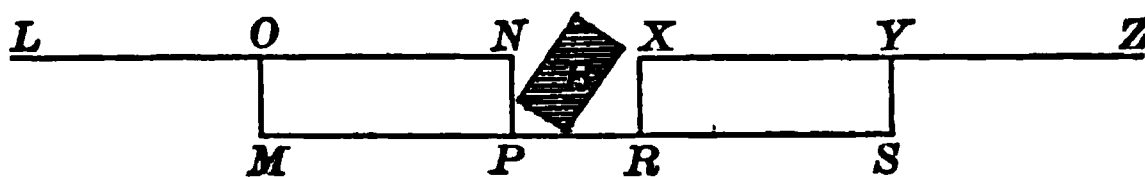


line MQ , so that $RQ = \frac{MR \cdot RP}{RO}$. A line passing through PQ will be the parallel required.

If R be taken at the middle point of OP , and QR be made equal to MR , the direction of the parallel PQ would be shown at once.

E. OBSTACLES TO ALIGNMENT.

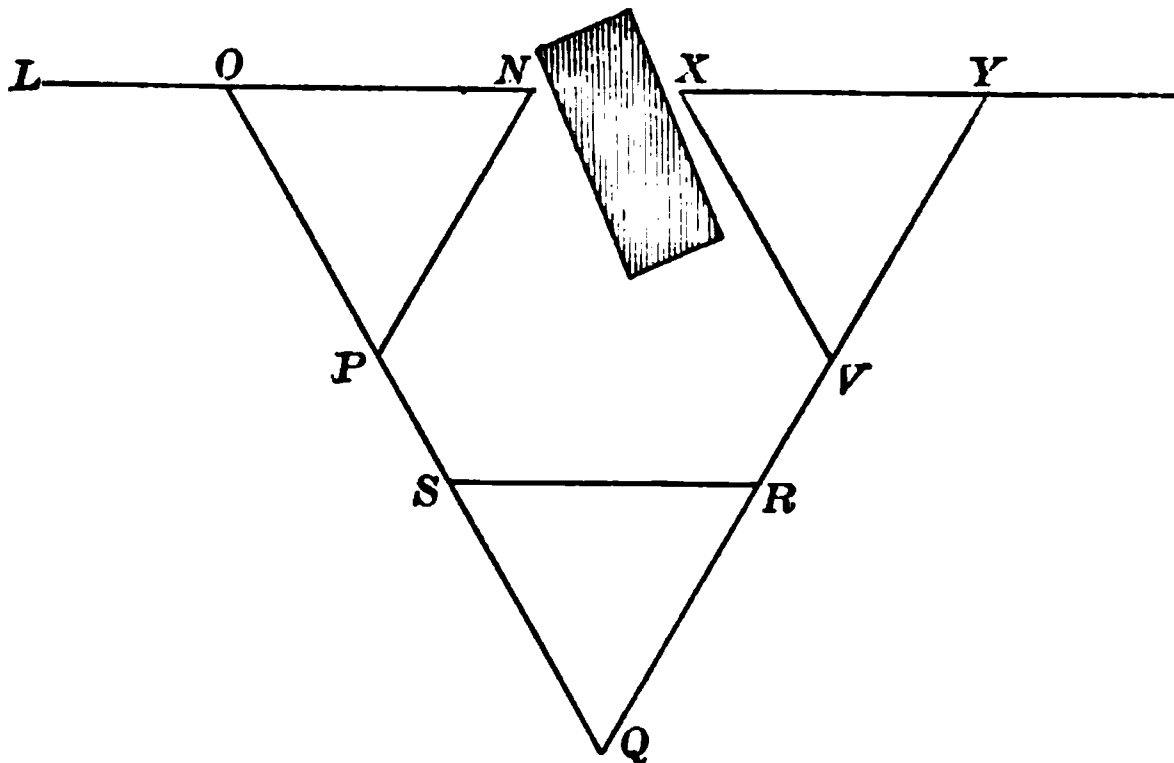
29. To prolong a line when an obstacle, as a tree or building, prevents direct sighting, we may proceed as follows :



By Perpendiculars. Let LN be the line which it is desired to prolong past a building B . At two points O and N in the

line, set off equal perpendiculars NP and OM , of such length that a line MP through these may be extended past the obstacle to some point S . At R and S set off perpendiculars to X and Y , of the same length as before, at O and N , and join XY ; it will be the prolongation of LN .

30. Otherwise: by Equilateral Triangles. On LN , the line to be prolonged, take a distance ON as a base, and construct on it an equilateral triangle NOP ; extend the side OP to some



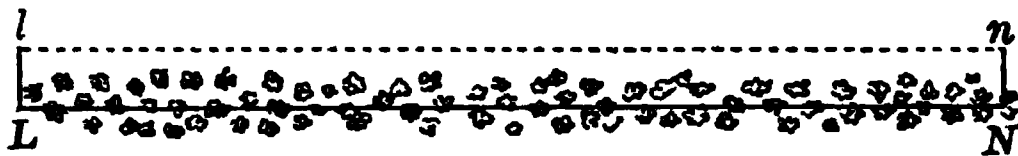
point Q . Describe an equilateral triangle QRS , and prolong the side QR to Y , making $QY = QO$; finally the construction of the equilateral triangle VXY will give XY the direction sought.

F. OBSTACLES TO MEASUREMENT.

31. a. When Both Ends of the Line are Accessible.

By Perpendiculars. For example, if it is desired to measure one side of a field or farm where a fence, hedge, or bushes prevent chaining on the line, set off perpendiculars, and measure the parallel line.

Let LN represent a line which, on account of fence and brush, it is impracticable to make the measurement exactly on the line.

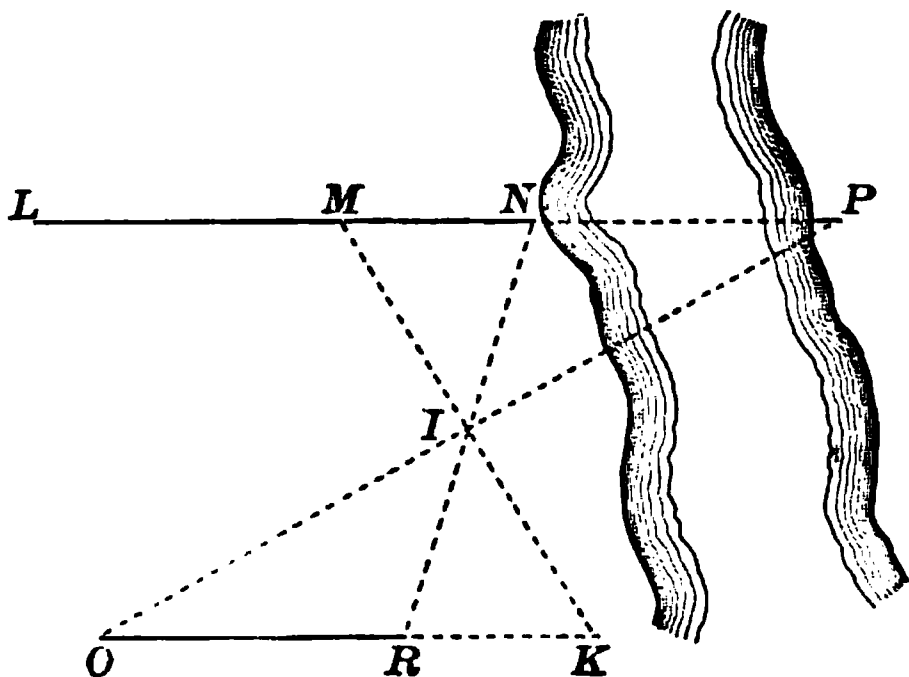


Erect at L and N perpendiculars Ll and Nn , of equal and sufficient length so that a line connecting l and n will clear the obstruction. Measure ln ; it will be the length of the required line.

32. b. When One End is Inaccessible.

By Symmetrical Triangles. Suppose LP the line, P the inaccessible end, visible, but on the opposite bank of a river.

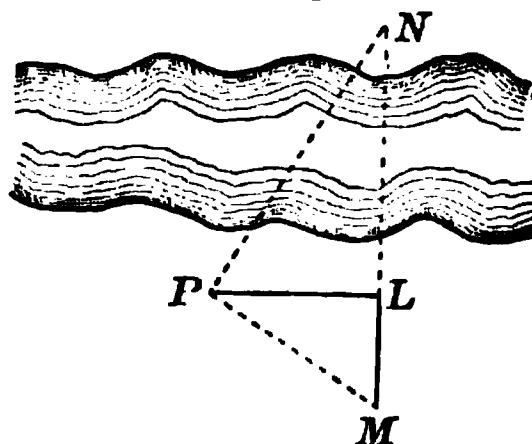
Measure from any point N near the river, in a direction diverging from its bank to R , making $NI = IR$. Through any other point M , in the line LN , measure through I to K , so $MI = IK$. If now a point O be found in the prolongation of RK , and in line with I and P , RO may be measured and taken for their distance NP .*



33. Otherwise. Measure from the line the perpendicular LP ; erect at P a perpendicular to PN , and extend it to a point M in the prolongation of LN . Measure LM ; then the proportion

$$ML : LP = LP : LN$$

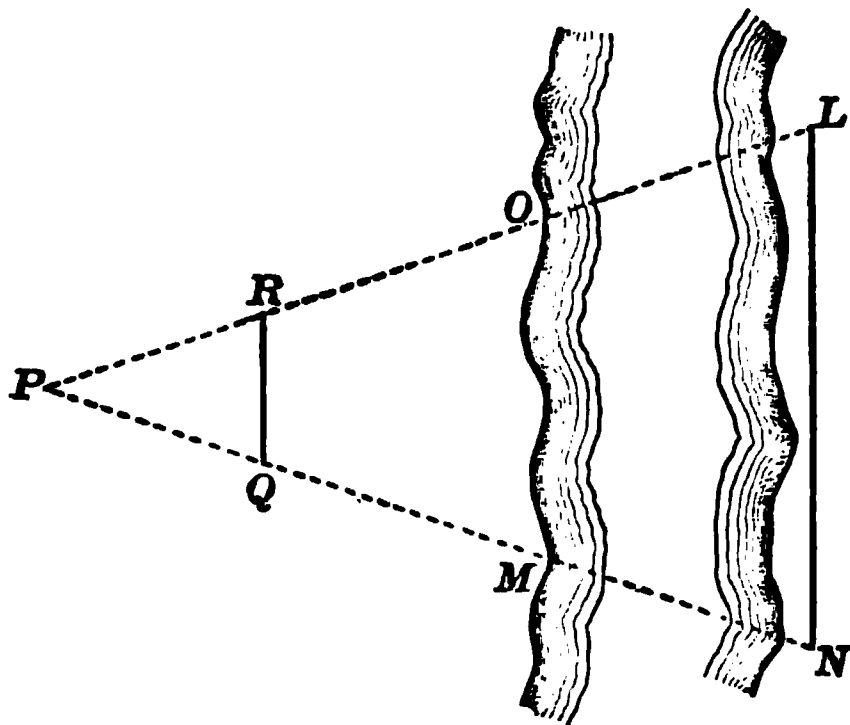
gives
$$LN = \frac{PL^2}{ML}.$$



* The student will show that ROI and NIP are symmetrical triangles, and NP and RO are homologous.

34. c. When Both Ends are Inaccessible.

By Symmetrical Triangles. Let LN be the line, the length of which it is required to determine. Take any point P , measure PO and PM , and find by one of the preceding methods OL ,

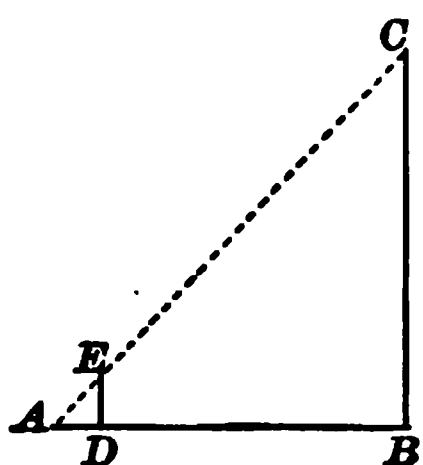


MN , and hence, the total length of PL and PN . Now take points R and Q in the lines PL and PN respectively, so that $PR : PQ = PL : PN$, and measure RQ ; then the required line LN may be calculated by the proportion $PQ : PN = RQ : LN$.

G. MEASUREMENT OF HEIGHTS.

35. To measure the height of a tree or a flag-staff. Let BC represent the height required. At a point D set up a staff of a known height so that, with the eye at A , C and E will be in line of sight; measure AD and DB ; then the similar triangles ADE and ABC give the proportion

$$AD : DE = AB : BC.$$



Whence $BC = \frac{DE \times AB}{AD}.$

EXAMPLES.

1. If the height of a staff is 4 feet, and the distance from it to a tree = 80 feet, AD being $4\frac{1}{8}$ feet, what is the height of the tree? *Ans.* $77\frac{1}{8}$ feet.

QUERIES. If the height of the staff is equal to AD , the length of neither being known, simply the distance AB given, could the height of the tree be ascertained?

If the ratio of the height of the staff to AD is known, but not the absolute length, could the required height be found by simply measuring AB ?

Is this method applicable on other than horizontal ground?

2. A liberty pole, whose height was 90 feet, standing on a horizontal plane, was broken off, and the extremity of the top struck the ground 28 feet from the bottom of the pole. Required the length of the broken part.

EXERCISES.

1. Set a stake 40 feet perpendicularly distant from a given point in a given line.

2. Through a given point 50 feet from a given line run a parallel 120 feet in length.

3. Prolong a line beyond a house or other obstacle.

4. Measure the width of a stream or pond without crossing it.

5. Run a line to the bank of a stream or lake, and let fall a perpendicular on the line near its extremity from a given point without it.

6. Measure the height of a tree, flagstaff, or church spire.

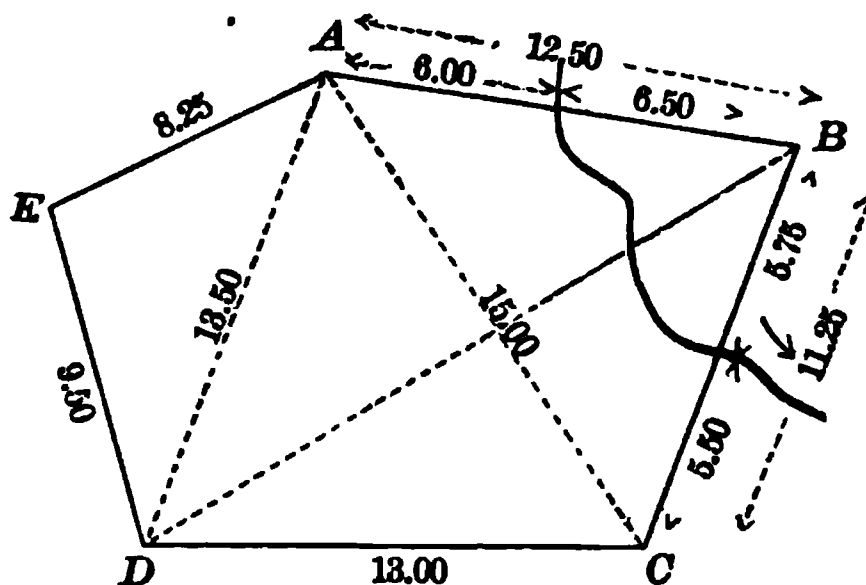
SECTION III.

RECORDING THE FIELD NOTES.

36. The Field Notes should be kept in a neat, concise, and intelligible manner, exhibiting a complete record of the work done, and the method of doing it, so that a surveyor unacquainted with the work, and having the record before him, could make a plot of the tract, or go on the field and readily ascertain the position of any point indicated in the notes.

Either of two methods may be employed, or a combination of them.

37. Sketch. One is to make a sketch of the tract as the survey progresses, writing the length of each line and indicating the intersection of fences, roads, streams, etc., as shown below.



For surveying a field or small tract of land, this is a good method, but if the tract is large, many sided, and numerous points to be noted in and near the side-lines and diagonals, it would be difficult if not impossible to decipher the sketch on a page of the ordinary field-book, and to make an intelligible record of the work would require a book or sheet inconveniently large to carry about the field.

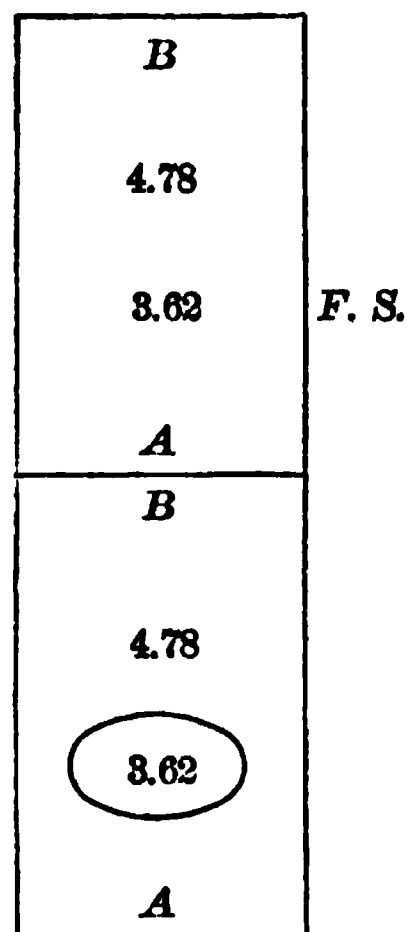
38. Columns. A method which will answer as well for complex as for simple surveys consists in drawing two parallel lines, about an inch apart, extending from top to bottom of the notebook, and near the middle of the left-hand page. Between the lines the distances and stations are to be recorded, commencing at the bottom of the page and proceeding upwards. Roads, fences, streams, etc., should be represented on either or both sides of the column as they naturally appear. The record of the measurements on any line being referred to the beginning of the line.

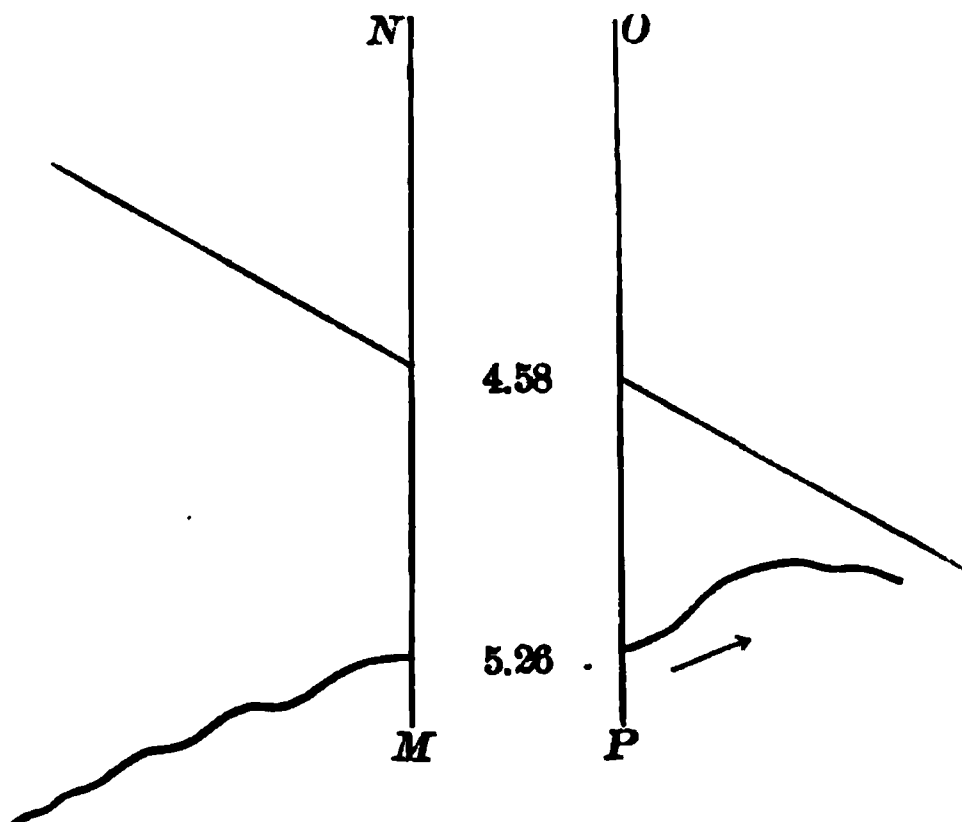
The right-hand page may be used for sketching any part of the survey to further elucidate, where necessary, the work done.

A station is indicated by a triangle (Δ) or a circle (\odot). If the station is at the end of a line it is usual to name it by the letter or number, designating that corner as station *A* or station 1, and the line extending from *A* to *B* is called the line *AB*, from 4 to 5, the line 4, 5; or a line may be designated by its length; a line that is 3 chains and 52 links long would be referred to as the line 352.

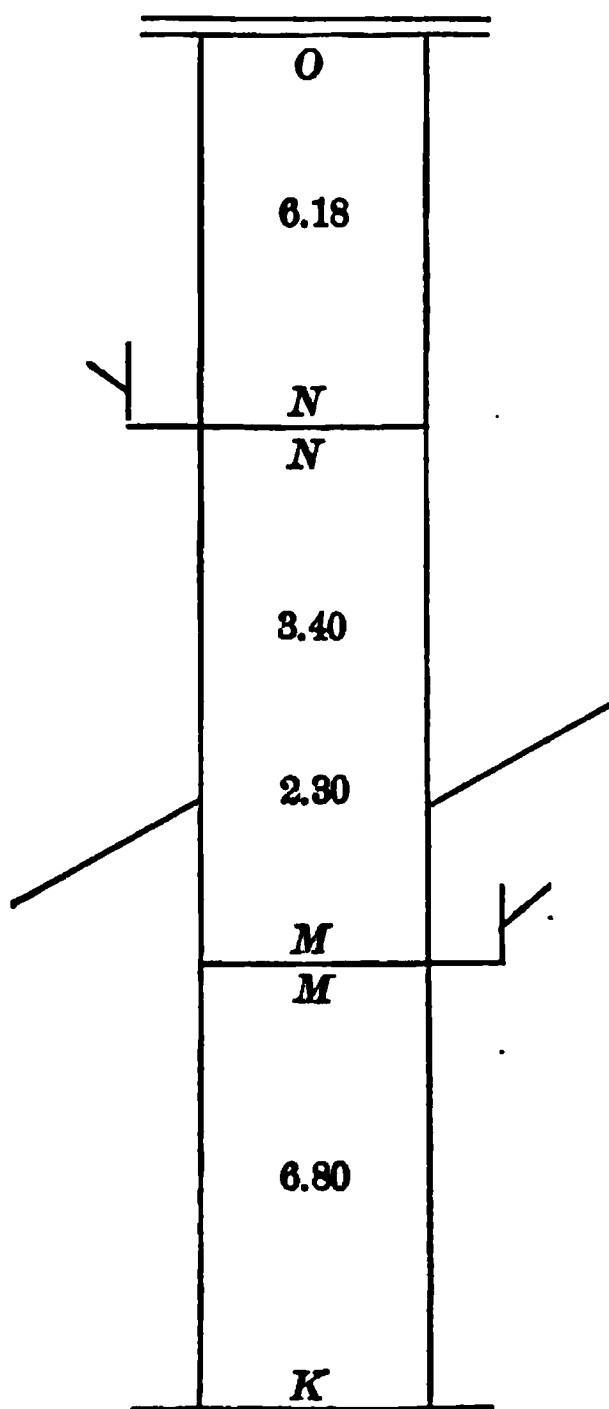
A false station is a point in a line whence other measurements are to be made either to the right or left, and are designated by enclosing in a curve its distance from the end of the measured line, or by writing *F. S.* opposite that distance, as per margin, which shows that there is a false station at a distance of 3.62 chains from *A* on the line *AB*.

A fence, brook, road, etc., intersecting the measured line, should be drawn so as to indicate, as nearly as possible, its inclination thereto, but not as a continuous line; the ends on each side being directly opposite, as at 4.58 and 5.26, so that if the vertical column





were to vanish by the two lines MN and OP coinciding, the fence or creek would be shown as continuous.



When the record of a line, as MN , is complete, and the measurement is continued from N , a horizontal line is drawn across the column as shown in the figure. But if the survey closes at the end of a line, as at O , or if for any reason the work is to proceed from some other point, *two* lines are drawn across the column.

A mark (\vee) or (Γ) placed at the beginning of a line indicates by shape, as well as position, that the line along which it stands bears to the right of the preceding; the reverse position of the angle (\vee or Γ) indicates a turn to the left.

In the figure, MN bears to the right of KM , and NO to the left of MN .

The record of the survey sketched in Article 37 would be represented by the column method as follows :

| | | | |
|-------|---|-------|---|
| Sides | | A | |
| | | 8.25 | |
| | | E | ✓ |
| | | E | . |
| | | 9.50 | |
| | | D | ✓ |
| | | D | |
| | | 18.00 | |
| | | C | ✓ |
| | | C | |
| | | 11.25 | |
| | ↖ | 5.75 | ↗ |
| | | B | ✓ |
| | | B | |
| | | 12.50 | |
| | ↖ | 6.00 | ↗ |
| | | A | |

| | | | |
|-----------|---|-------|--|
| Diagonals | | D | |
| | | 13.50 | |
| | ✓ | A | |
| | | A | |
| | | 15.00 | |
| | | C | |

SECTION IV.

MAPPING AND PLOTTING.

39. A Map of a survey is a correct representation or copy of the tract surveyed, exhibiting not only its boundaries, roads, streams, etc., in relative dimensions and positions, but also the irregularities and appearances of its surface.

A Plot (or *Plat*) is an outline map, in which, in general, only the boundaries, roads, streams, and important lines are delineated, but no attempt is made to indicate the *topography* of the tract. The surveyor usually makes a *plot* of a field or farm survey. The civil engineer makes a map of a proposed railroad.

INSTRUMENTS USEFUL FOR MAKING A PLOT OF A CHAIN SURVEY.

40. Drawing-Board, T-Square, Triangles, Dividers, Scale, Drawing Pen and Pencil.*

A Drawing-Board is a rectangular, smooth board to which the paper that is to contain the drawing is fastened. There are two patterns: one consists of a frame of walnut, or other hard wood, with a detachable centre of soft white pine. The paper, which should be somewhat larger than the detachable centre, being moistened and laid on it, becomes well stretched when the parts of the board are buttoned together and the paper dries. The other is simply a rectangular white pine board made of several pieces of wood laid in different directions to prevent warping. Both patterns are made of various dimensions.

41. A T-Square, as its name indicates, is a *square* or ruler with a cross-piece or head at one end, giving it the appearance

* Other instruments used in drawing are described in Chapter II. Section VIII.

of a letter T. There are two patterns of these, one with a head fixed at right angles to the ruler or *blade*; the other, in addition to the permanent head, has another head attached to it with a clamp screw, so that by properly setting the movable head, lines of any desired inclination may be drawn. The blade, being long and thin, should be tested occasionally by means of a metallic straight edge or another T-square to see whether or not it is perfectly straight. The correctness of the angles should also be tested; this may be done as indicated in the next article.

42. Triangles are made of hard wood, rubber, or metal, and are either solid or have an open centre. The angles are usually 30, 60, and 90 degrees, or 45, 45, and 90 degrees, and the longest side rarely exceeds 12 inches.

The T-square and triangles are frequently employed together to draw parallels, perpendiculars, and many of the oblique lines of a plot.*

The sides of triangles should be tested occasionally, to see if they are straight, by placing them against the edge of a metallic straight edge.

The right angle may be tested by placing one of its sides against a straight edge; mark the direction of the other side, reverse the triangle, but bring the same side against the straight edge, and having the right angle at the same point as before, mark the side again. If the two marks coincide, the angle is right; otherwise, it is not.

When correct, the right angle of the triangle may be used to test the correctness of the right angle of the T-square.

43. Dividers (or Compasses) are made of different sizes and numerous appendages. The surveyor will need at least one with a detachable leg, so that another leg, carrying a pen or

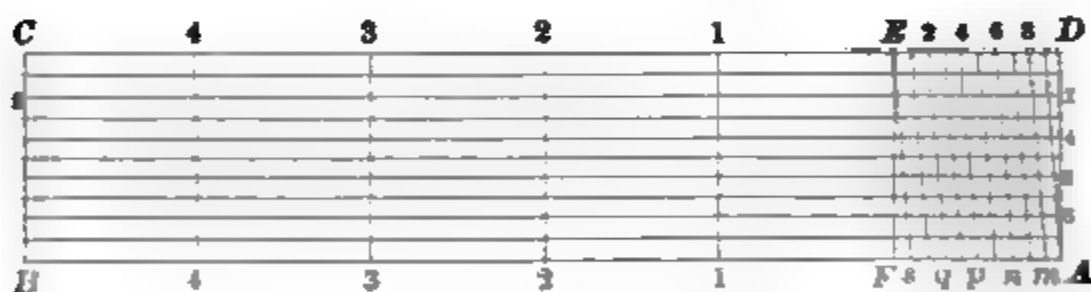
* The results are tolerably accurate within the limits usually required in a farm survey. It may be well, however, to caution the student not to rely too much upon the accuracy of a point located by means of and near the extremity of a thirty-inch T-square.

pencil point. may be inserted when necessary. These, it need hardly be said, are used for laying off lines, describing area, circles, etc.

44. Lead-Pencil. Fine quality, hard, used in outlining the work; and a *Drawing-Pen*, medium size, for inking in the drawing.

45. Scales are made of box-wood, metal, ivory, or paper, and are of various kinds. *Triangular* and *diagonal* are generally used for plotting chain surveys. The *triangular scale* for engineers and surveyors is usually 12 inches long, and made of good box-wood, each of the six bevelled faces being graduated with a single scale, viz.: one face contains 10 divisions to the inch, one 20, another 30, another 40, one 50, and one 60 divisions; and generally one inch on each face is subdivided so that an extremely small fraction of an inch may be set off or read. This is a very convenient scale: not only can very small divisions be readily transferred from it to a drawing, but by simply placing the instrument properly on a line of a drawing, the scale of which is known, its length may be directly determined.

The **Diagonal Scale** is usually six inches long, thin and flat, divided transversely into 6 equal parts of one inch each, and longitudinally into ten equal parts. At one end, as *AD*, one inch is divided by 10 oblique lines, as *s m*, *6 n*, etc., into 10 equal parts and numbered as shown in the figure.



Now *Fs* being .1, the next division between the perpendicular *FE* and the oblique line *sE* is .09, the next .08, and the last

division, or one nearest F , is .01. Hence the scale may be used to measure .01 of an inch, or one hundredth of any division taken as the unit. For example, to lay off 3.4, place one foot of the dividers at 3 on the line EC and extend the other foot to 4 between DE . To lay off 3.42, place one foot at the intersection of 3, 3, and 2, 2, and the other on the same line 2, 2, at its intersection with 4 p .

The diagonal scale usually found with a box of drawing instruments contains various graduations. The simplest are divided to inches, and halves, quarters, tenths, and twelfths of an inch; each quarter and half subdivided diagonally into tenths, so that a tenth of a quarter can be taken off at once; and even tenths of these are indicated on the scale — besides other divisions of more or less utility.

Paper scales are frequently employed, and regarding hygro-metric changes are better than the others, for the scale and the paper containing the drawing expand and contract more nearly alike. Generally, however, they are not divided with the same degree of accuracy.

46. Drawing to a Scale consists in drawing lines whose length shall be some fraction of the length of the line measured. Suppose, for example, a line is 13 chains long, and it is desired to draw it to a scale of 5 chains to an inch; then $2\frac{6}{5}$ inches will evidently be the distance to transfer from the scale to the paper to represent the length of the line.

A line 10 chains and 50 links in length drawn to a scale of 3 chains to an inch will be represented by a line $3\frac{1}{2}$ inches long, and so on. The length of the line divided by the number of units — chains, yards, feet, etc. — to the inch, always giving the distance to be taken off the scale. Obviously the converse of this is true; that is, the real length of a line may be ascertained when the scale is known, by multiplying the units in the length of the line in the drawing by the number of chains or feet which each unit represents. In the last example the length of the line being found $3\frac{1}{2}$ inches, and the scale 3 chains to an

inch, the true length = $3.5 \times 3 = 10.50$ chains. The scale should always be given on the drawing. It may be stated thus: Scale, 3 chains to an inch, 1000 feet to an inch, 2 miles to an inch, or fractionally, and thereby indicating the relative length of the lines in the drawing to those which they represent; as, 1 : 500, 1 : 2000, etc.

47. Size of Drawing or Scale to Adopt. In farm surveys of small extent, 1 or 2 chains to an inch may be used; for medium tracts 3 chains to an inch (1 : 2376) is perhaps the best. The shape of the farm, the length of the shortest and longest sides, as well as the object of the drawing, will, however, influence the surveyor in his decision of the scale.

48. Scale Unknown. If the area of a tract of land is known but the scale not given, it may be found by measuring the lines of the drawing referred to any convenient scale and computing the area from these determined lengths. Then, since the areas of similar figures are to each other as the squares of their homologous sides, the true scale may be obtained by the proportion,

$$\frac{\text{computed area}}{\text{known area}} = \frac{\text{square of assumed scale}}{\text{square of true scale}}.*$$

SECTION V.

ON AREAS, AND ILLUSTRATIVE EXAMPLES.

A. AREAS.

49. The following are geometrical truths with which the student is supposed to have an acquaintance, but are given here for convenience of reference.

*The protractor and other drawing-instruments used in connection with compass and transit surveying are described in Chapter II.

The Area of a Triangle is equal to one-half the product of its base and altitude.

In Terms of the Three Sides the area is equal to the square root of the continued product of one-half the sum of the sides, and the half-sum minus each side severally, or in symbols, where A = area, a, b, c , the three sides, and s their sum,

$$A = \sqrt{\frac{1}{2}s(\frac{1}{2}s-a)(\frac{1}{2}s-b)(\frac{1}{2}s-c)}.$$

If the triangle is equilateral and s = length of a side,

$$A = \frac{s^2}{4}\sqrt{3}.$$

50. The Area of a Rectangle is equal to the product of its length and breadth, or $A = bl$ where b = breadth and l = length.

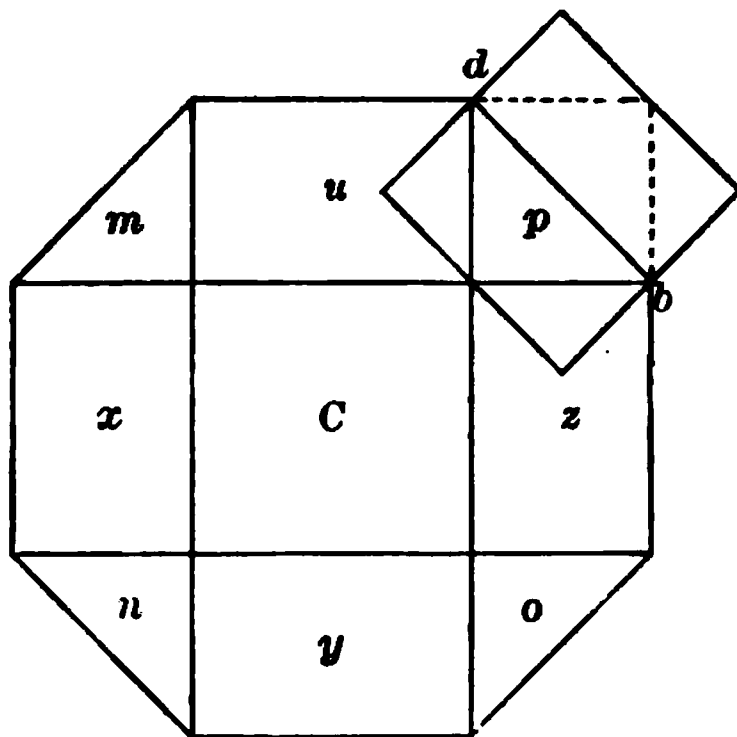
51. The Area of a Parallelogram is equal to the product of its base and altitude, or $A = bh$ where b = breadth and h = height.

52. The Area of a Trapezoid is equal to the product of one-half the sum of its parallel sides and the perpendicular distance between them, or $A = \frac{p}{2}(m+n)$ where m and n are the parallel sides, and p the perpendicular distance between them.

53. The Area of a Regular Hexagon, where s denotes the length of one of its sides, is $A = \frac{3}{2}s^2\sqrt{3}$, or it is equal to six equal equilateral triangles, having for each side the length of one side of the hexagon.

54. The Area of a Regular Octagon, each of its sides being unity, may be calculated by the rules of geometry, thus: Let the figure represent the octagon. It is evident that the area of the central square = 1. The sum of the areas of the four triangles $m, n, o, p = 1$, since their sum equals the square described on db .* Now, the dimensions of each of the four

* The square described on the diagonal of a square is double the given square.



remaining figures (rectangles) x , y , z , and u , are 1, and $\frac{1}{2}\sqrt{2}$; hence the sum of the areas of these four rectangles

$$= 4 \times \frac{1}{2}\sqrt{2} = 2\sqrt{2};$$

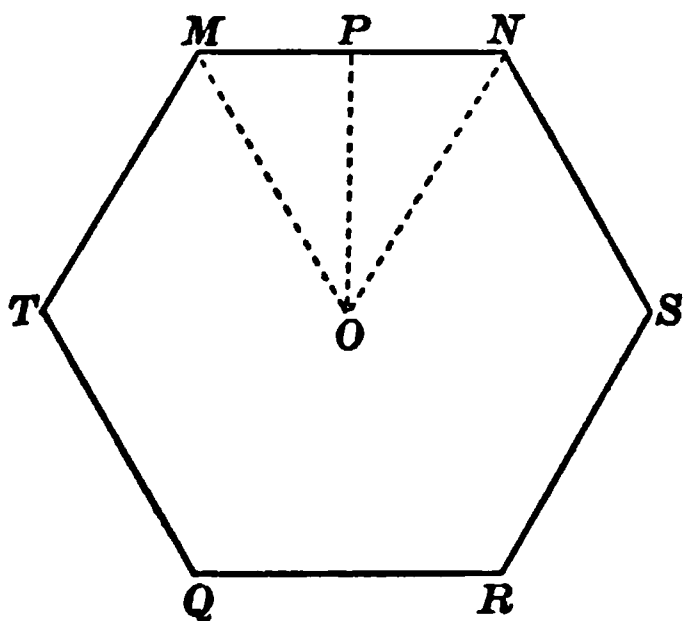
adding all the parts, there results

$$1 + 1 + 2\sqrt{2} = 2 + 2\sqrt{2}$$

for the area of the octagon.

55. The Area of a Regular Polygon in terms of the *perimeter* and *apothem*, or radius of inscribed circle, is equal to one-half the product of the perimeter and apothem, or $A = \frac{pr}{2}$; p denoting the perimeter, and r the radius of inscribed circle or apothem.

56. The Area of a Regular Polygon in terms of the *number of*



sides and *length* of *one side* may be determined as follows: Let $r = OP$ be the radius of the inscribed circle or apothem, l the length of each side, and n the number of sides, A the area, as before; then

$$r = \frac{l}{2} \cot \frac{180^\circ}{n},$$

and
$$A = \frac{nl}{2} \times \frac{l}{2} \cot \frac{180^\circ}{n} = \frac{nl^2}{4} \cot \frac{180^\circ}{n}.$$

If $l=1$, and $n=8$, the area of the polygon (octagon) becomes $2 \cot 22^\circ 30' = 2 + 2\sqrt{2}$, as before found.

57. By the application of the formulas just found, the following table may be constructed, showing the apothems and areas of some of the regular polygons, each of whose sides is unity.

| NAMES. | SIDES. | APOTHEMS. | AREAS. |
|-------------------|--------|-----------|------------|
| Triangle | 3 | 0.2886732 | 0.4330127 |
| Square | 4 | 0.5000000 | 1.0000000 |
| Pentagon | 5 | 0.6881910 | 1.7204774 |
| Hexagon | 6 | 0.8660254 | 2.5980762 |
| Heptagon | 7 | 1.0382601 | 3.6339124 |
| Octagon | 8 | 1.2071069 | 4.8284271 |
| Nonagon | 9 | 1.3737385 | 6.1818242 |
| Decagon | 10 | 1.5388418 | 7.6942088 |
| Hendecagon . . . | 11 | 1.7028439 | 9.3656399 |
| Dodecagon | 12 | 1.8660252 | 11.1961524 |

Now, since the areas of similar polygons are proportional to the squares on their homologous sides, this table may be used to find the area of any regular polygon named in the table, whatever may be the length of its side. Using the notation above, the principle just enunciated will be expressed as follows :

$$1^2 : \text{area in table} = l^2 : A, \text{ or } A = l^2 \times \text{area in table.}$$

That is, the area of a regular polygon is equal to the square of its side multiplied by the area of a similar polygon each of whose sides is 1.

EXAMPLE. The area of a regular pentagon, each side being 30,
 $= 30^2 \times 1.7204774 = 1548.43.$

58. The Area of a Circle is equal to π multiplied by the square of the radius, or one-half the product of the circumfer-

ence and radius. Let R represent the radius, C the circumference, and A the area; then

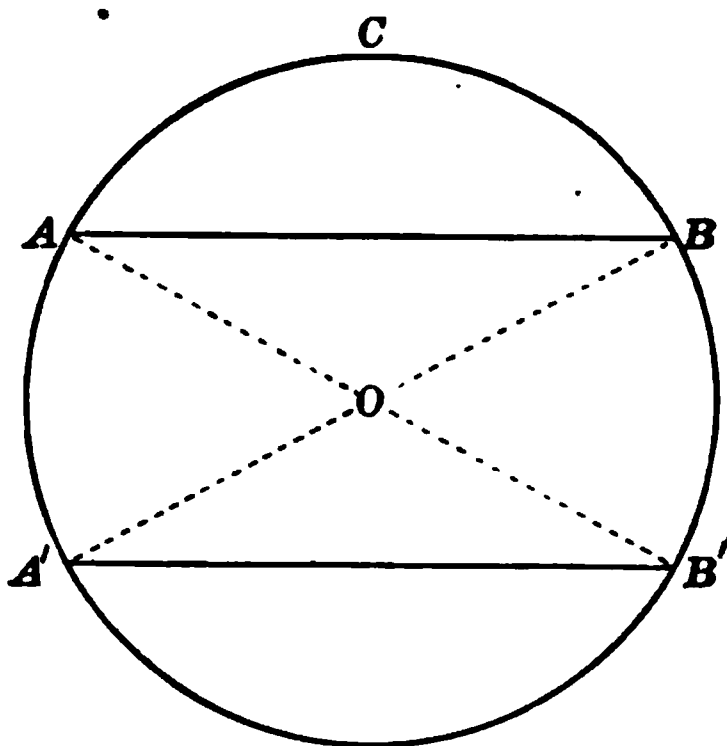
$$A = \pi R^2 = \frac{RC}{2}.$$

The area of a Quadrant $= \frac{\pi R^2}{4}$.

59. The Area of a Sextant $= \frac{\pi R^2}{6}$, and in general, the area of any sector of a circle $= \frac{n}{360} \times \pi R^2$, in which n denotes the number of degrees in the sector, or $A = \frac{Rl}{2}$, in which l denotes the length of the arc.

60. The Area of a Circular Ring is evidently the difference of the areas of the outer and inner circles; or, in symbols, if R and r equal the outer and inner radii, $A = \pi(R^2 - r^2)$.

61. The Area of a Segment of a circle, as ABC , is evidently equal to the area of the sector $AOBC$, minus the area of the



triangle AOB ; or, in symbols, since the area of the triangle $= \frac{R^2 \sin n}{2}$, and the area of the sector as given above,

$$A = \frac{n\pi R^2}{360} - \frac{R^2 \sin n}{2}.$$

If n is greater than 180° , as in the segment $A'B'BCA$, $\sin n$ becomes negative, thereby making the second term of the right-hand member positive, as it should; since in this case the segment is greater than the sector, and the triangle $A'OB'$ is additive.

If the lengths of arc and chord are given, denote by $2c$ the length of chord, the other notation as above; then

$$A = \frac{Rl}{2} \mp c\sqrt{R^2 - c^2};$$

the *minus* sign to be used when the segment is less than a semicircle, and the *plus* sign when the segment is greater than a semicircle.

62. The Area of an Ellipse is equal to πAB , in which A and B denote the semi-axes.

B. ILLUSTRATIVE EXAMPLES.

EXHIBITING VARIOUS METHODS EMPLOYED TO SURVEY LAND, TO PLOT THE SURVEY, AND TO CALCULATE THE AREA.

TRIANGLES.

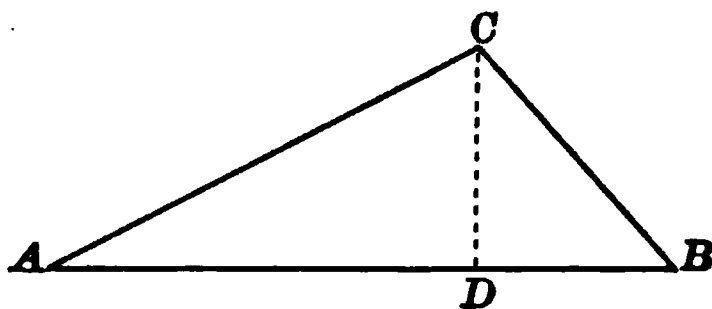
63. First Method. Measure the perpendicular CD , and the segments AD and DB , into which it divides the base; then

$$A = \frac{AB \times DC}{2}.$$

To Make the Plot. Draw AB according to any convenient scale, and locate D ; with the same scale erect at D a perpendicular $= DC$. Join CA and CB , and the triangle ABC will result.

EXAMPLES.

1. Required the area and plot of a triangular field, the perpendicular of which measures 4.86 chains, and divides the side on which it falls into segments measuring 5.80 chains and 3.16 chains, or a total length of 8.96 chains.



Calculation. $A = \frac{8.96 \times 4.86}{2} = 21.7728$ square chains.. Dividing by 10, since there are 10 square chains in an acre, their results $\frac{21.7728}{10} = 2.177 +$ acres.* (The student will make the plot.)

QUERIES. Could a correct plot of the tract be made if there were given simply the base and altitude?

Would there be, usually, any choice of side to take as the base?

2. A triangular field measures 12.18 chains on one side, and the perpendicular erected at a point 5.10 chains from one end measures 7.54 chains. Calculate the area and make the plot.

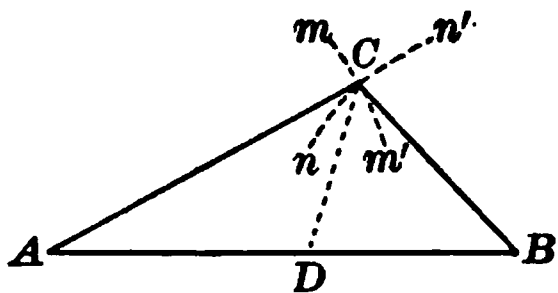
64. Second Method.† Measure all the sides, and calculate the area by the formula given above for that case.

EXAMPLES.

1. The lengths of the sides of a triangle are as follows: $AB = 40$ chains, $AC = 30$ chains, and $BC = 20$ chains. Required the area and plot.

$$A = \sqrt{45 \times 5 \times 15 \times 25} = 29.047 \text{ acres.}$$

To Make the Plot. Take 40 chains to any convenient scale in the dividers, and lay it off for the base AB ; then, with A



as a centre and 30 chains to the same scale in the dividers, describe an arc mm' ; also, with B as a centre and 20 chains for radius, describe the arc nn' .

The point C connected with A and B will give the triangle ABC required.

REMARK. It is customary when making a chain survey, to

* The area is usually expressed in acres and hundredths or thousandths of an acre.

† Other methods are given in Chapter II. Section IX.

measure a *proof** line such as CD , and this should always be constructed to test the accuracy of the work.

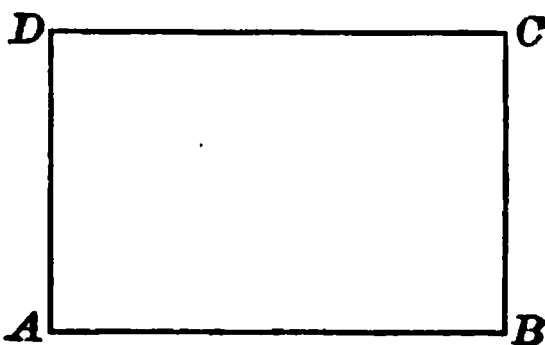
2. The three sides of a triangle measure 49, 50.25, and 25.69 chains. Find the area. *Ans.* 61.498 acres.

3. The sides of a triangular field are 24, 18, and 15 chains. A proof line, 12 chains in length, intersects the longest side or base at a point 10.25 chains from the angle formed by the two longest sides of the field. Required the area and plot. Test accuracy of latter by constructing proof line.

RECTANGLES.

65. Measure any two adjacent sides, as AB and BC . The area $= A = AB \cdot BC$.

To Plot. Lay off AB to any desired scale, and erect a perpendicular with the same scale at the extremities $= AD$ and BC ; connect D and C , and the required figure will be formed.



EXAMPLES.

1. The length and breadth of a rectangle are 12.32 and 7.16 chains respectively. Required the area. *Ans.* 8.82 acres.

2. The length of a rectangle is 1250 feet, and its breadth 840 feet. What is its area? *Ans.* 24.1 acres.

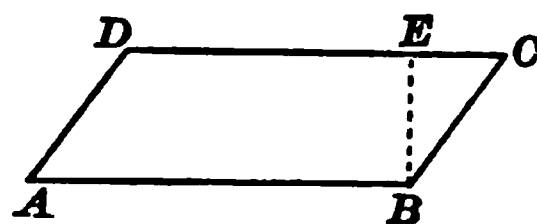
3. A road running across a farm is $\frac{3}{8}$ of a mile long and 3 rods wide. How much land does it occupy? *Ans.* $2\frac{1}{4}$ acres.

4. The length of a road on a hillside inclined to the horizon at an angle of 20° is 2310 feet, and its width $2\frac{3}{4}$ rods. At the rate of \$84 per acre, what must be paid to the owner across whose land the road runs? *Ans.* \$189.93.

* A line to check the measurement.

PARALLELOGRAMS.

66. Measure a side, as AB , the perpendicular distance, as BE , to the opposite side DC , and the distance CE . Then $A = AB \times BE$.



To Plot. Lay off the base AB , and at the extremity B erect a perpendicular equal BE . Through E draw DC equal to and parallel to AB , making EC its proper length. Join DA and CB , and the parallelogram $ABCD$ will be formed.

EXAMPLES.

1. The base of a parallelogram measures 10.54 chains. A perpendicular from one extremity of the base to the opposite side 5.16 chains, and the distance corresponding to EC in the last figure is 1.82 chains. Required the area and plot.

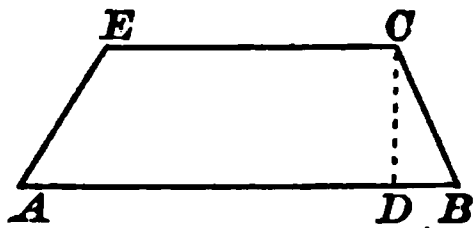
Ans. 5.439 acres.

2. A surveyor employed to determine the area of a rhombus, and knowing that the obtuse angles were double the acute, measured the shorter diagonal only, and found it 100 feet. Was the measurement sufficient? If so, give the area.

QUERIES. Can the area of a rhombus be ascertained if the lengths only of the two diagonals be given? If either diagonal and a side be given?

TRAPEZOIDS.

67. Measure EC , the perpendicular CD , and BA ; note where the perpendicular CD meets the base AB .



$$A = \frac{1}{2}(AB + EC) CD.$$

To Plot. Lay off the base AB to the desired scale, and at D erect a perpendicular thereto equal to DC . Through C draw CE of the required length and parallel to AB . Join EA and CB . The figure resulting will be the trapezoid required.

EXAMPLES.

1. The base of a trapezoid measures 12.62 chains, the parallel side 8.14 chains, and the perpendicular 7.44 chains. The distance corresponding to DB in the last figure is 1.12 chains. Required the area and plot. Area = 7.723 acres.

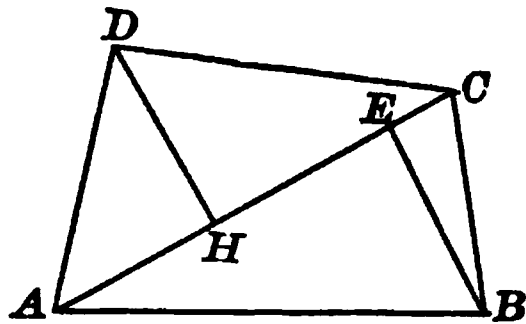
2. A railroad embankment extends 3240 feet perpendicularly across a farm intersecting parallel sides. At one end its base is 96 feet wide, and at the other 60 feet. Supposing the property line is 10 feet from the embankment on each side, how much of the farm is taken for railroad purposes?

TRAPEZIUMS.

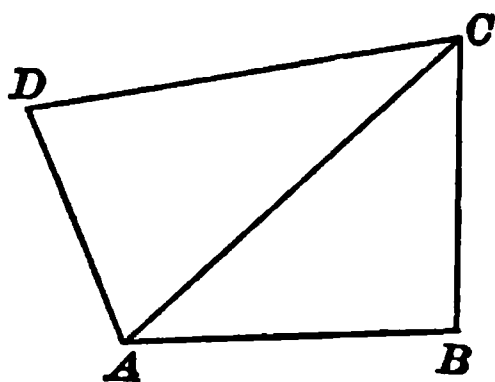
68. First Method. Measure either diagonal, and the perpendiculars thereto from the opposite angles, noting the distances AH and EC .

$$A = \frac{1}{2} AC(DH + EB)$$

To Plot. Draw the diagonal AC to the desired scale, and fix the points H and E . At these points erect perpendiculars corresponding to the scale and measurements. Joining DA and DC , and BA and BC , will complete the plot required.



Second Method. Measure all the sides and a diagonal as shown in the figure, thereby dividing the trapezium into two triangles, all the sides of which are known; whence the area may be computed by the formula for the area of a triangle in terms of the three sides.



To Plot. Lay off the diagonal AC , and locate the points B and D by methods heretofore given. Connect the points $ABCD$ for the plot required.

EXAMPLES.

1. The diagonal of a trapezium measures 120 rods, and the two perpendiculars 30 and 40 rods; what is the area?

Ans. $26\frac{1}{4}$ acres.

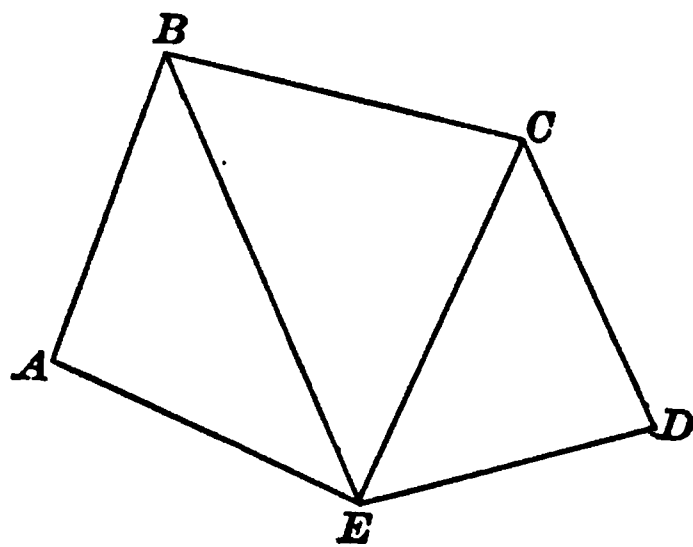
2. The sides of a trapezium taken in regular order are $AB = 5$, $BC = 9$, $CD = 11$, and $DA = 13$ chains, and the diagonal $AC = 12$ chains. Required the area and plot.

3. The sides of a trapezium are 18.10, 22.14, 28.16, and 34.62 chains, and the diagonal from the first to the third corner is 30.76 chains. Determine the area.

POLYGONS.

Regular or irregular, five or more sides.

69. First Method. Measure all the sides and the diagonals, thus dividing the tract into three or more triangles. The area will equal the sum of the areas of the triangles thus formed.

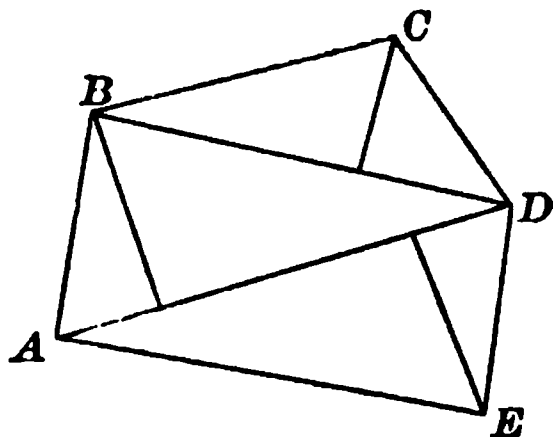


To Plot. Draw a line representing the diagonal BE , and construct the triangle ABE on it; on the other side of BE construct BCE ; if a pentagon, the plot will be completed by adding CDE .

If a hexagon, there must be measured another diagonal giving four triangles, and generally, for any number of sides n , there will be $n - 3$ diagonals and $n - 2$ triangles, the area of the tract being equal to the sum of the areas of the $n - 2$ triangles.

If the tract be a regular polygon, the measurement of one side by the aid of the table in (57) will be sufficient to determine the area.

70. Second Method.* Measure one or more diagonals, and perpendiculars from these to the opposite angles, or corners, thereby dividing the tract into right triangles, or right triangles and trapezoids. The sum of the areas of these figures will equal the area of the polygon.



EXAMPLES.

1. The sides of a pentagon taken in regular order are, 6.80, 4.20, 5.30, 8.90, and 9.62 chains. The diagonals from the fifth corner to the second and third are each 10 chains. Find the area,† and make a plot.

2. A side of a regular heptagon measures 4.25 chains. What is the area?

Given the following field notes to calculate the areas and make the plots. The distances are in chains.

| | D | | | D | |
|--------|-------|--------|--|--------|--------|
| | 16.75 | | | 12.50 | |
| C 4.50 | 18.50 | | | 7.80 | 4.60 E |
| | 12.90 | 4.50 E | | C 2.80 | 5.90 |
| | 9.00 | 3.80 F | | B 5.50 | 3.20 |
| B 4.50 | 4.80 | | | | 2.60 |
| | 3.20 | 6.25 G | | | 3.00 F |
| | A | | | A | |

* Other methods are given in Chapter II.

† The work may be abridged by using logarithms.

DATE RECEIVED

Exercice 1

111

—

1
 2
 3
 4
 5
 6
 7
 8
 9
 10
 11
 12
 13
 14
 15
 16
 17
 18
 19
 20
 21
 22
 23
 24
 25
 26
 27
 28
 29
 30
 31
 32
 33
 34
 35
 36
 37
 38
 39
 40
 41
 42
 43
 44
 45
 46
 47
 48
 49
 50
 51
 52
 53
 54
 55
 56
 57
 58
 59
 60
 61
 62
 63
 64
 65
 66
 67
 68
 69
 70
 71
 72
 73
 74
 75
 76
 77
 78
 79
 80
 81
 82
 83
 84
 85
 86
 87
 88
 89
 90
 91
 92
 93
 94
 95
 96
 97
 98
 99
 100
 101
 102
 103
 104
 105
 106
 107
 108
 109
 110
 111
 112
 113
 114
 115
 116
 117
 118
 119
 120
 121
 122
 123
 124
 125
 126
 127
 128
 129
 130
 131
 132
 133
 134
 135
 136
 137
 138
 139
 140
 141
 142
 143
 144
 145
 146
 147
 148
 149
 150
 151
 152
 153
 154
 155
 156
 157
 158
 159
 160
 161
 162
 163
 164
 165
 166
 167
 168
 169
 170
 171
 172
 173
 174
 175
 176
 177
 178
 179
 180
 181
 182
 183
 184
 185
 186
 187
 188
 189
 190
 191
 192
 193
 194
 195
 196
 197
 198
 199
 200
 201
 202
 203
 204
 205
 206
 207
 208
 209
 210
 211
 212
 213
 214
 215
 216
 217
 218
 219
 220
 221
 222
 223
 224
 225
 226
 227
 228
 229
 230
 231
 232
 233
 234
 235
 236
 237
 238
 239
 240
 241
 242
 243
 244
 245
 246
 247
 248
 249
 250
 251
 252
 253
 254
 255
 256
 257
 258
 259
 260
 261
 262
 263
 264
 265
 266
 267
 268
 269
 270
 271
 272
 273
 274
 275
 276
 277
 278
 279
 280
 281
 282
 283
 284
 285
 286
 287
 288
 289
 290
 291
 292
 293
 294
 295
 296
 297
 298
 299
 300
 301
 302
 303
 304
 305
 306
 307
 308
 309
 310
 311
 312
 313
 314
 315
 316
 317
 318
 319
 320
 321
 322
 323
 324
 325
 326
 327
 328
 329
 330
 331
 332
 333
 334
 335
 336
 337
 338
 339
 340
 341
 342
 343
 344
 345
 346
 347
 348
 349
 350
 351
 352
 353
 354
 355
 356
 357
 358
 359
 360
 361
 362
 363
 364
 365
 366
 367
 368
 369
 370
 371
 372
 373
 374
 375
 376
 377
 378
 379
 380
 381
 382
 383
 384
 385
 386
 387
 388
 389
 390
 391
 392
 393
 394
 395
 396
 397
 398
 399
 400
 401
 402
 403
 404
 405
 406
 407
 408
 409
 410
 411
 412
 413
 414
 415
 416
 417
 418
 419
 420
 421
 422
 423
 424
 425
 426
 427
 428
 429
 430
 431
 432
 433
 434
 435
 436
 437
 438
 439
 440
 441
 442
 443
 444
 445
 446
 447
 448
 449
 450
 451
 452
 453
 454
 455
 456
 457
 458
 459
 460
 461
 462
 463
 464
 465
 466
 467
 468
 469
 470
 471
 472
 473
 474
 475
 476
 477
 478
 479
 480
 481
 482
 483
 484
 485
 486
 487
 488
 489
 490
 491
 492
 493
 494
 495
 496
 497
 498
 499
 500
 501
 502
 503
 504
 505
 506
 507
 508
 509
 510
 511
 512
 513
 514
 515
 516
 517
 518
 519
 520
 521
 522
 523
 524
 525

11

(147A)

10

11

三

[illegible]

CIRCLES AND CIRCULAR RINGS.

71. Measure the radius or diameter of a circle, and the radii or diameters of a circular ring.

$$\text{The area of the former} = \pi R^2 = \frac{\pi D^2}{4}.$$

$$\text{The area of the latter} = \pi(R^2 - r^2).$$

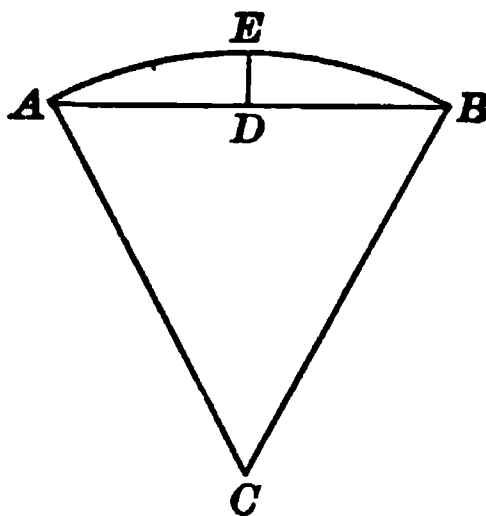
EXAMPLES.

1. The diameter of a circle is 10.16 chains. What is the area?

2. What is the area of a circular ring, the outer and inner radii measuring respectively 20 and 12 rods?

SECTORS AND SEGMENTS.

72. Measure the chord AB , and the perpendicular distance or height of arc DE , from the centre of AB to the arc AEB . From these data the radius and the angle at the centre may be found; and hence the area obtained. See (59) and (61). Otherwise, measure the radius BC , and by short chords the arc AEB ; whence the area may be computed. (The student will supply the details for both cases.)



EXAMPLES.

1. If the length of the arc of a sector is 500 feet and the radius 1000 feet, how many acres does it contain? *Ans.* 5.739.

2. If the chord AB (last figure) = 40 feet, and the height of arc DE = 10 feet, what is the area of the segment?

Ans. 279.558 square feet.

3. Given the radius, which is bisected by the chord, = 100 feet. Required the area of sector and segment.

SECTION VI.

OFFSETS AND TIE-LINES.

73. When any portion of the boundary of a tract of land is irregular, as, for example, when it is a stream or crooked road, the survey along such sides is best effected by measuring a straight line, as LN , and setting off short perpendiculars $m'm$, $o'o$, and $p'p$ at points m' , o' , and p' , and extending them to the boundary line. Such short perpendiculars are called offsets, and they should be so chosen that the part of the curve Lm , mo , op , etc., intercepted between any two consecutive ones may be considered straight; whence the area of the part lying between the straight and curved lines may be obtained by adding together the area of the triangles and trapezoids into which it is thus divided.

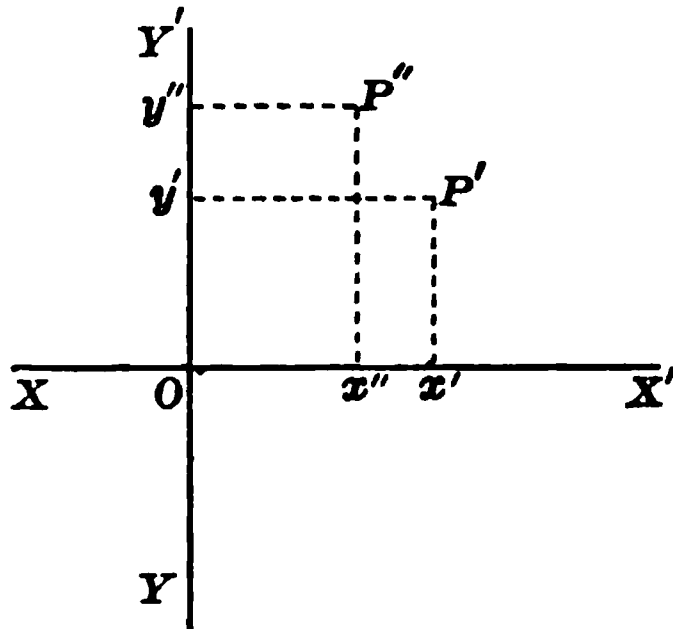
If the field notes corresponding to the above figure are as below :

| | |
|------|----------|
| N | |
| 9.60 | 0 |
| 7.00 | 1.30 p |
| 2.50 | 1.40 o |
| 1.20 | 1.00 m |
| L | 0 |

The area between straight line and boundary

| | | | | | |
|---|---|----------------|------------|--------------|---------------|
| = | { | Area triangle | Lmm' , | 6000 | square links. |
| | | Area trapezoid | $mm'oo'$, | 15600 | “ “ |
| | | Area trapezoid | $oo'pp'$, | 60750 | “ “ |
| | | Area triangle | $p'pN$, | 16900 | “ “ |
| | | | | | |
| | | | | <u>99250</u> | “ “ |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

74. Rectangular Co-ordinates. Let XX' and YY' be two straight lines intersecting each other at right angles at O , and $P'P''$, points in their plane. Then if perpendiculars be drawn through these points to the lines XX' and YY' , the distances cut off on the former are called *abscissas*, and those on the latter *ordinates*. The abscissa and ordinate referring to one point, as P' , are termed the co-ordinates of that point.



The lines to which the measurements are referred are called the *axes*; XX' being called the axis of abscissas or axis of X , and YY' the axis of ordinates or axis of Y .

The axes being at right angles, the system is called the *rectangular system of co-ordinates*. O is the *origin*.* Designating the ordinates measured from the axis of X upward, and the abscissas measured to the right of the axis of Y , as *plus*, and those downward from the X -axis and to the left of the Y -axis, as *minus*, it is evident that a point can be located in either quadrant very readily by this method.

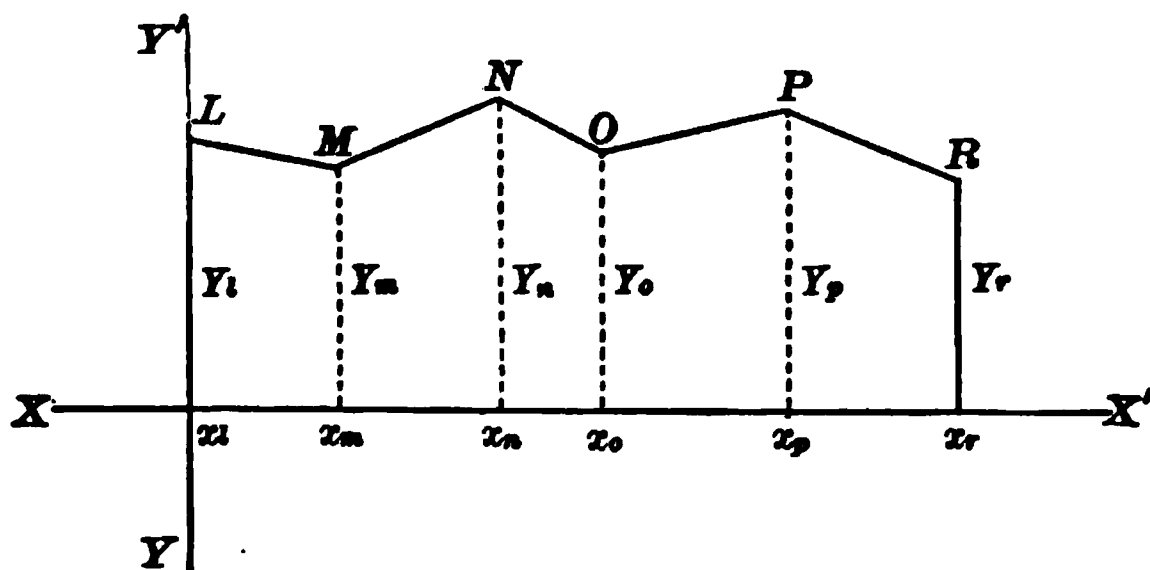
If the co-ordinates of P' are $x = 6$ and $y = 4$, it means simply that $Ox' = 6$ and $Oy' = 4$, and the point may be located by drawing the lines as indicated. If $x = -5$ and $y = 3$, the point is five units to the left of the Y -axis, and three units above the X -axis, etc.

75. Application of Rectangular Co-ordinates to the Computation of Areas.

Suppose it is required to find the area of any number of trapezoids formed by a broken line, and perpendiculars from its angles upon a straight line as indicated in the figure. XX' , the

* Axes inclined to each other are called *oblique*.

straight line, may be taken as the axis of X , and YY' the axis of Y . Let x_1, x_m, x_n , etc., y_1, y_m, y_n , etc., denote respectively the abscissas and ordinates of the points L, M, N , etc.



The area required

$$= \frac{1}{2} [x_m(y_1 + y_m) + (x_n - x_m)(y_m + y_n) + (x_o - x_n)(y_n + y_o) + (x_p - x_o)(y_o + y_p) + (x_r - x_p)(y_p + y_r)].^*$$

By expanding and simplifying there results

$$\frac{1}{2} [x_m(y_1 - y_n) + x_n(y_m - y_o) + x_o(y_n - y_p) + x_p(y_o - y_r) + x_r(y_p + y_r)].$$

Whence for calculating the area of a tract of land included between a straight line and a broken line, whose angles are given by their co-ordinates upon the straight line as base, we have the following

RULE.

Multiply the difference between each ordinate and the second succeeding one by the abscissa of the intervening ordinate.

Multiply also the sum of the last two ordinates by the last abscissa.

The half of the algebraic sum of these several products will be the area.

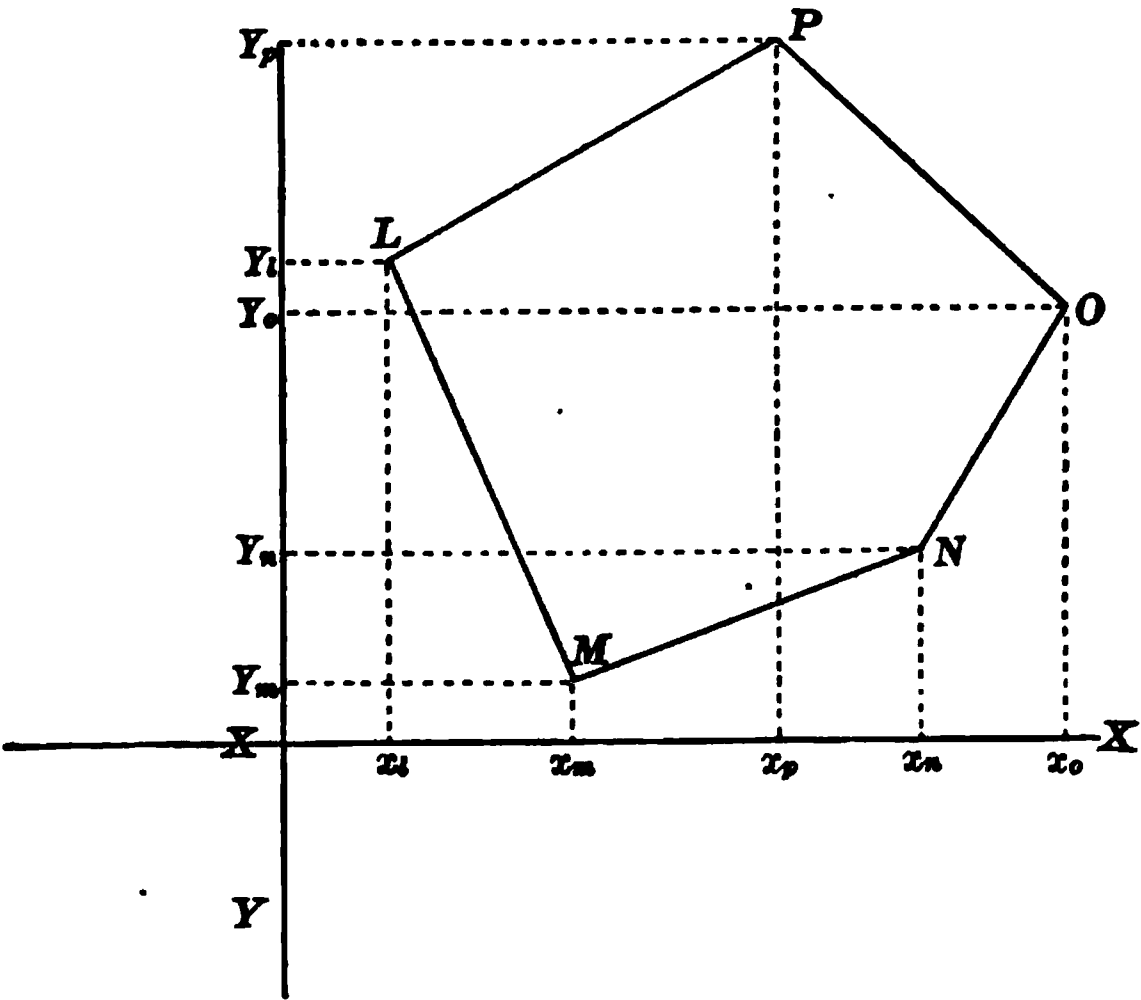
EXAMPLES.

Calculate the areas, and make the plots from the following field notes; the distances are in chains.

* A similar expression could evidently be found for any number of trapezoids.

| | | | |
|------|------|------|------|
| 1.20 | .90 | 2.20 | 1.60 |
| 2.60 | 1.50 | 4.30 | 2.00 |
| 4.00 | 2.10 | 5.00 | 2.40 |
| 3.00 | 1.60 | 3.20 | 1.50 |
| 1.80 | 1.00 | 2.50 | 1.00 |
| 1.00 | .60 | 1.70 | .20 |
| | | 0 | 0 |

76. A slight modification of the rule just given will make it applicable to the case where a broken line encloses a tract or forms the boundary of a polygon.



Let the tract enclosed be represented by the figures, then the area

$$\begin{aligned}
 A = \frac{1}{2} [& y_1 - y_n \quad x_n - x_1 + y_2 - y_n \quad x_n - x_2 \\
 & + y_3 - y_1 \quad x_1 - x_2 + y_3 - y_2 \quad x_2 - x_1 \\
 & + \dots + y_n - y_{n-1} \quad x_{n-1} - x_n]
 \end{aligned}$$

By expanding, cancelling, and factoring, we may obtain either of the following expressions:

$$\begin{aligned}
 A = \frac{1}{2} [& x_1 y_n - y_1 x_n + x_2 y_n - y_2 x_n + x_3 y_1 - y_3 x_1 \\
 & + x_4 y_2 - y_4 x_2 + \dots + x_n y_{n-1} - y_n x_{n-1}] : \quad (1)
 \end{aligned}$$

$$\text{or. } A = -\frac{1}{2} [& y_1 x_n - x_1 y_n + y_2 x_n - x_2 y_2 + y_3 x_1 - x_3 y_1 \\
 & + y_4 x_2 - x_4 y_2 + \dots + y_n x_{n-1} - x_n y_{n-1}] \quad (2)$$

Whence, for the area of a polygon whose corners are given by their co-ordinates, we have the following

RULE.

*Take one-half the sum of the products of each ^{abscissa}ordinate and the difference of its adjacent ^{ordinates}abscissas, always making the subtraction in the same direction round the plot.**

EXAMPLES.

1. Given the abscissas of the several corners of a field, *L, M, N, O, P*, respectively:

2.00, 5.50, 12.00, 15.00, and 8.60 chains.

The corresponding ordinates:

10.20, 1.80, 4.00, 9.40, and 14.00 chains:

to compute the area.

* The work of computation may be abridged when the abscissas are greater than the ordinates, by making the differences of the abscissas the factors with the ordinates; and when the ordinates are greater than the abscissas, taking the differences of the ordinates with the abscissas. If the axis of ordinates pass through *L*, the abscissa of that point would vanish. Regard must, in all cases, be had to the resulting signs.

The form of reduction is as follows :

| CORNERS. | ORDINATES. | ABSCISSAS. | DIFFERENCE BETWEEN ALTERNATE ABSCISSAS. | DOUBLE AREAS. |
|----------|------------|------------|--|--|
| <i>L</i> | 10.20 | 2.00 | 3.10 | 31.6200 |
| <i>M</i> | 1.80 | 5.50 | — 10.00 | — 18.0000 |
| <i>N</i> | 4.00 | 12.00 | — 9.50 | — 38.0000 |
| <i>O</i> | 9.40 | 15.00 | 3.40 | 31.9600 |
| <i>P</i> | 14.00 | 8.60 | 13.00 | 182.0000 |
| | | | | 245.5800 — 56. <hr/> 2)189.5800 10)94.79 sq. chs. 9.479 acres. |

2. Given the abscissas of the several corners of a field, *L*, *M*, *N*, *O*, *P*, *Q*, *R*, respectively :

0, 6.50, 14.60, 22.80, 20.00, 16.70, 9.90 ;

and the corresponding ordinates :

13.20, 3.72, 4.40, 3.90, 17.24, 16.90, and 17.30,

all in chains ; to determine the area and make a plot.

3. Given the abscissas of the several corners of a field, *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, respectively :

100, 300, 360, 290, 400, 250, 120, 0 ;

and the corresponding ordinates :

0, 0, 160, 300, 380, 520, 520, and 330,

all in feet ; to determine the area, and make a plot.

4. Verify Example 3 by a method independent of that given on the preceding page.

5. Required the area and plot from the following field notes :

| | | | | | | | | |
|-----------------------|------|-------|----------------|----------|----------|-----------|------------|--|
| Sides | | | | | | | Proof line | |
| | | A | | | E | | | |
| | | 23.50 | | Diag. DF | 41.10 | | | |
| | | F | ✓ | | 25.80 | | | |
| | | | | | Diag. CF | 17.65 | | |
| | | F | | | B | | | |
| | | 21.90 | ✓ | | | | | |
| | | E | | | F | | | |
| | | 20.50 | ✓ | | 30.10 | | | |
| | | D | | | D | | | |
| | | | | | C | | | |
| | | D | | ✓ | 28.90 | | | |
| | | 17.40 | | | F | | | |
| | | 15.50 | | | 26.75 | | | |
| Offsets to river bank | 1.60 | 13.00 | | | | Diagonals | | |
| | 1.75 | 8.50 | | | | | | |
| | 1.00 | 6.00 | | | | | | |
| | 1.20 | 3.50 | | | | | | |
| | 0 | C | ✓ | | | | | |
| | | C | On river bank. | | | | | |
| | 0 | 18.00 | | | | | | |
| | 2.00 | 13.50 | | | | | | |
| | 3.50 | 10.00 | | | | | | |
| | 2.50 | 5.50 | | | | | | |
| | 0 | B | ✓ | | | | | |
| | | B | | | | | | |
| | | 18.50 | | | | | | |
| | | A | | | | | | |

Other examples containing offsets are given in Chapter II.

3. It is known that the base of an isosceles triangle is $\frac{4}{5}$ the length of one of its equal sides. The perpendicular measures 80 feet. The sides and area are required.

Ans. Each side, 100 feet; base, 120 feet.
Area, 4800 square feet.

4. Desiring to ascertain the radius of a railroad curve (it being the boundary of a field), a surveyor measured from centre to centre of tracks, a chord of 200 feet; also the perpendicular distance from the centre of chord to the middle of tracks, 4 feet. Show that these measurements indicate the radius = 1252 feet.

QUERY. How should the data obtained in Example 4 be employed to determine the area, assuming that the curve is concave to the field?

5. The circumference of a circle is 100 rods. How many acres does it contain? *Ans.* 4.974.

QUERY. Can Problem 5 be solved without first finding the radius or diameter?

6. If the number expressing the area of an equilateral triangle in square feet is the same as that showing the length of one of its sides in lineal inches, what is its area?

Ans. 332.55.

7. The chord of a circle measures 60 feet, and the height of arc, or versed sine, 10 feet. Find in the same circle the versed sine of a chord of 90 feet. *Ans.* 28.2 feet.

8. The lengths of two chords lying on the same side of the diameter of a circle are 96 and 60, and their distance apart 26. Required the area between them.

SUGGESTION. Let x = perpendicular distance from centre of short chord to the nearest point of circumference, and y = perpendicular distance from centre of long chord to the farthest point of circumference; that is, measured in the opposite direction from the first.

Then
$$\begin{aligned} x(y + 26) &= 900. \\ y(x + 26) &= 2304. \end{aligned}$$

Whence the diameter is readily determined and thence the area required.

9. Show that the area of the circumscribed hexagon is to the area of the circumscribed equilateral triangle as 2 is to 3.

10. Show that the area of a regular inscribed polygon of n sides $= \frac{n}{2} r^2 \sin \frac{360^\circ}{n}$.

11. Show that the area of a regular circumscribed polygon of n sides $= nr^2 \tan \frac{180^\circ}{n}$.

12. The distance between the centres of two circles, whose diameters are each 50, is equal to 30. What is the area common to the two circles? *Ans.* 559.15.

13. Three equal circles being tangent to each other externally enclose 40 rods. What is the radius of each circle? *Ans.* 15.75 rods.

EXERCISES.

1. Survey a polygon, measure all the sides and necessary diagonals, run test-lines, record the notes, make a plot, and compute the area.

2. Take the boundaries as found above, and complete the survey by measuring one diagonal and perpendicular offsets to the corners. Make record, plot, and computation.

3. Measure a field partly bounded by a creek or lake, rendering it necessary to take offsets thereto. Record the notes, plot, and calculate area.

4. Survey a pond or small lake by tie-lines and offsets.

CHAPTER II.

COMPASS AND TRANSIT SURVEYING.

SECTION I.

DEFINITIONS AND DESCRIPTION OF INSTRUMENTS.

78. The Axis of the earth is the imaginary line about which it rotates.

The Poles are the points where the axis pierces the earth : one the north pole, the other the south pole.

79. A Meridian Plane is a plane embracing the earth's axis.

80. A Meridian Line, or true meridian, is the intersection of a meridian plane with the surface of the earth.

In plane surveying the meridians passing through the extremities of lines surveyed are considered parallel.

81. The Magnetic Needle is a thin bar of strongly magnetized steel, balanced on a pivot, so that it may turn freely, and always come to rest in the direction of the magnetic meridian.

82. The Magnetic Meridian is indicated by the direction of a bar magnet, when horizontal, freely suspended and at rest. It does not in general coincide with the geographic meridian. The angle included between them is called the declination of the needle, or variation of the compass,* and the *change* in this angle is termed the *variation* of the declination.

* See Chapter III., on Declination of the Needle.

83. The Azimuth of a Line is the angle which the vertical plane containing it makes with the plane of the meridian.

84. The Bearing of a Line, called also the course, is the angle which it forms with the direction of the magnetic needle.

85. The Meridian Distance of a Point is its perpendicular distance from an assumed meridian.

86. The Meridian Distance of a Line is the meridian distance of the middle point of that line.

87. A Horizontal Angle is an angle included between two lines in a horizontal plane.

A Vertical Angle is an angle included between two lines in a vertical plane.

88. An Angle of Elevation is a vertical angle, one side of which is horizontal, and the other inclined upward from the angular point.

89. An Angle of Depression is a vertical angle, one side of which is horizontal, and the other inclined downward from the angular point.

In Compass and Transit Surveying, in addition to the measurement of lines, angles are observed; hence, besides the instruments previously described, we present the following:

THE SURVEYOR'S COMPASS.

90. The Surveyor's * Compass consists essentially of a brass plate carrying a horizontal graduated circle, in the centre of which is suspended, so as to turn freely, a magnetic needle; and at the extremities of the plate are attached vertically two flattened pieces of brass, called sights, having fine slits and

* The Solar Compass is described in Chapter VI.

circular openings in them, by which the instrument is directed upon any object or station.

In addition to the essentials named, this instrument usually has two small spirit levels set on the plate at right angles to each other, a vernier scale for setting off the declination of the needle, a tangent scale for reading vertical angles, and a brass head for mounting the instrument upon a tripod or a single staff called Jacob's Staff.

91. The graduated circle is divided into half-degrees, and is figured from 0 to 90 on each side of the centre line of zeros.

The magnetic needle is from 4 to 6 inches long in the different sizes of compasses, having set in its centre a piece of hardened steel highly polished, which, resting upon the hardened point of the centre-pin, allows the needle to turn freely, horizontally, and to take its direction in the magnetic meridian.

92. The needle is lifted from its support by a concealed spring actuated by a screw. The test of the delicacy of a magnetic needle is the number of vibrations which it will make in a certain arc before coming to rest.

When the compass is not in use, the needle should be screwed up against the glass, and the instrument set so that the north end of the needle points towards the north.

TO ADJUST THE COMPASS.

93. The Levels. First bring the bubbles into the centre, by the pressure of the hand on different parts of the plate, and then turn the compass half-way around; should the bubbles run to the end of the tubes, it would indicate that those ends were the highest: lower them by tightening the screws immediately under, and loosening those under the lowest ends until, by estimation, the error is half removed; level the plate again, and repeat the first operation until the bubbles will remain in the centre during an entire revolution of the compass.



SURVEYOR'S COMPASS.

94. The Sights may next be tested by observing through the slits a fine hair or thread, made exactly vertical by a plumb. Should the hair appear on one side of the slit, the sight must be adjusted by filing off its under surface on that side which seems the highest.

95. The Needle is adjusted in the following manner: Having the eye nearly in the same plane with the graduated rim of the compass-circle, with a small splinter of wood or a slender iron wire bring one end of the needle in line with any prominent division of the circle, as the zero or ninety-degree mark, and notice if the other end corresponds with the degree on the opposite side: if it does, the needle is said to "cut" opposite degrees; if not, bend the centre-pin by applying a small brass wrench, about one-eighth of an inch below the point of the pin, until the ends of the needle are brought into line with the opposite degrees.

Then, holding the needle in the same position, turn the compass half-way around, and note whether it now cuts opposite degrees; if not, correct half the error by bending the needle, and the remainder by bending the centre-pin.

The operation should be repeated until perfect reversion is secured in the first position.

This being obtained, it may be tried on another quarter of the circle; if any error is there manifested, the correction must be made in the centre-pin only, the needle being already straightened by the previous operation.

96. Electricity. A little caution is necessary in handling the compass, that the glass covering be not excited by the friction of cloth, silk, or the hand, so as to attract the needle to its under surface.

When, however, the glass becomes electric, the fluid may be removed by breathing upon it, or touching different parts of its surface with the moistened finger.

97. The Needle is remagnetized as follows :

The operator, being provided with an ordinary permanent magnet, and holding it before him, should pass with a gentle pressure each end of the needle from centre to extremity over the magnetic pole, describing before each pass a circle of about six inches radius, to which the surface of the pole is tangent, drawing the needle towards him, and taking care that the north and the south ends are applied to the *opposite* poles of the magnet.

Should the needle be returned in a path near the magnetic pole, the current induced by the contact of the needle and magnet, in the pass just described, would be reversed, and thus the magnetic virtue almost entirely neutralized at each operation.

When the needle has been passed about twenty-five times in succession, in the manner just described, it may be considered as fully charged.

A fine brass wire is wound in two or three coils on the south end of the needle, and may be moved back or forth in order to counterpoise the varying weight of the north end.

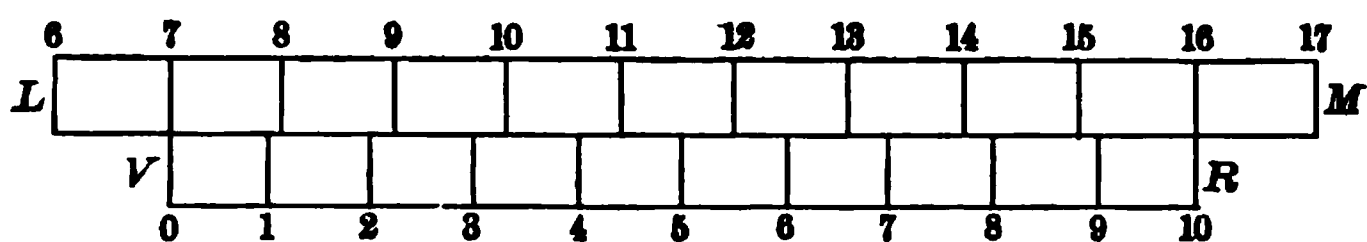
98. The Centre-Pin. This should occasionally be examined, and if much dulled, taken out with a brass wrench or with a pair of pliers, and sharpened on a hard oil-stone — the operator placing it in the end of a small stem of wood or a pin-vise, and delicately twirling it with the fingers as he moves it back and forth at an angle of about 30 degrees to the surface of the stone.

When the point is thus made so fine and sharp as to be invisible to the eye, it should be smoothed by rubbing it on the surface of a soft and clean piece of leather.

99. Weight. The average weights of the different sizes of compasses, including the brass head of the jacob-staff, beginning with the smallest, are respectively $5\frac{1}{2}$, $7\frac{1}{2}$, and $9\frac{1}{2}$ pounds.

THE VERNIER.

100. A Vernier is an auxiliary scale for measuring smaller divisions than those into which a graduated scale or limb is divided.* The smallest reading of the vernier, or *least count*, is the difference in length between one division on the graduated scale or limb, and one on the vernier. If the divisions on the vernier are smaller than those on the limb, the vernier is *direct*; if the reverse, *retrograde*.



Let LM represent any scale divided into tenths, and we wish to measure or read to tenths of these divisions, *i.e.* to $\frac{1}{100}$. Using a direct vernier, we should have 10 spaces on it equal to 9 on the scale, and each one of them equal to $\frac{9}{10}$ of $\frac{1}{10}$, or $\frac{9}{100}$, of the scale graduation; giving a *least count* of $\frac{1}{100} - \frac{9}{100} = \frac{1}{100}$, as desired. To read to twentieths of the divisions on the scale, we should have 20 divisions on the vernier corresponding to 19 on the scale, or each space on the vernier equal to $\frac{19}{20} \cdot \frac{1}{10} = \frac{19}{200}$, and giving a *least count* of $\frac{1}{100} - \frac{19}{200} = \frac{1}{200}$.

In general, if s = the smallest division of the scale or limb,
 v = the smallest division of the vernier,
 n = number of divisions on the vernier,

we shall have least count $= s - v = \frac{s}{n}$.

Or, the *least count* of a vernier is equal to the smallest division of the scale or limb divided by the number of divisions on the vernier.†

If $s = \frac{1}{2}$ degree, and $n = 30$, as ordinarily found on transit

* It derives its name from Peter Vernier, 1631.

† It is evidently immaterial whether LM be straight or curved.

plates, the least count will be $\frac{1}{2} \div 30 = \frac{1}{60}$ of a degree = one minute.

If $s = \frac{1}{2}$ degree, and $n = 40$, oftentimes found on vertical arcs to solar attachments, the smallest reading $= \frac{1}{2} \div 40 = \frac{1}{80}$ of a degree $= \frac{1}{2}$ minute.

To space a vernier for a given least count, say $10''$, on a limb graduated to $10'$, we must have $n = \frac{s}{s-v} = \frac{10}{\frac{1}{60}} = 60$ spaces, covering 59 spaces on the limb.

101. To read an Instrument having a vernier consists in determining the number of units and fractional parts thereof, into which its scale or limb may be divided, from the zero point on the limb, where the graduation begins, to the zero point of the vernier.

It is accomplished as follows: Take the reading of the scale, as shown by the last graduation preceding the zero of the vernier; then find a line on the vernier which coincides with a line on the scale. The number of this line, as indicated by the graduation on the vernier, shows how many units of the *least count* are to be added to the first reading.

EXERCISES.

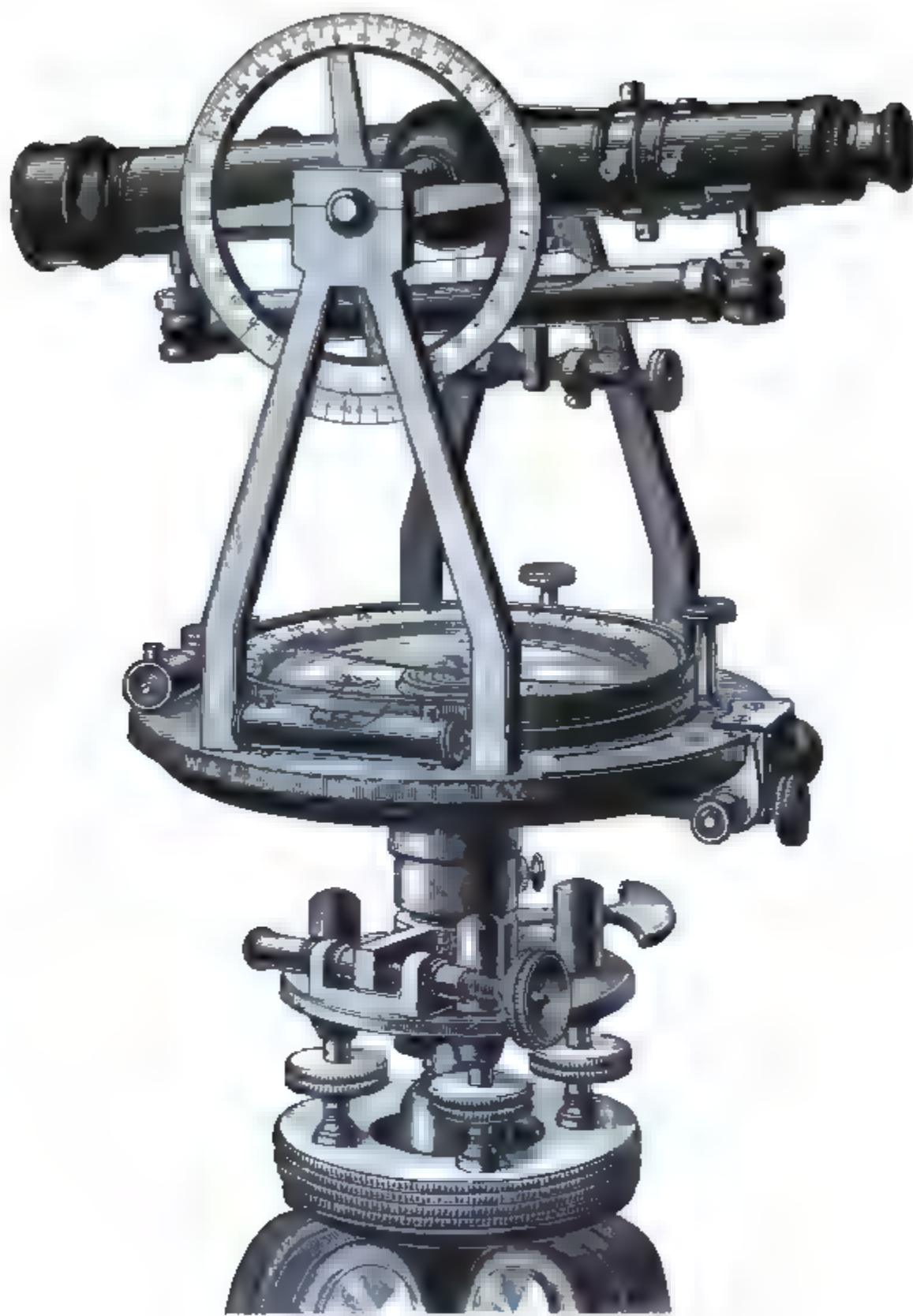
1. A levelling-rod is graduated into feet, tenths, and hundredths. It is required to space a direct vernier so that the rod may be read to thousandths of a foot.

2. An arc is graduated into quarter-degrees, and a vernier of 30 parts covers 29 parts of the arcs; find the least count.

3. A scale is divided into inches and tenths of an inch; plan a direct vernier by means of which the scale may be read to $\frac{1}{160}$ of an inch.

Plan a retrograde vernier to accomplish the same object.

4. Design a vernier which when applied to a limb graduated into $20'$ will give a least count of $20''$.



SURVEYOR'S TRANSIT.

NOTE. The principal part of the description of the Compass and Transit, and the plates for the engraving of these instruments, were kindly furnished by Messrs. W. & L. E. Gurley, Troy, N.Y.

SURVEYOR'S TRANSIT.

102. The essential parts of the Transit, as shown in the cut, are the *telescope* with its axis and two supports, the *circular plates* with their attachments, the *sockets* upon which the plates revolve, the *levelling-head*, and the *tripod* on which the whole instrument stands.

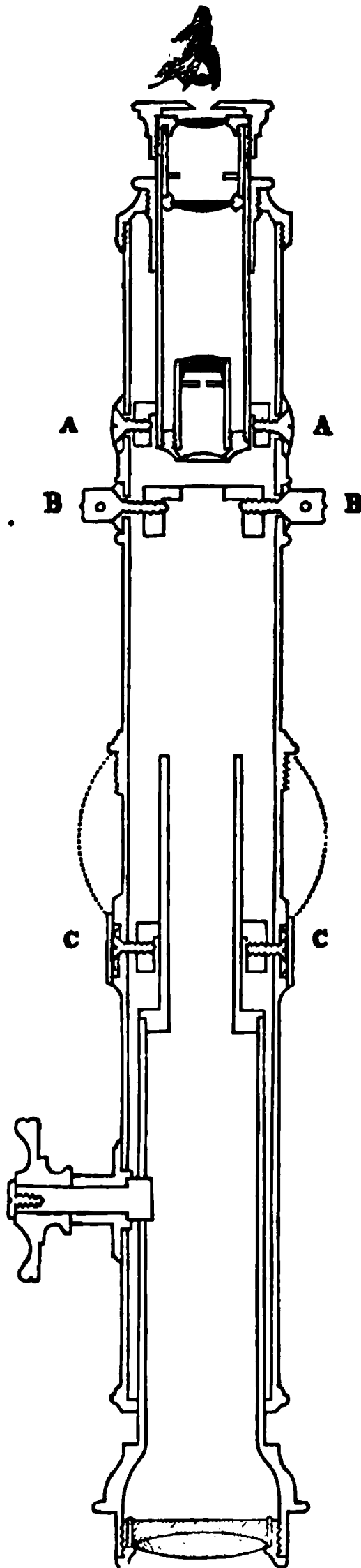
The *telescope* is from 10 to 11 inches long, firmly secured to an axis having its bearings nicely fitted in the standards, and thus enabling the telescope to be moved in either direction, or turned completely around if desired.

The different parts of the telescope are shown in the marginal figure.

The object-glass is composed of two lenses, so as to show objects without color or distortion, is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring *CC*, suspended within the tube by four screws, only two of which are shown in the cut.

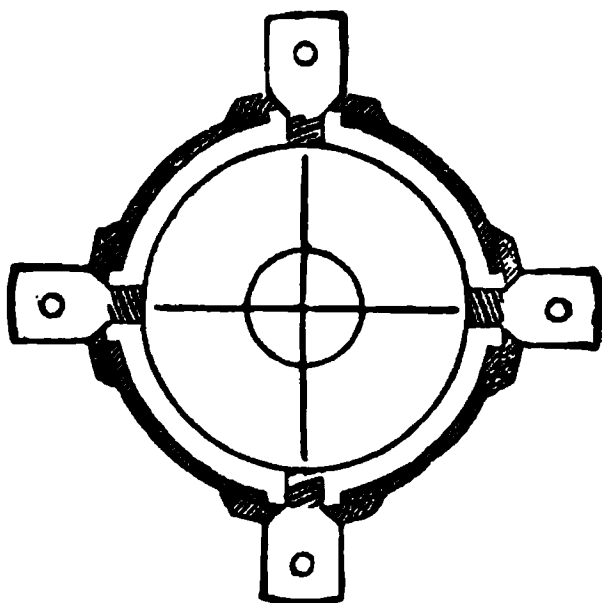
The object-glass is carried out or in by a pinion working in a rack attached to the slide, and thus adjusted to objects either near or remote as desired.

The eye-piece is made up of four plano-convex lenses, which, beginning at the eye-end, are called respectively the eye, the field, the amplifying, and the object-lenses, the whole forming a compound microscope having its focus in the plane of the cross-wire ring *BB*.



The eye-piece is brought to its proper focus usually by twisting its milled end, the spiral movement within carrying the eye-tube out or in as desired; sometimes a pinion, like that which focuses the object-glass, is employed for the same purpose.

103. The Cross-Wires are two fibres of spider-web or very fine platinum wire, cemented into the cuts on the surface of a



metal ring, at right angles to each other, so as to divide the open space in the centre into quadrants.

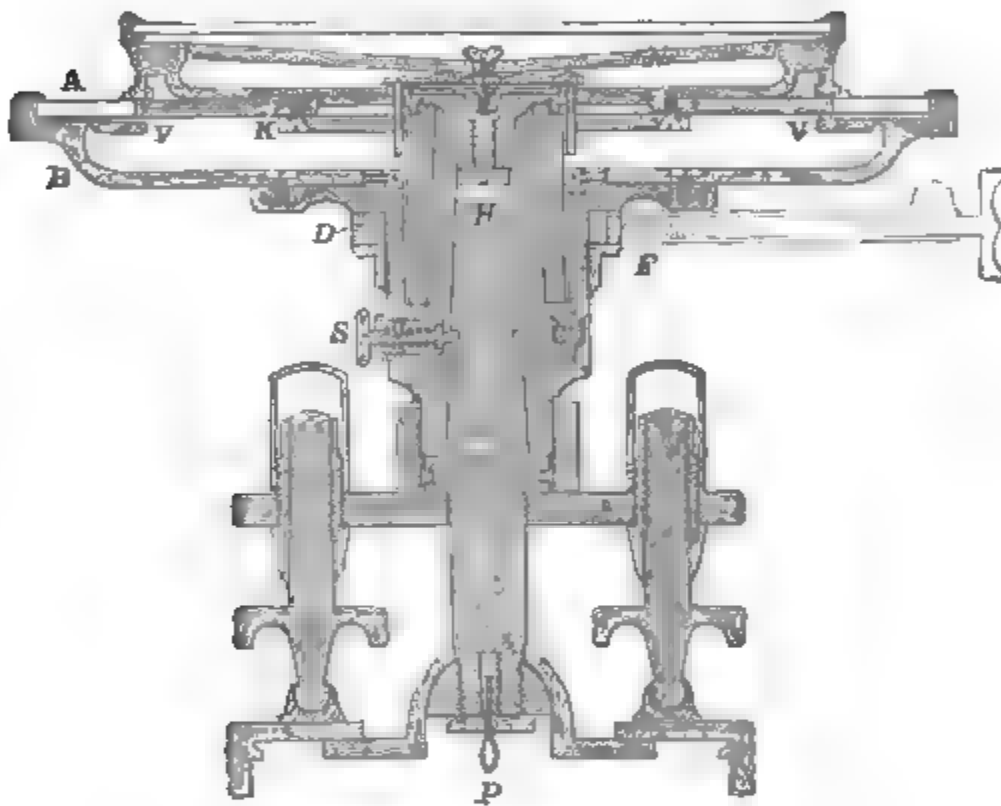
104. Optical Axis. The intersection of the wires forms a very minute point, which, when they are adjusted, determines the optical axis of the telescope, and enables the surveyor to fix it upon an object with the greatest precision.

The imaginary line passing through the optical axis of the telescope is termed the “line of collimation,” and the operation of bringing the intersection of the wires into the optical axis, is called the “adjustment of the line of collimation.” This will be hereafter described.

105. The Standards of the Transit are firmly attached by their expanded bases to the upper plate, one of them having near the top, as shown in the cut, a little movable box, actuated by a screw underneath, by which the telescope axis is made truly horizontal, as will be hereafter described.

The sectional view here given shows the interior construction of the sockets of the transit, the manner in which it is detached from the spindle, and the means by which it can be taken apart if desired.

In the figure, the limb *BB* is attached to the main socket *C*, which is itself carefully fitted to the conical spindle *H*, and held in place by the spring catch *S*.



The upper plate, *AA*, carrying the compass-circle, standards, etc., is fastened to the flanges of the socket *K*, which is fitted to the upper conical surface of the main socket *C*; the weight of all the parts being supported on the small bearings of the end of the socket, as shown, so as to turn with the least possible friction.

A small conical centre, in which from below is inserted a strong screw, is brought down firmly upon the upper end of the main socket *C*, and thus holds the two plates of the instrument securely together, while at the same time allowing them to move freely around each other in use.

A small disc above the conical centre contains the steel centre-pin upon which rests the needle, as shown; the disc is fastened to the upper plate by two small screws, as represented.

The main socket with all its parts is of the best bell-metal and is most carefully and thoroughly made, the long bearing of the sockets insuring their firm and easy movement, while at the same time they are entirely out of the reach of dust, or other source of wear.

When desired, the whole upper part of the instrument can be taken off from the spindle by pulling out the head of the spring catch at *S*, and when replaced will be secured by the self-acting spring of the catch.

The figure also shows the covers of the levelling-screws, the shifting centre of the lower levelling-plate, and the screw and loop for the attachment of the plummet.

The compass-box, containing the needle, etc., is covered by a glass to exclude the moisture and air; the circle is silvered, and is divided on its upper surface or rim into degrees and half-degrees, the degree marks being also cut down on its inner edge, and figured from 0 to 90 on each side of the centre or line of zero.

106. The Magnetic Needle is four to five inches long in the different sizes of transits, its brass cap having inserted in it a little socket or centre of hardened steel, perfectly polished, and this resting upon the hardened and polished point of the centre-pin, allows the needle to play freely in a horizontal direction, and thus take its direction in the magnetic meridian. The needle has its north end designated by a scallop or other mark, and on its south end a small coil of fine brass wire, easily moved, so as to bring both ends of the needle to the same level. The needle is lifted from the pin by a concealed spring underneath the upper plate, actuated by a screw shown above, thus raising the button so as to check the vibrations of the needle, or bring it up against the glass when not in use, to avoid the unnecessary wear of the pivot.

107. The Clamp and Tangent Movement, shown in the engraving, page 64, attached to the plates, serves to fasten the two plates together, so that by the tangent screw they can be slowly moved around each other in either direction, or loosened at will and moved by the hand, thus enabling one to direct the telescope rapidly and accurately to the point of sight.

The Two Levels are shown placed at right angles to each other so as to level the plate in all directions, and adjusted by turning the capstan-head screws at their ends, by a small steel adjusting-pin. The glass vials used in the levels are ground on their upper interior surface, so as to make the bubble move evenly and with great sensitiveness.

108. The Lower Plate, or Limb BB, is divided on its upper surface — usually into degrees and half-degrees — and generally figured in two rows; viz., from 0 to 360, and from 0 to 90 each way.

109. The Verniers are double, having on each side of the zero mark thirty equal divisions corresponding precisely with twenty-nine half-degrees of the limb; they thus read to single minutes, and the number passed over is counted in the same direction in which the vernier is moved.

The use of two opposite verniers in this and other instruments gives the means of “cross-questioning” the graduations, the perfection with which they are centred, and the dependence which can be placed upon the accuracy of the angles indicated.

Reflectors of silver or celluloid, as in the mountain transit, are often used to throw more light upon the divisions, and more rarely shades of ground glass are employed to give a clear but more subdued light.

110. The Graduations are made commonly on the brass surface of the limb, afterwards filled with black wax, and then finished and silvered. Many instruments, however, have a solid silver plate put over the brass, and the graduations made on the silver itself.

The last is more costly, but insures a finer graduation, with less liability to tarnish or change color.

III. The Sockets of the transit are compound; the interior spindle attached to the vernier plate, turning in the exterior socket *C* when an angle is taken on the limb; but when the plates are clamped together, the exterior socket itself, and with it the whole instrument, revolves in the socket of the levelling-head.

The sockets are made with the greatest care, the surfaces being truly concentric with each other, and the bell-metal or composition of which they are composed, of different degrees of hardness, so as to cause them to move upon each other easily and with the least possible wear.

The levelling-head also consists of two plates connected together by a socket, having at its end a hemispherical nut, fitting into a corresponding cavity in the lower plate.

The plates are inclined to each other or made parallel at will by four levelling-screws, of which only two are shown in the section.

The screws are of bronze or hard composition metal and fitted to long nuts of brass, screwed into the upper parallel plate; and, as will be noticed, have threads only on the upper ends, the lower part of their stems turning closely in the lower unthreaded part of the nuts.

By this arrangement dust is excluded from the lower end of the screws, while the brass cover above equally protects the other end.

The screws rest in little cups or sockets, which are secured to their ends and in which they turn without marring the surface of the lower plate, the cups also permitting the screws to be shifted from side to side, or turned around in either direction on the lower plate.

The clamp and tangent movement of the levelling-head serves to turn the whole instrument upon its sockets, so as to fix the telescope with precision upon any given point, and when un-

clamped allowing it to be directed approximately by hand. The tangent screws, as will be seen, press on opposite sides of the clamp-piece, and thus insure a very fine and solid movement of the instrument.

112. The Lower Levelling-Plate is made in two pieces — the upper one, which is screwed fast to the top of the tripod, having a large opening in its centre, in which the smaller lower one is shifted from side to side, or turned completely around.

By this simple arrangement, termed a *shifting centre*, the instrument is easily moved over the upper plate, and the plummet which hangs from the centre *P*, set precisely over a point, without moving the tripod.

113. The Levelling-Head of the engineer's transit is attached to the sockets by a screw and washer below; it can be removed for cleaning, oiling, etc., but should be in place when the instrument is in use, or packed for transportation.

114. The Tripod has three mahogany legs, the upper ends of which are pressed firmly on each side of a strong tenon on the solid bronze head by a bolt and nut on opposite sides of the leg; the nut can also be screwed up at will by a wrench furnished for the purpose, and thus kept firm.

The lower end of the leg has a brass shoe with iron point, securely fastened and riveted to the wood.

115. To Adjust the Transit. Every instrument should leave the hands of the maker in complete adjustment; but all are so liable to derangement by accident or careless use, that we deem it necessary to describe particularly those which are most likely to need attention.

The principal adjustments of the transit are :

1. *The Levels.*
2. *The Line of Collimation.*
3. *The Standards.*

116. To Adjust the Levels. Set up the instrument upon its tripod as nearly level as may be, and having unclamped the plates, bring the two levels above and on a line with the two pairs of levelling-screws; then, with the thumb and first finger of each hand clasp the heads of two, opposite; and, turning both thumbs in or out, as may be needed, bring the bubble of the level directly over the screws, exactly to the centre of the opening. Without moving the instrument, proceed in the same manner to bring the other bubble to its centre; after doing this, the level first corrected may be thrown a little out; bring it in again; and when both are in place, turn the instrument half-way around: if the bubbles both come to the centre, they would need no correction, but if not, with the adjusting-pin turn the small screws at the end of the levels until the bubbles are moved over half the error; then bring the bubbles again into the centre by the levelling-screws, and repeat the operation until the bubbles will remain in the centre during a complete revolution of the instrument, and the adjustment will be complete.

117. To Adjust the Line of Collimation. To make this adjustment, — which is, in other words, to bring the intersection of the wires into the optical axis of the telescope, so that the instrument, when placed in the middle of a straight line, will, by the revolution of the telescope, cut its extremities, — proceed as follows:

Set the instrument firmly on the ground and level it carefully; and then, having brought the wires into the focus of the eye-piece, adjust the object-glass on some well-defined point, as the edge of a chimney or other object, at a distance of from 200 to 500 feet; determine if the vertical wire is plumb, by clamping the instrument firmly and applying the wire to the vertical edge of a building, or observing if it will move parallel to a point taken a little to one side: should any deviation be manifested, loosen the cross-wire screws, and by the pressure of the hand on the head outside the tube, move the ring around until the error is corrected.

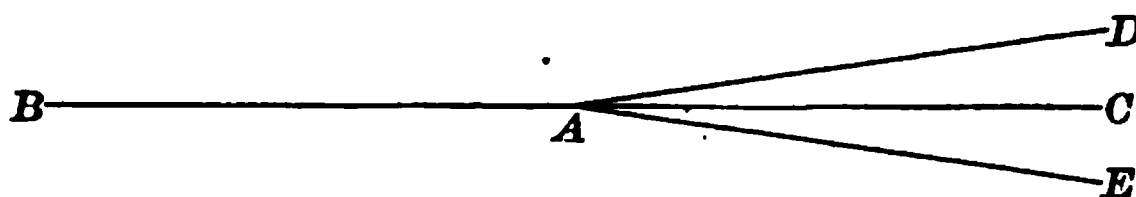
The wires being thus made respectively horizontal and vertical, fix their point of intersection on the object selected; clamp the instrument to the spindle, and having revolved the telescope, find or place some good object in the opposite direction, and at about the same distance from the instrument as the first object assumed.

Great care should always be taken in turning the telescope, that the position of the instrument upon the spindle is not in the slightest degree disturbed.

Now, having found or placed an object which the vertical wire bisects, unclamp the instrument, turn it half-way around, and direct the telescope to the first object selected; having bisected this with the wires, again clamp the instrument, revolve the telescope, and note if the vertical wire bisects the second object observed.

Should this happen, it will indicate that the wires are in adjustment, and the points bisected are with that of the centre of the instrument, in the same straight line.

If not, however, the space which separates the wires from the second point observed, will be double the deviation of that point from a true straight line, which may be conceived as drawn through the first point and the centre of the instrument, since the error is the result of two observations, made with the wires when they are out of the optical axis of the telescope.



For, as in the diagram, let *A* represent the centre of the instrument, and *BC* the imaginary straight line, upon the extremities of which the line of collimation is to be adjusted.

B represents the object first selected, and *D* the point which the wires bisected, when the telescope was made to revolve.

When the instrument is turned half around, and the telescope again directed to *B*, and once more revolved, the wires will

bisect an object E , situated as far to one side of the true line as the point D is on the other side.

The space DE , is therefore the sum of two deviations of the wires from a true straight line, and the error is made very apparent.

In order to correct it, use the two capstan-head screws on the sides of the telescope, these being the ones which affect the position of the vertical wire.

Remember that the eye-piece inverts the position of the wires, and therefore, that in loosening one of the screws and tightening the other on the opposite side, the operator must proceed as if to increase the error observed. Having in this manner moved back the vertical wire until, by estimation, one-quarter of the space DE has been passed over, return the instrument to the point B , revolve the telescope, and if the correction has been carefully made, the wires will now bisect a point C , situated midway between D and E , and in the prolongation of the imaginary line, passing through the point B and the centre of the instrument.

To ascertain if such is the case, turn the instrument half around, fix the telescope upon B , clamp to the spindle, and again revolve the telescope towards C . If the wires again bisect it, it will prove that they are in adjustment, and that the points B , A , C , all lie in the same straight line.

Should the vertical wire strike to one side of C , the error must be corrected precisely as above described, until it is entirely removed.

118. To Adjust the Standards. In order that the wires may trace a vertical line as the telescope is moved up or down, it is necessary that both the standards of the telescope should be of precisely the same height.

To ascertain this and make the correction if needed, proceed as follows :

Having the line of collimation previously adjusted, set up the instrument in a position where points of observation, such as

the point and base of a lofty spire, can be selected, giving a long range in a vertical direction.

Level the instrument, fix the wires on the top of the object, and clamp to the spindle; then bring the telescope down, until the wires bisect some good point, either found or marked at the base; turn the instrument half around, fix the wires on the lower point, clamp to the spindle, and raise the telescope to the highest object.

If the wires bisect it, the vertical adjustment is effected; if they are thrown to either side, this would prove that the standard opposite that side was the highest, the apparent error being double that actually due to this cause.

To correct it, one of the bearings of the axis is made movable, so that by turning a screw underneath this sliding piece, as well as the screws which hold on the cap of the standard, the adjustment is made with the utmost precision.

OTHER ADJUSTMENTS OF THE TRANSIT.

Besides the three adjustments already described — which are all that the surveyor will ordinarily have to make — there are those of the needle and the object-glass slide which may sometimes be required.

The first is given with the description of the compass; the last will now be described.

119. To Adjust the Object-Slide. Having set up and levelled the instrument, the line of collimation being also adjusted for objects from 300 to 500 feet distant, clamp the plates securely, and fix the vertical cross-wire upon an object as distant as may be distinctly seen; then, without disturbing the instrument, throw out the object-glass, so as to bring the vertical wire upon an object as near as the range of the telescope will allow. Having this clearly in mind, unclamp the limb, turn the instrument half-way around, reverse the eye-end of the telescope, clamp the limb, and with the tangent-screw bring the vertical

wire again upon the near object; then draw in the object-glass slide until the distant object first sighted upon is brought into distinct vision. If the vertical wire strikes the same line as at first, the slide is correct for both near and remote objects; and, being itself straight, for all distances.

But if there be an error, proceed as follows: first, with the thumb and forefinger twist off the thin brass tube that covers the screws *CC* shown in the sectional view of the telescope, p. 65. Next, with the screw-driver, turn the two screws *CC* on the opposite *sides* of the telescope, loosening one and tightening the other, so as apparently to increase the error, making, by estimation, one-half the correction required.

Then go over the usual adjustment of the line of collimation, and having it completed, repeat the operation above described; first sighting upon the distant object, then finding a near one in line, and then reversing, making correction, etc., until the adjustment is complete.

120. To Use the Transit. The instrument should be set up firmly, the tripod legs being pressed into the ground, so as to bring the plates as nearly level as convenient; the plates should then be carefully levelled and properly clamped, the zeros of the verniers and limb brought into line by the upper tangent-screw, and the telescope directed to the object by the tangent-screws of levelling-head.

The angles taken are then read off upon the limb, without subtracting from those given by the verniers, in any other position.

Before an observation is made with the telescope, the eyepiece should be moved in or out, until the wires appear distinct to the eye of the operator; the object-glass is then adjusted by turning the pinion-head until the object is seen clear and well-defined, and the wires appear as if fastened to its surface.

The intersection of the wires, being the means by which the optical axis of the telescope is defined, should be brought precisely upon the centre of the object to which the instrument is directed.

The needle is used, as in the compass, to give the bearing of lines, and as a rough check upon the angles obtained by the verniers and limb; but its employment is only subsidiary to the general purposes of the transit.

121. Attachments of Transits. The engraving of the Surveyor's Transit represents the attachments often applied to the Engineer's Transit, viz.: vertical circle, level on telescope, and clamp and tangent to telescope axis. They are of use where approximate levelling and vertical angles are to be taken in connection with the ordinary use of the transit, and with their adjustments, etc., will now be described.

122. The Vertical Circle firmly secured to the axis of the telescope is $4\frac{1}{2}$ inches in diameter, plated with silver, divided to half-degrees, and with its vernier enables the surveyor to obtain vertical angles to single minutes.

123. The Level on Telescope consists of a brass tube about $6\frac{1}{2}$ inches long, each end of which is held between two capstan-nuts connected with a screw or stem attached to the under side of the telescope tube.

124. The Clamp and Tangent consists of an arm at one end encircling the telescope axis, and at the other connected with the tangent-screw; the clamp is fastened at will to the axis by a clamp-screw, inserted at one side of the ring, and then by turning the tangent-screw the telescope is raised or lowered as desired.

125. To Adjust the Vertical Circle. Having the instrument firmly set up and carefully leveled, bring into line the zeros of the circle and vernier, and with the telescope find or place some well-defined point or line, from 200 to 300 feet distant, which is cut by the horizontal wire.

Turn the instrument half-way around, revolve the telescope, and fixing the wire upon the same point as before, note if the zeros are again in line.

If not, loosen the capstan-head screws, which fasten the vernier, and move the zero of the vernier over half the error ; * bring the zeros again into coincidence, and proceed precisely as at first, until the error is entirely corrected, when the adjustment will be complete.

This method is not applicable when only an *arc* of a circle is attached. The adjustment may then be made as follows : Observe successively from each of the two points to the other, and as before use half the error in adjusting the vernier. Verify by repetition.

A slight error may be most readily removed by putting the zeros in line and then moving the wire itself over half the interval.

126. The Level is Adjusted by bringing the bubble carefully into the centre by the nuts at each end ; and when there is a vertical circle on the instrument, this should be done when the zeros of circle and vernier are in line and in adjustment ; when there is no vertical circle, proceed as follows :

127. To Adjust the Level on Telescope. Choose a piece of ground nearly level, and having set the instrument firmly, level the plates carefully, and bring the bubble of the telescope into the centre with the tangent-screw. Measure in any direction from the instrument, from 100 to 300 feet, and drive a stake, and on the stake set a staff, and note the height cut by the horizontal wire ; then take the same distance from the instrument in an opposite direction, and drive another stake.

On that stake set the staff, and note the height cut by the wire when the telescope is turned in that direction.

The difference of the two observations is evidently the difference of level of the two stakes.

Set the instrument over the lowest stake, or that upon which

* Called *Index Error*. It may be rectified as here shown, or each observation corrected by this amount.

the greatest height was indicated, and bring the levels on the plates and telescope into adjustment as at first.

Then, with the staff, measure the perpendicular distance from the top of the stake to the centre of one of the horizontal cross-wire screw-heads; from that distance subtract the difference of level between the two stakes and mark the point on the staff thus found; place the staff on the other stake, and with the tangent-screw bring the horizontal wire to the mark just found, and the line will be level.

The telescope now being level, bring the bubble of the level into the centre, by turning the little nuts at the end of the tube, and noting again if the wires cut the point on the staff; screw up the nuts firmly and the adjustment will be completed.

128. To Take Apart the Surveyor's Transit. When it is necessary to separate the plates of the transit, proceed as follows:

(1) Remove the clamp-screw and take off the head of the pinion, both on the north end and outside the compass circle.

(2) Unscrew the bezel ring containing the glass cover of the compass, remove the needle and button beneath it, and take out the two small screws so as to remove the disc.

(3) Take the instrument from its spindle, and with a large screw-driver take out the screw from the underside of the conical centre (see figure, p. 67).

(4) Drive out the centre from below by a round piece of wood, holding the instrument vertical so that the centre will not bruise the circle.

(5) Set the instrument again upon its spindle, take out the clamp-screw to the tangent movement of the limb, and the work is complete. To put the transit together again, proceed exactly the reverse of the operation thus described.

129. The Solar Attachment is essentially the solar apparatus of Burt placed upon the cross-bar of the ordinary transit, the polar axis only being directed above instead of below, as in the solar compass.

A little circular disc of an inch and a half diameter, and having a short, round pivot projecting above its upper surface, is first securely screwed to the telescope axis.

Upon this pivot rests the enlarged base of the polar axis, which is also firmly connected with the disc by four capstan-head screws passing from the under side of the disc into the base already named.

These screws serve to adjust the polar axis, as will be explained hereafter.

130. The Hour Circle surrounding the base of the polar axis is easily movable about it, and can be fastened at any point desired by two flat-head screws above. It is divided to five minutes of time; is figured from I. to XII., and is read by a small index fixed to the declination circle, and moving with it.

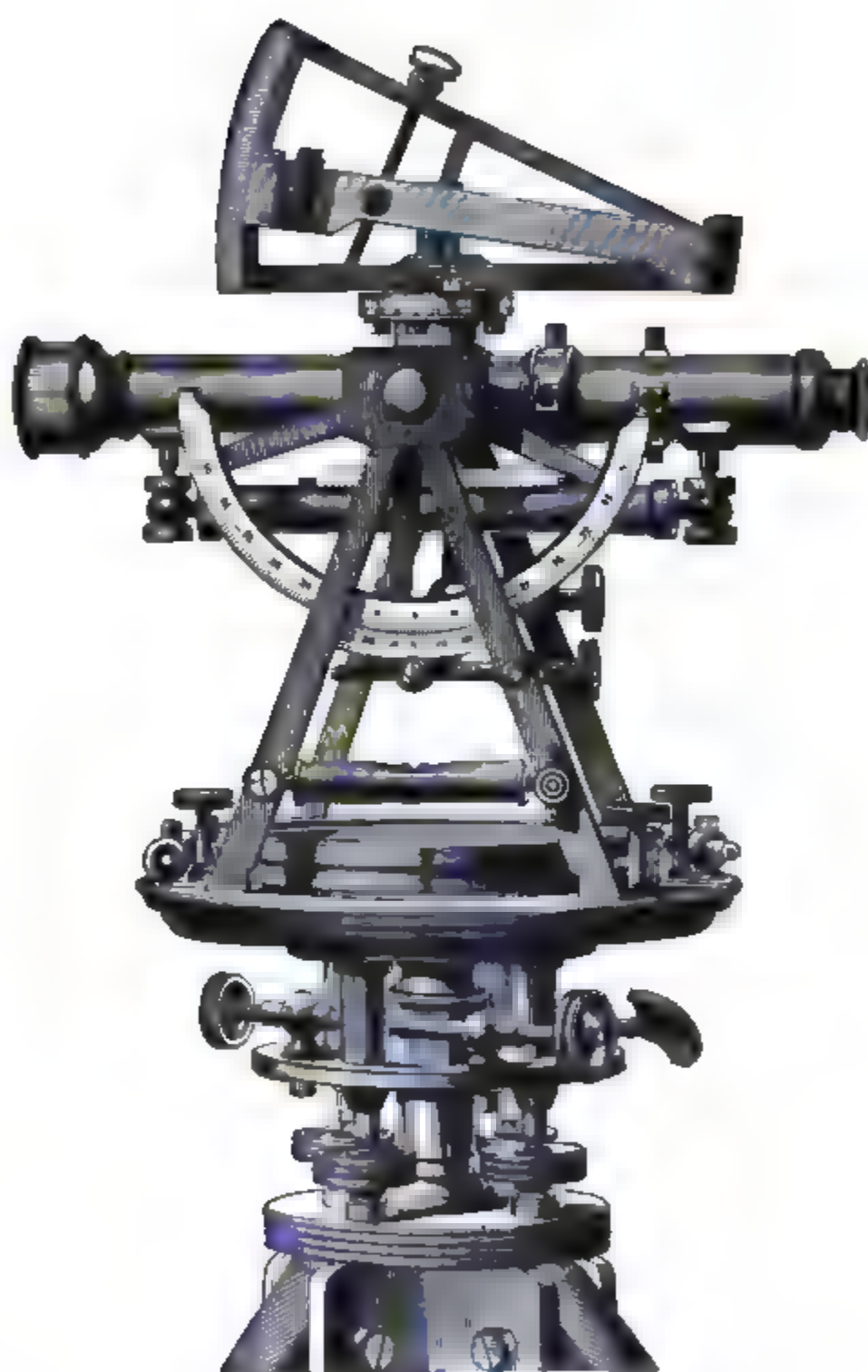
A hollow cone, or socket, fitting closely to the polar axis, and made to move snugly upon it, or clamped at any point desired by a milled-head screw on top, furnishes by its two expanded arms below a firm support for the declination arc, which is securely fastened to it by two large screws, as shown.

131. The Declination Arc is of about 5 inches radius, is divided to quarter degrees, and reads by its vernier to single minutes of arc, the divisions of both vernier and limb being in the same plane.

The declination arm has the usual lenses and silver plates on the two opposite blocks, made precisely like those of the ordinary solar compass, but its vernier is outside the block, and more easily read.

The declination arm has also a clamp and tangent movement, as shown in the cut. The arc of the declination limb is turned on its axis, and one of the other solar lens used, as the sun is north or south of the equator; the cut shows its position when it is north.

The Latitude is set off by means of a large vertical limb having a radius of $2\frac{1}{2}$ inches; the arc is divided to twenty minutes,



TRANSIT WITH SOLAR ATTACHMENT.

is figured from the centre, each way, up to 80° , and is read by its vernier to single minutes.

It has also a clamp-screw inserted near its centre, by which it can be set fast to the telescope axis in any desired position.

The vernier of the vertical limb is made movable by the tangent-screw attached, so that its zero and that of the limb are readily made to coincide when, in adjusting the limb to the level of the telescope, the arc is clamped to the axis.

The usual tangent movement to the telescope axis serves, of course, to bring the vertical limb to the proper elevation, as hereafter described.

A level on the under side of the telescope, with ground vial and scale, is indispensable in the use of the solar attachment.

The divided arcs, verniers, and hour circle, are all on silver plate, and are thus easily read and preserved from tarnishing.

THE ADJUSTMENTS.

132. The Solar Lenses and Lines are adjusted precisely like those of the ordinary solar, the declination arm being first detached by removing the clamp and tangent screws, and the conical centre with its two small screws, by which the arm is attached to the arc.

The adjuster, which is a short bar furnished with every instrument, is then substituted for the declination arm, the conical centre screwed into its place at one end, and the clamp-screw into the other, being inserted through the hole left by the removal of the tangent-screw, thus securing the adjuster firmly to the arc.

The arm is then turned to the sun, as described in the article on the Solar Compass, and reversed by the opposite faces of the blocks upon the adjuster, until the image will remain in the centre of the equatorial lines. This adjustment is very rarely needed, as the lenses are cemented in their cells, and the plates securely fastened.

133. The Vernier of the Declination Arc is adjusted by setting the vernier at zero, and then raising or lowering the telescope by the tangent-screw, until the sun's image appears exactly between the equatorial lines.

Having the telescope axis clamped firmly, carefully revolve the arm until the image appears on the other plate.

If precisely between the lines, the adjustment is complete; if not, move the declination arm by its tangent-screw, until the image will come precisely between the lines on the two opposite plates; clamp the arm and remove the index error by loosening two flat-head screws on the back, which fasten the movable arc to the declination limb; place the zero of the limb and vernier in exact coincidence and the adjustment is finished.

134. To Adjust the Polar Axis. First level the instrument carefully by the long level of the telescope, using in the operation the tangent movement of the telescope axis in connection with the levelling screws of the parallel plates, until the bubble will remain in the centre during a complete revolution of the instrument upon its axis.

Place the equatorial sights on the top of the blocks as closely as is practicable with the distinct view of a distant object; and having previously set the declination arm at zero, sight through the interval between the equatorial sights and the blocks at some definite point or object, the declination arm being placed over either pair of the capstan-head screws on the under side of the disc.

Keeping the declination arm upon the object with one hand, with the other turn the instrument half around on its axis, and sight upon the same object as before. If the sight strikes either above or below, move the two capstan-head screws immediately under the arm, loosening one and tightening the other as may be needed, until half the error is removed.

Sight again and repeat the operation, if needed, until the sight will strike the same object in both positions of the instrument, when the adjustment of the axis in one direction will be complete.

Now turn the instrument at right angles, keeping the sight still upon the same object as before ; if it strikes the same point when sighted through, the axis will be truly vertical in the second position of the instrument.

If not, bring the sight upon the same point by the other pair of capstan-head screws now under the declination arc, reverse as before, and continue the operation until the same object will keep in the sight in all positions, when the polar axis will be made precisely at right angles to the level and to the line of collimation of the transit.

It should here be noted that as this is by far the most delicate and important adjustment of the solar attachment, it should be made with the greatest care, the bubble kept perfectly in the centre and frequently inspected in the course of the operation.

135. To Adjust the Hour Arc. Whenever the instrument is set in the meridian, as will be hereafter described, the index of the hour arc should read apparent time.

If not, loosen the two flat-head screws on the top of the hour circle, and with the hand turn the circle around until it does, fasten the screws again, and the adjustment will be complete.

To obtain mean time, of course the correction of the equation for the given day, as given in the Nautical Almanac, must always be applied.

136. To Find the Latitude. First level the instrument very carefully, using, as before, the level of the telescope until the bubble will remain in the centre during a complete revolution of the instrument, the tangent movement of the telescope being used in connection with the levelling screws of the parallel plates, and the axis of the telescope firmly clamped.

Next clamp the vertical arc so that its zero and that of its vernier coincide as near as may be, and then bring them into exact line by the tangent-screw of the vernier.

Then, having the declination of the sun for 12 o'clock of the given day as affected by the meridional refraction carefully set

off upon the declination arc, note also the equation of time and fifteen or twenty minutes before noon, the telescope being directed to the north, and the object-end lowered until, by moving the instrument upon its spindle and the declination arc from side to side, the sun's image is brought nearly into position between the equatorial lines. Now bring the declination arc directly in line with the telescope, clamp the axis firmly, and with the tangent-screw bring the image precisely between the lines and keep it there with the tangent-screw, raising it as long as it runs below the lower equatorial line, or, in other words, as long as the sun continues to rise in the heavens.

When the sun reaches the meridian the image will remain stationary for an instant, and then begin to rise on the plate.

The moment the image ceases to run below is of course apparent noon, when the index of the hour arc should indicate XII, and the latitude be determined by the reading of the vertical arc.

It must be remembered, however, that the angle through which the polar axis has moved in the operation just described is measured from the zenith instead of the horizon, as in the ordinary solar, so that the angle read on the vertical limb is the complement of the latitude.

The latitude itself is readily found by subtracting this angle from 90° ; thus at Troy, the reading of the limb being found as above directed to be $47^\circ 16'$, the latitude will be

$$90^\circ - 47^\circ 16' = 42^\circ 44'.$$

It will be noticed that with this apparatus the latitude of any place can be most easily ascertained without any index error, as in the usual solar compass.

137. To Use the Solar Attachment. From the foregoing description it will be readily understood that good results cannot be obtained from the solar attachment unless the transit is of good construction,—furnished with the appliances of a level on telescope, clamp and tangent movement to axis, and vertical

arc with adjustable vernier, and the sockets or centres in such condition that the level of the telescope will remain in the centre when the instrument is revolved upon either socket.

138. To Run Lines with the Solar Attachment. Having set off the complement of the latitude of the place on the vertical arc, and the declination for the given day and hour as in the solar, the instrument being also carefully levelled by the telescope bubble, set the horizontal limb at zero, and clamp the plates together, loosen the lower clamp so that the transit moves easily upon its lower socket, set the instrument approximately north and south, the object-end of the telescope pointing to the north, turn the proper solar lens to the sun, and, with one hand on the plates and the other on the revolving arm, move them from side to side, until the sun's image is brought between the equatorial lines on the silver plate.

The lower clamp of the instrument should now be fastened, and any further lateral movement be made by the tangent-screw of the tripod. The necessary allowance being made for refraction, the telescope will be in the true meridian, and being unclamped, may be used like the sights of the ordinary solar compass, but with far greater accuracy and satisfaction in establishing meridian lines. Of course when the upper or vernier plate is unclamped from the limb, any angle read by the verniers is an angle from the meridian, and thus parallels of latitude or any other angles from the true meridian may be established as with the solar compass.

The bearing of the needle, when the telescope is on the meridian, will also give the variation of the needle at the point of observation.

The declination of the needle being set off, and the needle kept then at zero, or "with the sun," lines may be run by the needle alone when the sun is obscured.

Though when not inconsistent with the remarks following the table on page 95, the sun should be observed for direction at every station.

THE SAEGMULLER ATTACHMENT.

139. As seen in the engraving on the opposite page, it consists essentially of a small telescope and level, the telescope being mounted in standards, in which it can be elevated or depressed. The standards revolve around an axis, called the polar axis, which is fastened to the telescope axis of the transit instrument. The telescope, called the "Solar Telescope," can thus be moved in altitude and azimuth. Two pointers, attached to the solar telescope to approximately set the instrument, are so adjusted that when the shadow of the one is thrown upon the other the sun will appear in the field of view.

140. Adjustments. When the apparatus is attached to the transit, which instrument must be in good adjustment, its polar axis *should be at right angles both to the horizontal axis of the main telescope and to the line of collimation.*

TEST. Level the transit, and bring the bubble of each telescope to the centre of its run. Revolve the solar telescope about its polar axis, and if its bubble remains central, this adjustment is complete. If not, correct half the movement by the adjusting screws at the base of the polar axis, and the other by revolving the solar telescope on its horizontal axis.

141. Second. *The line of collimation of the solar telescope and the axis of its attached level must be parallel.*

TEST. Bring the telescopes into the same vertical plane, and the large bubble to the middle of its run. Direct then the transit telescope to a mark at a convenient distance away, say 100 feet; point also the "solar" to a mark above this equal to the distance between their axes. If now the bubble of the solar telescope is not in the middle of the tube, make it so by the adjusting screws, and the instrument will be in adjustment.

When the combined instrument is in proper adjustment the bubbles of the telescopes and plates will be in the middle of their tubes, and the lines of collimation parallel.



TRANSIT WITH SOLAR ATTACHMENT,
AS MADE BY FAUTH & CO., WASHINGTON, D.C.

All the adjustments, including those of the transit, should be *frequently* examined, and kept as nearly perfect as possible.

142. The advantages of solar attachments over the ordinary solar compass consist principally in the telescopic sight, and the use of a vertical limb to set off declination and co-latitude.

LATITUDE.

By the Sun. — With Saegmuller's Attachment.

143. Level the transit carefully, point the telescope south, and elevate or depress the object-end, according as the declination of the sun is south or north, an amount equal to the declination.* Bring the solar telescope into the vertical plane of the main telescope, level it carefully, and clamp it. With the solar telescope observe the sun a few minutes before his culmination, bring the horizontal middle wire tangent to the upper limb by moving the transit telescope in altitude and azimuth, and keep it so by the slow-motion screws until the sun ceases to rise. Then take the reading of the vertical arc, correct for index error, if any, for refraction due to altitude,† as per table below; diminish the result by the sun's semi-diameter, and subtract the result from 90° for the latitude.

* For declination, consult a nautical almanac.

† Corrected for index error, the arc reading would be the sum of the co-latitude and refraction. The refraction being due to the meridian altitude of the sun, which altitude in the United States is equal to the algebraic sum of the declination and co-latitude.

TABLE OF MEAN REFRACTIONS OF CELESTIAL OBJECTS FOR TEMPERATURE 50°, AND BAROMETER 29.6 INCHES.

| ALTITUDE. | REFRACTION. | ALTITUDE. | REFRACTION. |
|-----------|-------------|-----------|-------------|
| 10° | 5' 15'' | 20° | 2' 35'' |
| 11 | 4 47 | 25 | 2 02 |
| 12 | 4 23 | 30 | 1 38 |
| 13 | 4 03 | 35 | 1 21 |
| 14 | 3 45 | 40 | 1 08 |
| 15 | 3 30 | 45 | 0 57 |
| 16 | 3 17 | 50 | 0 48 |
| 17 | 3 04 | 60 | 0 38 |
| 18 | 2 54 | 70 | 0 21 |
| 19 | 2 44 | 80 | 0 10 |

By interpolation, the refraction due to any altitude within the limits of the table may be found.

LATITUDE BY CIRCUMPOLAR STAR.

144. The arc measuring the angle of elevation of the pole at any station indicates the latitude of that station. If, then, the place of the pole were indicated by a heavenly body, its altitude measured and corrected for refraction would give at once the latitude.

There being no such body, a circumpolar star may be used. Take its altitude at either culmination, subtract refraction due to altitude, and the remainder, increased or diminished by the polar distance according as the lower or upper culmination was observed, will give the latitude.

Better, when practicable, to observe both culminations, correct for refraction, and take the arithmetical mean of the result. Still greater accuracy would be obtained by taking the mean of observations at upper and lower transit of several circumpolar stars.

If A and A' respectively denote the angles measuring, from the north, the altitudes of a circumpolar star at its upper and lower culminations, and r and r' the corresponding refractions, then,

$$\text{latitude} = \frac{1}{2} [A + A' - (r + r')].$$

TO FIND THE MERIDIAN AND DECLINATION OF THE NEEDLE,
USING THE ATTACHMENT.*

145. First. Take the declination of the sun as given in the Nautical Almanac for the given day, and correct it for refraction and hourly change. Incline the *transit telescope* until this amount is indicated by its vertical arc. If the declination of the sun is north, depress the object-end; if south, elevate it. Without disturbing the position of the transit telescope, bring the solar telescope into the same vertical plane, and make it horizontal by means of its level. The two telescopes will then form an angle which equals the amount of the declination, and the inclination of the solar telescope to its polar axis will be equal to the polar distance of the sun.

Second. Without disturbing the *relative* positions of the two telescopes, incline them and set the vernier to the co-latitude of the place.

By moving the transit and the solar attachment around their respective *vertical* axes, the image of the sun will be brought into the field of the solar telescope, and after accurately bisecting it the *transit telescope must be in the meridian, and the compass-needle indicates its deviation at that place.*

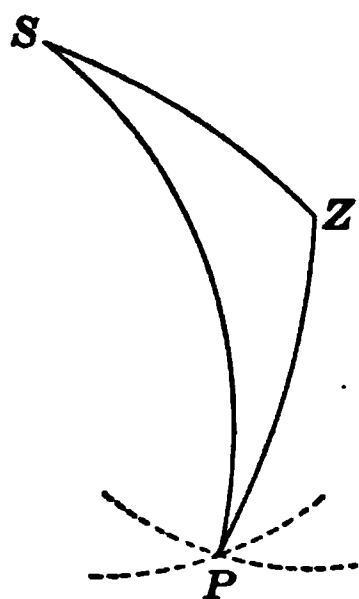
The vertical axis of the solar attachment will then point to the pole, the apparatus being in fact a small equatorial. Revolve the main telescope on its horizontal axis, and set a mark at a convenient distance,—1000 feet if practicable.

Make a *reverse* observation as follows: Turn the transit 180° in azimuth, and set off the declination, elevating or depressing now the *eye-end*, according as the declination is south or north; bring the object-end of the solar telescope to point in the direction of the eye-end of that of the main instrument, and level it. Set the vertical arc to the co-latitude of the place, and complete the observation as before. Reverse the large telescope on its

* For other methods, see Chapter III., p. 218, and Chapter VI., Solar Compass.

horizontal axis, and see if it points to the mark set by the *direct* observation ; if it do not, take the mean of the two pointings for the meridian.

If greater accuracy is required, make other observations at different hours of the day, under different conditions of the atmosphere, and compare results with those given in Chapters III. and VI.



146. Time and azimuth are calculated from an observed altitude of the sun by solving the spherical triangle formed by the sun, the pole, and the zenith of the place. The three sides, SP , PZ , ZS , complements respectively of the declination, latitude, and altitude are given, and we hence deduce SPZ , the hour angle, from apparent noon, and PZS the azimuth of the sun.*

The "Solar Attachment" solves the same spherical triangle by construction, for the second process brings the vertical axis of the solar telescope to the required distance ZP from the zenith, while the first brings it to the required distance SP from the sun.

If the two telescopes, both being in position — one in the meridian, and the other pointing to the sun — are now turned on their *horizontal* axes, the vertical remaining undisturbed, until each is level, the angle between their directions — found by sighting on a distant object — is SPZ , the time from apparent noon.

This gives an easy observation for correction of time-piece.

147. An error either in the declination or latitude will cause an error in the azimuth.

These errors in azimuth corresponding to one-minute error in declination or latitude, for various hours and half-hours of the

* A Table of Equation of Time is given at the end of this book which will be useful in solving analytically the spherical triangle PZS for time.

day, and for different latitudes, have been computed and tabulated.*

The following table exhibits these errors in latitude 40° .

For latitude 50° the errors are one-fifth greater, and for latitude 30° the errors are about one-ninth less than those given.†

By interpolation, those corresponding to other latitudes and fractional parts of the hour may be obtained.

TABLE OF ERRORS IN AZIMUTH FOR ONE MINUTE ERROR IN LATITUDE OR DECLINATION ON THE PARALLEL OF 40° .

| Hours | 11.30 A.M. | 11 A.M. | 10 A.M. | 9 A.M. | 8 A.M. | 7 A.M. | 6 A.M. |
|------------------------------|------------|---------|---------|--------|--------|--------|--------|
| | 12.30 P.M. | 1 P.M. | 2 P.M. | 3 P.M. | 4 P.M. | 5 P.M. | 6 P.M. |
| For one min. error in dec. } | 10.00' | 5.05' | 2.61' | 1.85' | 1.51' | 1.35' | 1.30' |
| For one min. error in lat. } | 9.92' | 4.87' | 2.26' | 1.30' | 0.75' | 0.35' | 0.00' |

The table indicates the best time to observe the sun for meridian, or to determine the true bearing of a line, to be soon after sunrise or just before sunset.

However, on account of refraction at these times being great and very uncertain, it is best in general not to make the observation when the sun is nearer the horizon than about 15 degrees. Moreover, the solar apparatus should not be relied on for very accurate work between 10 A.M. and 2 P.M.

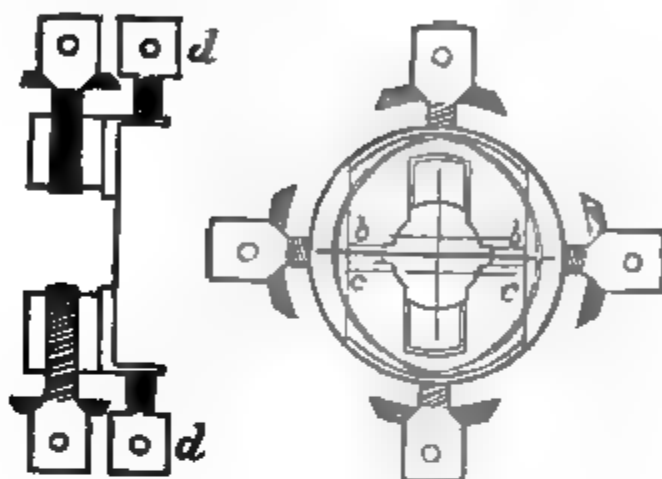
An error in latitude does not cause an error in azimuth when the sun is in the pole of the meridian.

148. The Stadia, or Micrometer, is a compound cross-wire ring or diaphragm, shown below, having three horizontal wires, of which the middle one is cemented to the ring as usual, while the others, *bb* and *cc*, are fastened to small slides, held apart by

* By Professor Johnson, C.E., Washington University, Mo.; and by R. T. Stewart, C.E., Instructor in Mathematics and Engineering, Western University of Pennsylvania.

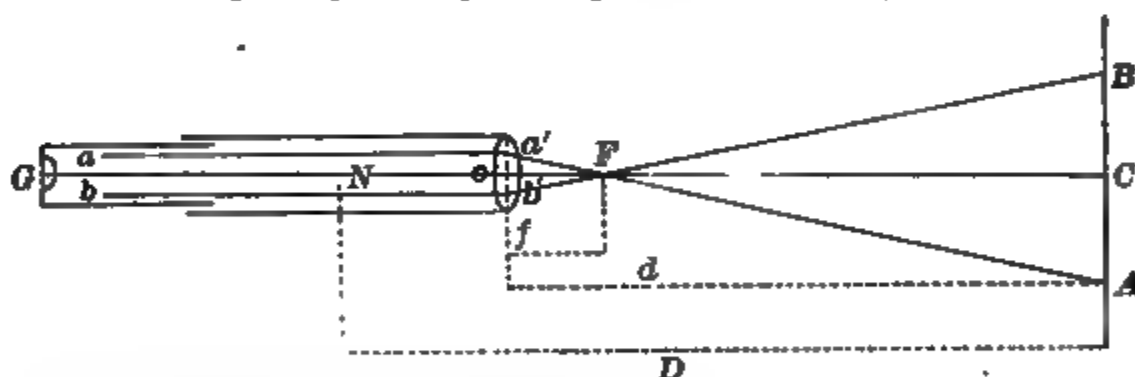
† More accurately, $11\frac{1}{2}$ per cent.

a slender brass spring hoop, and actuated by independent screws dd , by which the distance between the two movable wires can be adjusted to include a given space; as, 1 foot on a rod 100 feet distant. These wires will in the same manner include 2 feet on



a rod 200 feet distant, or half a foot at a distance of 50 feet, and so on in the same proportion; thus furnishing a means of measuring distances — especially over broken ground — much more easily, and even more accurately, than with a tape or chain.

149. Its principles may be explained more fully as follows :



Let the above figure represent a section of a common telescope with but two lenses, between which the diaphragm with the stadia wires is placed, and assume that

- f = the focal distance of the object-glass ;
- p = the distance of the stadia wires a and b from each other ;
- d = the horizontal distance of the object-glass to the stadia ;

a = stadia reading (BA) ;

D = horizontal distance from middle of instrument to stadia.

The telescope is levelled and sighted to a levelling or stadia rod, which is held vertically, hence at right angles with the line of sight. According to a principle of optics, rays parallel to the axis of the lens meet, after being refracted, in the focus of the lens. Suppose the two stadia wires are the sources of those rays, we have, from the similarity of the two triangles $a'b'F$ and FAB , the proportion

$$d - f : a = f : p.$$

The quotient $f : p$ is, or at least can be made, constant, and may be designated by k ; hence we may write

$$d - f = FC = ka.$$

To get the distance from the centre N of the instrument there must be added to FC the value

$$c = OF + ON.$$

ON is mostly equal to half the focal length of the object-glass ; hence,

$$c = 1.5 f.$$

Therefore the formula for the distance of the stadia from the centre of instrument, when that stadia is at right angles to the level line of sight, is

$$D = ka + c. \quad (1)$$

150. When the line of sight is not level, it is impracticable, especially in long distances, to hold the rod in a vertical plane, and at the same time perpendicular to the line of sight ; hence it is customary to hold the rod vertical, as in the preceding case, and obtain the true distance by applying a correction depending upon the angle of inclination of the sight.

This correction is deduced as follows :

Let $AGB = 2m$;

n = the angle of inclination ;

Substituting this value of GF in the equation above, we obtain

$$a = \frac{CD \cos m \sin m \cos (n + m) + \cos (n - m)}{2 \sin m \cos (n + m) \cos (n - m)};$$

or, $CD = a \frac{\cos^2 n \cos^2 m - \sin^2 n \sin^2 m}{\cos n \cos^2 m},$

and $D' = c + ka \frac{\cos^2 n \cos^2 m - \sin^2 n \sin^2 m}{\cos n \cos^2 m}.$

Whence,

$$D = c \cos n + ka \cos^2 n - ka \sin^2 n \tan^2 m.$$

The third member of this equation may be safely neglected, as it is very small, even for long distances and large angles of elevation (for 1500', $n = 45^\circ$ and $k = 100$, it is but 0.07'); therefore the final formula for distances, with a stadia kept vertical, and with wires equidistant from the centre wire, is the following:

$$D = c \cos n + ak \cos^2 n. \quad (2)$$

The value of $c \cos n$ is usually neglected, as it amounts to but 1 or 1.5 feet; it is exact enough to add always 1.25' to the distance as derived from the formula

$$D = ak \cos^2 n. \quad (2a)^*$$

151. The focal length f of the object-glass may be found by focussing the instrument upon some distant object, say a heavenly body, and measuring then the distance between the plane of the cross-wires and that of the objective. ON , being equal to the distance between the objective and the intersection of a plumb-line with the horizontal axis of the telescope, may be obtained by direct measurement.

The distance p , between the stadia wires, may be determined as follows:

Set up the instrument on level ground, or nearly so, and measure forward from the plumb-line a distance equal to c , and

* The above explanation of the stadia is substantially that given by Mr. G. J. Specht, published by Van Nostrand, 1884, though corrected and simplified.

mark the point; measure onward from the mark any convenient distance d , 400 or 500 feet, as a base. The telescope being level, observe carefully the space a intercepted by the stadia wires on a levelling-rod held vertically at the farther extremity of the base.

Then from the proportion $d - f : a = f : p$ the required distance p may be obtained.

EXAMPLES.

1. Given $f = 8$ inches, base = 500 feet, and $a = 5.25$ feet. Find $p = .084$ inches.

2. At what fractional part of the focal length must the stadia wires be separated so that one foot on the rod will correspond to 100 feet base? State also the distance between the wires in terms of the focal length, when one foot on rod corresponds to 66 feet base.

3. Measure with a stadia one or more sides of a field, also the distance across a valley, or from one ridge to another, and compare the results with chain measurement between the same points.

4. Measure with the stadia up or down a hillside, and chain between the same points. Compare results.

GRADIENTER.

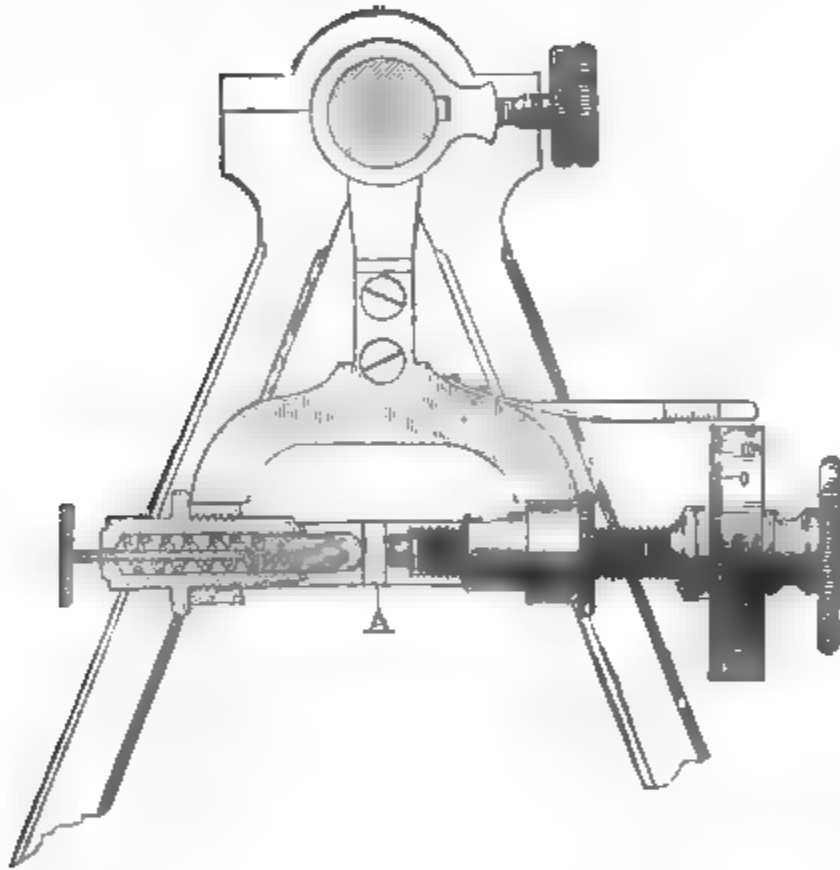
152. This attachment, as shown on next page, is often used with transits for fixing grades, determining distances, etc.

It consists mainly of a screw attached to the semicircular expanded arm of the ordinary clamp of the telescope axis; the screw is accurately cut to a given number of threads, and passing through a nut in one side of the arm, presses against a little stud A fixed to the inside surface of the right-hand standard.

In the other side of the semicircular arm is inserted a hollow cylinder containing a pin actuated by a strong spiral spring, the end of the pin pressing against the side of the stud opposite that in contact with the screw.

Near the other end of the screw, and turning with it, is a wheel, or micrometer, the rim of which is plated with silver, and divided into one hundred equal parts.

A small silver scale, attached to the arm and just above the micrometer wheel, is divided into spaces, each of which is just equal to one revolution of the screw; so that by comparing the edge of the wheel with the divisions of the scale, the number of complete revolutions of the screw can be easily counted.



It will be seen that when the clamp is made fast to the axis by the clamp-screw, and the gradienter-screw turned, it will move the telescope vertically, precisely like the tangent-screw ordinarily used.

And as the value of a thread is such that a complete revolution of the screw will move the horizontal cross-wire of the telescope over a space of one foot on a rod at a distance of one hundred feet, it is clear that when the screw is turned through fifty spaces on the graduated head, the wire will pass over fifty one-hundredths, or one-half a foot on the rod, and so on in the same proportion.

In this way the gradienter can be used in the measurement of distances, precisely like the stadia just described.

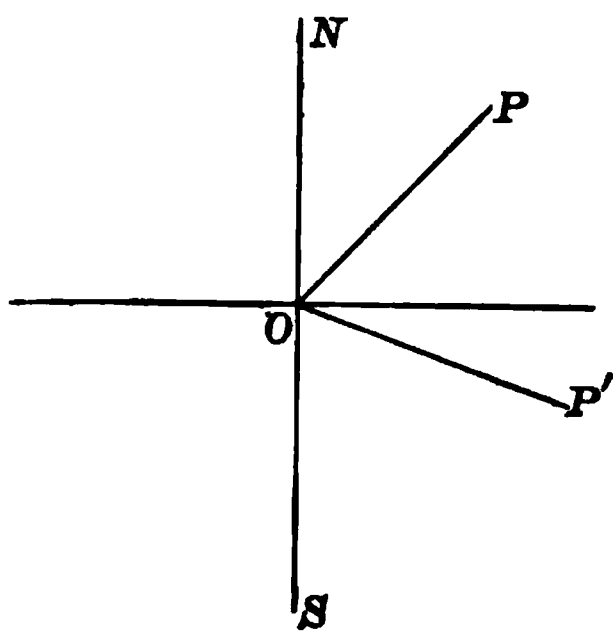
Grades can also be established, with great facility, as follows: First, level the instrument; bring the telescope level to its centre by the clamp and gradienter screw; move the graduated head until its zero is brought to the edge of the scale; and then turn off as many spaces on the head as there are hundredths of feet to the hundred in the grade to be established.

SECTION II.

A. BEARINGS WITH COMPASS.

153. To Obtain the Bearing of a Line. At one end of the line, or at any other point in it, set up and level the compass, loosen the needle, and direct the sights toward the other end. The degree on which the needle comes to rest will indicate the angle between the magnetic meridian and the direction of the line, or the bearing.

For example, if the line lies between the north and east points, as OP , and the angle NOP being, say 42 degrees, the bearing



of the line OP is written, N. 42° E., and read, "north forty-two degrees east." If, as OP' , it lies between south and east, and the angle SOP' is, say 74 degrees, it is written, S. 74° E., and read, "south seventy-four degrees east"; in like manner for lines in other quadrants.

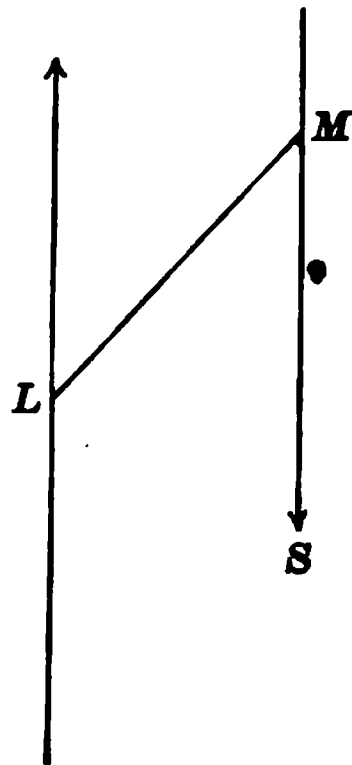
It will be observed that the bearing of a line does not exceed 90°.

A line which might be read "N. 90° W." or "S. 90° W." is recorded as *west*. The bearing can be read most accurately by placing the eye over one end of the needle and taking the reading from the other end.

Since the graduations are usually made to half-degrees, the bearing can be taken quite accurately to quarter-degrees, and by practice, even closer, without the use of the vernier. In fact, the principal use of the vernier on a compass is to facilitate the running of lines from old deeds, where, when the declination is ascertained, it is turned off on the vernier, and the surveyor may use then the bearings as given in the deed by which he is surveying the tract, without making a calculation for the bearing of each line. The vernier cannot be relied on to read bearings to minutes, on account of the difficulty of accurately manipulating it.

154. Reverse Bearings. Since in plane surveying the meridians passing through the extremities of a line are considered parallel, the direct and reverse bearings should indicate the same angle. That is to say, a line, as LM , the bearing of which, taken at L , called also *fore-sight*, is N. 40° E., when taken at M , *back-sight*, should be S. 40° W.; the degrees being the same, the letters indicating the opposite cardinal points.

When surveying a tract of land with the compass, the instrument should be set up at every corner, and the bearing and reverse bearing of every line taken, as a check on the observer's reading and the working of the needle, since a disagreement in the angle thus measured would be evidence sufficient to warrant a review of the work.



155. Local Attraction. If the readings of the needle of the fore-sight and back-sight have been correctly made, and there is found a disagreement, local attraction exists. It is usually caused by the presence of ferruginous matter. It may exist at both stations or at only one of them.

Assuming that the direct and reverse bearings of the preceding line agree, then the difference in the reading at the two ends

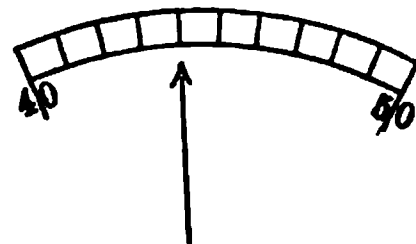
of the line, when the attraction exists, will show the local variation at the last station, and this correction must be applied to the reading of the needle for the bearing of the next line. If, however, the needle will not reverse on the first line of a survey, then it will be necessary to *set up* at some other point of the tract; or, if this is impracticable, select one or more stations near the suspected points, and by taking the bearings of these from the stations, and also the reverse bearings, the intensity and position of the attraction may be determined.

156. Proof Bearings and Tests of Accuracy. In any important compass survey it is well to check the work by sighting to distant prominent objects, such as buildings, trees, etc., and noting the readings. Since *two* bearings are required to locate each object, — and until it is located it cannot serve as a check, — it will be necessary to take at least *three* bearings to each. If, then, when plotting, the three lines intersect in a point, a proof is given of the correctness of the measurements thus connected. The lengths and bearings of diagonals of the tract may likewise be taken as checks on the accuracy of the work; also, when in plotting, if the last bearing and distance *close* the survey, it is considered a proof of the work. The best test, however, of the accuracy of the survey is by Latitudes and Departures, which is explained in Section VI. Articles 207 and 208.

It may be well to caution the student against the fallacy of a test sometimes given, — that if the sum of the interior angles, determined from the bearings, equals twice as many right angles, less four, as the figure has sides, it proves the work. This “test,” while it furnishes proof for a transit survey in which the interior *angles* have been measured, will *not* show that the *bearings* of a tract have been correctly taken. The student will readily perceive the truth of this statement if he makes or imagines a plot of a field with a certain side the meridian, then conceives the whole plot turned around so that another side comes to the meridian, it will be evident that

though the bearings are changed, the sum of the interior angles is unaffected. The *so-called* test would prove the work in either case.

157. Suggestions. Test frequently to see that the instrument is in proper adjustment. Keep the same end ahead. Read from the same end of needle. Sight as low on the flagstaff as possible. Make the line of sight as nearly horizontal as practicable. When reading near the cardinal points, be careful that the bearing is not read in the wrong quadrant, also that the common error of reading 56° for 44° is not committed. See that the instrument is set precisely over the station from which the measurements are to be made; that the flagstaff is exactly on the proper point, and that it is held plumb. Level the instrument carefully; especially see that it is level across the line of sight. Take the bearing and measure the distance on the *true* line when practicable; when not, because of a high fence, bushes, etc., set off the least perpendicular distance therefrom at both ends which will afford a clear view, and take the bearing and distance of the extremities of these perpendiculars.



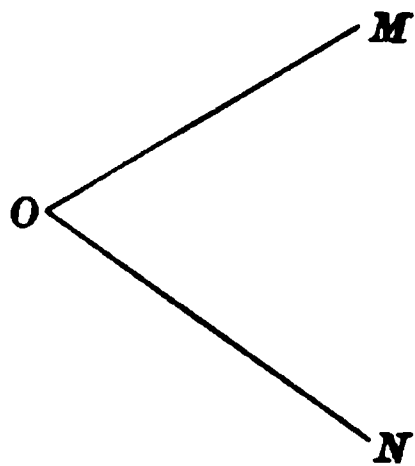
EXERCISES.

1. With a surveyor's compass, by a constant and direct bearing only, run a line, say 40 chains in length, over hilly ground, and part of it, if possible, through brush; then return, using the reverse bearing only.
2. With the same instrument run another line equally difficult, using both direct and reverse bearings forward and back.
3. Make a survey of a lot one side of which is near to a railroad track. If local attraction is found, determine its intensity.
4. Determine the magnetic bearing of each part of a broken line of several turns along a railroad track, or where local attraction is known to exist.

B. ANGLES WITH TRANSIT.

158. With the Transit the survey of a line or the measurement of an angle can be made with greater accuracy than with the compass, since the reading of the plates to minutes supplants the reading of the needle to quarter or half-quarter degrees, and the pointing power of the transit greatly exceeds that of the compass.

159. To measure a horizontal angle, as MON . Set up the instrument precisely at O ; level it and direct the intersection of the wires to either point, say N . Clamp the instrument firmly to the spindle, note the reading of the vernier, then loosen the vernier plate and bring the telescope quite near the other line so that its extremity M is in the field of view. Clamp the plate, and with its tangent or slow-motion screw bring the line of collimation precisely on M . Again take the reading.



The difference of the two readings will be the angle required. It is more convenient to make the first sight, ON , with the zero of the limb and plate coincident, since then the reading of the plates after observing M gives at once the angle. If at each observation but one vernier is read, it is best to read every time from the same one; it is better at each observation, though, to read both verniers and take the mean of these, thereby eliminating eccentricity. If, however, great accuracy is required, the measurement of the angles should be taken more than once, by the method of repetition or by series.

160. By Repetition. Make an observation upon any point, and read both verniers; clamp the lower plate to the spindle, direct the telescope to another point, and, as a check, again read the verniers.

Now, keeping the index at the last reading, turn both plates

back, and observe again on the first point; clamp, as before, the lower plate, and turn the upper one so as to sight on the second point. It is perceived that by this operation the angle has been measured twice, but on different parts of the limb. An angle may obviously be repeated any number of times: the mean of the several readings gives more nearly than a single measurement the true angle. The reading at each observation serves as a check on the work. An angle may be repeated by simply noting the reading at the first and last observation, taking their difference, and dividing by the number of repetitions. It must be noted, however, how often, if at all, the 360° point is passed. Now, if the telescope is plunged, the plates turned 180° in azimuth, and repetitions of the angle again be made, beginning at the second point, the mean of the two sets of readings will give still more nearly the true angle, since the errors of adjustment and twist of station are thus lessened and those of observation reduced.

161. By Series. Observe as before upon any point, and read the verniers, clamp the lower plate, turn the vernier plate until the telescope may be fixed upon another point, and again read; thus continue to make observations upon each point desired in their order, sweeping round the horizon, and make the last observation upon the first point. The last reading should be the same as the first. Plunge the telescope, move the plates in azimuth, and observe on the points again, proceeding in the contrary direction. Several series of observations may thus be made, as in the method by repetition. The magnitude of each angle is obtained from the mean of its reading.

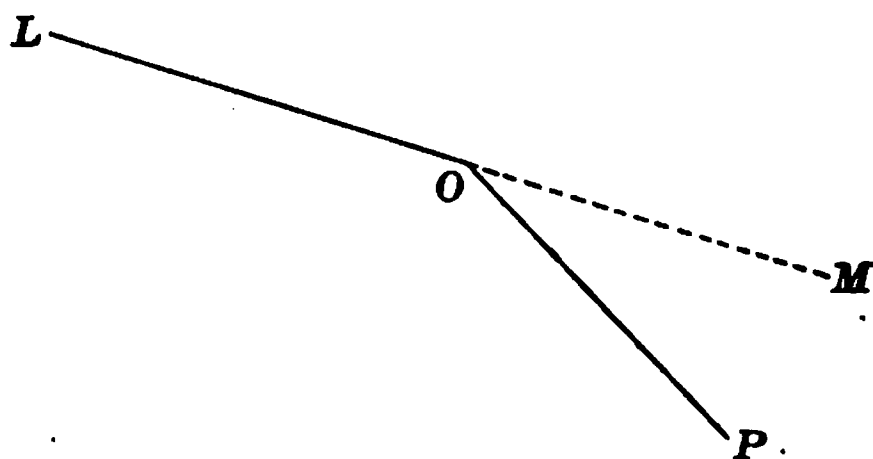
REMARK. Care should be exercised to have the instrument properly centred, that is, set precisely over the centre of the station, especially if the object sighted is near the observer. The error arising from an eccentric setting is inversely as the distance of the object sighted; an eccentric setting of one inch producing an error of nearly three ($3'$) minutes of arc in sight-

ing 100 feet, while the error arising from a sight of 900 feet is less than one-third ($\frac{1}{3}$) of a minute.

Read both verniers to eliminate eccentricity. See that the reading is not made from the wrong end of the vernier, and that a half-degree is not omitted, calling the reading, say, $36^{\circ} 15'$, instead of $36^{\circ} 45'$. If great accuracy is required when running a straight or broken line, lessen errors of adjustment by reversing the instrument in altitude and azimuth, making two sets of observations at each station, and take the mean of their readings. See Article 157.

If it is desired to locate the lines surveyed with reference to the meridian, the bearing of one of them should be taken by the needle of the instrument; the bearings of the others may be deduced therefrom. See Article 167.

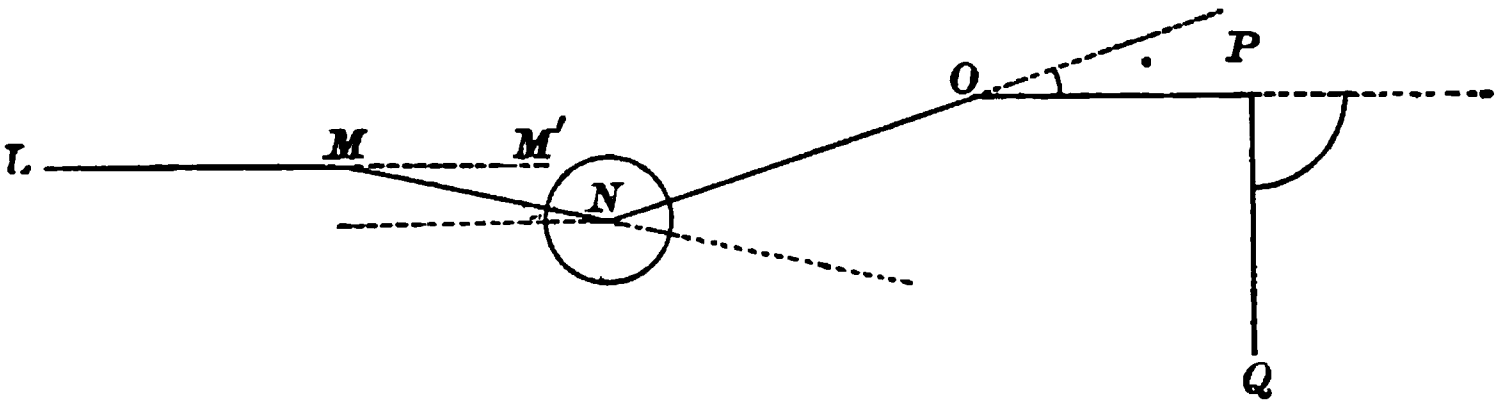
162. Angle of Deflection. The amount of divergence which a line makes with the preceding is called the deflection, and the angle which measures it is termed the deflection angle.



In the figure POM is the deflection angle: it is evidently the supplement of LOP . To measure it, set the transit at O , sight to L , clamp the limb to the spindle and the plates together, then plunge the telescope: it will point to M . Take the reading, unclamp the vernier-plate and move it until the wires intersect P . The difference between the reading now and the first reading is the deflection angle. If, when making the first observation, the vernier was at zero, the reading, after sighting P , would indicate at once the angle.

163. Traversing, or surveying by the back angle, is a method

of surveying by which the direction of each line of a survey is compared with the first as a meridian or reference line. It is effected as follows :



Let it be required to traverse the broken line *LMNOPQ*. Set up the instrument at *M*, clamp the vernier at zero, for convenience, and, with the lower motion, sight *L*, clamp below, transit the telescope, loosen above and observe *N*: the reading will show the angle *M'MN* which the line *MN* forms with *LM*. Clamp the plates, move to *N*, plunge the telescope, and, with the lower motion, sight *M*, the index remaining as at *M*; then clamp below, loosen above, transit the telescope, and direct it to *O*: the index will show the angle which the line *NO* makes with *LM*. And so continue until the end of the line.

To guard against mistakes in reading, and to avoid recording whether the deflection is right or left, it is well to assume all angles measured in the same direction. In the figure the readings are all to the right, or clockwise, as indicated by the circular arcs, and the record is as follows :

| STATIONS. | AZIMUTHS WITH <i>LM</i> . | BEARINGS WITH <i>LM</i> . | MAGNETIC BEARINGS ASSUMING BEARING OF <i>LM N. 50° E.</i> |
|-----------|------------------------------|------------------------------|---|
| <i>L</i> | 0° | North. | N. 50° E. |
| <i>M</i> | 18° | N. 18° E. | N. 68° E. |
| <i>N</i> | 340° | N. 20° W. | N. 30° E. |
| <i>O</i> | 360° or 0° | North. | N. 50° E. |
| <i>P</i> | 90° | East. | S. 40° E. |

From the nature of the operation it may be perceived that, algebraically, the azimuth of any line is equal to its deflection

plus the azimuth of the preceding line. This method is particularly adapted to surveying roads, streets, water courses, etc., and even in farm surveying it possesses an advantage over the survey by interior angles, on account of the readiness it affords in obtaining the bearings from the azimuths, and the greater rapidity with which the work may be plotted, since the angle which each line makes with the assumed meridian, or reference line, is taken at once from the field notes.

Suppose LM in the figure to be the meridian of the survey, and the azimuths of the several lines as recorded in the table. Now, assuming the direction of LM to be north, it is evident that MN will be in the northeast quadrant 18° from the north point, or N. 18° E; NO will be 20° to the west of north, or N. 20° W.; OP , making no angle with the meridian, will have a bearing north, and PQ east.

So that, in general,

When the azimuth is less than 90° , it equals the bearing, and the line is in the northeast quadrant.

When the azimuth is between 90° and 180° , the bearing is southeast, and is the supplement of the azimuth.

When the azimuth is between 180° and 270° , the bearing is southwesterly, and may be found by subtracting 180° from the azimuth.

When the azimuth is between 270° and 360° , the bearing is northwesterly, and is the difference between 360° and the azimuth.

When the azimuth is 90° , the bearing is due east.

When the azimuth is 180° , the bearing is due south.

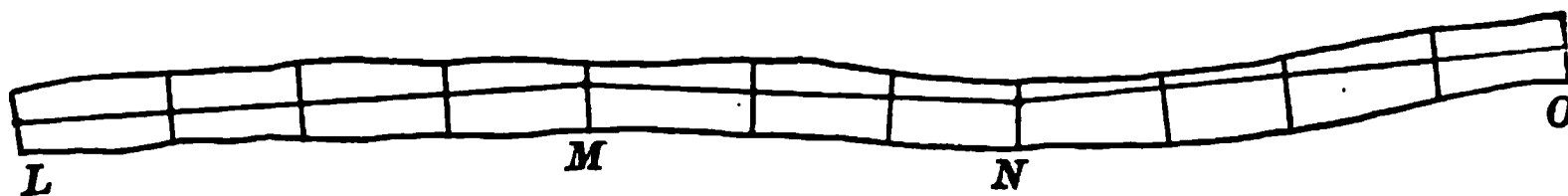
When the azimuth is 270° , the bearing is due west.

When the azimuth is 360° , the bearing is due north.

If it is required to find the magnetic or true bearing of any or all the lines, take the magnetic or true bearing of the meridian of the survey and apply it, by addition or subtraction, according as the bearing of the assumed meridian, or standard line, is northeast or southwest. In the example given, suppose the bearing of the assumed meridian LM to be N. 50° E. : then the bearing

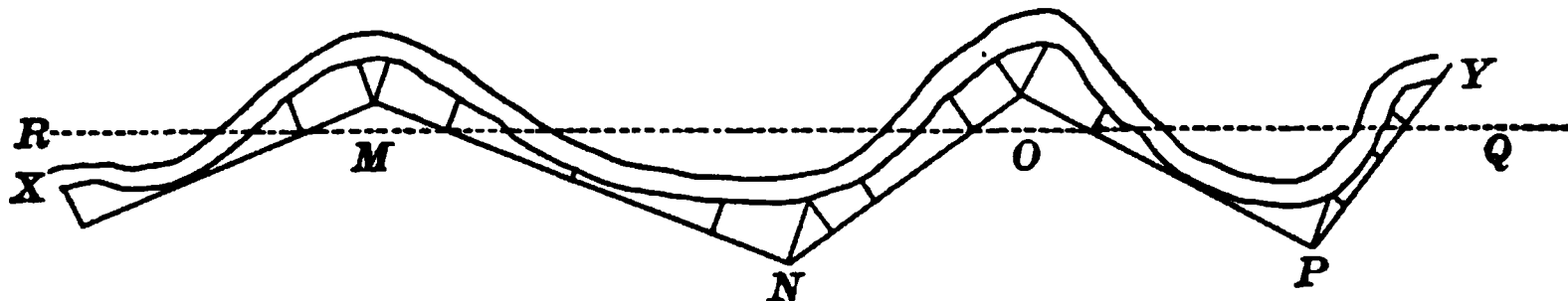
of the second line MN will be recorded 18° to the east of the reference line, or N. 68° E. ; the line NO , having a deflection of 20° to the left of the reference line will be recorded N. 30° E. ; and OP , N. 50° E. Thus the fourth column is added to the table.

164. To Traverse a Road, as $LMNO$. Proceed as indicated in the last article, and in addition measure the lines LM , MN , NO , and perpendicular offsets thereto, at proper distances.



If the road deviates much from a straight line, it will be necessary, in order to obtain more correctly the area, to take two offsets at M , one perpendicular to LM , the other to MN ; and also two at N , one perpendicular to MN , and the other perpendicular to NO .*

Likewise to Survey a Small Stream. Traverse and measure the distances between assumed stations, as L , M , N , O , P , so chosen as to make no more of them than is consistent with few and short offsets to the various bends of the stream. If the



stream is small, not exceeding 10 feet in width, or even wider if shallow, and it is desired to survey it between X and Y , a good plan is to run a straight line between these points and measure offsets therefrom to the stream ; or, if such a line will make the offsets rather long, run RQ , and measure offsets from it to X and Y and intermediate points. If, however, the stream is wide

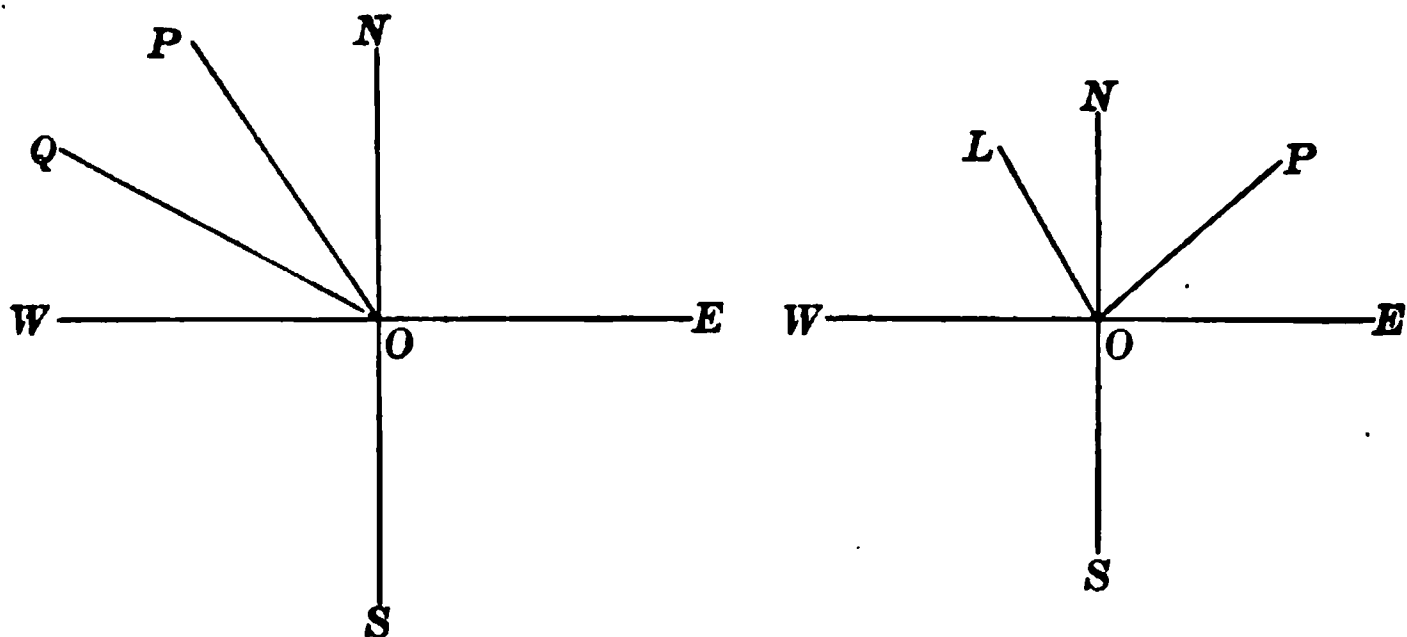
* Article 234.

and the crossing difficult, it will probably be better to use more stations, as shown in the figure. If a compass is used, the bearings may be taken instead of the angles.

If a river of considerable width is to be surveyed, it will be necessary, in addition to the measurement of broken lines on each side from which offsets are taken, to make a series of angular measurements connecting the lines on one side with those on the other, and thence by trigonometrical calculations determine their relative positions, and ultimately the surface of the river.

C. PROBLEMS ON ANGLES AND BEARINGS.

165. Angles between Lines. To determine the angle between two lines, meeting at a point, given by their bearings.



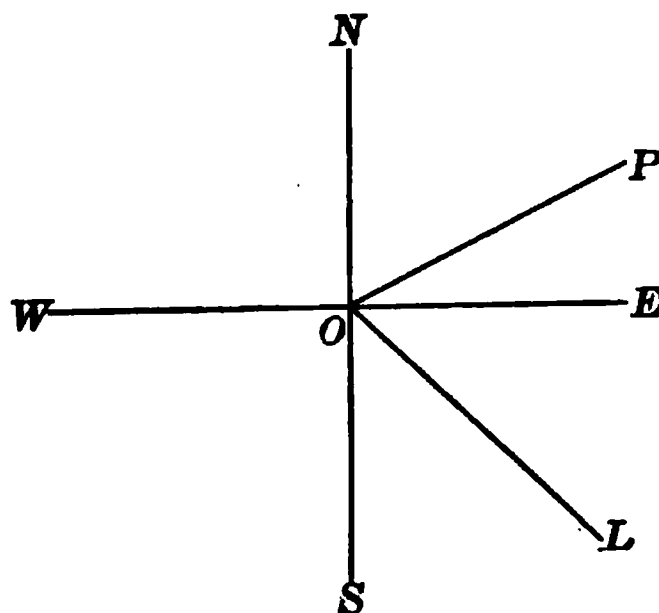
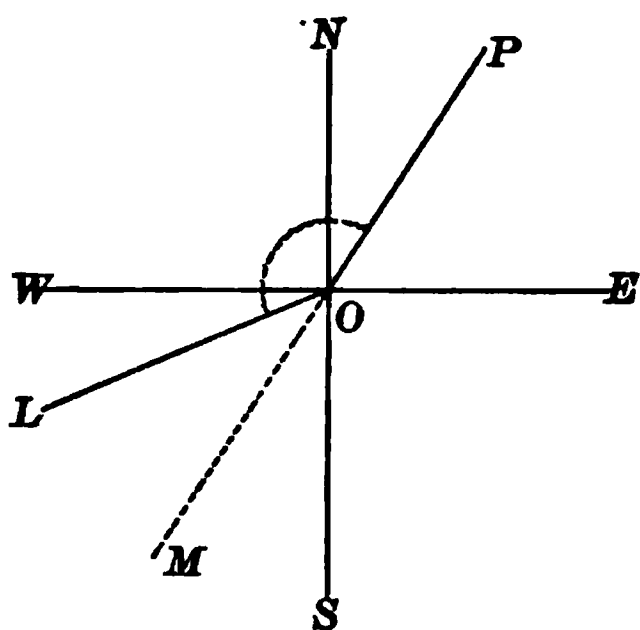
1. If the lines run between the same cardinal points, that is, in the same quadrant, take the difference of their bearings.

Suppose the bearing of OP is N. 32° W. and that of OQ N. 60° W.; the angle between them is obviously $NOQ - NOP$; or, $60^\circ - 32^\circ = 28^\circ$.

2. When the lines run in different quadrants and both above or both below the horizontal or E. and W. line, take the sum of their bearings. If OP bears N. 60° E. and OL N. 20° W., the angle $POL = PON + NOL = 60^\circ + 20^\circ = 80^\circ$.

3. If the lines run in diagonally opposite quadrants, subtract the difference of the bearings from 180° . Assuming the bearing of OP N. 28° E. and of OL S. 58° W., the angle

$$POL = 180^\circ - LOM = 180^\circ - (58^\circ - 28^\circ) = 150^\circ.$$



4. When the lines are in different quadrants, and both to the right or both to the left of the vertical or N. and S. line, subtract the sum of the bearings from 180° . If OP bears N. 65° E. and OL S. 42° E., the angle

$$POL = 180^\circ - (NOP + SOL) = 180^\circ - (65^\circ + 42^\circ) = 73^\circ.$$

ADDITIONAL EXAMPLES.

1. A line OP bears N. 40° W. and OL N. 40° E.; required the angle POL .

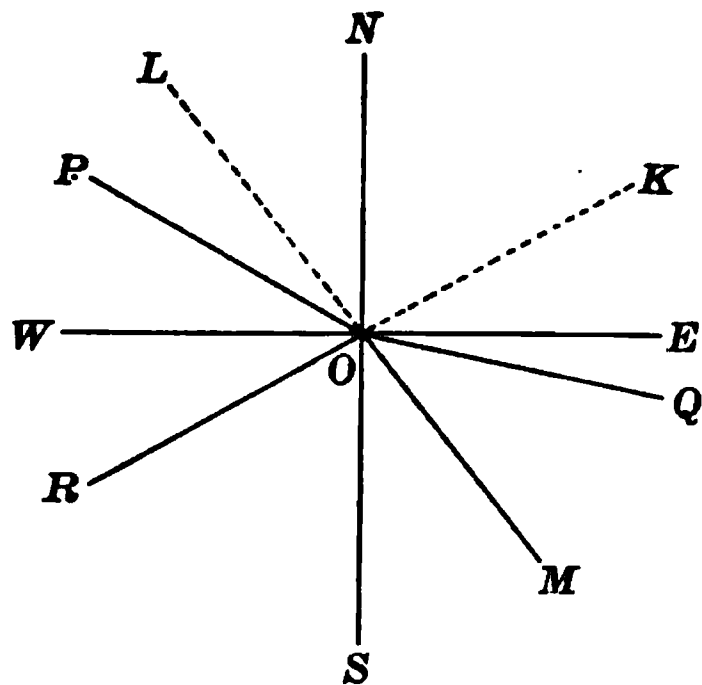
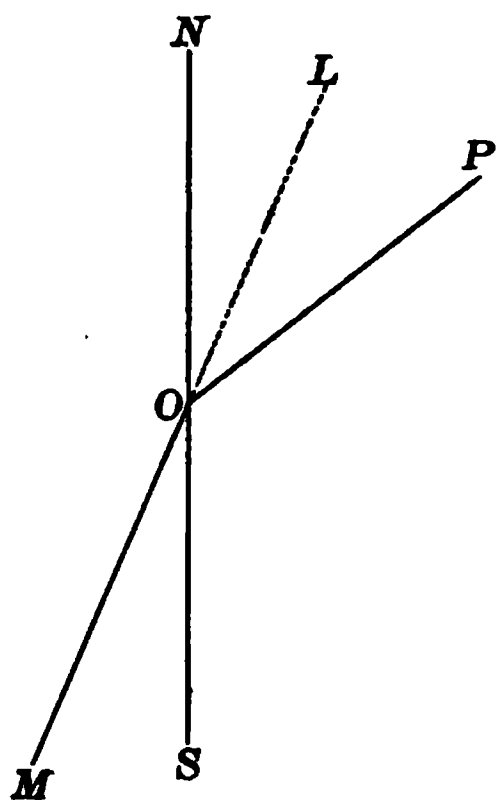
2. Find the angle POL , when OP bears S. 50° E. and OL N. 89° E.

3. Required the angle at O , when OP bears N. 80° W. and OL S. 79° E.

4. What is the angle O , if OP runs S. $89\frac{3}{4}^\circ$ W. and OL N. $89\frac{1}{2}^\circ$ E.?

5. A line OP runs S. 70° W. and OL S. 45° W. Find the angle O .

166. There may be given the bearing of a line, as MO , and the deflection angle LOP , to the right or left of the direction of MO , to find the bearing of OP ; or, the bearings of MO and OP may be given to determine the magnitude of the deflection angle LOP .

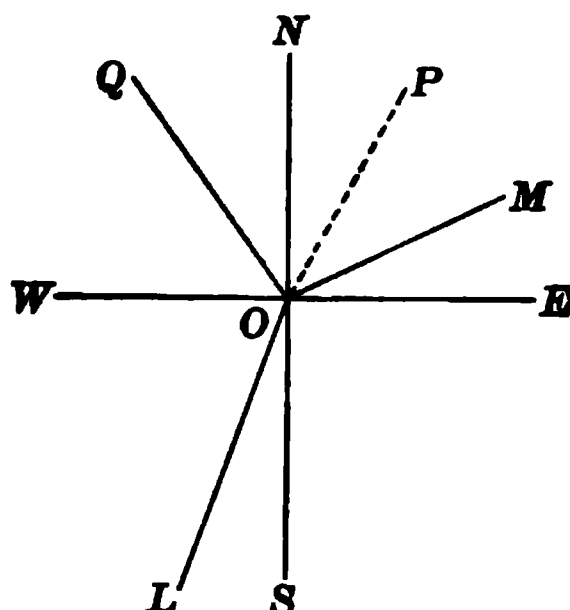


a. Given the bearing of a line and the deflection of the next, to find its bearing.

Suppose MO bears N. 32° W., and the deflection of $OP = 20^\circ$ to the left; the bearing of OP is evidently 20° farther towards the west than MO or its prolongation OL . It is therefore N. 52° W. Again, assuming RO bears N. 60° E. and the deflection of OQ 40° to the right, it is evident that OQ is in the southeast quadrant, 10° from the east point; or, its bearing is S. 80° E.

b. When the bearings of the lines are given, to determine the deflection.

Suppose LO (p. 115) bears N. 20° E. and OM N. 70° E.; the deflection of OM from LO , or its prolongation OP , is evidently $70^\circ - 20^\circ = 50^\circ$ to the right. Again, the bearing of LO remaining the same, and that of OQ N. 30° W., then it is readily seen that the deflection angle is $20^\circ + 30^\circ = 50^\circ$ to the left.



General rules might be given for the cases under the above heads, corresponding to those in the preceding article, but they are deemed unnecessary, as a little reflection will enable the student to determine the required bearing, or angle, in any given case.

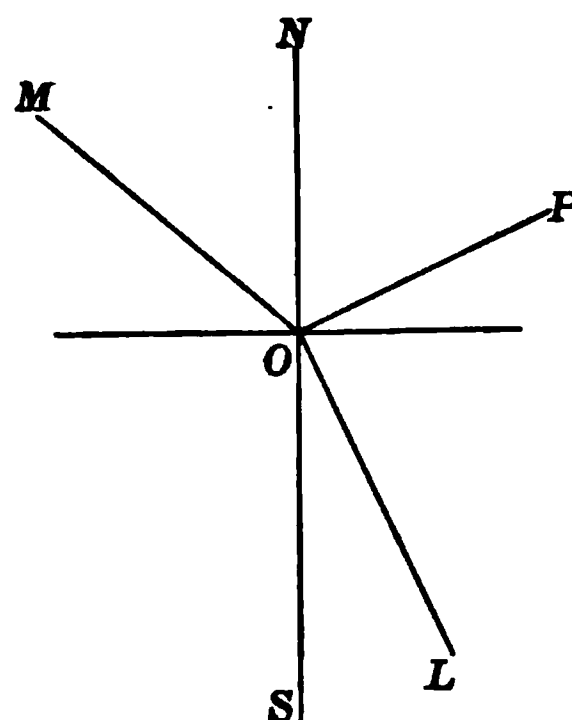
167. *Given the angle between two lines, and the bearing of one line, to find the bearing of the other.*

The solution of this problem is ordinarily required in transit surveying, for, when surveying with that instrument, it is common to take the bearing of only one line, and deduce the courses of the others from that bearing and the measured angles. Suppose LO bears N. 24° W. and the angle $LOP = 82^\circ$, to find the bearing of OP . It is evident that the bearing of OP or the angle NOP , which gives the degrees in the bearing,

$$\begin{aligned} &= 180^\circ - (SOL + LOP) \\ &= 180^\circ - (24^\circ + 82^\circ) = 74^\circ. \end{aligned}$$

Hence the bearing of OP is N. 74° E.

Assume the angle $POM = 100^\circ$, and the bearing of OP as found above; then, since there are $100^\circ - 74^\circ$, or 26° , more in the angle than lies between OP and the



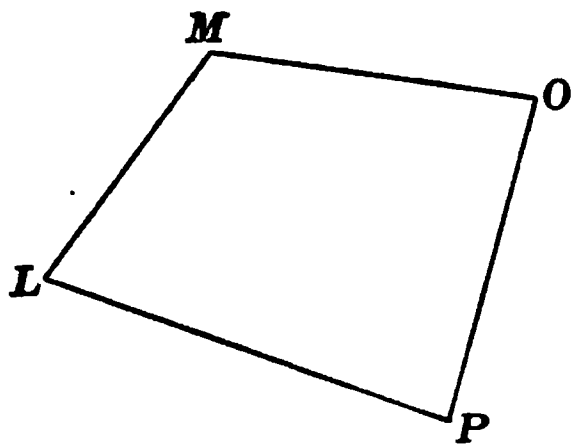
north point, the position of OM is to the west of north 26° , or its bearing is N. 26° W.

Some simple combinations, as indicated in the illustrations given, will enable the student, unencumbered with *rules*, to readily solve any of the problems coming under this head.

EXAMPLES.

1. A line bears S. $89^\circ 15'$ W. What is the bearing of a line perpendicular to it? Also, the bearing of a line making an angle of 135° with it? Is there more than one answer to the last?

2. If OP bears S. 36° W., and the angle $OPL = 68^\circ$, what is the bearing of PL ? Ans. N. 32° W.



SUGGESTION. Pass a meridian through the angle, and consider the given bearing reversed.

3. The angles L , M , O , P , of the trapezium are respectively 62° , 130° , 80° , and 88° , and the bearing of LM N. 70° E.; find the other bearings.*

168. To Change the Bearings of the Sides of a Survey. It is sometimes desirable to change the bearings of a survey so that a particular side shall become a meridian. The whole plat is conceived to revolve through an angle sufficient to make the desired side the meridian; the relative position of the sides remains unaltered. The following rule is substantially that given by Gummere, who states that the method was communicated to him by Prof. Robert Patterson, late of Philadelphia.

RULE.

Subtract the bearing of the side that is to be made a meridian from those bearings that are between the same points that it is,

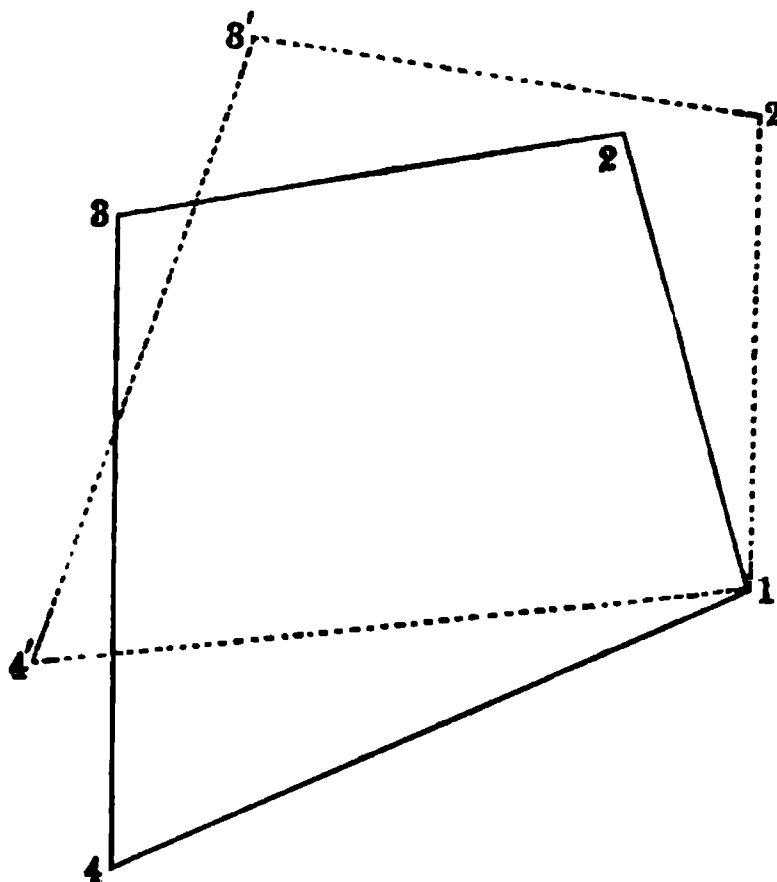
* The calculation may be tested, after having deduced the bearings of all the sides, by taking the last bearing found, as PL , applying the angle L , and observing if it gives the proper bearing of LM .

and also from those that are between points directly opposite to them. If it is greater than any of those bearings, take the difference, and change west to east or east to west.

Add the bearing of the side which is to be made a meridian to those bearings which are neither between the same points that it is nor between the points directly opposite to them. If either of the sums exceed 90° , take the supplement, and change south to north or north to south.

The accompanying diagram of full and dotted lines exhibits the positions of the sides of the following described farm, before and after turning through $16\frac{1}{4}^\circ$ to the right:

(1) N. $16\frac{1}{4}^\circ$ W., 24.63 chains; (3) S. $\frac{1}{4}^\circ$ W., 34.28 chains;
(2) S. 79° W., 27.00 chains; (4) N. 65° E., 37.20 chains,
to the place of beginning. The bearings are changed so as to make the first side a meridian.



EXAMPLES.

1. Given the bearings of a tract of land:

- (1) S. 10° E.; (2) S. 30° W.; (3) N. 60° W.;
(4) N. 20° W.; (5) N. 80° E.,

to the place of beginning. Required the changed bearings that the fourth side may be a meridian.

| | |
|----------------------------|----------------------------|
| (1) S. 10° E. | (4) North. |
| 20 | |
| Changed bearing, S. 10° W. | |
| (2) S. 30° W. | (5) N. 80° E. |
| 20 | 20 |
| Changed bearing, S. 50° W. | 100 |
| (3) N. 60° W. | 180 |
| 20 | Changed bearing, S. 80° E. |
| Changed bearing, N. 40° W. | |

The student who avails himself of the hints and methods referring to the manipulation of angles and bearings as given in the preceding articles, will have no difficulty in determining the changed bearings direct from the data, without the use of *rules*. Thus in the example above it will be observed that each line is turned through 20° to the right; that is, the fourth course is made due north. The next side to it going round to the right, N. 80° E., will be turned the same number of degrees (20), which places it 10° from the east point in the south-east quarter, or its bearing is S. 80° E.; the first side turning through the same angle (20°) will be thrown 10° west of the south point, or S. 10° W.; the second course will be 20° farther to the southwest, or S. 50° W.; and the third course turned toward the north point 20° will be N. 40° W.

2. Find the bearings of all the sides of the following described tract of land when the second side is made a meridian:

- (1) N. 68½° E., 8.42 chains; (3) S. 78¼° W., 4.90 chains;
 (2) N. 27° W., 10.25 chains; (4) S. ½° E., 4.40 chains;
 (5) S. 12° E., 7.04 chains,

to the place of beginning.

3. Given the bearings of a tract of land as follows :

- (1) S. $39\frac{1}{2}^\circ$ W.; (3) N. 15° W.; (5) N. 2° E.;
 (2) East; (4) N. $79\frac{1}{4}^\circ$ E.; (6) S. $73\frac{3}{4}^\circ$ W.,

to find the bearings of all the sides when the first becomes a meridian.

4. Given the bearings of a tract of land as follows :

- (1) S. 79° W.; (3) N. $89\frac{1}{2}^\circ$ E.; (5) S. $80\frac{3}{4}^\circ$ E.;
 (2) S. $\frac{1}{4}^\circ$ W.; (4) N. $1\frac{3}{4}^\circ$ E.; (6) S. $58\frac{1}{2}^\circ$ E.;
 (7) N. 39° E.; (8) N. $16\frac{1}{4}^\circ$ W.,

to find the bearings when the eighth side becomes a meridian.

EXERCISES.

1. With a transit, using back and fore sights, run a tangent forward and back over hilly and brush land requiring six or eight settings of the instrument. The last two points set forward will give the direction back. Note the distance, if any, between the corresponding positions occupied by the instrument.

2. Traverse, or survey by the back angle, a broken line of six stations, using the first line as the meridian, or reference line, of the survey. Record the notes, indicating the azimuthal angles and bearings.

3. Measure the three angles of a triangular piece of land, the corners being visible from each other; see how much, if any, their sum differs from two right angles.

4. Traverse a pentagonal field, the index at the beginning being set at zero, and see if, when finally sighting on the station first occupied, the reading is zero.

SECTION III.

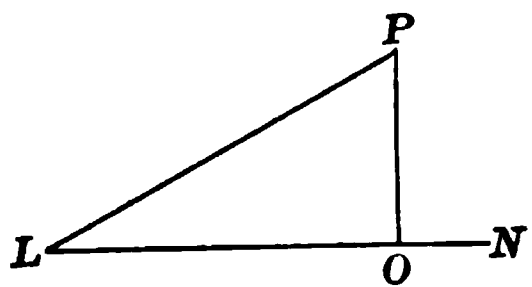
OBSTACLES.

A. PROBLEMS ON PERPENDICULARS AND PARALLELS.

169. The Obstacles which occur in field work are more easily and expeditiously overcome with the compass, or transit, and chain, than with the chain alone. Methods for the latter were given and illustrated in Chapter I. Section II., Chain Surveying.

To erect a perpendicular to a line at any given point. Set up the instrument over the point; if a compass is used, take the bearing of the line, and then move the instrument in azimuth until a bearing differing 90° from the first is observed. The line of sights will then indicate the direction of the required perpendicular. If a transit is employed, centre on the point, sight to a point in the line, clamp to spindle, and turn the vernier plate 90° either way; then the line of collimation will show the direction of the perpendicular sought. Of course by the methods explained above, a line can be run with either instrument from any given point and making *any* given angle thereat with a line.

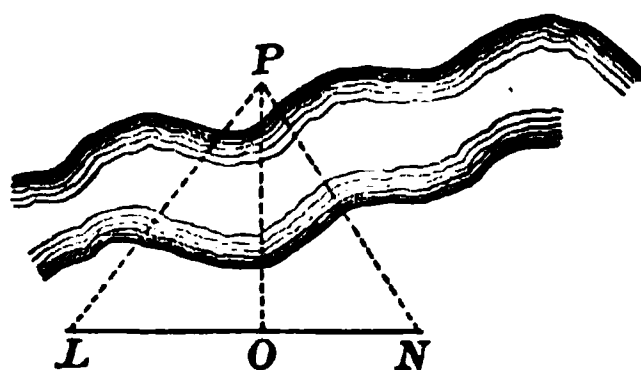
170. *To let fall a perpendicular from a given point to a line.* Let P be the point, and LN the line. If the compass is used,



take the bearing of LN , remove the instrument to P , and with a bearing differing 90° from the first, run PO for the required perpendicular. With a transit centre on L , measure the angle OLP , remove to P , and make the angle LPO equal to the complement of L ; the line of sight of the instrument will then be in the direction of the required perpendicular.

171. To let fall a perpendicular to a line from an inaccessible point. Measure the distance between any two points, as L and N , in the line; also the angles PLN and LNP . Then in the triangle PLN we have given the side LN and the angles to find PL or PN . Computing PL , the distance

$$LO = PL \cos PLO.$$



Or we may deduce an expression for LO in terms of the measured line and the observed angles, thus :

$$LO = PO \cot PLO.$$

$$NO = PO \cot PNO.$$

Hence $LO : NO = \cot PLO : \cot PNO$,

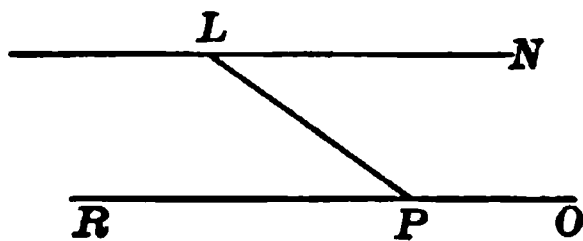
and $LO : LO + NO = \cot PLO : \cot PLO + \cot PNO$;

but $LO + NO = LN$,

$$\text{therefore } LO = \frac{LN \cot PLO}{\cot PLO + \cot PNO}.$$

QUERY. Could a line be run not perpendicular as above through an inaccessible point, making any angle with the given line?

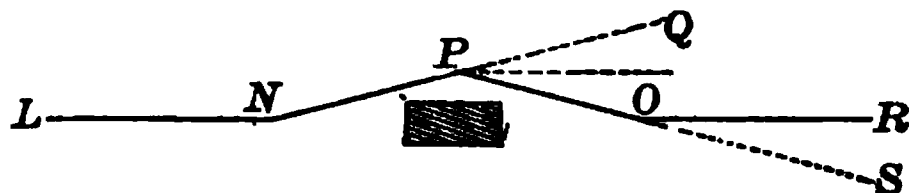
172. To run a line through a given point parallel to a given line. With the compass obtain the bearing of the line, and then from the given point run a line with the same bearing. With the transit, LN being the line and P the point, centre on L , measure the angle NLP , remove to P , and make the angle LPR equal to NLP ; the line of collimation will then be in the required parallel.



B. PROBLEMS ON ALIGNMENT.

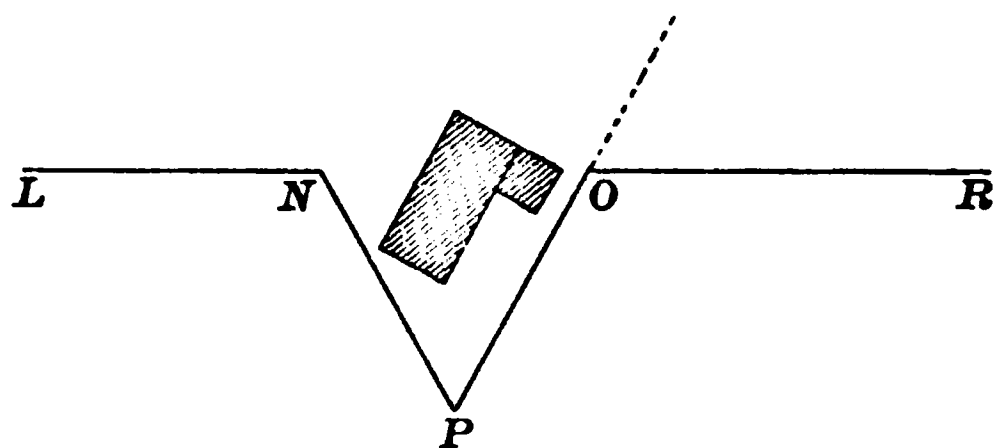
173. To prolong a line, as LN , beyond a tree, a building, or any obstacle.

First Method. By Deflection Angles. Set up the instrument at any point of the line, as N , and deflect, sufficient to pass the obstacle, to any point P . Measure NP , remove to P , deflect to O , making the angle QPO double the angle at N .



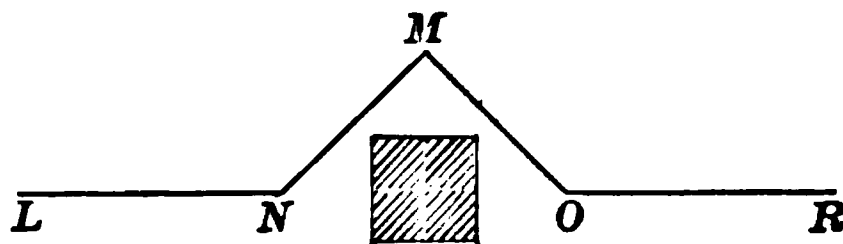
Measure $PO = PN$, place the instrument at O , observe P , plunge the telescope and deflect to R , so that $SOR = \frac{1}{2} OPQ$; the telescope will then be in the prolongation of LN .

174. Second Method. By Equilateral Triangle. Deflect 60° from the direction of the line at N ; measure to P a distance



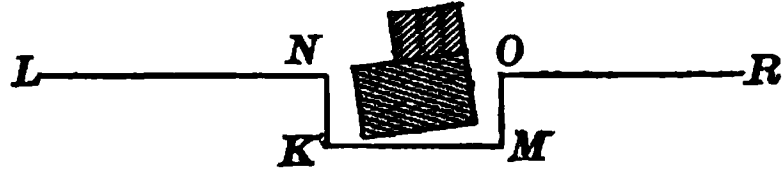
sufficient that PO , making an angle of 60° with PN , will clear the obstacle. Measure $PO = PN$, and turn the telescope in the direction of OR , the prolongation of LN , by deflecting 60° from the direction of PO .

175. Third Method. By Isosceles Triangle. Deflect at N 45° to M , measure NM , make NMO a right angle, and MO



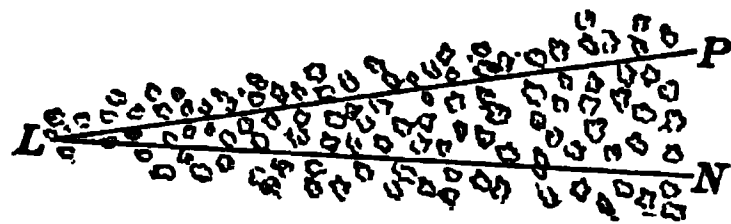
$= MN$; at O turn into OR by deflecting from the direction of OM 45° .

176. Fourth Method. By Perpendiculars. Erect a perpendicular NK of sufficient length that a line passing through



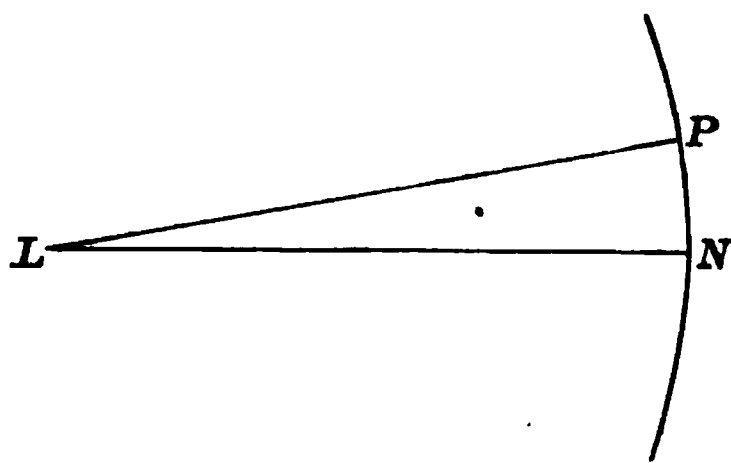
K parallel to LN will clear the obstacle; run KM ; lay off $MO = NK$, and a right angle turned from MO will indicate the direction of LN , or its prolongation OR .

177. Random Line. When brush, wood, or any obstruction prevents N being seen from L , run a line LP as nearly as may



be judged in the direction of LN : when opposite N , as at P , measure the shortest distance from P to N , call it d ; then the angle PLN in degrees = $\frac{57.3 \times d}{LP}$.

Setting up again at L , and applying the correction thus found in a proper manner to the angle or bearing before used, the line LN may be traced.



Demonstration. When the distance PN does not exceed 5 per cent of the length of PL , PN and PL may be regarded as radii of a circle, and PN coincident with the arc which subtends the angle PLN ; then

$$2\pi LP : 360 = PN : PLN,$$

or
$$PLN = \frac{360 \times PN}{2\pi \times LP} = \frac{57.3 \times PN}{LP}.$$

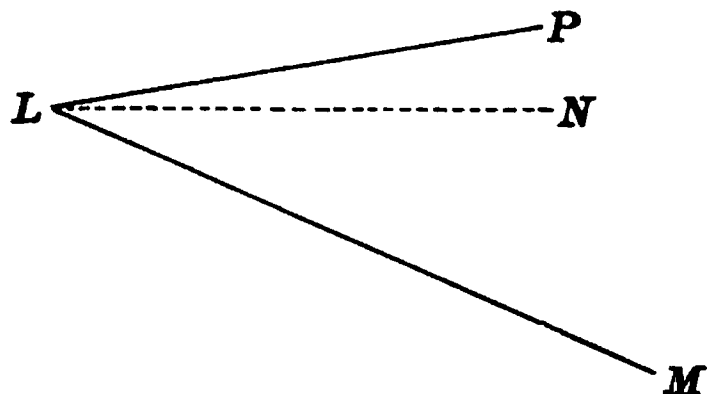
When PN exceeds the limit stated, the angle PLN should be found by measuring PN perpendicularly from PL , and dividing this by the length LP for the tangent of the angle PLN .

EXAMPLES.

1. A random line was run N. $41^\circ 15'$ E. 18.34 chains, when the nearest distance to the desired corner, which was to the left, was found to be 16 links. Required the correction and the bearing of the true line. *Ans.* Cor. $30'$; bearing of line, N. $40^\circ 45'$ E.

2. A random line was run S. $89^\circ 45'$ W. 24.80 chains, when the corner was found 22 links to the right. Find the correction and the bearing of the line.

3. The length of a random line is 16.64 chains, and a perpendicular from its extremity to the desired point equals 96 links. What correction is needed?



4. A random line LP , 25.12 chains long, run by transit, makes an angle of 27° with LM , and the point P is 18 links to the left of N ; LN being the true line. Determine the proper angle to turn off at L with which to trace LN .

C. PROBLEMS ON MEASUREMENT.

178. *a.* When the Ends of the Line are Accessible and Visible from Each Other.

The methods indicated in Problems on Alignment will be found useful in many instances for the determination of the lengths of lines, the direct measurements of which are impracticable. Thus, in the figure in Article 176, the distance NO will be found by measuring KM .

In figure accompanying Article 174 the measurement of either NP or PO will give the side NO .

Otherwise (Article 175). Measure NM , and multiply it by $\sqrt{2}$, or extract the square root of twice the square of NM for the required length NO .

By random line, as in Article 177, when the shortest distance PN is taken, the length of the true line will equal the measured or random line.

If the perpendicular from P is used, then the length of the true line will equal the square root of the sum of the squares of LP and PN ; that is,

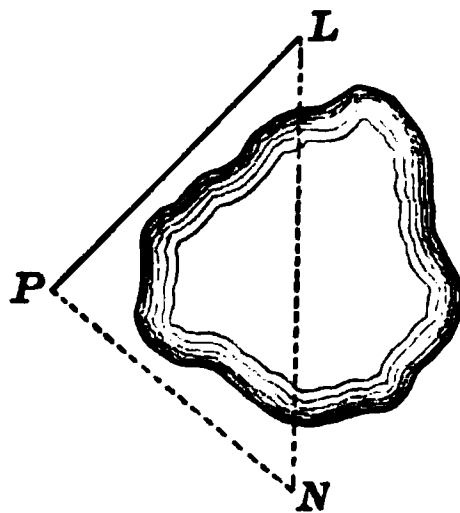
$$LN = \sqrt{LP^2 + PN^2}.$$

To ascertain the horizontal measurement of a hillside, take the angle of its slope, measure up or down it (preferably down), and the product of this distance and the cosine of the angle will be the horizontal distance required.*

179. By Triangulation. Measure LP and the angles L and P ; the sine proportion may then be employed to determine

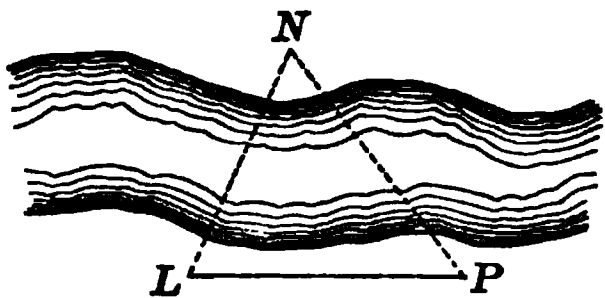
$$LN = \frac{LP \times \sin P}{\sin (L + P)}.$$

180. Otherwise. Measure LP , PN , and the angle P . Then having two sides and the included angle of the triangle, the third side LN may be computed.



* When measuring the angle of elevation, the surveyor should sight to a point on the rod a distance above ground equal to the height of the line of collimation of his instrument.

181. b. When One End of the Line is Inaccessible. Let N

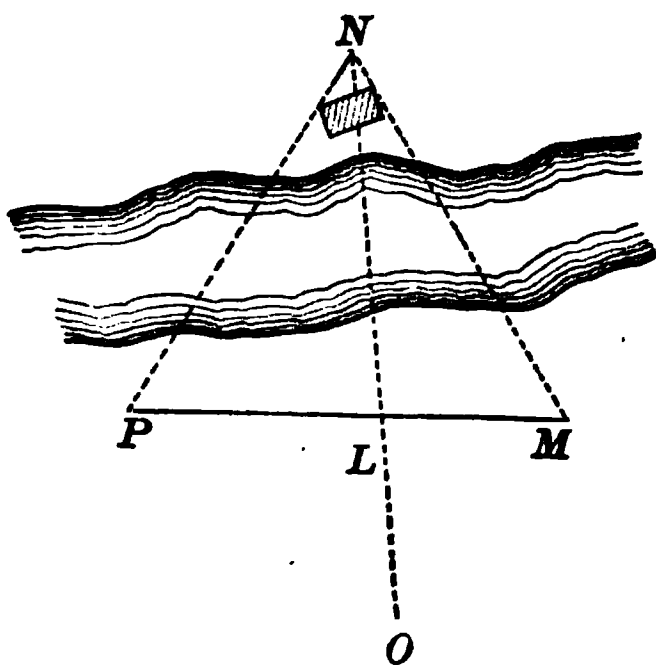


represent the inaccessible but visible end of the line LN , the length of which is desired. Measure LP of such length, if possible, that none of the angles will be less than 30° ; the nearer LNP is equi-

lateral, the better. Observe the angles L and P . Then, by the sine proportion,

$$LN = \frac{LP \times \sin P}{\sin (L + P)}.$$

182. When the Points are not Visible from Each Other. In the figure let N represent the invisible point in the line LN ,

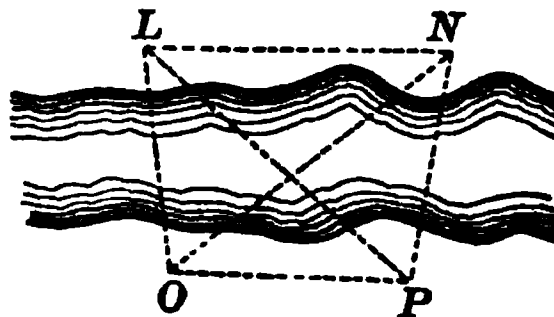


the length of which is required. Measure a line in any convenient direction through L , as MP , noting the distances ML and LP , of such a length that the point N may be seen from each extremity. Observe the angles P and M . In the triangle PMN , find, by the sine proportion, the length of PN . Then in PNL are known two sides and the included angle, with which may be found LN .

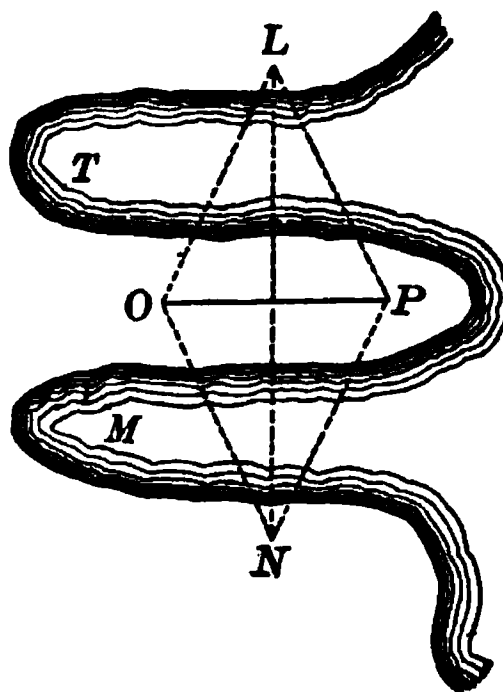
It will be observed that the problem requires the measurement of the distance between *two points*, L and N , invisible from each other, and direction unknown. If it were simply to determine the distance from L to an *invisible point* in the *prolongation of* OL , we should measure perpendicularly from OL to a point P , from which the point N could be seen, observe the angle LPN ; then $LN = PL \times \tan LPN$.

QUERY. What would be the best method of solving the problem under the last supposition, if it were impracticable to measure a perpendicular from OL ?

183. c. When the Ends of the Line are Inaccessible. Let it be required to determine the length of the inaccessible line LN . Measure OP , and observe the angles LON , NOP , OPL , and LPN ; then in the triangle LOP compute LO , and in NOP , ON . There will then be given two sides and the included angle of the triangle LON to find LN .



184. The same general method would apply if the base intersected the line the length of which is desired. Suppose it is required to determine the distance between L and N , points on opposite sides of two inlets, M and T . Measure OP and take the angles at the extremities on both sides of the base. There will then be data sufficient to find OL and ON , and finally LN .



QUERY. Would it be practicable in any case to make OP perpendicular to LN ? If so, would it be necessary to measure the distance OP and all the angles, as above? Why?

EXAMPLES.

1. To determine the distance between two points L and N , on opposite banks of a stream, I measured a base $LP = 300$ feet, and observed the angles which N made with L and P to be $58^\circ 45'$ and $64^\circ 50'$, respectively. Required LN .

2. If LP in Example 1 were taken at right angles to LN , the angle P being $40^\circ 30'$, what would be the length of LN ?

3. To ascertain the distance LN between two inaccessible points invisible from each other, I measured a line MP through

L , from the extremities of which N could be seen. $ML = 160$ feet; $LP = 200$ feet; angle at $M = 65^\circ 30'$; angle $P = 69^\circ 15'$. What is the length of LN ?

4. To determine the distance between two points L and N , situated on the side of a river opposite to where I was, a base line OP 400 feet long was measured, and the following angles observed: $LON = 68^\circ 30'$; $NOP = 32^\circ 45'$; $NPL = 50^\circ 30'$; $LPO = 40^\circ 15'$. Required LN .

EXERCISES.

1. Prolong a line beyond a house, tree, or other obstruction, using any one of the methods herein given. Return, pass the obstruction by some other method. See how near the starting-point is reached.

2. Run a trial line of considerable length through a wood, with a view of sighting a stake previously set. Make the proper measurements and calculation to correct the angle and re-run the line. Note the distance, if any, from the stake after the second trial.

3. Triangulate across a creek or small lake. Use at least two methods. See how near the results agree.

4. By triangulation determine the distance between two points without going near them. Verify the result by subsequent measurement.

5. Measure the distance between two points in a given line, invisible and assumed inaccessible from each other. Compare the results of two methods. Verify subsequently by direct measurement.

6. Run a trial line between two points which are invisible from each other, on account of an intervening ridge. Correct the angle and re-run the line. If the proper point is not reached, should the angle be again corrected?

SECTION IV.

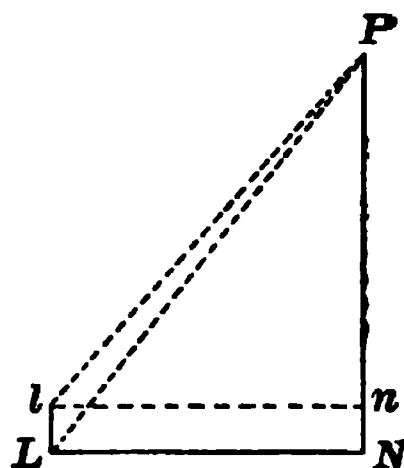
HEIGHTS AND DISTANCES.

A. ACCESSIBLE HEIGHTS.

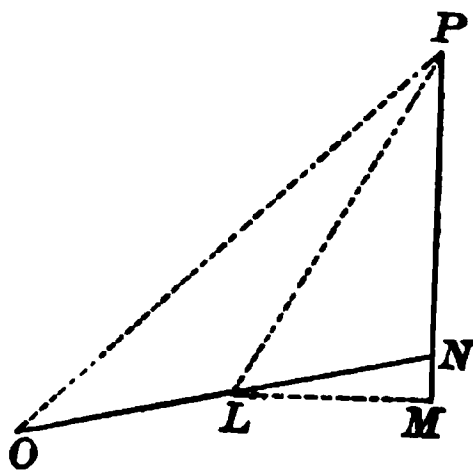
185. Let it be required to determine the height P above a horizontal plane LN . Measure the distance LN and the angle of elevation L . Then

$$PN = LN \tan L.$$

If the ground is level, or nearly so, the telescope cannot be placed at L , in the horizontal plane with N , but at some point l , and the angle Pln is measured instead of PLN . In such a case Nn must be added to the calculated height.



186. Let it be required to find the height of an object standing on an inclined plane ON . Measure the distances NL and LO , and the angles NLP and NOP . In the triangle OLP , by the sine proportion, find PL . Then in the triangle PLN , having two sides and the included angle, PN may be determined.



187. Otherwise. Measure NL , and at L the angles of elevation of N and P . Then the projection of LN on the horizontal plane

$$= LM = LN \cos NLM,$$

and $MN = LN \sin NLM;$

$$PM = LM \tan PLM;$$

whence $PN = PM - NM;$ or, expressed in a single equation,

$$PN = LN \times \cos NLM \times \tan PLM - LN \times \sin NLM.$$

EXAMPLES.

1. At 120 feet distance from the centre of the foot of a liberty pole, the angle of elevation of its top was $38^{\circ} 40'$. Required its height.

2. The distance LN (see Article 185) measures 90 feet, the angle of elevation l is $42^{\circ} 30'$, the telescope being 4.8' above the horizontal plane LN . Determine height of the point P .

3. To determine the height of an object on an inclined plane, two stations, L and O (marginal figure, Article 186), were selected, one 50 feet and the other 110 feet, measured on the slope from N . The angle $NLP = 40^{\circ} 15'$, and $NOP = 22^{\circ} 30'$. Required the height.

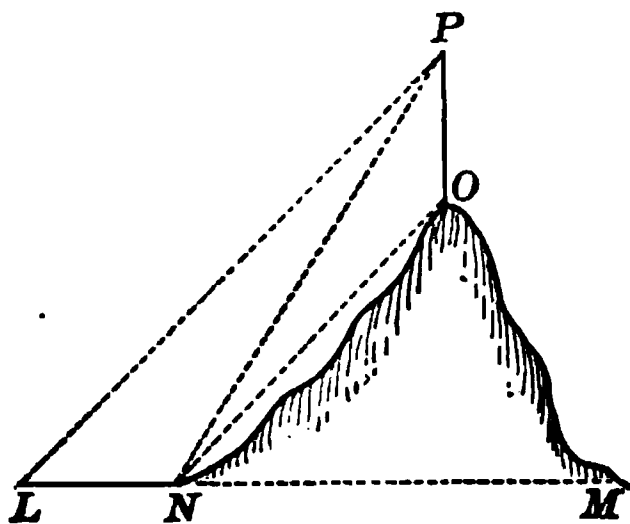
QUERIES. Practically, is it necessary to know the height of instrument * in such cases?

If there was a change of slope at L , would any other measurement be necessary to calculate the required height?

4. Suppose NL (figure, Article 186) measures 60 feet, and the angles of elevation at L , of N and P , are respectively $12^{\circ} 30'$ and $59^{\circ} 20'$. Determine the height of P above N .

B. INACCESSIBLE HEIGHTS.

188. To determine the height of an object situated on an inaccessible hill.



Measure in the same vertical plane with P a horizontal line LN , and observe at N the angles of elevation of the points O and P , and at L the angle of elevation of P . In the triangle LNP , by the sine proportion, calculate PN .

By the same method, find NO from the triangle PON . Then

* Height of instrument is the height of the line of sight above the ground, or any other assumed horizontal plane.

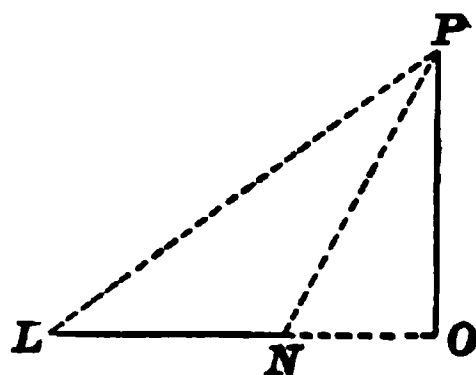
$$PO = PN \sin PNM - NO \sin ONM.$$

The student may show, after finding PN and NO as above, a different method of finding PO than that indicated.

Ex. At a certain station the angle of elevation of the base of a tower on a hill-top was $38^\circ 40'$, and that of the top $50^\circ 15'$; 190 feet more remote, the angle to the top was $36^\circ 20'$. The stations being in the same horizontal plane, required the height of tower and of the hill.

189. Let PO be an object whose height is required. Measure in the same vertical plane with P a horizontal base line LN , and observe the angles of elevation PLN and PNO . Then, by the sine proportion, find PN , and

$$PO = PN \sin PNO.$$



190. Otherwise. $PO \cot L = LO$,

$$PO \cot N = NO,$$

$$LO - NO = LN;$$

$$\text{or, } PO (\cot L - \cot N) = LN.$$

$$\therefore PO = \frac{LN}{\cot L - \cot N}.$$

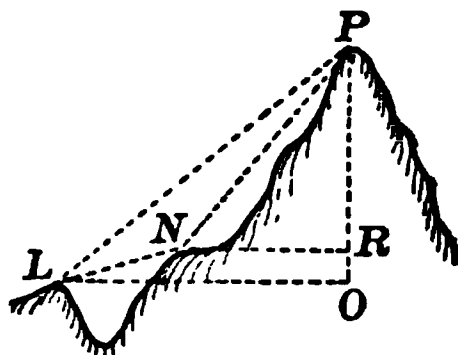
Ex. If $LN = 120$ feet, and the angles at L and N respectively $27^\circ 50'$ and $45^\circ 19'$,

$$PO = \frac{120}{\cot 27^\circ 50' - \cot 45^\circ 19'} = 136.6 \text{ feet. } \text{Ans.}$$

191. If it is impracticable to locate the base line in a horizontal plane, measure from L in the direction of P any line LN , and at L take the angles of elevation of N and P . Observe also the angle at N . By the sine proportion obtain LP . Then

$$PO = LP \sin PLO,$$

$$\text{and } PR = PO - RO = LP \sin PLO - LN \sin NLO.$$



QUERY. May the observed angle at N be either LNP or PNR ?

192. Otherwise. L and N being in different planes, measure the horizontal distance between them. Observe the angle of

elevation PLO and the horizontal angles OLN and ONL . By the sine proportion find LO . Then

$$PO = LO \tan PLO,$$

or, expressed in a single equation,

$$PO = \frac{LN \sin LNO \tan PLO}{\sin NOL},$$

which equals the height of P above the horizontal plane through L .

If it is required to find the height of P above the horizontal plane through N , proceed as follows: Assuming N to be below * L , observe at N the angle of elevation of P ; then find the horizontal distance between N and O by the sine proportion, using the triangle NLO ; thus, $\sin O : \sin L = LN : \text{fourth term}$. This fourth term will not be NO , since the measurement of the distance and angles employed in the computation is referred to a horizontal plane, and hence the fourth term will express the *horizontal* distance between N and O , which equals NR , R being a point in the prolongation of the vertical PO . Whence,

$$PR = NR \tan PNR.$$

EXAMPLES.

1. At a certain station the angle of elevation of the top of an inaccessible object situated on a horizontal plane was $60^\circ 50'$, and 120 feet farther away the angle was $29^\circ 10'$. Required the height of the object and its distance from the first station.

2. Suppose LN (figure, Article 191) is 140 feet, the angles of elevation at L , of N and P , are respectively $9^\circ 25'$ and $30^\circ 16'$,

* It may obviously be above or below L ; the same reasoning will hold.

and the angle $PNR = 42^\circ$. Find the height of P above O and R .

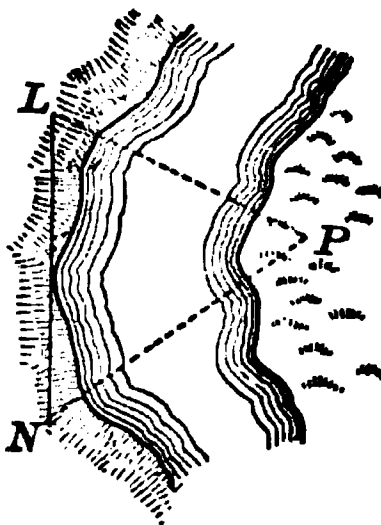
3. In figure, Article 192, suppose

$$\begin{array}{ll} LN = 1000 \text{ feet;} & \text{angle } PLO = 26^\circ 18'; \\ \text{angle } OLN = 36^\circ 20'; & \text{angle } PNR = 55^\circ 10'. \\ \text{angle } ONL = 95^\circ 40'; & \end{array}$$

Find PO and RO .

193. To determine the perpendicular distance from a given horizontal plane of an inaccessible object situated below it.

Let P be the point whose perpendicular distance from a horizontal plane through L is required. Select two points L and N visible from each other, and from which P can be seen. Measure the horizontal distance between them; observe also the horizontal angles PLN and PNL , and the angle of depression of the point P , at L . By the sine proportion calculate the horizontal distance from L to P ; this multiplied by the tangent of the angle of depression observed at L will give the perpendicular distance required.

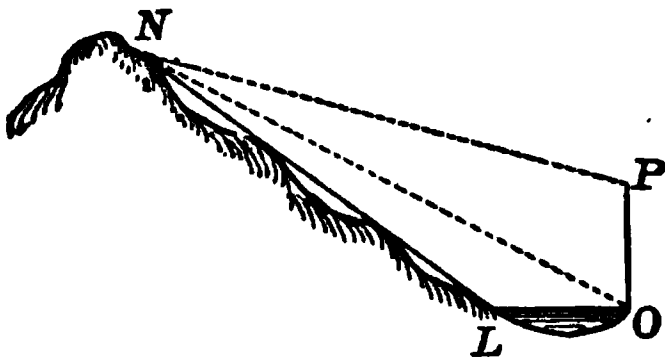


If L and N are not in the same horizontal plane, observe at N the angle of depression of P , and calculate as above the perpendicular distance between the point and the horizontal plane through N . The difference of these perpendicular distances will also give the difference in height of L and N . A check on the work may be had by determining from more direct methods already given the difference in elevation of L and N .

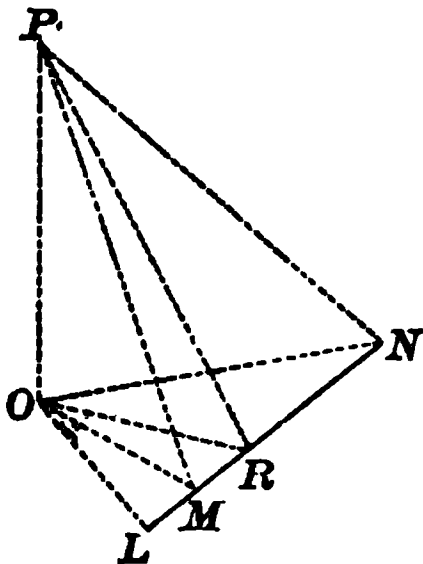
EXAMPLES.

1. At L and N (last figure) the horizontal angles measure respectively $67^\circ 40'$ and $43^\circ 10'$; and sighting P , the angles of depression taken in the same order are $32^\circ 18'$ and $21^\circ 42'$. The distance between the stations being 1200 feet; required the difference in height of P , L , and N .

2. To find the height of an object, PO , standing on the edge of a lake and inaccessible to L , a station on the opposite rocky shore, a distance of 500 feet was measured from L up the slope to N , where the angles of depression of L , O , and P were observed respectively, $39^\circ 40'$, $25^\circ 20'$, and $21^\circ 32'$. Required the height of PO .



194. To determine the height of an object, and its distance from three observing-stations situated in a straight line and in the horizontal plane through the foot of the object.



Let PO represent the required height ; L , R , and N the stations ; the angles of elevation of P taken at each and in the order named α , β , and θ . The distance $LR = a$, $RN = b$, and the unknown height $= x$. It is evident that the triangles POL , POR , and PON are right-angled at O , and therefore

$$OL = x \times \cot \alpha.$$

$$OR = x \times \cot \beta.$$

$$ON = x \times \cot \theta.$$

Again, drawing OM perpendicular to LN , we shall have from the acute-angled triangle LOR ,

$$\overline{OL}^2 = \overline{OR}^2 + \overline{RL}^2 - 2RL \times RM,$$

and from the obtuse-angled triangle NOR ,

$$\overline{ON}^2 = \overline{OR}^2 + \overline{RN}^2 + 2RN \times RM;$$

or, substituting the proper values for the lines represented, we shall have

$$x^2 \cot^2 \alpha = x^2 \cot^2 \beta + a^2 - 2a MR,$$

$$x^2 \cot^2 \theta = x^2 \cot^2 \beta + b^2 + 2b MR.$$

Eliminating MR by multiplying the first by b , the second by a , adding and factoring, we obtain

$$\begin{aligned} x^2(b \cot^2 \alpha + a \cot^2 \theta) \\ = x^2 \cot^2 \beta (a + b) + ab(a + b). \end{aligned}$$

Whence
$$x = \sqrt{\frac{ab(a + b)}{b \cot^2 \alpha + a \cot^2 \theta - \cot^2 \beta (a + b)}}$$

If the stations are equidistant, the formula reduces to

$$x = \sqrt{\frac{2a^2}{\cot^2 \alpha + \cot^2 \theta - 2 \cot^2 \beta}}$$

Or,
$$x = \frac{a}{\sqrt{\frac{\cot^2 \alpha}{2} + \frac{\cot^2 \theta}{2} - \cot^2 \beta}}.$$

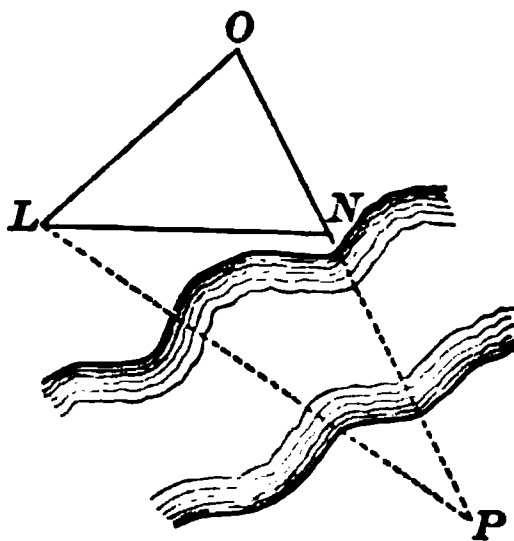
Having obtained the height of P above the plane, the horizontal distance from the object to either station may be determined by multiplying this height by the cotangent of the angle of elevation at the station. The oblique distance from either station to P is given by the product of PO and the cosecant of the angle of elevation at the station.

INACCESSIBLE DISTANCES.

195. *The distance apart of three objects, L , O , and N , inaccessible from P are known, viz.: $LO = 2000$ feet, $ON = 1800$ feet, and $LN = 2400$ feet. At P , situated in the prolongation of ON , the observed angle $= 21^\circ 48'$; how far is it from station P to each object?*

First calculate angle O ; then in the triangle POL there will be known all the angles and one side, whence the required distances may be readily found.

Usually the station P cannot be chosen so as to fall in ON or OL produced; then the measurement of *two* angles will generally be sufficient, with the known distances to locate the



point of observation. For example, suppose the distances and angles are as follows :

$$NO = l = 3000 \text{ feet ;}$$

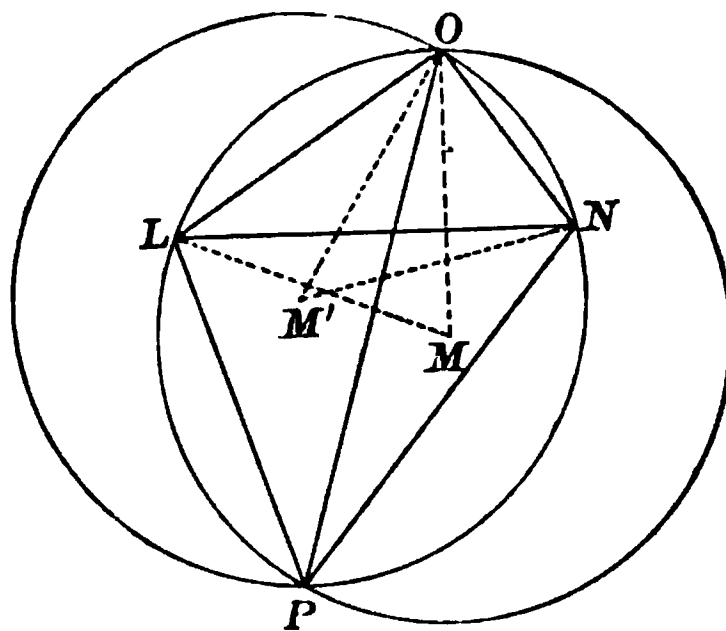
$$OL = n = 3600 \text{ feet ;}$$

$$LN = o = 4800 \text{ feet ;}$$

$$\text{angle } NPO = \alpha = 23^\circ 40' ;$$

$$\text{angle } LPO = \beta = 29^\circ 50'.$$

By construction, the point P may be found as follows : Subtract from 180° $2 LPO$, and from LO lay off at L and O the angles LOM and OLM , each equal to half the remainder. From the point M thus determined as a centre, and with a



radius LM , describe the circumference OLP . The angle LPO will then be contained in the segment LPO , and the point P must be somewhere in the circumference OLP . In like manner, by means of the angle OPN , find another circumference ONP , in which the point P must be situated. The intersection of these circumferences indicates its position.

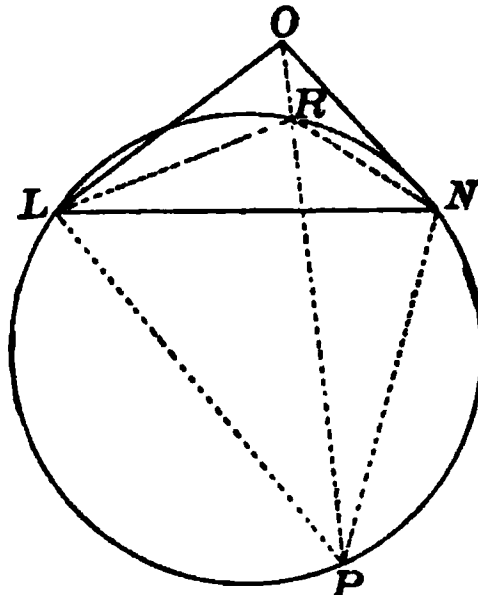
The angle at the circumference being half that at the centre, the angle LMO , subtended by the same chord as LPO , will be $2 LPO$, and the angles OLM and LOM being equal and together the supplement of LMO , each angle will

$$= \frac{180^\circ - 2 LPO}{2} = 90^\circ - LPO.$$

Otherwise. Construct an angle NLR equal to OPN ; also LNR equal to OPL , and describe a circumference through the points L , R , and N . The point P must lie in the circumference, and also in the line drawn from O through R . Their point of intersection therefore will indicate its position.

The student may give the reason.

196. By Calculation. Pass a circle through the points L , N , P , and join L and N with R , thus forming a triangle in which the angles RLN and RNL are equal, respectively, to the observed angles RPN and RPL , and these, with the known side LN , furnish data sufficient to compute the sides LR and RN . Next calculate the angle ONL , whence, by subtraction, the angle ONR is found. Now, in the triangle NOR there are given two sides and the included angle to find NOR and ORN , or its supplement PRN , and by means of the sine proportion and the triangles PON and POL the distances PN , PO , and PL may be obtained.



Otherwise. After finding the angle O , obtain an expression for either OLP or ONP , and then, by the sine proportion, the required distances.

Denote the angle OLP by ϕ , ONP by ψ , and the other parts as before; then

$$\sin \beta : \sin \phi = n : OP, \text{ or } OP = \frac{n \sin \phi}{\sin \beta};$$

$$\sin \alpha : \sin \psi = l : OP, \text{ or } OP = \frac{l \sin \psi}{\sin \alpha}.$$

$$\text{Whence } \frac{n \sin \phi}{\sin \beta} = \frac{l \sin \psi}{\sin \alpha};$$

$$\text{and } \sin \phi = \frac{l \sin \psi \sin \beta}{n \sin \alpha}.$$

Again, $\phi = 360 - \alpha - \beta - O - \psi$;

or, putting $360 - \alpha - \beta - O = \theta$,

$\phi = \theta - \psi$, in which θ is known;

and $\sin(\theta - \psi) = \frac{l \sin \psi \sin \beta}{n \sin \alpha}$.

Developing the left-hand member, dividing through by $\cos \psi$, and simplifying, there results

$$\sin^+ \tan \psi = \frac{n \sin \alpha \cos \theta^*}{l \sin \beta + n \sin \alpha \cos \theta};$$

or, $\cot \psi = \frac{l \sin \beta}{n \sin \alpha \sin \theta} + \cot \theta$.

There are therefore but three steps in the solution:

1. Calculate the angle O , and thence obtain θ .
2. Find $\tan \psi$, or $\cot \psi$.
3. By sine proportion, calculate PN , PO , and PL .

In the example given, since the sides are in the proportion 5 : 6 : 8, the angle O may be readily found from the well-known formula for the cosine of an angle,

$$\cos O = \frac{25 + 36 - 64}{60} = -.05 = 92^\circ 52',$$

and $\theta = 213^\circ 38'$;

whence $\psi = 109^\circ 53'$,

$\phi = 103^\circ 45'$.

| | |
|------------------------|--------------------|
| $\sin 23^\circ 40'$ | Ar. co. = 0.396406 |
| : $\sin 109^\circ 53'$ | = 9.973307 |
| :: 3000 | = 3.477121 |
| : $PO = 7028$ | = 3.846834 |

* Regard must be given to the signs of the trigonometrical functions.

$$\begin{array}{rcl}
 \sin 23^\circ 40' & \text{Ar. co.} & = 0.396406 \\
 : \sin 46^\circ 27' & & = 9.860202 \\
 :: 3000 & & = 3.477121 \\
 : PN = 5417 & & = \underline{3.733729}
 \end{array}$$

$$\begin{array}{rcl}
 \sin 29^\circ 50' & \text{Ar. co.} & = 0.303225 \\
 : \sin 46^\circ 25' & & = 9.859962 \\
 :: 3600 & & = 3.556303 \\
 : PL = 5242 & & = \underline{3.719490}
 \end{array}$$

If the supplement of the observed angles at P equals the angle at O , the circle will pass through the three points L , N , and O , and P may be *anywhere* on the circumference, and hence its distance is indeterminate by the first method given above; and, substituting in the formula the proper values to find $\cot \psi$ by the second method, the numerator of the fraction will become infinite, as also the $\cot \theta$; hence, such an observation will fail in both cases to locate the point P .

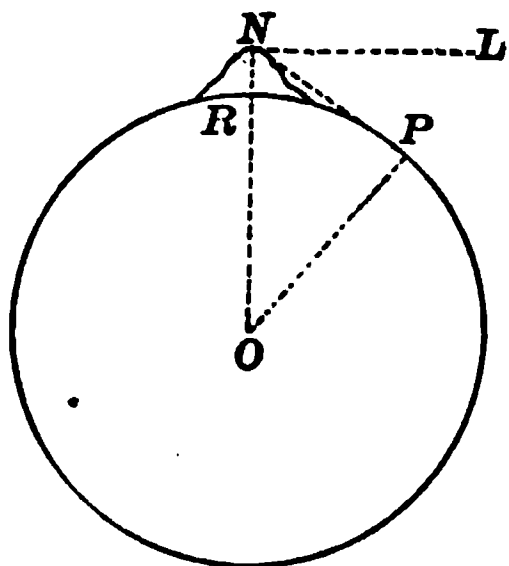
EXAMPLE.

Suppose $LN = 960$ rods, $NO = 576$ rods, $LO = 640$ rods, the angle $LPO = 19^\circ$, and $NPO = 25^\circ$. Find the distances PO , PN , and PL .

Ans. $PL = 758$ rods; $PO = 1310$ rods; $PN = 1350$ rods.

197. From the top of a mountain m miles high the angle of depression of a line tangent to the earth's surface is a degrees; it is required thence to find an expression for the radius of the earth, assuming it to be a sphere.

Let O represent the centre of the earth; N the mountain top; P the point of tangency; OP and OR radii of the earth; RN the height of mountain and prolongation of OR .



Draw NL perpendicular to ON , and denote the radius of the earth by r ; then, since NL and NP are respectively perpendicular to NO and OP , the angle $NOP =$ the angle of depression $LNP = a$.

Hence

$$(r + m) \cos a = r.$$

$$\therefore r = \frac{m \cos a}{1 - \cos a}. \text{ Ans.}$$

MISCELLANEOUS PROBLEMS.

1. Determine the height of a hill, knowing that the angle of elevation of its top from a certain station $= 50^\circ$, and at a station 800 feet more remote the angle of elevation $= 36^\circ 20'$.

2. The angle of depression, taken from a balloon to a station whose horizontal distance is known $= 18^\circ 40'$. Find the height of the balloon.

3. Two war vessels, desiring to ascertain their distances from a fort, remove from each other 2000 feet, and measure the angle between each other and the fort; the angles being $79^\circ 40'$ and $82^\circ 20'$, what were their distances?

4. Two observers on the same horizontal plane, 1500 feet apart, and in a vertical plane with a balloon, observe its angles of elevation to be $62^\circ 40'$ and $71^\circ 10'$. Required the height of the balloon.

5. The passage between two objects L and N being obstructed by a swamp, the lines $LP = 420$ feet, and $PN = 540$ feet, were measured, and the angle LPN observed $= 86^\circ 42'$. Find the distance LN .

6. What distance can a person whose eye is $5\frac{1}{2}$ feet above the ocean see its surface? Assume radius $= 3960$ miles.

7. If the sun subtend an angle of $32^\circ 2'$, and his distance from the earth is 93,000,000 miles, what is his diameter?

8. What is the altitude of the sun when the shadow of a staff cast on a horizontal plane is to the height of the staff as 7 to 5?

9. If the horizontal parallax * of the moon be $56' 50''$ and the diameter of the earth 7920 miles, what is the distance of the moon from the earth?

10. If the moon subtend an angle of $31' 14''$, when its distance is 240,000 miles, what is its diameter?

11. When the meridian altitude of the sun is 50° , the shadow cast by the peak of a mountain reaches a certain point on a horizontal plain; but when his mid-day altitude is 60° , the shadow strikes a point 2000 feet nearer the base of the mountain. Determine the height of the mountain above the plain.

QUERIES. If on the same day two observations were made on the sun for altitude, one or both when he was not on the meridian, and the length of the shadow measured as in Ex. 11, would sufficient data be thus obtained to determine the height of the mountain?

Would it be possible with data obtained, as in the first query, to ascertain the height of the mountain if the sun was vertical over the mountain at noon?

12. If the height of a mountain is m miles and its top is visible d miles, find an expression for the diameter of the earth, assuming it to be a sphere.

13. The angle of depression taken on the top of Peak of Teneriffe, which is two and a half miles high, to the farthest visible point was $2^\circ 2'$. It is required to determine the observed distance and the diameter of the earth, assuming it to be a sphere. Dist., 140,876 miles; Diam., 7936 miles. *Ans.*

EXERCISES.

1. Measure the height of a flagstaff or church spire above the street.

* The angle at the moon, or other heavenly body, subtended by the semi-diameter of the earth.

2. Measure the height of a monument, tower, or some other prominent building upon a hill, without obtaining the distance to the foot of the object. Also, if practicable, measure the distance to the foot of the object and the proper angles. Compute and compare results with each other, and with the actual height, if it can be ascertained.

SECTION V.

RECORDING THE FIELD NOTES.

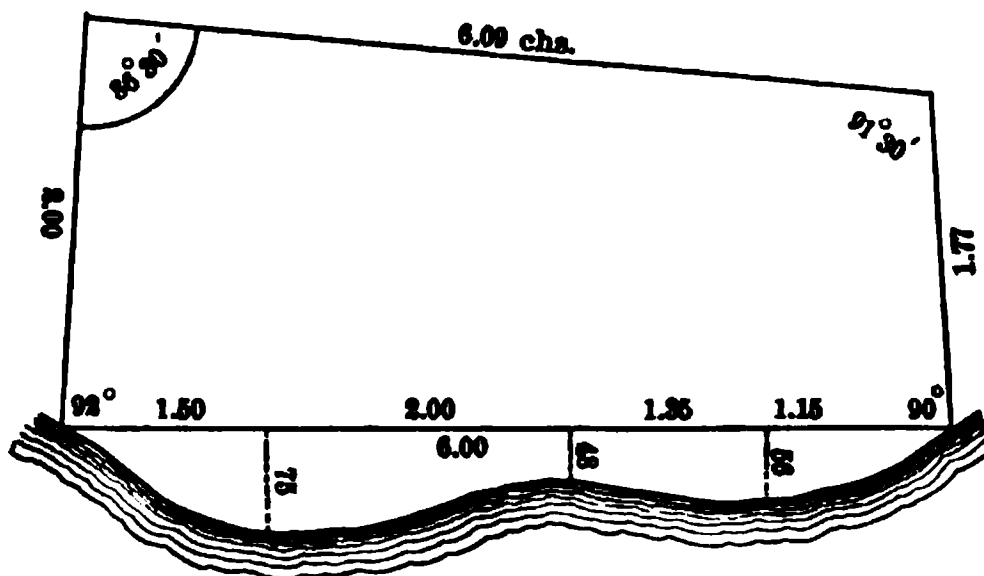
198. The Field Notes may be recorded in various ways, depending upon the instrument used, and the extent and intricacy of the survey.

First Method. If the compass is employed, the bearings simply to be taken, distances measured, and the tract bounded by straight lines (no offsets), the simplest, most compact, and also most convenient form for the subsequent calculation of the area is to write the stations, bearings, and distances in three columns, thus :

| STATIONS. | BEARINGS. | DISTANCES. | REMARKS. |
|-----------|---------------|------------|-----------------------|
| 1 | S. 21° 53' E. | 13.11 | To a maple. |
| 2 | N. 48° 12' E. | 13.70 | " birch. |
| 3 | N. 43° 40' W. | 4.73 | " stake and stones. |
| 4 | N. 45° 08' W. | 4.75 | " white oak. |
| 5 | S. 51½° W. | 2.53 | " sandstone. |
| 6 | S. 72½° W. | 6.53 | " red oak, beginning. |

199. *Second Method.* If the tract is not large, and there are offsets in addition to the bearings and distances, or if simply the angles and distances are measured, a very good method, especially for a beginner, is to make a rough plat of the survey,

and indicate in their corresponding places on the sketch the bearings, or angles, and the lengths of the lines and offsets, as shown below :



The above is a sketch of a small field, showing offsets to stream, etc. The following are hasty surveys of boundaries, etc., of land for proposed park in City of Wilmington, Del., July, August, and September, 1885 :

Instruments : Transit. Chesterman's 100-foot steel tape.

Work :

Lines run with transit, and carefully measured with steel tape from station to station.

Angles between these lines taken, always from left to right.

Magnetic bearings of lines taken.

Stations numbered or lettered in regular order.

Offsets (sometimes angles and distances) taken to locate houses, corners of fences, etc., offsets made at right angles with lines joining stations.

Notes :

Taken free-hand in small note-books (size $5\frac{3}{4}'' \times 3\frac{1}{2}''$).

Sketches made to suit the page and to make the matter clear for plotting.

The usual checks used on field and office work.

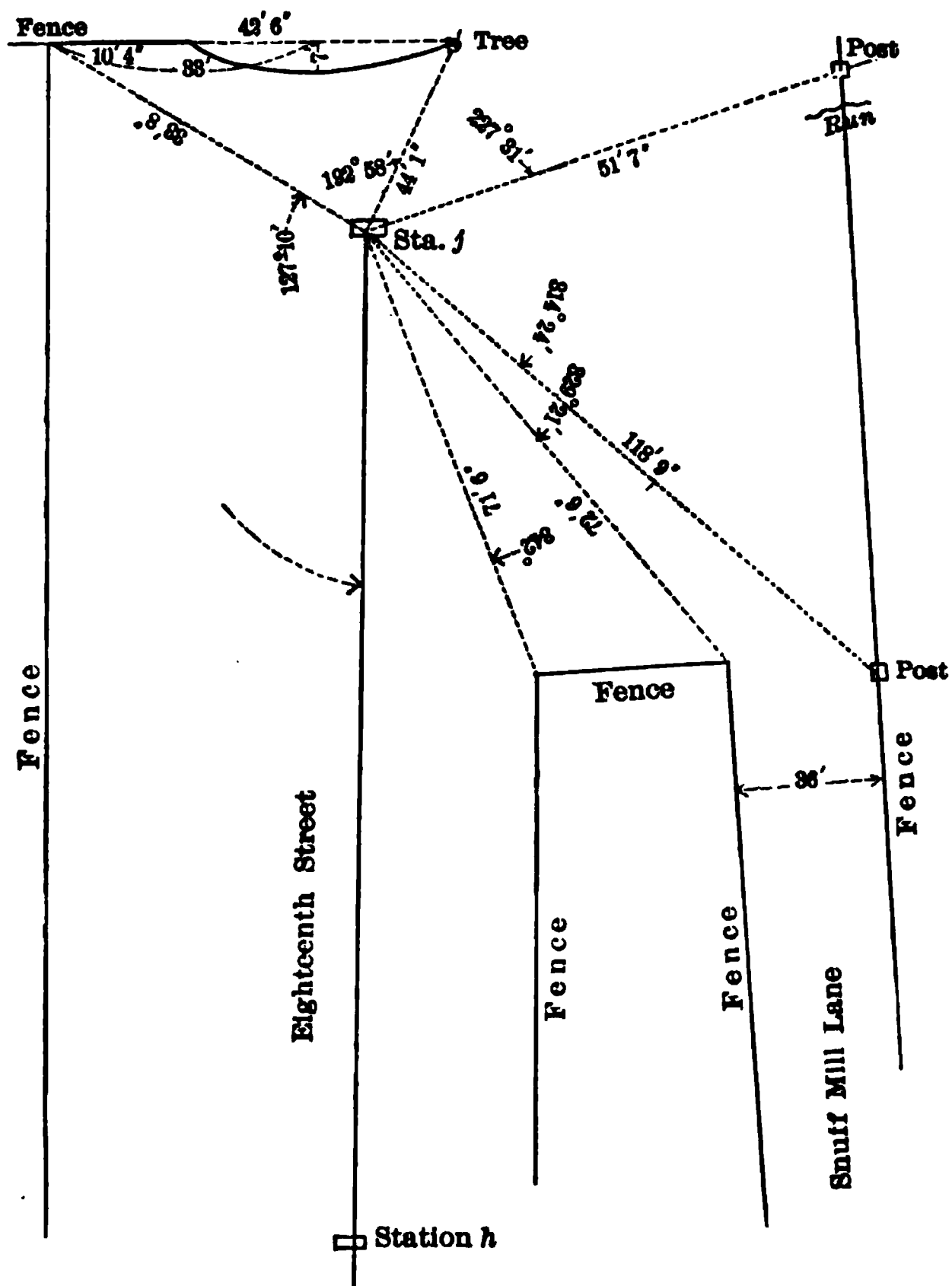
Explanation of Sketches :

No. 1. Single page of note-book. Location of fences on boundary of land proposed for park.

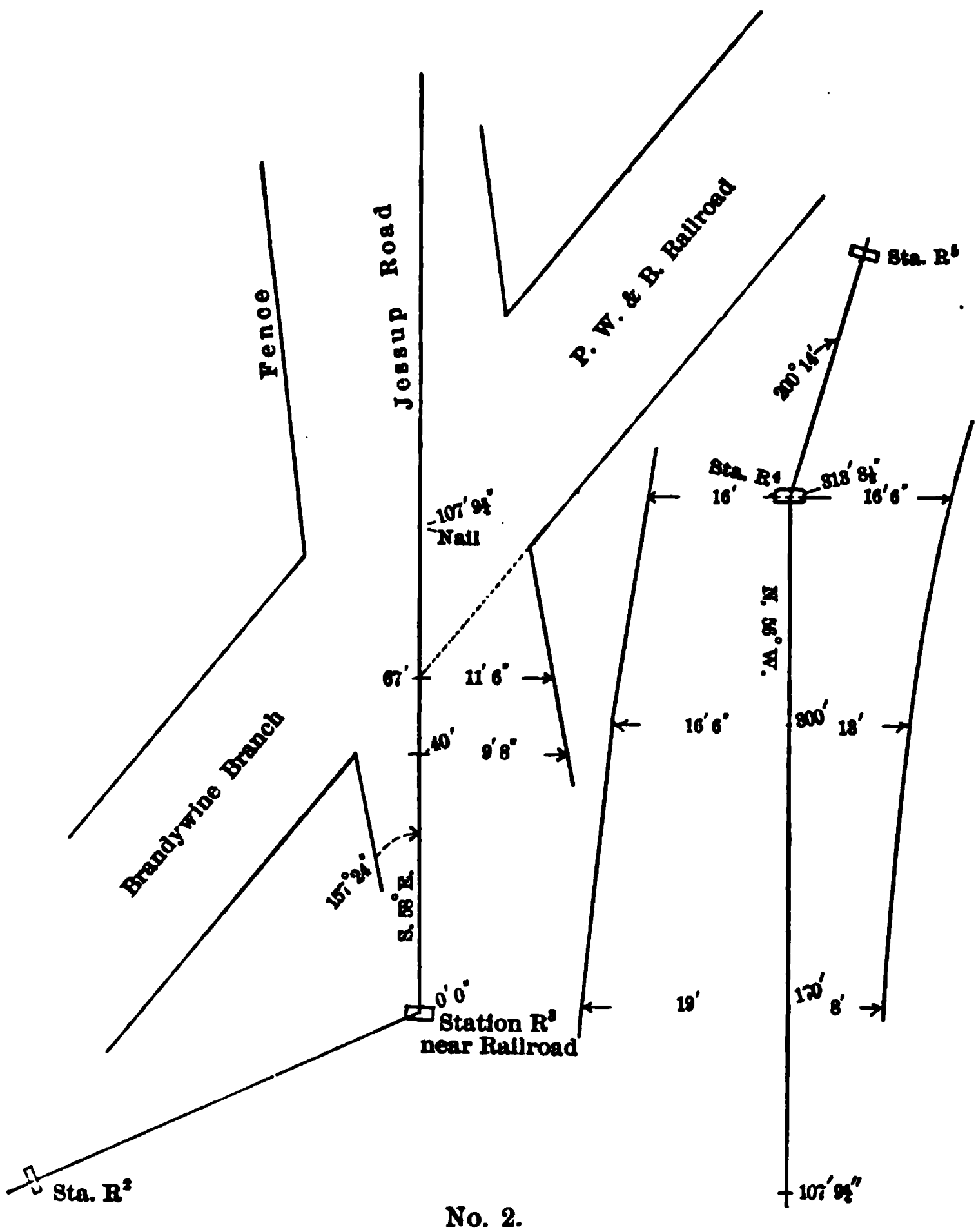
No. 2. Two opposite pages of note-book. Location of road through land proposed for park, showing railroad crossing.

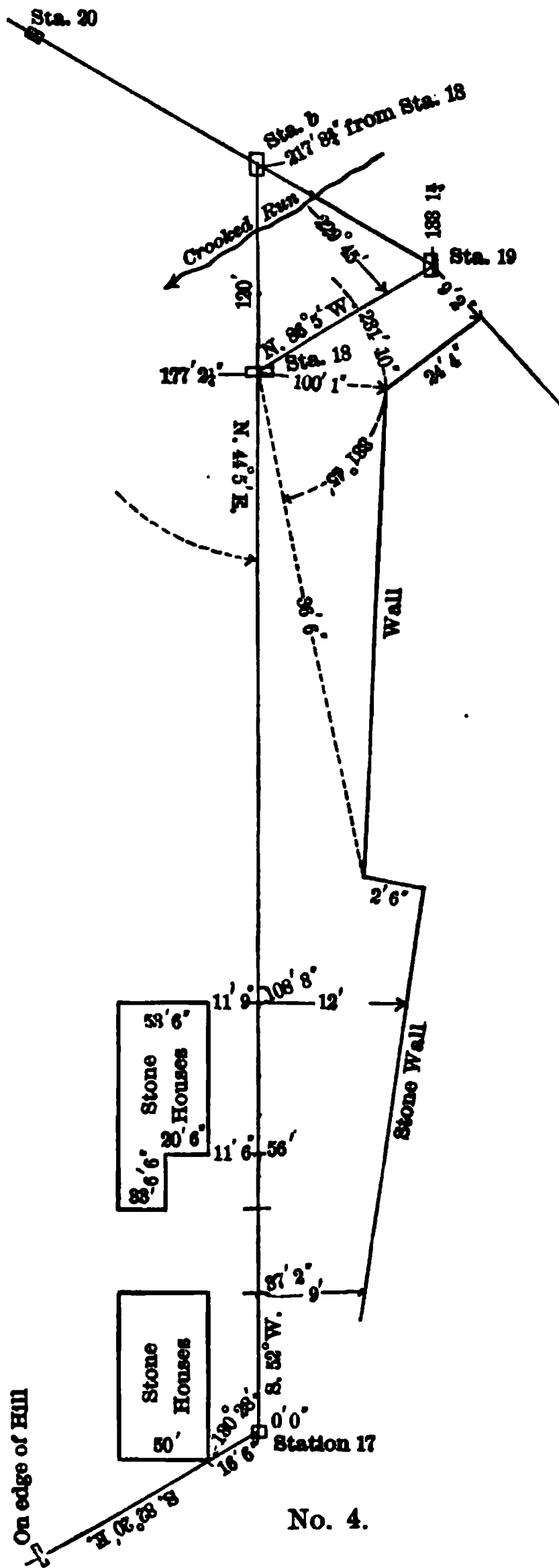
No. 3. Two opposite pages of note-book. Location of run between two adjoining owners of land proposed for park.

No. 4. Two opposite pages of note-book. Location of houses, etc., in land proposed for park.



No. 1.

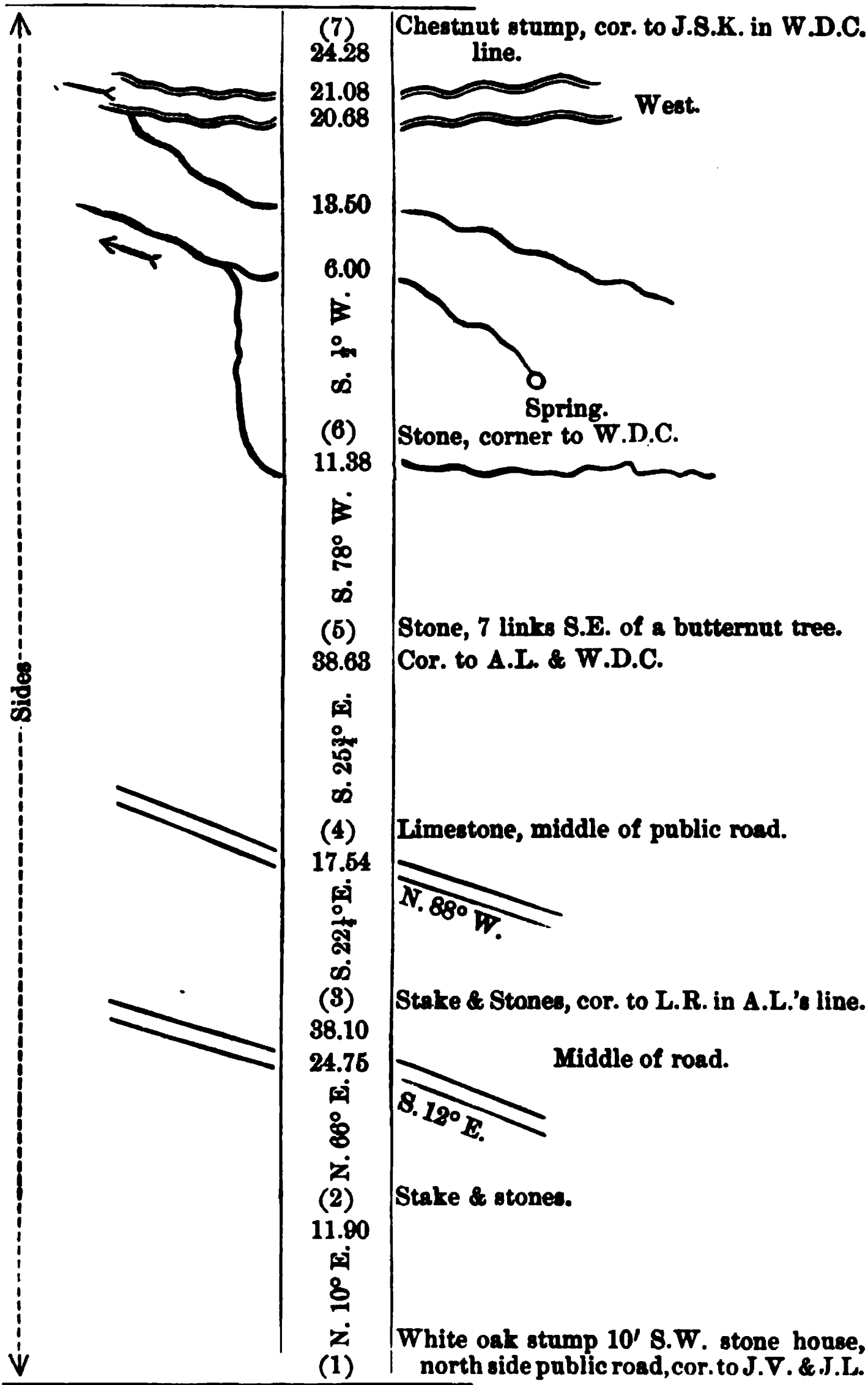








200. Third Method.

The column method, analogous to that shown in Article 38, Chain Surveying, is, however, the most general. If the bearings are taken, they may be inserted in the column either vertically or diagonally; if only the angles are observed, they should be placed at the stations which indicate where the measurements were made. The objects to which offsets are measured may be designated or delineated on the marginal side of the line as they naturally appear. Where streams, roads, fences, etc., cross the line, representations of them are made, indicating approximately their direction; or, if desirable, their bearings, or angular deviations from the line, may be taken and recorded.

The following notes will more fully explain the method under consideration :



| | | | |
|-------|--|---------------|---|
| Sides | | (1) | |
| | | 53.39 | |
| | | 14.52 | |
| | | 14.12 | |
| | | N. 1 1/4° E. | |
| | | (12) | Limestone, cor. to W.V. in C.C.'s line. |
| | | 10.47 | |
| | | S. 89 1/4° W. | |
| | | (11) | Limestone. |
| | | 8.00 | |
| | | N. 1 1/4° E. | |
| | | (10) | Limestone, cor. to J.S.K. in C.C.'s line. |
| | | 30.60 | |
| | | S. 89 1/4° W. | |
| | | (9) | Limestone. |
| | | 8.31 | |
| | | S. 1 1/4° W. | |
| | | (8) | Limestone. |
| | | 4.45 | |
| | | N. 88 1/4° W. | |
| | | (7) | |

| | | | |
|-----------------------|---|------------|--|
| ↑ Public Road ↓ | | (4) | Limestone. Sta. (4) in foregoing description. |
| | | 10.49 | |
| | Stone house 100' from line.  | 9.00 | |
| | | S. 88° E. | |
| | | (C) | Limestone. |
| | N. 12° W. | 10.41 | Lane leading to dwelling, S. 11° E. |
| | Road | 9.00 |  Barn. |
| | | 6.40 |  House 60' from line. |
| | | N. 85½° E. | |
| | | (B) | |
| | | 20.38 | |
| | | N. 65½° E. | |
| | Stone house near corner.  | (1) | White oak stump. Sta. (1) in foregoing description. |

The bearing and distance of proof-line from *P* to Station (11) = S. 62½° W. 19.10.

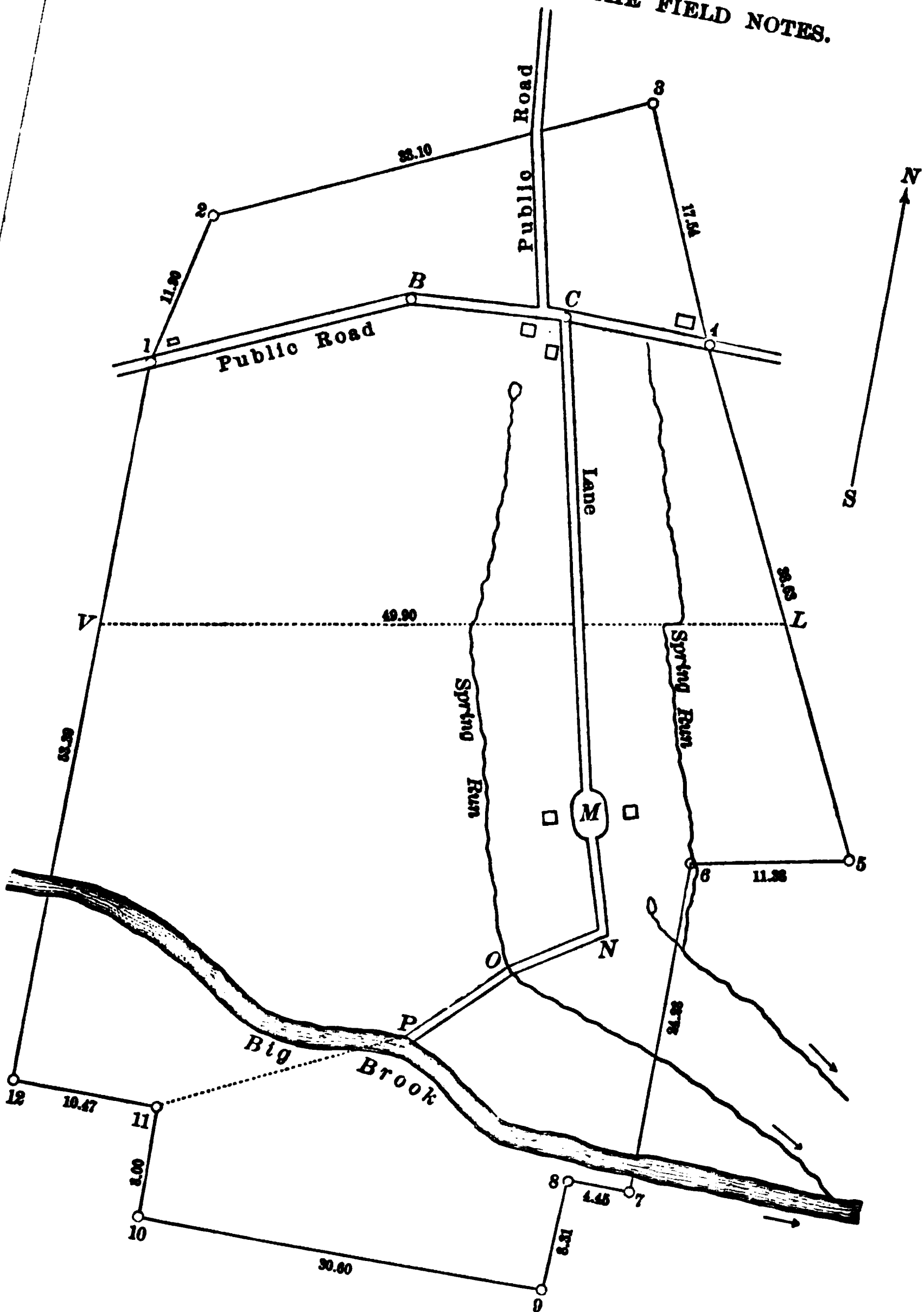
| | | |
|---|------------|--|
| <p>↑</p> <p>-----Lane from Public Road to Dwelling and to "Big Brook"-----</p> <p>↙</p> <p>S. 69° E.</p> <p>↕</p> <p>Spring Run.</p> <p>House 73'</p> <p>N. 79° 10' E. 6.00 to Spring Run</p> <p>and 15.82 to East line of Survey.</p> <p>↘</p> | (P) | Limestone at end of lane on north bank of "Big Brook." |
| | 9.20 | |
| | S. 39° W. | |
| | (O) | Limestone on bank of Spring Run. |
| | 5.75 | |
| | S. 57½° W. | N. 16¼° W. |
| <p>↕</p> <p>Spring Run.</p> <p>Sta. betw'n house and barn.</p> <p>(M)</p> <p>36.20</p> <p>71' Barn</p> <p>S. 79° 10' W. 7.08 to Spring Run</p> <p>and 34.08 to West line.</p> <p>S. 11° E.</p> <p>(C)</p> <p>↘</p> | (N) | Point in lane. |
| | 8.80 | |
| | S. 16° E. | |
| | (M) | |
| | 36.20 | |
| | 71' Barn | |
| <p>↕</p> <p>Spring Run.</p> <p>↘</p> | (C) | Limestone in middle of public road at end of lane. |

The notes show that the sides of the tract were first surveyed; which, with their bearings and distances, include also the location and general direction of road-crossings, streams, etc., a description of the corners, and the names of owners of property adjoining the survey. Next to traversing the bounding lines, the survey of the public road, crossing the farm from east to west, was made. This road enters the tract at station (1); at 6.40 chains from (*B*) it passes a house which is 60 feet to the right; at 9.00 chains a road to the left, the bearing of which is given; at 10.41 chains is a corner at end of lane leading to dwelling; near the east end of road a stone house is located, at 100 feet north of the line; and at 10.49, station (4) of sides survey is reached, at which point the road leaves the farm. The survey of the lane to the dwelling, and thence to the creek, is next recorded. Here are noted the intersection of a line S. $79^{\circ} 10'$ W., and the distances on this, east and west, to spring runs, as well as the distances to the east and west sides of the tract;* the dwelling and barn are located, and the limestone on the north bank of Big Brook reached. A line was run from this last point to station (11), which, in connection with the survey of the lanes, the public road, and the cross-line from *L* to *V*, gave proof of the accuracy of the work.

* This line was made a boundary in the subsequent division of the land.

RECORDING THE FIELD NOTES.

153



SECTION VI.

LATITUDES AND DEPARTURES.

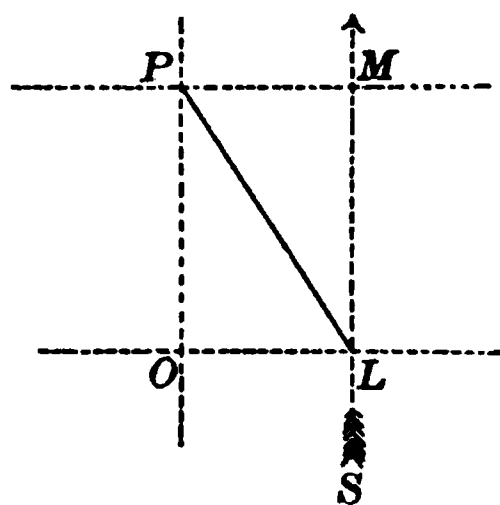
201. The **Difference of Latitude** of the two ends of a line is the perpendicular distance between the parallels of latitude which pass through them, and is reckoned north or south, according as the bearing is northerly or southerly.

The **Difference of Longitude** of the two ends of a line is the perpendicular distance between the meridians which pass through them, and is reckoned east or west, according as the bearing is easterly or westerly.

The difference of latitude of a line is often called briefly the *latitude*, or *northing* or *southing*; and the difference of departure, its *departure*, or *easting* or *westing*.

It will be perceived from the definitions just given that, when a line bears either due north or south, the distance equals the latitude, and the departure is nothing; but if the bearing is east or west, the distance and departure are equal, and the latitude is zero. Furthermore, it will be seen that in all other cases except those just cited, the latitude, departure, and distance form the three sides of a right triangle: the distance being the hypotenuse, and the latitude and departure the sides about the right angle.

Let LP represent a line given by its bearing and distance; it is required to determine its latitude and departure.



Let OL and PM represent parallels of latitude, and LM and OP meridians. The lengths of $LM = OP$ and $LO = MP$ are required.

The problem stated simply is: Given in a right triangle LMP the hypotenuse LP (distance), the angle L (bearing), to find the side LM (latitude), and MP (departure).

From Trigonometry, $LM = LP \cos L$,

$$MP = LP \sin L.$$

So it is seen that the latitude of a line is obtained by taking the product of the distance and the *cosine* of the bearing, and the departure is equal to the product of the distance and *sine* of the bearing.

202. The case just treated is the principal one which the surveyor will use, since it is necessary — as will subsequently be seen — in computing areas, to determine the latitudes and departures; and by these formulas he will generally obtain them, having taken in the field the bearings, or angles, and distances.

Other cases, however, will occur in practice referring to the triangle LMP , and for convenience they are here subjoined.

Designating the length of the line, or distance, by s , the bearing by b , the latitude and departure respectively by l and d , then we may write the following formulas:

| CASE. | GIVEN. | REQUIRED. | FORMULAS. |
|-------|---------|-----------|--|
| 1 | $b, s.$ | $l, d.$ | $l = s \cos b, \quad d = s \sin b.$ |
| 2 | $b, l.$ | $s, d.$ | $s = \frac{l}{\cos b} = l \sec b, \quad d = l \tan b.$ |
| 3 | $b, d.$ | $s, l.$ | $s = \frac{d}{\sin b}, \quad l = \frac{d}{\tan b} = d \cot b.$ |
| 4 | $s, l.$ | $b, d.$ | $\cos b = \frac{l}{s}, \quad d = \sqrt{s^2 - l^2}.$ |
| 5 | $s, d.$ | $b, l.$ | $\sin b = \frac{d}{s}, \quad l = \sqrt{s^2 - d^2}.$ |
| 6 | $l, d.$ | $b, s.$ | $\tan b = \frac{d}{l}, \quad s = \sqrt{l^2 + d^2}.$ |

EXAMPLES.

1. Given the bearing and distance of a line, N. $23^\circ 54'$ W. 18.25 chains; required its latitude and departure.

2. Given the bearing of a line N. $87^{\circ} 40'$ E., and the departure 2640 feet; find its distance and latitude.

3. Given the length of a line 24.60 chains, and the departure 17.40; find its bearing and latitude.

4. Given the latitude 23.76 chains south, and the departure 0.94 chains west; required the bearing and distance.

5. Given the distance 1886 feet, and the latitude 943; determine its bearing and departure.

6. It is required to find the distance and departure of a line, given the bearing S. $30'$ W., and latitude 10.80 chains.

203. The Traverse Table. By the use of Formula 1, last article, latitudes and departures have been calculated for every quarter-degree of the quadrant, corresponding to distances from 1 to 10, and even from 1 to 100; these results tabulated constitute the *traverse table*. Such a table was considered quite indispensable when the compass was the principal surveying instrument, but since the more accurate transit has to a great extent superceded the compass, and surveyors are now reading to *minutes* instead of *quarter-degrees*, the common traverse table reading only to quarter-degrees is of little practical value.

When, therefore, the bearings are read to minutes, the latitudes and departures are generally best obtained from a table of natural sines and cosines.*

However, for the benefit of those engaged in compass surveying, and for those who, though reading to minutes, prefer to obtain by interpolation the latitudes and departures from the traverse table, one is given near the end of this volume.

* A traverse table in which the calculations are made to every *minute* of bearing for distances from 1 to 10 and extending to five decimal places, would answer the purpose admirably. Such a table is in existence, but it is not common. The common tables of natural sines and cosines are simply tables of latitudes and departures corresponding to a unit's distance. With a distance 2, the latitude and departure are twice those in the table; when the distance is 3, three times; when n , n times.

Explanation of the Traverse Table. The number of degrees in the bearing if it does not exceed 45 is found in the left-hand column of the page, and the latitudes and departures, as indicated at the top, may be taken under the proper distance; if the number of degrees is greater than 45, it is found in the right-hand column of the page, and the columns of latitudes and departures are indicated at the bottom. For example:

1. Let it be required to find the latitude and departure corresponding to a bearing N. $34^{\circ} 30'$ E. and distance 5 chains.

We find in the table, opposite $34^{\circ} 30'$ and under "distance 5," in the column headed "Lat.," 4.121, and in the column headed "Dep.," 2.832. Hence the latitude and departure are respectively 4.12 N. and 2.83 E.

2. Required the latitude and departure of a line bearing N. $72\frac{1}{4}^{\circ}$ W. 9 chains.

Looking in the column at the right of the page for $72^{\circ} 15'$, and under "distance 9," we find, reading at bottom,

in the Lat. column, 2.744;

in the Dep. column, 8.572.

Hence the latitude is 2.74 chains N., and the departure 8.57 chains W.

204. The table may be used to find the latitude and departure for any distance however great. If, in first example above, we suppose the bearing to remain the same, but the distance to be 50 chains; then, since for the same bearing the latitudes and departures vary directly as the distances, the latitude, or departure, for 50 chains is 10 times that for 5; and, as multiplying by 10 is in effect removing the decimal point one place to the right, we may take directly from the table opposite 5 the latitude and departure of 50, or 41.21 N. and 28.32 E.

If the distance is not a multiple of 10, but made up of units and tens, we may take out of the table the latitude and departure for the *units*, and for the *tens* as indicated above. The sum of these will evidently be the latitude and departure required.

3. Let it be required to find the latitude and departure of a line S. 40° E. 34 chains.

Looking in the table opposite 40° and under "distance 3," take out at once, by conceiving the decimal point removed one place to the right.

| | | | |
|------|-------------------|----------------------|----------------------|
| | For 30 chains, | Lat. 22.98 | Dep. 19.28 |
| Then | " 4 " | " 3.06 | " 2.57 |
| | <u>34 chains,</u> | <u>Lat. 26.04 S.</u> | <u>Dep. 21.85 E.</u> |

By an extension of the above principle, the table may be used to obtain the latitude and departure when the distance is composed of chains and links.

4. Given the bearing of a line S. $28^\circ 45'$ W. 26.58 chains, to find its latitude and departure.

| | | | |
|--------------|----------------|------------------------|------------------------|
| For 20 | chains, | Lat. = 17.53 | Dep. = 9.62 |
| 6 | " | " = 5.26 | " = 2.89 |
| .5 | " | " = .44 | " = .24 |
| .08 | " | " = .07 | " = .04 |
| <u>26.58</u> | <u>chains,</u> | <u>Lat. = 23.30 S.</u> | <u>Dep. = 12.79 W.</u> |

5. Find by the traverse table the latitude and departure of a line bearing N. $41^\circ 45'$ E. 17.29 chains.

6. Given the bearing of a line S. $\frac{1}{2}^\circ$ W., distance 23.48 chains, to find its latitude and departure.

7. What are the latitude and departure of a line bearing S. $85^\circ 30'$ E. 135.42 chains?

8. If the bearing and distance are N. $89\frac{1}{4}^\circ$ W. 20.09 chains, what are the latitude and departure?

205. By means of interpolation the traverse table may be used to find the latitude and departure when the bearing is given to minutes. Thus, the bearing being N. $34^\circ 20'$ E. any given distance, take out the latitude and departure corre-

sponding to $34^{\circ} 15'$ and the given distance, and *add* * to that departure $\frac{5}{15}$, or $\frac{1}{3}$, of the difference between it and that corresponding to $34^{\circ} 30'$ and the given distance, for the departure required. Likewise obtain $\frac{5}{15}$ of the difference between the latitudes corresponding to $34^{\circ} 15'$ and $34^{\circ} 30'$ and the distance, and *subtract* * from the latitude first found for the latitude required.

For a bearing $34^{\circ} 23'$, the fractional part to be taken of the difference between $34^{\circ} 15'$ and $34^{\circ} 30'$ would be $\frac{8}{15}$; the numerator being the excess in minutes above the quarter, and the denominator 15.

206. In the absence of a traverse table calculated to minutes, the table of natural sines and cosines, as before stated, is the best to use when the bearings are given to minutes.

It is shown in Article 201 that the cosine of the bearing multiplied by the distance gives the latitude, and the product of the distance and sine of bearing gives the departure.

EXAMPLES.

1. The bearing and distance of a line are N. $37^{\circ} 43'$ W. 24.29 chains; required its latitude and departure.

Four places of decimals from the table will usually be sufficient.

The cosine of $37^{\circ} 43'$ true to four places = .7911.

The sine of $37^{\circ} 43'$ true to four places = .6118.

$$.7911 \times 24.29 = 19.21 \text{ N. Lat.}$$

$$.6118 \times 24.29 = 14.86 \text{ W. Dep.}$$

The following contracted form of multiplication, using five decimal places, gives practically the same result:

Cosine of bearing = .79105; sine of bearing = .61176.

* The departure increases with an increase of the bearing; the latitude diminishes.

| | | | | | |
|-----------|---|-------|---------|-----------------|-----------------|
| Distances | { | 20 | chains, | Lat. = 15.8210 | Dep. = 12.2352 |
| | | 4 | " | " = 3.1642 | " = 2.4470 |
| | | .2 | " | " = .1582 | " = .1224 |
| | | .09 | " | " = .0712 | " = .0551 |
| | | <hr/> | | <hr/> | |
| | | 24.29 | chains, | Lat. = 19.21 N. | Dep. = 14.86 W. |

2. Find the latitude and departure of a line bearing S. $62^{\circ} 17'$ E. 37.18 chains.

3. Required the latitude and departure of a line N. $88^{\circ} 57'$ W. 28.97 chains.

4. Required the latitude and departure of a line bearing S. $\frac{1}{2}^{\circ}$ E. 2640 feet.

5. Given the bearings and distances of two lines running from the same point P , as follows: PO , N. $38^{\circ} 37'$ E. 1760 feet, and PL , N. $71^{\circ} 54'$ E. 1320 feet; to find by means of latitudes and departures the distance OL .

6. Assuming PO bears N. $48^{\circ} 17'$ W. 27.42 chains, and PL S. $36^{\circ} 28'$ W. 19.24 chains, find, as in the last example, the distance OL between the extremities of the lines.

207. Testing a Survey. It is evident that when a surveyor has passed completely round a tract of land and returned to the place of beginning, he has gone in a northerly direction just as far as he has gone in a southerly direction, and as far easterly as westerly. Hence the sum of the north latitudes should equal the sum of the south latitudes, and the sum of the east departures equal the sum of the west departures.*

In practice, this degree of accuracy is seldom attained, for various causes incident to the manipulation of the instruments, their inherent defects, imperfect chaining, etc.

* If the survey is effected by traversing (Article 163), the reading at the last station should be 360° or 0° . If the interior angles are measured, their sum should equal twice as many right angles less four as the figure has sides. If a small error exists, it must be distributed evenly among the angles, unless on account of the difficulty of observing one or more of the angles, these should have a larger share of the error. See, also, Article 156.

On account of the varying conditions in different surveys, it is impracticable to state precisely how great an error should be allowed without a re-survey of the tract. A rule usually followed by compass surveyors is to allow an error of 1 link for every 5 chains, 1 : 500.

This is perhaps a fair average for ordinary farm surveying. If the ground is exceptionally clear, and quite level, an error of 1:1000 is not too great; if, on the other hand, the ground is uneven, rocky, and brushy, 1:300, or even 1:200, might be allowed. The error resulting from a transit survey of the same ground should be much less. For the average case given above, instead of 1:500 it should not be much less than 1:1200.

The above rules are cited simply as guides to the young surveyor to aid him in forming a standard for himself, based on his own experience.

208. Correcting Latitudes and Departures, or Balancing the Survey. (1) A survey is *balanced* when the northings equal the southings, and the eastings equal the westings. When these equalities do not exist, the *error* is distributed among the lines, proportioned to their lengths. This operation is called *correcting the latitudes and departures*. It is best illustrated by an example :

| STATIONS. | BEARINGS. | DISTS. | LATITUDES. | | DEPART- URES. | | CORREC- TIONS. | | CORRECTED LATITUDES. | | CORRECTED DEPART'S. | |
|-----------|---------------|--------|------------|-------|------------------|-------|-------------------|------|-------------------------|-------|------------------------|-------|
| | | | N. | S. | ■ | W. | Lat. | Dep. | N. | ■ | E. | W. |
| 1 | S. 20° 53' E. | 13.11 | ... | 12.25 | 4.07 | ... | 2 | 1 | ... | 12.27 | 4.08 | ... |
| 2 | N. 43° 10' E. | 13.62 | 9.08 | ... | 10.16 | ... | 2 | 1 | 9.06 | ... | 10.16 | ... |
| 3 | N. 43° 40' W. | 4.73 | 3.42 | ... | ... | 3.26 | 1 | .. | 3.41 | ... | ... | 3.26 |
| 4 | N. 45° 08' W. | 4.75 | 3.35 | ... | ... | 3.36 | 1 | 1 | 3.34 | ... | ... | 3.35 |
| 5 | S. 51° 30' W. | 2.53 | ... | 1.57 | ... | 1.96 | .. | .. | ... | 1.57 | ... | 1.96 |
| 6 | S. 72° 30' W. | 6.56 | ... | 1.96 | ... | 6.26 | 1 | 1 | ... | 1.97 | ... | 6.25 |
| | | 45.30 | 15.85 | 15.78 | 14.82 | 14.96 | 7 | 4 | 15.81 | 15.61 | 14.84 | 14.84 |
| | | | 15.79 | | | 14.82 | | | | | | |

Error in latitude, 7 Hnks. 4 Hnks, Error in departure.

In the table the latitudes and departures corresponding to the several bearings and distances are obtained by means of a table of sines and cosines, and placed in their proper columns.

The first course being between the south and east, the latitude found is written in the column headed *S.*, the departure in column *E.*, and so on, the letters of the course indicating the columns in which to place the latitudes and departures. The difference of the sums in the latitude columns is then taken, and found to be 7 links: this is the error in latitude.

The error in departure, found in a corresponding manner, is 4 links.

The total distance round the field is shown by the footing of the distance column to be 45.30 chains. The distribution of the error is effected then by the proportions:

For the Latitude.

Sum of the sides : length of any side = error : correction for that side.

$$45.30 : 13.11 = 7 : 2$$

$$45.30 : 13.62 = 7 : 2$$

For the Departure.

$$45.30 : 13.11 = 4 : 1$$

It is unnecessary usually to make but one proportion each for the latitude and departure correction, since the error for any other side may be found mentally by comparing its length with that of the side used in the proportion. Whole links only are used. The latitude correction for the second side is a little greater than 2, but it is nearer 2 than 3, and is therefore called 2.

The corrections thus found are written in their proper columns, headed "Correction, Lat. Dep.," opposite the sides to which they refer, and are so applied by addition or subtraction as may be required to reduce the errors to zero. The quantities thus obtained are placed in the columns of corrected latitudes and departures to the right of the corrections.

Since the southings are too small, the correction 2 is added to 12.25, making 12.27, for the first entry in the column of corrected latitudes. The eastings being too small, the correction 1 is added to 4.67, making 4.68, to be written under *E.* in the corrected departures; and so on for the rest.

If the corrections have been properly applied, the northings will equal the southings, and the eastings the westings, and the survey is *balanced*.

In the example just given, the difference of latitude is 7 and the departure 4 links; hence, the length of a line to *close* the survey $= \sqrt{7^2 + 4^2}$ = about 8 links; and as the perimeter of the tract = 45.30 chains, the "error of the survey," or "error of closure," = 1 link for 5.66 chains, or 1 : 566.

Some surveyors prefer a more compact table than that given above, and instead of a double set of latitudes and departures, use but one, and write in ink of different colors the *corrected* latitudes and departures over the first. Others, again, prefer *two* columns instead of *four* for the latitudes and departures, using the plus (+) sign to indicate north latitudes and east departures, and the minus (−) sign to indicate south latitudes and west departures.

The form given above is, however, preferable to either, since a mistake in the application of the corrections is in that more easily detected, the footings are more expeditiously and accurately obtained, and the subsequent part of the work referring to the area is thereby facilitated.

If a side of the survey passes over very rough ground, or through a dense wood, or for any reason it is rendered more difficult to measure than any of the others, the surveyor should exercise his judgment in deciding how much more of the error than the rule would indicate should be applied to that side.

Regard must also be had to the probability of error in the bearings; hence, when a side of considerable length is aligned through a thicket, or over very uneven ground, and where oftentimes the observations are made to top of rod, if it is found that a slight change in the bearing will diminish materially the error, the change should be made.

The diurnal variation of the needle is not unfrequently a source of error in compass surveys. A range of 10 minutes is quite common, and even 15 minutes is occasionally noted. This error may be avoided by measuring the *angles* of the tract, or testing the compass every two or three hours by setting up and sighting on some line as standard.

Some authors and surveyors affirm that when the bearing of a line is due east or due west, the error in latitude is nothing, and therefore such a line needs no correction. Likewise a due north and south line has no error in departure. The writer does not concur in this view; for the errors in compass work are not confined to the chaining, and in transit surveying there is frequently considerable error in the angles. In the application of the rule these facts are assumed; indeed, as soon as a correction, made in the usual manner, is applied to any side, a change of bearing results, for the corrected latitudes and departures no longer belong to the original bearing, but to some other. Moreover, there is no more reason for supposing a line runs due *north* because it is so read than that a line runs $N. \frac{1}{4}^{\circ} E.$ or $N. 89\frac{3}{4}^{\circ} E.$ being so read; yet no surveyor would hesitate to apply the rule to either of these, thus assuming that an error in bearing as well as in chaining was committed; and this is the correct assumption on which, without excepting any side, the distribution of the error, except as follows, should be made.

(2) If, however, a survey is made with a transit in good adjustment, the angles, either interior or deflection, being carefully observed, and the test hereinbefore mentioned when applied giving the inference that the angles were accurately measured, and the error of closure therefore due to erroneous chaining, then the correction which should be applied is obtained as follows:

Add up the columns of latitudes, and also those of departures, and say, as the arithmetical sum of all the $\left\{ \begin{array}{l} \text{latitudes} \\ \text{departures} \end{array} \right\}$ *is to any particular* $\left\{ \begin{array}{l} \text{latitude} \\ \text{departure} \end{array} \right\}$ *, so is the error in* $\left\{ \begin{array}{l} \text{latitude} \\ \text{departure} \end{array} \right\}$ *to the correction to be applied to that* $\left\{ \begin{array}{l} \text{latitude} \\ \text{departure} \end{array} \right\}$.

(3) If greater accuracy is required than can be attained by the preceding methods, each side should be weighted; that is to say, the surveyor determines the relative difficulties in measurement and alignment of the boundaries, considering some one side the standard. Calling the error probably made in the side chosen as standard one (1), another side, which in the judgment of the surveyor was, per unit, twice as difficult to measure, would be multiplied by 2, or, as it is termed, have a weight of 2; another multiplied by 3, or $1\frac{1}{2}$, etc. Then, instead of taking the perimeter for the divisor, as was done in the first case above, the sum of the sides thus multiplied or weighted is used, and the proportion is as follows:

*As the sum of the multiplied distances is to any particular multiplied distance, so is the error in { latitude
departure } to the correction
to be applied to that { latitude
departure }.**

The following illustrates the method of balancing a survey when the sides are weighted:

| STATIONS. | BEARINGS. | DISTANCES. | WEIGHTS. | MULTIPLIED DISTANCES. | LATITUDES. | | DEPART- URES. | | CORREC- TIONS. | | CORRECTED DISTANCES. | | CORRECTED DEPART'S. | |
|---|------------|------------|----------|--------------------------|------------|-------|------------------|-------|-------------------|------|-------------------------|-------|------------------------|-------|
| | | | | | N. | S. | E. | W. | Lat. | Dep. | N. | S. | E. | W. |
| 1 | N. 9° W. | 15.50 | 1 | 15.50 | 15.81 | ... | ... | 2.43 | 1 | 2 | 15.82 | ... | ... | 2.41 |
| 2 | N. 31° E. | 25.40 | 3 | 76.20 | 21.77 | ... | 13.09 | ... | 6 | 9 | 21.83 | ... | 13.18 | ... |
| 3 | S. 71° E. | 10.00 | 3 | 30.00 | ... | 3.17 | 9.48 | ... | 3 | 4 | ... | 3.14 | 9.52 | ... |
| 4 | S. 10½° E. | 19.70 | 2 | 39.40 | ... | 19.37 | 3.59 | ... | 3 | 5 | ... | 19.34 | 3.64 | ... |
| 5 | S. 10½° W. | 14.60 | 1½ | 21.90 | ... | 14.34 | ... | 2.72 | 2 | 2 | ... | 14.32 | ... | 2.70 |
| 6 | S. 89° W. | 21.25 | 1 | 21.25 | ... | 0.37 | ... | 21.25 | 2 | 2 | ... | 0.35 | ... | 21.23 |
| | | | | 204.00 | 37.08 | 37.25 | 26.16 | 26.40 | | | 37.15 | 37.15 | 26.34 | 26.34 |
| | | | | | | 37.08 | | 26.16 | | | | | | |
| Error in latitude, 17 links. 24 links, error in departure. | | | | | | | | | | | | | | |

* Weights could be applied to the correction of the chaining in the second case, by multiplying the latitudes and departures instead of the lengths of the sides.

EXAMPLES.

Correct the latitudes and departures in the following examples by the first method :

| 1. | 2. |
|--|-------------------------------|
| (1) S. $\frac{1}{4}^{\circ}$ E. 22.45 chains ; | (1) South 22.45 chains ; |
| (2) N. $89\frac{3}{4}^{\circ}$ E. 67.10 “ | (2) East 67.10 “ |
| (3) N. $\frac{1}{4}^{\circ}$ W. 23.85 “ | (3) North 23.85 “ |
| (4) S. $89\frac{3}{4}^{\circ}$ W. 66.30 “ | (4) West 66.30 “ |
| (5) S. $21\frac{1}{4}^{\circ}$ W. 1.30 “ | (5) S. 22° W. 1.30 “ |

EXERCISES.

A few surveys should now be made, and the methods above given employed in balancing.

SECTION VII.

SUPPLYING OMISSIONS.

209. When, for any cause, it is impracticable to obtain the direction or the length, or both, of a side of a tract of land, these may be obtained by calculation. Even the lengths or bearings of *two* sides may in general be supplied.*

The determination, however, of these sides or bearings is based upon the measurements of the other bounding lines and angles ; but as these are not usually *precisely correct*, and as there are no means of testing them in their application to the solution of problems under this head, it is earnestly recommended that *all measurements*, if possible, be made.

There are four cases.

* If the two omitted sides are parallel and equal, their bearings cannot be supplied ; or if they are parallel and of equal or unequal lengths, their distances cannot be computed.

CASE I.

210. *Given the bearings and distances of all the sides of a tract of land except the bearing and distance of one side, to determine these.*

Find the latitudes and departures of the given sides. The difference of the northings and southings will show the latitude of the line omitted, and the difference of the eastings and westings its departure. Then

$$\begin{aligned} \text{Length of line} &= \sqrt{\text{lat.}^2 + \text{dep.}^2} \\ \text{Tan angle of bearing of line} &= \frac{\text{dep.}}{\text{lat.}} \end{aligned}$$

The cardinal points between which the line runs are indicated by the deficiency in the latitude and departure columns.

EXAMPLES.

1. Given (1) N. $24\frac{1}{2}^\circ$ E. 23.75 chains ;
 (2) S. $81\frac{1}{4}^\circ$ E. 11.70 "
 (3) S. 1° E. 12.64 "
 (4) S. $11\frac{1}{2}^\circ$ W. 14.50 "

To find the length and bearing of a line connecting the extremity of the fourth side with the first corner.

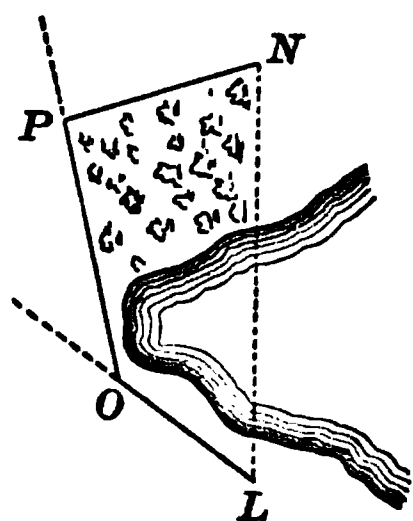
| STA- TIONS. | BEARINGS. | DISTS. | N. | S. | E. | W. |
|------------------|-----------------------------|--------|-------|-------|-------|------|
| 1 | N. $24\frac{1}{2}^\circ$ E. | 23.75 | 21.61 | ... | 9.85 | ... |
| 2 | S. $81\frac{1}{4}^\circ$ E. | 11.70 | ... | 1.78 | 11.56 | ... |
| 3 | S. 1° E. | 12.64 | ... | 12.64 | .22 | ... |
| 4 | S. $11\frac{1}{2}^\circ$ W. | 14.50 | ... | 14.21 | ... | 2.89 |
| | | | 21.61 | 28.63 | 21.63 | 2.89 |
| | | | | 21.61 | 2.89 | |
| 7.02 N. 18.74 W. | | | | | | |

Length of line = $\sqrt{(7.02)^2 + (18.74)^2}$ = 20.01 chains.

$$\begin{aligned}\text{Tan bearing} &= \frac{18.74}{7.02} \\ \text{Bearing} &= \text{N. } 69^{\circ} 28' \text{ W.}\end{aligned}$$

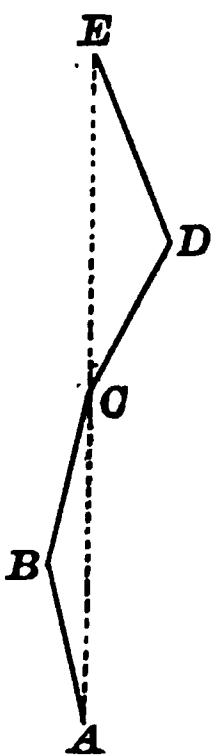
2. Given the bearings and distances of the sides of a tract of land as follows ; it is required to find the length and bearing of the fourth side.*

- (1) N. $11\frac{3}{4}^{\circ}$ E. 12.69 chains ;
- (2) S. $87\frac{3}{4}^{\circ}$ W. 8.50 “
- (3) N. $85\frac{1}{2}^{\circ}$ W. 11.70 “
- (5) S. $82\frac{1}{2}^{\circ}$ E. 10.53 “



The foregoing case may be employed to overcome an obstacle in a line, as LN . Thus, surveying $LOPN$, there will be given all the sides except LN , which can be determined as above. If it is desired to straighten an old road, the length and direction of the new road may be computed from the distances and deflections, or bearings of the old.

For example, let $ABCDE$ be a crooked road which it is desired to replace by a straight one, AE . The bearings and distances being as follows, the length and bearing of AE are required.



- AB , N. 12.70 chains ;
- BC , N. 20° E. 13.25 “
- CD , N. 35° E. 12.75 “
- DE , N. 10° W. 16.90 “

Ans. N. $9^{\circ} 41'$ E. 52.98 chains.

EXAMPLE 2. Given the following as the bearings and distances of a road, it is desired to straighten, to find the length and bearing of the new road.

* In practice, the result should be checked by making a plot of the field.

- (1) N. 12° W. 13.10 chains;
- (2) N. 8° E. 16.20 “
- (3) N. 24° W. 14.40 “
- (4) N. $40\frac{1}{2}^\circ$ E. 15.08 “
- (5) N. $60\frac{1}{2}^\circ$ W. 16.12 “

EXAMPLE 3. In last figure but one, suppose LO bears N. $44^\circ 20'$ W., distance 3.95 chains. Deflection at O from OL 30° , and $OP = 6.90$ chains. Deflection at P from OL 100° , and $PN = 5.40$ chains. It is required to find the length and bearing of NL . *Ans.* Bearing south. Length, 12.55 chains.

CASE II.

211. *Given the bearings and distances of all the sides of a tract of land, except the distances of two sides not parallel, to determine these.*

By Article 168, change all the bearings so that one of the sides, whose direction only is known, shall become a meridian. Tabulate the latitudes and departures corresponding to the changed position of the sides. The side made meridian will have no departure, and the difference of the eastings and westings, therefore, will be the departure of the other unknown side. Now with this departure and the changed bearing the distance and difference of latitude of this side may be found, and should be inserted in their proper places in the table. Then the difference between the northings and southings will be the latitude, or length of the side made a meridian.*

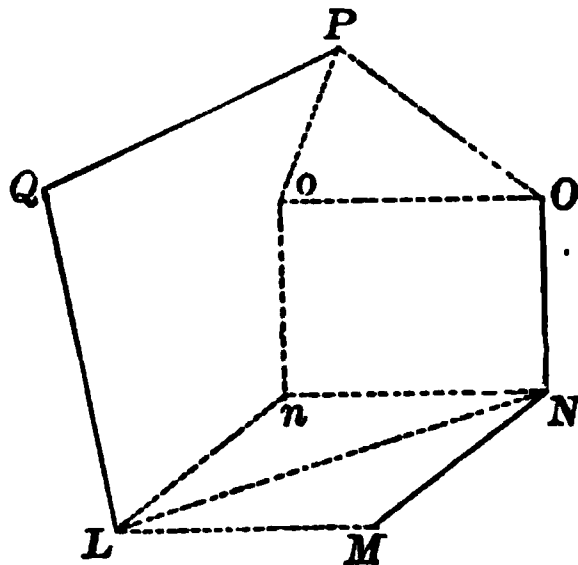
212. *Otherwise. If the deficient sides adjoin.*

If a line† be drawn connecting L and N , a figure, $LNOPQ$, will be shown, in which all the sides are given except LN : the bearing and distance of this side may, therefore, be calculated by the preceding case. This line and the two sides, LM

* It is immaterial whether or not the deficient sides adjoin.

† Called a *closing line* since it closes the survey $LQPON$.

and MN , whose bearings only are given, will form a triangle, in which will be known one side and all the angles, whence the unknown distances, LM and MN , may be computed.



213. *If the sides do not adjoin.*

In the figure suppose that the distances LM and PO are wanting. Draw Ln and no parallel and equal respectively to MN and NO . Then by joining oP , a closed figure will be formed, all the bearings and distances of which are known except the bearing and distance of the closing line, Po , and these may be found by Case I. Po thus determined, there will be known in the triangle PoO all the angles and one side, to find PO , and Oo , which is equal to LM .

EXAMPLES.

1. Given the following bearings and distances of the sides of a tract of land, to find the length of the 3d and 6th sides. (See last figure.)

- (1) N. $6\frac{1}{2}^\circ$ W. 9.38 chains ;
- (2) N. $65\frac{1}{2}^\circ$ E. 8.25 “
- (3) S. 39° E. Unknown ;
- (4) S. 2° W. 4.45 chains ;
- (5) S. 46° W. 5.00 “
- (6) N. 88° W. Unknown.

| STA-TIONS. | BEARINGS. | DISTS. | N. | S. | E. | W. |
|------------|------------|--------|-------|------|------|------|
| 1 | N. 6½° W. | 9.38 | 9.32 | ... | ... | 1.06 |
| 2 | N. 65½° E. | 8.25 | 3.42 | ... | 7.51 | ... |
| 3 | S. 39° E. | ... | ... | ... | ... | ... |
| 4 | S. 2° W. | 4.45 | ... | 4.45 | ... | 0.16 |
| 5 | S. 46° W. | 5.00 | ... | 3.47 | ... | 3.60 |
| | | | 12.74 | 7.92 | 7.51 | 4.82 |
| | | | 7.92 | | 4.82 | |
| | | | 4.82 | | 2.69 | |

Tan bearing = $\frac{2.69}{4.82}$, or *Po* bears S. 29° 10' W.

Length of *Po* = $\sqrt{(4.82)^2 + (2.69)^2} = 5.52$.

Angle *P* therefore = 68° 10'.

Angle *O* " = 49°.

Angle *o* " = 62° 50'.

sin. 49° Ar. co. = 0.122220
: sin. 62° 50' = 9.949235
:: 5.52 = 0.741939
: *PO* (3d side) = 6.51 = 0.813394

sin. 49° Ar. co. = 0.122220
: sin. 68° 10' = 9.967674
:: 5.52 = 0.741939
: *Oo* = *LM* (6th side) = 6.79 = 0.831833

EXAMPLE 2. Given the following data to supply the omissions.

- (1) N. 8½° E. 9.80 chains ;
- (2) N. 31½° E. Unknown ;
- (3) S. 70° E. "
- (4) S. 5½° W. 17.70 chains ;
- (5) N. 87° W. 18.75 chains, to the beginning.

EXAMPLE 3. In the last example insert the distances found, and suppose the first and fourth sides are wanting; determine these by either or both methods.

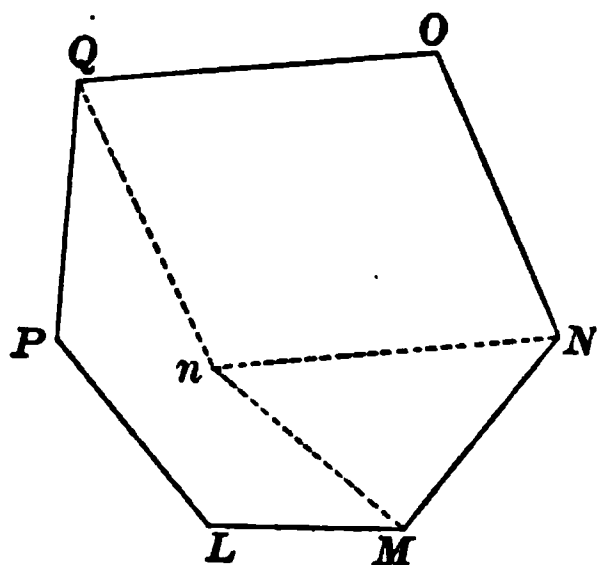
CASE III.

214. *Given the bearings and distances of all the sides of a tract of land, except the bearings of two sides, to determine these.*

Tabulate the latitudes and departures of the sides completely given; obtain the difference of the northings and southings, and of the eastings and westings. These differences will be the latitude and departure of a *closing line*.

The bearing and distance of the closing line may hence be computed; then in the triangle formed by this line and the two sides whose distances are given, determine the angles; and thence, with a proper application of them to the bearing of the closing line, the wanting bearings may be found.

In the figure let $PQONML$ represent a tract of land in which



all the bearings and distances are known except the bearings of QO and NM .

Drawing nN parallel and equal to QO , and joining Qn and nM , a closed figure, $PQnMLP$, will be formed, in which the bearing and distance of nM , the closing line, may be calculated by Case I. Then in the triangle MnN , having all the sides, the angles are readily found, and by proper application of these with the bearing of Mn the bearings of NM , and $nN = QO$ may be obtained.

NOTE. — If the sides whose bearings are required adjoin, the reasoning is evident. If they do not adjoin, a transposition of some of the sides may be made, as in the preceding case, without changing the direction or length of any of them, making the unknown sides adjoin, and with the closing line form the triangle referred to in the last paragraph. The rule is, therefore, applicable to either.

EXAMPLES.

1. Given the following data of a survey, to supply the omissions. Referring to the last figure :

the bearing of *PQ*, N. 3° E. dist. 4.57 chains.
“ “ *QO*, “ 6.25 “
“ “ *ON*, S. 23½° E. “ 5.50 “
“ “ *NM*, “ 4.33 “
“ “ *ML*, N. 87° W. “ 2.97 “
“ “ *LP*, N. 43° W. “ 3.33 “

| STA-TIONS. | LINES. | BEARINGS. | DISTS. | N. | S. | E. | W. |
|---------------------|-----------|------------|--------|---------------------|------|------|------|
| <i>P</i> | <i>PQ</i> | N. 3° E. | 4.57 | 4.56 | ... | 0.24 | ... |
| <i>Q</i> | <i>QO</i> | | 6.25 | ... | ... | ... | ... |
| <i>O</i> | <i>ON</i> | S. 23½° E. | 5.50 | ... | 5.05 | 2.19 | ... |
| <i>N</i> | <i>NM</i> | | 4.33 | ... | ... | ... | ... |
| <i>M</i> | <i>ML</i> | N. 87° W. | 2.97 | 0.15 | ... | ... | 2.97 |
| <i>L</i> | <i>LP</i> | N. 43° W. | 3.33 | 2.43 | ... | ... | 2.28 |
| | | | | 7.14 | 5.05 | 2.43 | 5.25 |
| | | | | 5.05 | | | 2.43 |
| Deficiency, 2.09 S. | | | | Deficiency, 2.82 E. | | | |

Tan of bearing of *nM* = $\frac{2.82}{2.09}$, and bearing = S. 53° 28' E.

Dist. *nM* = $\sqrt{(2.09)^2 + (2.82)^2}$ = 3.51.

To find the angle of *nMN*:

log 4.33 Ar. co. = 9.363512
log 3.51 “ = 9.454693
log 7.045 = 0.847881
log .795 = 1.900367
2)19.566453
log cosine ½ *nMN* = 9.783226
and ½ < = 52° 37'
2
> *nMN* = 105° 14'

Now, since Mn bears N. $53^\circ 28'$ W., and the angle $nMN = 105^\circ 14'$, the line MN is in the northeast quadrant, and makes an angle with the meridian $= 105^\circ 14' - 53^\circ 28' = 51^\circ 46'$, or its bearing is N. $51^\circ 46'$ E.; and hence, reading in the order the measurements were made, the bearing of $NM = S. 51^\circ 46' W.$

To find the angle nNM , and thence the bearing of QO :

| | |
|--|----------------------------------|
| 6.25 | Ar. co. = 9.204120 |
| : 3.51 | = 0.545307 |
| :: $\sin 105^\circ 14'$ | = 9.984466 |
| : $\sin 32^\circ 48' (< nNM)$ | = 9.733893 |
| Bearing of NM | = S. $51^\circ 46'$ W. |
| $< nNM 32^\circ 48'$ on west side, add | <u>$32^\circ 48'$</u> |
| Bearing of $Nn = OQ$ | = S. $84^\circ 34'$ W., |
| or bearing of QO | = N. $84^\circ 34'$ E. |

2. Supply the omissions from the following data:

- (1) N. 34° W. 13.00 chains;
- (2) S. $41\frac{1}{2}^\circ$ W. 12.90 "
- (3) S. 50° E. 8.20 "
- (4) 2.56 "
- (5) 6.90 "
- (6) N. 26° E. 9.95 "

CASE IV.

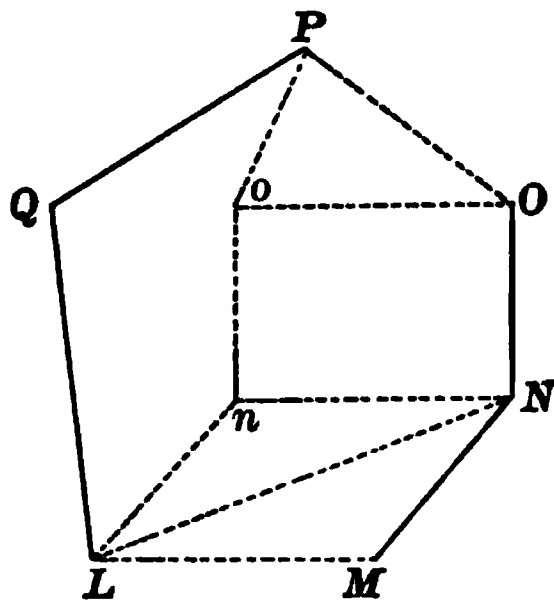
215. *Given the bearings and distances of all the sides of a tract of land except two, one of which has only its bearing given, and the other the distance, to supply the omissions.*

Make a meridian the side whose bearing only is given. Tabulate the latitudes and departures corresponding to the changed position of the survey. The side made meridian will have no departure, and the difference of the eastings and westings, therefore, will be the departure of the side whose bearing is unknown. With the given distance and this departure the

changed bearing and difference of latitude of this side may be found, and should be inserted in their proper places in the table. Then the difference of the northings and southings will be the latitude, or length, of the side made a meridian.

216. *Otherwise. When the deficient sides adjoin.*

Let the bearing of MN and the distance LM be wanting. Calculate by Case I. the direction and length of the closing line LN . A triangle, LMN , may then be formed in which will be given the lengths of LN and MN , and the angle NLM . The distance LM and the angle N may therefore be computed, and the angle N thus found properly applied to the bearing of the closing line will give the bearing of MN .



217. *When the deficient sides do not adjoin.*

Referring to the same figure as before, suppose the bearing of LM and the distance OP wanting. Transpose the sides as there shown, and calculate, as in Case I., the direction and length of the closing line PO . Then, as in the preceding article, there will be given a triangle, OPo , in which are known two sides PO and Oo , and the angle P , whence the bearing of Oo , or LM , and the distance PO , may be determined.

EXAMPLES.

1. Given the following notes, to supply the omissions.

| | | | |
|--------|--------------------------|-----|---------------|
| QP . | N. 10° | E. | 13.71 chains; |
| PO . | S. $88\frac{1}{2}^\circ$ | E. | 18.75 " |
| ON . | S. $16\frac{1}{2}^\circ$ | E. | 16.50 " |
| NM . | S. | W. | 13.00 " |
| ML . | N. 80° | W.; | |
| LQ . | N. 36° | W. | 10.00 chains. |

| STA-TIONS. | LINES. | BEARINGS. | DISTS. | N. | S. | E. | W. |
|---------------------|-----------|------------|--------|----------------|-------|-------|------|
| <i>Q</i> | <i>QP</i> | N. 10° E. | 13.71 | 13.50 | ... | 2.38 | ... |
| <i>P</i> | <i>PO</i> | S. 88½° E. | 18.75 | ... | 0.49 | 18.74 | ... |
| <i>O</i> | <i>ON</i> | S. 16½° E. | 16.50 | ... | 15.82 | 4.68 | ... |
| <i>N</i> | <i>NM</i> | | 13.00 | ... | ... | ... | ... |
| <i>M</i> | <i>ML</i> | N. 80° W. | ... | ... | ... | ... | ... |
| <i>L</i> | <i>LQ</i> | N. 36° W. | 10.00 | 8.09 | ... | ... | 5.88 |
| | | | | 21.59 | 16.31 | 25.80 | 5.88 |
| | | | | 16.31 | | 5.88 | |
| Deficiency, 5.28 S. | | | | Def., 19.92 W. | | | |

Tan of bearing of closing line = $\frac{19.92}{5.28}$,
or bearing of *NL*, S 75° 09' W.

Length of *NL* = $\sqrt{(5.28)^2 + (19.92)^2} = 20.61$,
and angle *MLN* = 24° 51'.

To find angle *LMN*:

13.00 (*NM*) Ar. co. = 8.886057
: sin. 24° 51' (<*L*) = 9.628502
:: 20.61 (*LN*) = 1.314078
: sin. 41° 47' = 9.823637

Angle *LMN* = 180° - 41° 47' = 138° 13' (see note).
Angle *LMN* = 180° - (138° 13' + 24° 51') = 16° 56'.

NOTE. — When the side *MN*, whose length only is given, is longer than the closing line *LN*, the angle *M* must be acute; if shorter, the angle *M* may be acute or obtuse, depending upon the length of the side *LM*, the bearing of which only is known. Hence, when this last relation obtains, it is necessary, in the application of this case, to remove ambiguity, that enough be known concerning the length of the side, whose bearing only is given, to indicate whether the angle *M* is greater or less than a right angle.

In the example, *LM* is known to be shorter than *NM*, and hence angle *M* is obtuse. The ambiguity is not removed by employing the method given in Article 215.

The bearing of *NM*, S. $75^{\circ} 09' W - 16^{\circ} 56' = S. 58^{\circ} 13' W$.

To find the length of *LM*:

$$\begin{array}{rcl} \sin. 24^{\circ} 51' & \text{Ar. co.} & = 0.376498 \\ : \sin. 16^{\circ} 56' & & = 9.464279 \\ :: 13.00 & & = 1.118943 \\ : 9.01 (LM) & & = 0.954720 \end{array}$$

The student may verify by the method in Article 215.

EXAMPLE. As an exercise, from any of the preceding problems strike out from two sides that do not adjoin the bearing of one and the distance of another, and compute them.

SECTION VIII.

PLOTTING A COMPASS OR TRANSIT SURVEY.

218. In addition to the drawing-instruments explained in chain surveying, the draughtsman will now find very convenient an instrument for measuring angles, or,

A Protractor. It is made of metal* or paper, usually in the form of a semi-circle, the arc of which is divided into 180 equal parts, or degrees, subdivided and numbered both ways.

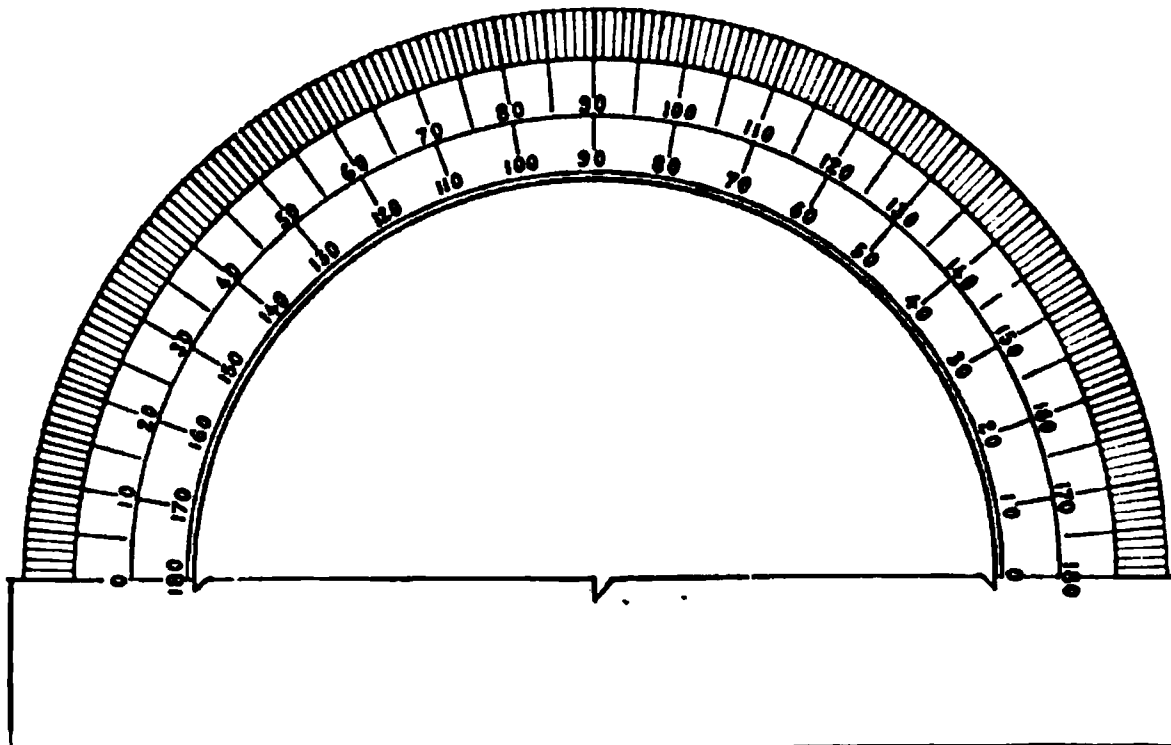
To draw a line making a given angle with another at a certain point. Bring the diameter of the protractor to coincide with the given line, its centre with the point, and the arch lying in the direction of the desired line; then with a sharp pencil or fine needle prick off the required number of degrees; joining the point thus fixed and the given point completes the problem.

Some plain scales are graduated to degrees on three edges so

* For more accurate work there is attached a movable arm or ruler, extending beyond the circumference and carrying a vernier.

12-inch protractors,—complete circle,—made of heavy paper, on which are printed the divisions to quarter-degrees, are quite reliable.

as to be used like a protractor, but are objectionable on account of the obliquity of the divisions and their varying lengths.



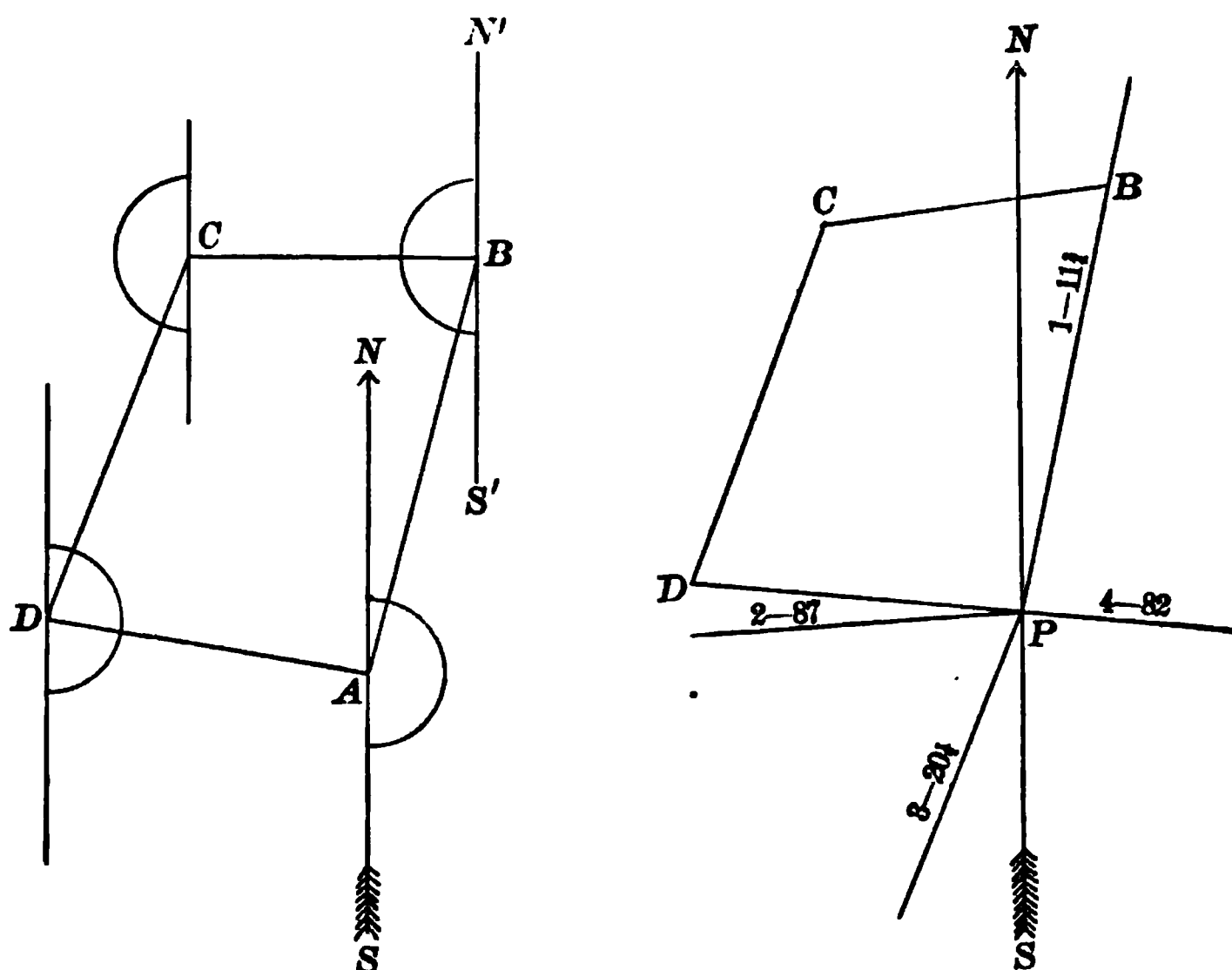
219. Illustration. To plot a survey the record of which is as follows :

- (1) N. $11\frac{3}{4}^{\circ}$ E. 13.19 chains ;
- (2) S. 87° W. 8.50 “
- (3) S. $20\frac{1}{2}^{\circ}$ W. 11.75 “
- (4) S. 82° E. 10.03 “

With a Protractor. First Method. Represent the meridian by drawing on the paper a line so situated that there will be sufficient room on either or both sides of it, as the case may be, to complete the drawing. Fix upon a point in this line to indicate a corner of the tract, usually “the place of beginning.” In this particular example the first corner is the easterly boundary, and as it runs northerly, we will draw our meridian near the lower right-hand side of the paper, as at *A*. Prick off the angle $11\frac{3}{4}^{\circ}$ from the north end of the protractor-arch to the right, and draw the line 13.19 chains (*AB*) to any convenient scale, say 2 chains to an inch, or 6.6 inches. Pass another meridian *N'S'* through *B*; and since the bearing is south-westerly, we prick off the degrees, 87 from the south point, and draw the line 8.50 (*BC*) to the same scale. In a similar man-

ner draw the line CD , and finally DA , which should end at A . If it does not end precisely at A , an error in plotting, or inaccuracy in the survey, would thereby be indicated.

An error in plotting a line by this method would affect the *position*, but not the *direction* of the following lines.



220. Another Method. By laying off the angles from one point, or from one position of a protractor having a complete circle. With the protractor at any convenient point, P , in the meridian NS , prick off the degrees shown by the bearings, and indicate each, and the side to which it belongs, as per figure. Then, by instruments used for drawing parallel lines, transfer them to their proper places, and make the lengths correspond to the scale adopted. The point P , from which all the angles were set off, may or may not be one of the corners of the field. The figure shows that it saves one transfer if so taken.

EXAMPLES.

1. Plot a triangle, given two sides and the included angle.

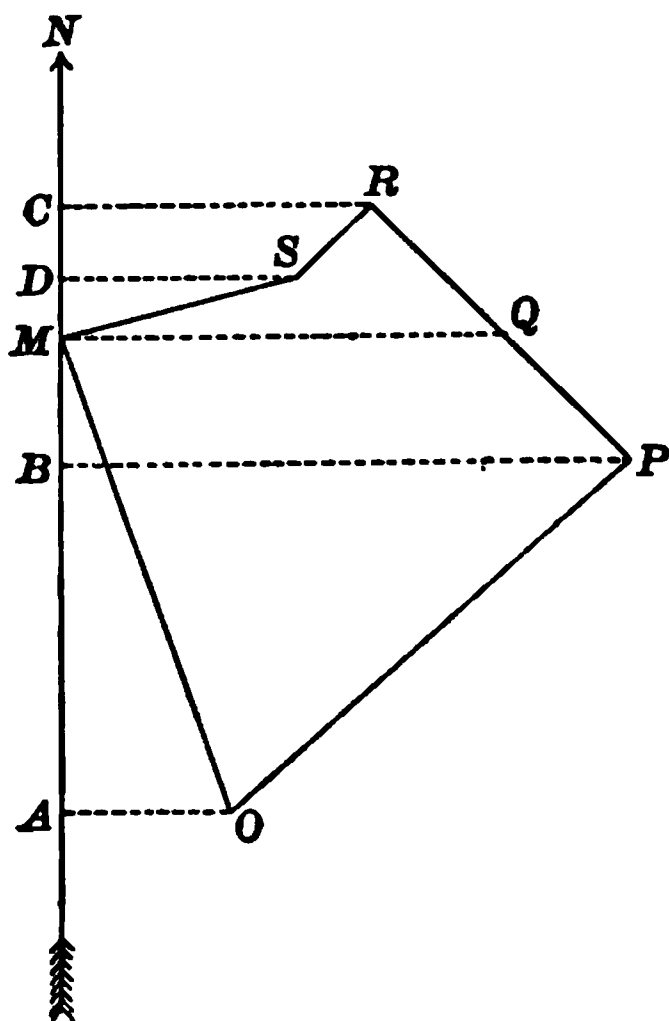
2. Given two angles and the included side, to plot the triangle.

3. Given three sides and two included angles, to plot a trapezium.

QUERY. Can a trapezium be plotted when there are given all the sides and one angle?

221. By Latitudes and Departures. The survey being *balanced*, this is the most accurate method, and is equally applicable to a compass or transit survey.

Taking the record of the survey in Article 208, and, using the corrected latitudes and departures, let us make a plot of it.



Draw through the first station* (*M*) a meridian, and find, by algebraic additions, from the columns of corrected latitudes and departures, the distance each corner is north or south from this station, called total latitude, and east or west from the meridian called total departure. These distances may be ascertained mentally as we proceed with the drawing, but to avoid error it is best to tabulate them, using three columns, as follows: + indicates distance north or east, and —, south or west, from the references.

* Any station will answer, but the one through which the meridian is supposed to pass in calculating the area is preferable.

| STATIONS. | TOTAL LATITUDES FROM STATION <i>M</i> . | TOTAL DEPARTURES FROM MERIDIAN <i>NS</i> . |
|-----------|---|--|
| 1 | 0 | 0 |
| 2 | -12.27 | + 4.68 |
| 3 | - 3.21 | +14.84 |
| 4 | + 0.20 | +11.58 |
| 5 | + 3.54 | + 8.23 |
| 6 | + 1.97 | + 6.25 |
| 1 | 0 | 0 |

The total latitude of the last station is the latitude of the last line with its sign changed. The same is true regarding the total departure of last station. A check is thus had on the work.

From *M* lay off on the meridian negatively, or to the south, 12.27 chains according to the scale adopted, to *A*; from *A* set off perpendicularly to the east, with the same scale, 4.68 chains, to *O*; connect *M* and *O*, showing the first line. Set off from *M*, again to the south, 3.21 chains, to *B*; thence perpendicularly to the right, or east, 14.68 chains, to *P*.

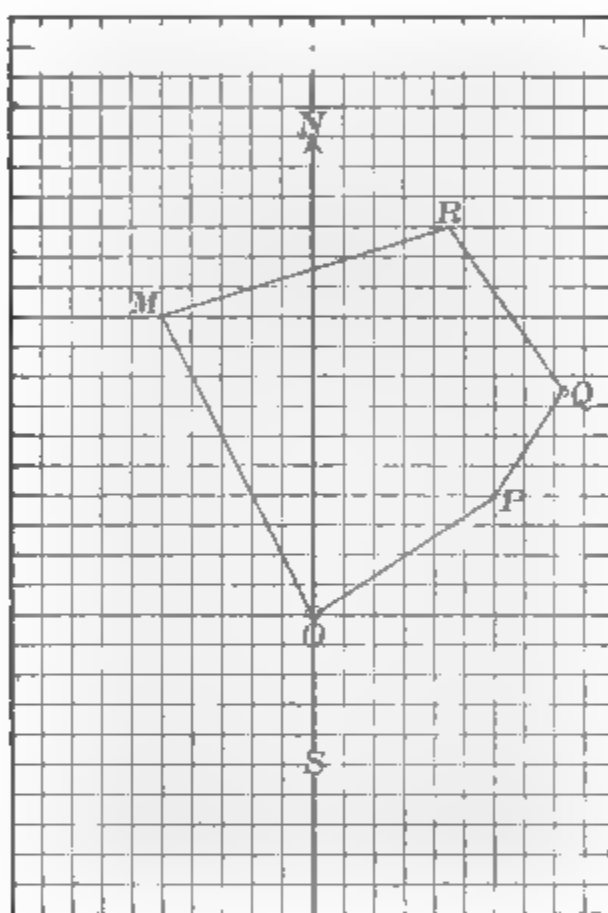
OP represents the second line of the survey. Next lay off 20 links to the north from *M*, thence 11.58 chains to the east, and join *PQ* for the third line, and so on, the last line, *SM*, requiring a distance laid off on the meridian north = 1.97; and a perpendicular thereto, = 6.25 east, when drawn closes the survey, thus proving the correctness of the work.

A variation of the method just given is to draw two lines, one representing the meridian, the other an east and west line. On the first lay off, as before, the latitudes of the sides, and on the second the corresponding departures; then, by means of dividers, obtain the intersection of co-ordinates, and joining these points shows the plot.

For plots of ordinary farm surveys the method given above, being equally accurate and more expeditious, is recommended;

for plots of extraordinary size, extending over a large drawing-board or made on a large table, the *variation* noted should be adopted.

222. Using Cross-Section Paper* and the latitudes and departures, a tolerably accurate plot may be made with great facility. The vertical and horizontal lines of the paper may



represent respectively meridians, and east and west lines. Assume any convenient point *O* as the beginning of the survey, and suppose the latitude of the first line = 4.00 chains N., the departure = 6.00 chains E. Count from *O* northward four spaces, thence eastward six spaces, to *P*; join *OP*, thus delineating the first side. Suppose the latitude and departure of the

* Note-books may be procured having the alternate pages ruled in small squares, like cross-section paper.

second side = respectively 3.50 chains N. and 2.25 chains E. ; count off, as before (estimating the fractions of chain), three and a half spaces north and two and a quarter east ; connect the points *P* and *Q* for the second side, and so on to the place of beginning.

SECTION IX.

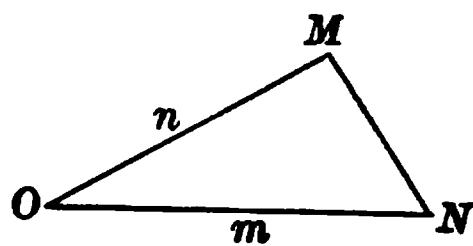
ON DETERMINING AREAS.

A. PARTICULAR FORMS AND CASES.

TRIANGLES.*

223. First Method. Measure two sides, as *m* and *n*, and the included angle *O*. Then the

$$\text{area} = A = \frac{m \times n \sin O}{2}.$$



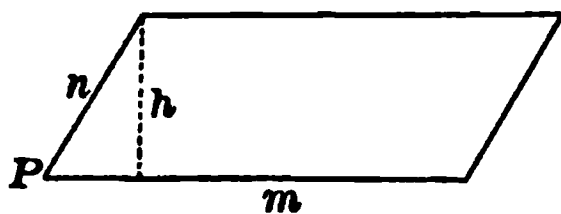
224. Second Method. Measure two angles, as *O* and *N*, and the included side *m*. Then

$$A = \frac{m^2 \sin N \sin O}{2 \sin(N + O)}.$$

PARALLELOGRAMS.

225. Measure two adjacent sides, *m* and *n*, and their included angle, *P*. Then *h* denoting the altitude,

$$A = mh = m \times n \sin P.$$



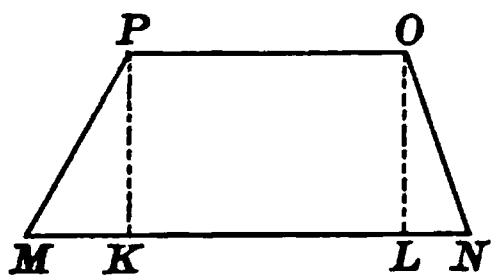
* For other methods than those found in this section of surveying triangles, quadrilaterals, and other polygons, see Chain Surveying, Articles 63 to 70.

EXAMPLES.

1. Two sides of a triangle measure 756 feet and 1024 feet, and their included angle $42^{\circ} 45'$; determine the area in acres.
2. Two angles of a triangle are $59^{\circ} 29'$ and $65^{\circ} 18'$, and their included side 932 feet. How many acres does it contain? Plot.
3. Two sides of a triangle measure 15.24 chains and 13.18 chains, and the angle opposite the first $54^{\circ} 25'$. Find the area.
4. Two adjacent sides of a parallelogram are 856 feet and 1252 feet, and their included angle $75^{\circ} 48'$. Compute the area.

TRAPEZOIDS.

- 226.** Measure three sides, say PM , MN , and NO , and one of the included angles, as N . From the data thus obtained compute the altitude, $OL = PK$, and the parallel side, PO . Then



$$A = \frac{MN + PO}{2} \times PK.$$

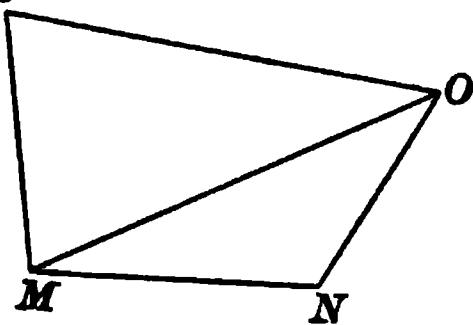
Or, instead of measuring the inclined sides, if it is equally convenient measure the parallel sides, and one of the other sides and an angle as before; then

$$A = \frac{MN + PO}{2} \times NO \sin N.$$

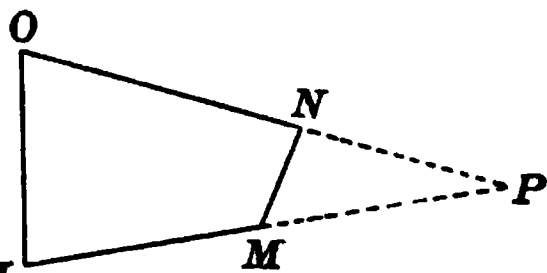
TRAPEZIUMS.

- 227.** Measure all the sides and one angle. With the data calculate the length of a diagonal dividing the tract into two triangles, in one of which two sides and the included angle will be given, and in the other three sides, whence the area may be found.

228. Or, measure three sides, PM , MN , and ON , and the included angles N and PMN . Draw PO , calculate the area of the triangle MNO , the diagonal MO , and the angle OMN . Subtract OMN from PMN ; then, having two sides and the included angle in the triangle PMO , its area may be computed, which added to the area of MNO gives the required content.



229. Otherwise. Measure two opposite sides, as OL and MN , and three angles, as O , L , and M . Conceive the sides ON and LM to be prolonged to meet in some point, P . From the data calculate the areas of the triangles POL and PMN . The difference will give the area sought.*



EXAMPLES.

1. Given in a trapezoid (see figure, Article 226) $PM=33$ rods, $MN=68$ rods, $NO=30$ rods, and the angle $N=70^\circ$; to find the area, and make a plot.

2. Given in a trapezium $PMNO$ (see figure, Article 228) $PM=7$ chains, $MN=7.50$ chains, $NO=6$ chains, the angle $N=120^\circ$, and $M=108^\circ$; to find the area, and make a plot.

3. Given in a trapezium $LMNO$ (see last figure) $LO=8$ chains, $MN=5$ chains, and the angles L , M , and N respectively 87° , 70° , and 80° ; to find the area, and make a plot.

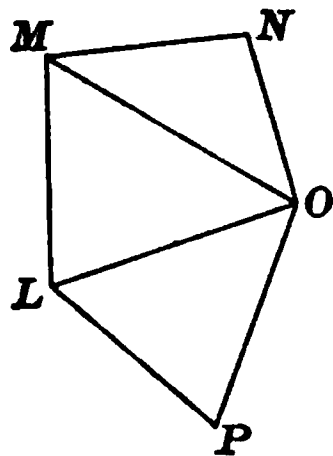
4. Given in a trapezium the angle M a right angle, the sides MN , NO , OP , and PM respectively 20, 12, 30, and 15 rods; also a perpendicular to MN from N extending to $PO=10$ rods; to find the area.

QUERY. Could the area be found without NO ?

* If practicable, observe all the angles, and thereby obtain a check on the measurements.

POLYGONS.

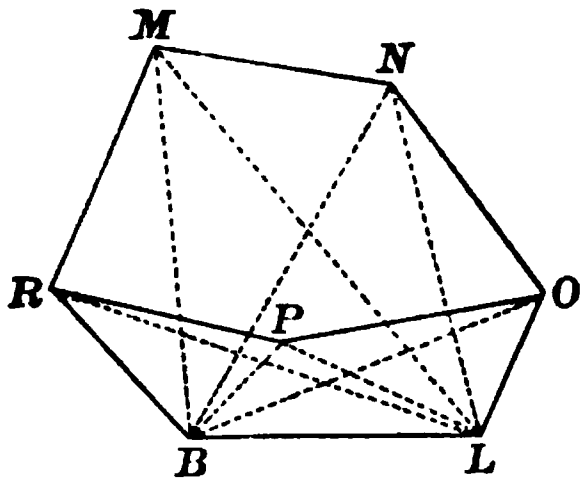
230. To find the area of an irregular pentagonal field $LMNOP$, when all the corners can be seen from one corner, as O . Measure the sides ON , OP , the diagonals OL , OM , and the three angles at O . Then two sides and the included angle of each triangle thus formed will be given, whence their areas may be calculated and, by addition, the area of the required polygon may be obtained. In like manner, a survey of any small irregular polygonal lot, in which all the corners are visible from one corner, may be effected. If there are n sides, measure from one corner two sides and $n - 3$ diagonals, observing from the same point the $n - 2$ angles which are formed by these diagonals and the two sides. Then, as above, the tract will be divided into $n - 2$ triangles, the area of each may be calculated, and the sum of these areas taken for the area of the polygon.



231. Or, measure from some point within or without the field radial lines to all the corners, and observe at the same point the angle which these lines make with each other.

There will thus be given two sides and the included angle of a series of triangles, whence the bounding lines and area may be computed.

232. Otherwise. Measure a *base line* within or without the tract, or use a portion or all of one side as a base line, and observe from each extremity of this line the angles formed by it and a visual line through each corner of the tract. There will thus be known two angles and the included side of a series of triangles, whence the bounding lines and area may be calculated.



The marginal figure represents the

case where the base line BL is taken outside the tract. It will be noticed that it is possible by this method to survey a farm without entering upon it.

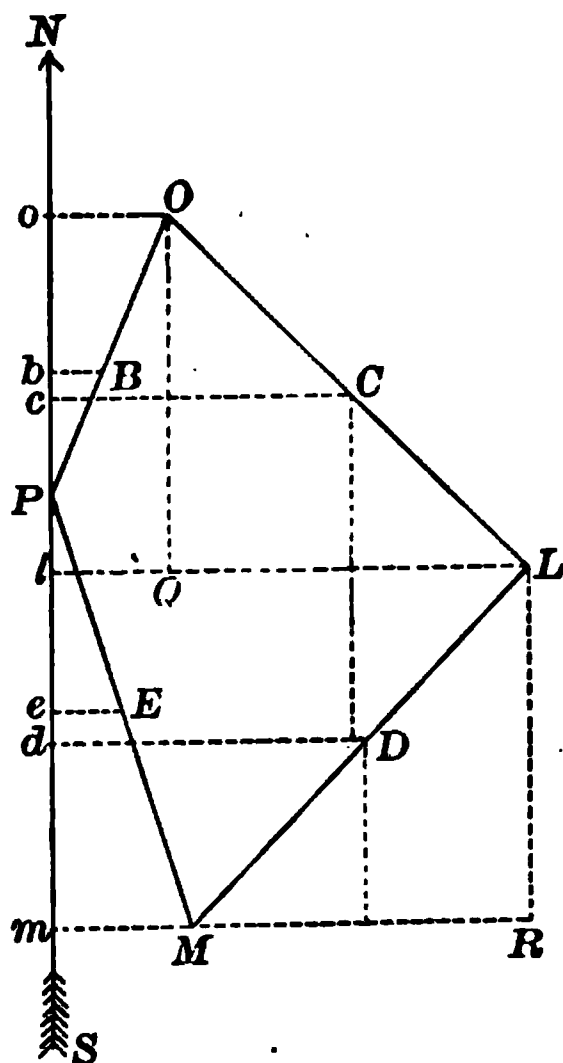
B. GENERAL METHOD.

233. The methods given in the last three articles are, however, quite limited in their application, since it rarely happens in a tract of considerable magnitude that all the corners are visible from any one corner, or from any point within or without the field.

The following method of determining the area by means of latitudes and departures is applicable to all right-lined figures, and is the most general and accurate.

Let $POLM$ represent a tract of land, the area of which is desired. Measure all the sides and angles, interior or deflection, with a single bearing, if the transit is used, or take all the bearings with a compass. Distribute the angular error, if any made by transit (see note, Article 207). Obtain the latitudes and departures, and balance the survey.

Let NS represent a meridian passing through P , the most westerly station of the tract, and Bb , Cc , and Dd meridian distances. Now, if perpendiculars be dropped from the angles O , L , and M to the meridian, it will readily appear that the area of $POLMP$ = area $oOLMmo$ minus the sum of the areas of the triangles PoO and PmM , or $POLMP$ = trapezoids ($OoLl + LlmM$) - triangles ($PoO + PmM$)



$$= Cc \times OQ + Dd \times LR - Bb \times Po - Ee \times Pm.$$

The computation, then, involves the latitudes and departures, and meridian distances; the former having been already explained, we shall now indicate how the latter may be obtained, or rather how the double meridian distances are found, since in order to lessen fractions the double lengths are used.

The double meridian distance, or D.M.D., of the side

$$PO = 2 Bb = Oo, \text{ its departure.}$$

$$\begin{aligned} \text{The D.M.D. of } OL &= 2 Cc = Oo + Ll = Oo + Ql + QL \\ &= 2 Bb + Oo + QL. \end{aligned}$$

$$\text{The D.M.D. of } LM = 2 Dd = 2 Cc + QL - MR.$$

$$\begin{aligned} \text{The D.M.D. of } MP &= 2 Ee = 2 Dd - MR - Mm \\ &= Mm = \text{its departure.} \end{aligned}$$

It is evident that, in a corresponding manner, the double meridian distances of the bounding lines of a tract may be found, no matter what the number of sides or magnitude of the angles. Hence, considering east departures plus (+), and west departures minus (−), the above deductions may be expressed in

A GENERAL RULE FOR OBTAINING DOUBLE MERIDIAN DISTANCES.

The double meridian distance of the first side is equal to its departure.

The double meridian distance of the second side is equal to the double meridian distance of the first side, plus its departure, plus the departure of the second side.

The double meridian distance of any side is equal to the double meridian distance of the preceding side, plus its departure, plus the departure of the side itself.

The double meridian distance of the last side deduced by the

foregoing rule should equal its departure, and will serve as a check on this part of the work.*

234. Continuing now the work of computing areas and referring to the last figure, we may form the following table:

| STATIONS. | LINES. | N. LAT. | S. LAT. | E. DEP. | W. DEP. | D.M.D. | NORTH DOUBLE AREAS. | SOUTH DOUBLE AREAS. |
|-----------|-----------|-----------|-----------|-----------|-----------|-------------|---------------------|---------------------|
| 1 | <i>PO</i> | <i>Po</i> | .. | <i>Oo</i> | .. | <i>2 Bb</i> | <i>2 Bb × Po</i> | |
| 2 | <i>OL</i> | .. | <i>OQ</i> | <i>QL</i> | .. | <i>2 Cc</i> | | <i>2 Cc × OQ</i> |
| 3 | <i>LM</i> | .. | <i>LR</i> | .. | <i>MR</i> | <i>2 Dd</i> | | <i>2 Dd × LR</i> |
| 4 | <i>MP</i> | <i>Pm</i> | .. | .. | <i>Mm</i> | <i>2 Ee</i> | <i>2 Ee × Pm</i> | |

The double meridian distances are placed in the column headed D.M.D. In the column headed North Double Areas are placed $2 Bb \times Po$ and $2 Ee \times Pm$, the product of the first and fourth double meridian distances and their corresponding latitudes. In the south double area column we find $2 Cc \times OQ$ and $2 Dd \times LR$, or the product of the double meridian distances of the second and third sides, and their respective latitudes. In other words, the column in which each of the products of double meridian distance and latitude is to be placed is indicated by the latitude employed in the multiplication.

Now, twice the area of the triangles POo and PMm , or the subtractive portion of the figure $oOLMmo$, is given in the north double area column, and twice the area of the trapezoids $OoLl$ and $LlMm$, which include the triangles named, is given in the column of south double areas. Half the difference, therefore,

* The position of the meridian (NS) may be assumed to pass through any other corner, or even through a point outside the survey. A slight modification of the rule just given would make it applicable to any of these cases. For convenience, it is generally assumed to pass through the most westerly station. When a survey is made with the transit, and the area only required, it is most convenient to consider one of the sides of the tract the meridian.

between these sums is the area *POLMP* required. The reasoning being general, and independent of the number of sides, we have for finding the area of any rectilineal figure, the bearings and distances of all the sides being known, the following

RULE.

1. *Prepare a table as exhibited below.*
2. *Find, and place in their proper columns, the latitudes and departures of the several sides of the tract.*
3. *Balance the survey (if necessary).*
4. *Find the double meridian distances, with reference to a meridian passing through the most westerly * station, and place them in the D.M.D. column.*
5. *Multiply each double meridian distance by its corresponding corrected latitude, and place the product in the column of double areas indicated by the latitude.*
6. *One-half the difference of the sums of the columns of double areas will be the required area.*

Let us now take the field notes given in Article 198, and compute the area of the tract.

The student will perceive that the meridian is assumed to pass through the most westerly station, that the double meridian distances are found as directed in 233, that each is multiplied by its corresponding latitude, and the resulting double area product placed in the column of the same name as the latitude.

Lastly, the difference of the two columns of double areas is taken, the remainder divided by two, giving the number of square chains in the tract, and the result divided by 10 = 12,032 acres, which is the area sought.

On account of the meridian passing through the most westerly station, and because the field is to the left,† the areas of

* For convenience simply, see note, preceding article.

† In the last figure, if the bearings are taken or recited in the order *PM*, *ML*, *LO*, etc., the tract is considered on the left; if this order is reversed, the tract is on the right.

DETERMINING AREAS.

| STATIONS. | BEARINGS. | DISTANCES. | LATITUDES. | | DEPARTURES. | | CORRECTIONS. | | CORRECTED LATITUDES. | | CORRECTED DEPARTURES. | | D.M.D. | NORTH DOUBLE AREAS. | SOUTH DOUBLE AREAS. |
|-----------|---------------|------------|------------|-------|-------------|-------|--------------|------|----------------------|-------|-----------------------|-------|--------|---------------------|---------------------|
| | | | N. | S. | E. | W. | Lat. | Dep. | N. | S. | E. | W. | | | |
| 1 | S. 20° 53' E. | 13.11 | ... | 12.25 | 4.67 | ... | 2 | 1 | ... | 12.27 | 4.68 | ... | 4.68 | ... | 57.4236 |
| 2 | N. 48° 10' E. | 13.62 | 9.08 | ... | 10.15 | ... | 2 | 1 | 9.08 | ... | 10.16 | ... | 19.52 | 176.8512 | ... |
| 3 | N. 43° 40' W. | 4.73 | 3.42 | ... | ... | 3.26 | 1 | ... | 3.41 | ... | ... | 3.26 | 26.42 | 90.0922 | ... |
| 4 | N. 45° 08' W. | 4.75 | 3.35 | ... | ... | 3.36 | 1 | 1 | 3.34 | ... | ... | 3.35 | 19.81 | 66.1664 | ... |
| 5 | S. 51° 30' W. | 2.53 | ... | 1.57 | ... | 1.98 | ... | ... | ... | 1.57 | ... | 1.98 | 14.48 | ... | 22.7336 |
| 6 | S. 72° 30' W. | 6.56 | ... | 1.96 | ... | 6.26 | 1 | 1 | ... | 1.97 | ... | 6.25 | 6.25 | ... | 12.3125 |
| | | | 15.85 | 15.78 | 14.82 | 14.86 | 7 | 4 | 15.81 | 15.81 | 14.84 | 14.84 | | 333.1098 | 92.4697 |
| | | | 15.78 | | | 14.82 | | | | | | | | | |

Error 7 links.

Error 4 links.

2) 240.6401

10) 120.32005

12.032 acres.

the trapezoids are north, and those of the triangles south. If we had assumed the meridian to pass through the most easterly corner, the areas of the trapezoids then formed would be south, and those of the triangles north.

If the bearings of the lines were reversed, or the survey made with the field to the right, the reverse of the preceding statement would be true.

In either case, however, one-half the difference of the sums of the double areas will give the contents.

As an exercise the student may obtain an expression for the area of *POLMP*, last figure, assuming the meridian to pass through *L*, and passing round by *MP*, etc., that is, keeping the field to the right. He may also, with the meridian through *P*, and keeping the field to the left, obtain an expression for the area.

As a further exercise he may verify the result in the last example solved, taking the meridian through the most easterly station.

Calculate the areas from the following notes; also make a plot of each:

1.

- (1) N. 9° W. 15.50 chains;
- (2) N. 31° E. 25.40 "
- (3) S. 69° E. 10.00 "
- (4) S. $10\frac{1}{2}^{\circ}$ E. 19.70 "
- (5) S. $10\frac{3}{4}^{\circ}$ W. 14.60 "
- (6) N. 89° W. 21.00 "

2.

| STA- TIONS. | LINES. | DISTS. | AZIMUTH WITH <i>LM</i> . |
|----------------|-----------|--------|------------------------------|
| <i>L</i> | <i>LM</i> | 22.45 | 0° |
| <i>M</i> | <i>MN</i> | 1.30 | 22° |
| <i>N</i> | <i>NO</i> | 66.30 | 90° |
| <i>O</i> | <i>OP</i> | 23.85 | 180° |
| <i>P</i> | <i>PL</i> | 67.10 | 270° |
| <i>L</i> | <i>LM</i> | . . . | 360° or 0° |

3.

- (1) N. $11\frac{1}{4}^{\circ}$ E. 13.19 chains ;
- (2) S. 87° W. 8.50 “
- (3) S. $20\frac{1}{2}^{\circ}$ W. 11.75 “
- (4) S. 82° E. 10.03 “

Ans. $11\frac{175}{1000}$ acres.

If in Article 76 we substitute respectively for abscissa and ordinate of a corner of a tract, departure and latitude of the side ending at said corner, the rule there given may be applied to surveys made with an angular instrument.

To illustrate, take the example given in the last article :

| CORRECTED LATITUDES. | | CORRECTED DEPARTURES. | | TOTAL LATITUDES. | TOTAL DEPARTURES. | DIFFER. BETWEEN ALTERNATE DEPARTS. | DOUBLE AREAS. |
|----------------------|-------|-----------------------|------|------------------|-------------------|------------------------------------|-------------------|
| N. | S. | E. | W. | | | | |
| .. | 12.27 | 4.68 | .. | 0.00 | .. | .. | |
| 9.06 | .. | 10.16 | .. | —12.27 | 4.68 | —14.84 | 182.0868 |
| 3.41 | .. | .. | 3.26 | — 3.21 | 14.84 | — 6.90 | 22.1490 |
| 3.34 | .. | .. | 3.35 | .20 | 11.58 | 6.61 | 1.3220 |
| .. | 1.37 | .. | 1.98 | 3.54 | 8.23 | 5.33 | 18.8682 |
| .. | 1.97 | .. | 6.25 | 1.97 | 6.25 | 8.23 | 16.2131 |
| | | | | | | | 2)240.6391 |
| | | | | | | | 10)120.32 sq. ch. |
| | | | | | | | 12.032 acres. |

In this case the axes were taken through the most westerly station, thereby making the total departures all plus, but giving both plus and minus total latitudes. On account of the signs the double areas are all plus. The axis of ordinates passing through the most westerly station makes the total latitude of that station zero, and consequently there is one less multiplication to be performed. The same would be the case if the Y axis passed through the most easterly corner.

Since the difference of the *alternate total* departures is equal to the sum of the *adjacent* departures, the rule just given may be stated as follows :

Multiply the total latitude of each station by the sum of the departures of the adjacent sides, and take half the sum of these products for the area.

To illustrate, take the following example :

| STATIONS. | BEARINGS. | DISTANCES. | N. | S. | E. | W. | TOTAL LATITUDES. | ADJACENT DEPARTS. | DOUBLE AREAS. |
|-----------|---------------|------------|-----|-----|-----|-----|---------------------|----------------------|---|
| 1 | N. 25° E. | 433 | 393 | .. | 183 | .. | 000 | .. | |
| 2 | N. 76° 55' E. | 191 | 43 | .. | 186 | .. | 393 | 369 | 145017 |
| 3 | S. 6° 41' W. | 539 | .. | 535 | .. | 62 | 436 | 124 | 54064 |
| 4 | S. 25° W. | 40 | .. | 36 | .. | 17 | — 99 | — 79 | 7821 |
| 5 | N. 65° W. | 320 | 135 | .. | .. | 290 | —135 | —307 | 41445 |
| | | | | | | | | | <div>2)248347</div> <div>43560)124173.5 sq. ft.</div> <div>2.852 acres.</div> |

The student may verify the preceding example by this method.

2. Given the bearings and distances of the sides of a field, as follows, to find the area by each of the two preceding methods. Ascertain, also, the error of the survey.

- (1) N. $6\frac{1}{2}^{\circ}$ W. 9.38 chains ;
- (2) N. $65\frac{1}{2}^{\circ}$ E. 8.25 “
- (3) S. 39° E. 6.51 “
- (4) S. 2° W. 4.45 “
- (5) S. 46° W. 5.00 “
- (6) N. 88° W. 6.79 “

3. Given the boundaries of a tract of land with the corresponding weights, as follows, to determine the area by double

meridian distances, using the weights in balancing the survey as indicated in 3°, Article 208. Determine, also, the error of the survey.

| | | | | | |
|-----|----|------------------------|-----|---------------------|---------------------|
| (1) | S. | 79° 10' | W., | dist. 27.00 chains, | weight, 1 ; |
| (2) | S. | $\frac{1}{2}^\circ$ | W., | " 34.08 " | " 3 ; |
| (3) | N. | 89 $\frac{1}{2}^\circ$ | E., | " 10.47 " | " 1 $\frac{1}{2}$; |
| (4) | N. | 1° 55' | E., | " 15.30 " | " 2 ; |
| (5) | S. | 80 $\frac{3}{4}^\circ$ | E., | " 7.15 " | " 2 ; |
| (6) | S. | 58 $\frac{1}{2}^\circ$ | E., | " 11.50 " | " 2 $\frac{1}{2}$; |
| (7) | N. | 39° | E., | " 9.20 " | " 1 ; |
| (8) | N. | 16 $\frac{1}{4}^\circ$ | W., | " 24.63 " | " 1. |

4. The distances and interior angles of a farm, together with the bearing of one line, are given below. The angles were measured very accurately. It is required to calculate the area, by either of the preceding methods, balancing the survey by (2°) Second Case, Article 208. Also make a plot.

Angle L , 90°; side LM , 28.00 chains.

| | | | | | | |
|---|-------|---------------------------|---|--------|-------|---|
| " | M , | 148 $\frac{1}{2}^\circ$; | " | MN , | 25.20 | " |
| " | N , | 81 $\frac{1}{2}^\circ$; | " | NO , | 14.70 | " |
| " | O , | 220°; | " | OP , | 12.48 | " |
| " | P , | 90°; | " | PQ , | 27.96 | " |
| " | Q , | 90°; | " | QR , | 15.16 | " |
| " | R , | 270°; | " | RS , | 11.90 | " |
| " | S , | 90°; | " | SL , | 21.60 | " |

Bearing of LM , N. 10° E.

5. The notes of a survey are given below; it is required to determine the area by double meridian distances after correcting the latitudes and departures by a combination of 2° and 3°, Article 208. See also note in same article.

The interior angles were observed.

Angle L , 91° 44'; side LM , 17.16 chains; weight, 2.

| | | | | | | | | |
|---|-------|-----------|---|--------|-------|---|---|-------------------|
| " | M , | 168° 20'; | " | MN , | 9.48 | " | " | 1. |
| " | N , | 104° 49'; | " | NO , | 8.39 | " | " | 1 $\frac{1}{2}$. |
| " | O , | 179° 30'; | " | OP , | 15.28 | " | " | 2. |

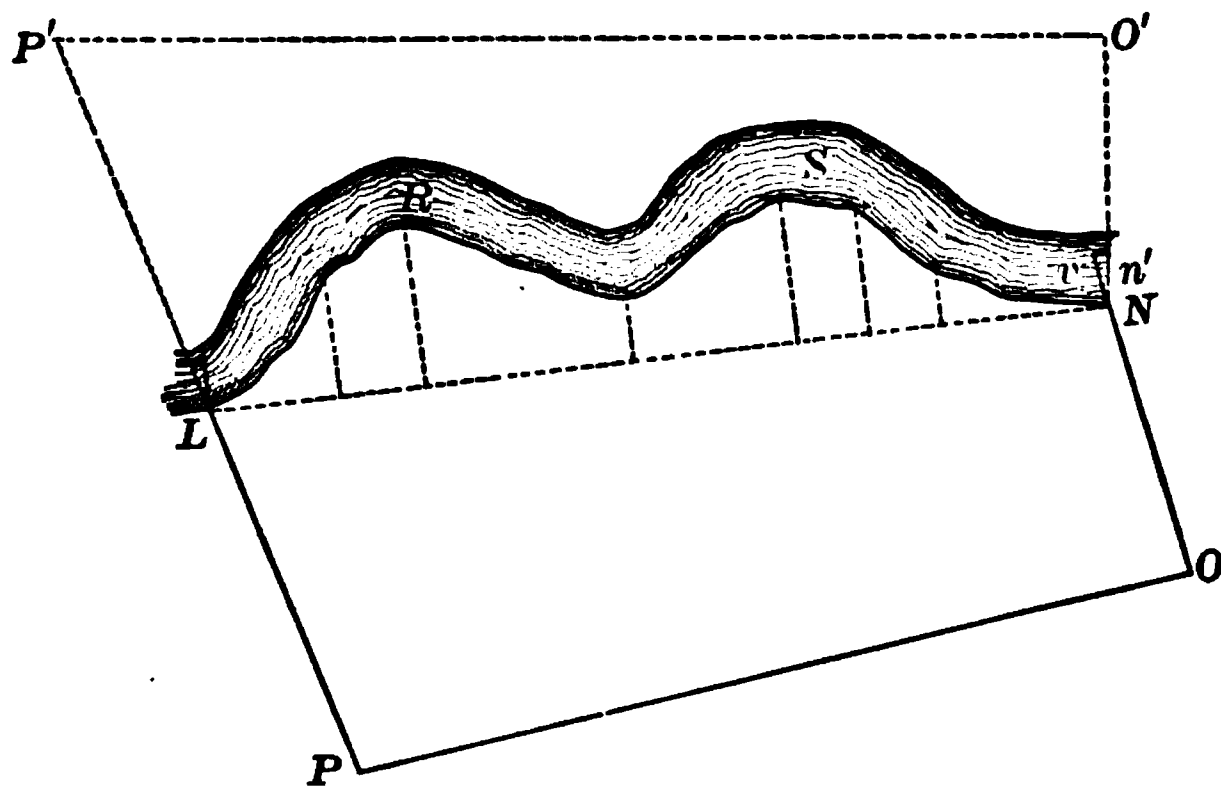
Angle P , $90^\circ 19'$; side PQ , 16.05 chains; weight, $2\frac{1}{2}$.
 " Q , $90^\circ 05'$; " QR , 15.68 " " 3.
 " R , $283^\circ 49'$; " RS , 11.40 " " 1.
 " S , $71^\circ 24'$; " SL , 13.80 " " 1.

6. Select a tract of land, some of the sides being much more difficult than the others to align and measure, survey it, weight the sides, balance the latitudes and departures according to the weights, and calculate the area.

7. Let one party of students survey a tract of uneven or hilly land of considerable magnitude, by means of transit and stadia and rectangular co-ordinates; another party at the same time, or the same party subsequently, survey the same tract in the usual way. Compare results.

C. WHEN OFFSETS ARE TAKEN.

235. Let the annexed figure represent the case. The property lines are NO , OP , PL , and the centre of the creek * RS . Obtain sufficient data to compute the area of the rectilinear

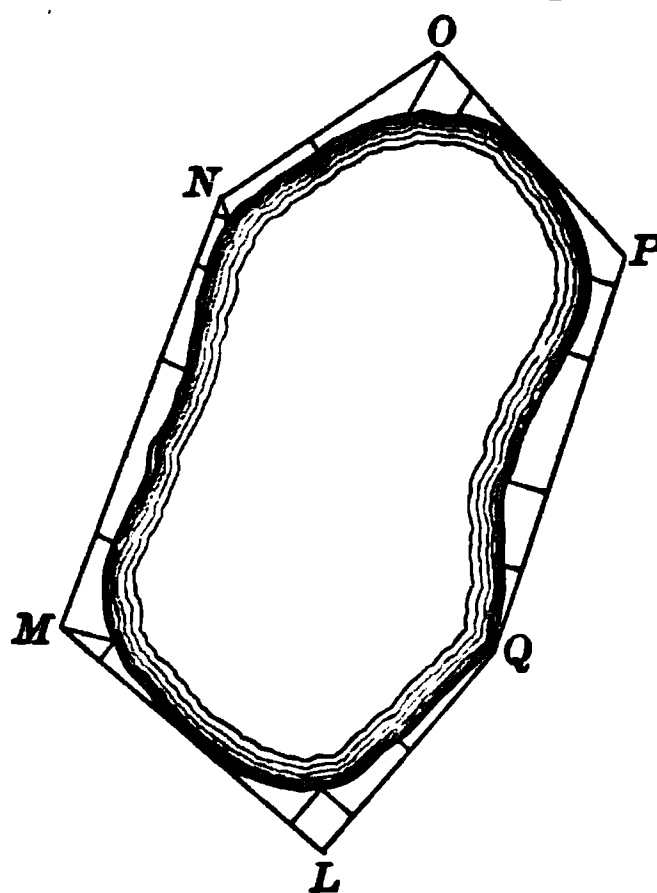


* When a non-navigable stream forms a boundary of a tract of land, the middle of it is considered the property line, unless otherwise specified. In navigable rivers and tidal waters, the boundary is low-water mark.

figure $LNOP$, and take offsets from the line LN to the middle of the stream, as directed in Offsets and Tie-Lines, Article 73; and in Traversing, Article 164. Calculate the area of $LNOP$ by one of the preceding methods; to this add* the sum of the areas of the trapezoids, and triangles formed by the offsets from the line LN to the middle of the creek. If the width of the stream is considerable, and especially if great accuracy is required, the surveyor must not ignore the small triangles† formed at L and N .

236. To Find the Area of a Pond or Small Lake, traverse, or take the bearings of the sides LM , MN , NO , etc., and measure them; also take offsets, at proper points, from these lines to the edge of the water.

Calculate the area included between the right lines, and subtract therefrom the area found by the offsets; the remainder will be the area required.



EXERCISES.

1. Let one party survey a field with compass and chain, taking bearings and distances of all the sides; another party survey the same field, using transit and chain, and observing

* If the base line LN is without the tract, as in $LNO'P'$, the area included between the middle of the stream and LN must be *subtracted* from that of $LNO'P'$.

† Other things being equal, the areas of these small triangles depend upon the obliquity of PL and ON . There will be none formed when PL and NO are perpendicular to the base LN . In the case presented, the area of the triangle at L is to be added, and that of $Nn'v$ subtracted from the sum of the areas of the trapezoids, to obtain the correct content between LN and the middle of the creek.

the interior or deflection angles ; a third party, using the chain only. Each party should use proof lines, make record, plot, and calculate the area. Compare results.

2. With a transit, survey a field, a part of which is bounded by a creek, lake, or some crooked line requiring offsets to be taken ; make a plot, and compute the area.

3. Triangulate a portion of a river or small lake ; make a plot, and compute the area.

4. Make the necessary measurements to write a description, to make a plot, and to compute the area of a portion of a crooked road.

5. Observe all the bearings and measure all the sides of a polygonal field, except the bearing and distance of one side. Compute the area, and length and bearing of omitted side. Subsequently observe the bearing and distance, and note, if any disagreement, how much the area is affected thereby.

CHAPTER III.

DECLINATION OF THE MAGNETIC NEEDLE, OR VARIATION OF THE COMPASS.

237. It has been already remarked (Article 82) that the magnetic and geographic meridian do not in general coincide. The angle included by the vertical planes containing these lines, or the angle which the direction of the needle makes with the geographic meridian, is the declination of the needle, sometimes called the variation of the compass. It is different at different places, and is a variable quantity at any place.

The declination is termed *east* or *west*, according as the north end of the needle points to the east or west of the geographic, or true meridian.

The magnetic declinations of a few places for the year 1885 are given below :

| | | | |
|----------------------|------------|-----------------------|------------|
| Eastport, Me., | 19° 10' W. | Sitka, Alaska, | 28° 50' E. |
| Albany, N.Y., | 10° 11' W. | Milledgeville, Ga., | 2° 32' E. |
| Pittsburg, Pa.,* | 2° 52' W. | New Orleans, La., | 6° 11' E. |
| Omaha, Neb., | 10° 06' E. | City of Mexico, Mex., | 7° 24' E. |
| San Francisco, Cal., | 16° 34' E. | | |

238. Irregular Changes. The magnetic needle is subject to disturbances during a thunder storm, or an exhibition of aurora, solar changes, and sometimes it is considerably agitated without any apparent cause, but probably on account of magnetic or electric disturbances more or less remote.

The changes, however, which especially concern the surveyor, are the diurnal and secular.

* At this place, September, 1887, the magnetic declination = 3° 01' W.

239. The Diurnal Variation. It has been ascertained, by repeated observations at various places, that the magnetic needle is subject to daily changes; that at a time varying from two to three hours after sunrise the north end of the needle attains its maximum deviation to the east, or, as it is called, its eastern elongation; from this time it is deflected westward, attaining its western elongation between 1 and 2 o'clock P.M., whence it retrogrades towards the east. There is sometimes an interruption of the motion at night, but generally a small reversed movement is exhibited, the magnetic meridian being crossed a second time between 7 and 9 P.M. The times at which these limits are reached vary with the seasons: during the north declination of the sun the averages for eastern and western elongations, respectively, are about 7.30 A.M. and 1.15 P.M.; for the remainder of the year, about 8.45 A.M. and 1.45 P.M.

The average daily direction or mean magnetic meridian is reached in summer about 10.15 A.M., and in winter about 10.45 A.M., at Philadelphia, and generally within half an hour of these times at other places.

The angular range between these limits is not constant, but, as may be seen by the table subjoined, it is considerably greater in summer than in winter, amounting at Philadelphia to 10' 30" in August, and only 6' in November, or a yearly average of 8', while at Key West, Florida, the average for the year is about 5' 30"; in higher magnetic latitudes the average being more than in the lower. It is least in years of minimum sun spots (as in 1878, for instance), and greatest in years of maximum sun spots (as in 1870), the ratio being about as 7 to 13 of the average amount of these years respectively. The daily variation is at times interrupted, at others enfeebled, and frequently in the winter there are days on which it cannot be recognized. On account of the daily movement of the needle, its variable range during the year, and disturbances from atmospheric phenomena, it is well, when taking the bearing of any important line, to record the date, time of day, and condition of the atmosphere, using the subjoined table as far as practicable.

240. For reducing the direction of the needle observed at other hours to the mean magnetic meridian, the following table (taken from instructions to United States Deputy Surveyors), is furnished. It gives to the nearest minute the variations of the needle from its average position during the day, for each hour in the day, for the four seasons of the year.

TABLE FOR REDUCING THE OBSERVED DECLINATION TO THE MEAN DECLINATION OF THE DAY.

| | A.M. | A.M. | A.M. | A.M. | A.M. | A.M. | M. | P.M. | P.M. | P.M. | P.M. | P.M. | P.M. |
|--------|------|------|------|------|------|------|----|------|------|------|------|------|------|
| Hour | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 1 | 2 | 3 | 4 | 5 | 6 |
| Spring | 3' | 4' | 4' | 3' | 1' | 1' | 4' | 5' | 5' | 4' | 3' | 2' | 1' |
| Summer | 4' | 5' | 5' | 4' | 1' | 2' | 4' | 6' | 5' | 4' | 3' | 2' | 1' |
| Autumn | 2' | 3' | 3' | 2' | 0' | 2' | 3' | 4' | 3' | 2' | 1' | 1' | 0' |
| Winter | 1' | 1' | 2' | 2' | 1' | 0' | 2' | 3' | 3' | 2' | 1' | 1' | 0' |

241. The Secular Variation. Observations extending through many years, at various places, indicate a continual change taking place in the declination of the needle; that these changes are not continuous in direction nor uniform in intensity; that in this country the movement which, at the end of the last century, was *eastward* is now *westward* at all places east of the Rocky Mountains, and that a period of 250 or 300 years may elapse before the needle will again resume the position it now occupies.*

242. The Line of no Declination,† or Agonic Line, is the locus of all points on the earth where the direction of the needle is

* The explanation of the secular change must ultimately be referred to forces of a periodic character, acting for centuries with great regularity. So far no approach has yet been made towards the discovery of the cause of the motion. . . . The study of the variation of the declination so far would seem to indicate a secular change cycle for stations in the United States, extending over, or varying between, the limits of about 220 or 360 years. The data, however, are very uncertain. (U. S. C. & G. S., 1879.)

† Sometimes called the *Line of no Variation*.

coincident with the geographic meridian. At all places on the American continent situated to the east of this line the declination is *west*, and at all places to the west of it, the declination is *east*.

The line of no declination has been moving westward during the present century. From a chart published by Professor Loomis, in the *American Journal of Science*, 1840, it appears that the lines of equal declination, or isogonic lines, crossed the United States in a N.N.W. direction; the deflection towards the west being greatest in Maine. The line of no declination at that time entered North Carolina about midway between Newbern and Wilmington, passed through the middle of Virginia, and into Lake Erie at a point nearly equidistant from Erie, Pa., and Cleveland, Ohio.

In 1885 the Agonic Line entered the United States a little to the east of Beach Inlet, S.C., thence through Greensboro, N.C., Christiansburg, Va., Point Pleasant, W.Va., St. Clairsville, Ohio, a short distance west of Detroit, and a few miles east of Fort Mackinac, Mich.

In the year 1700 the declination at Philadelphia, Pa., was $8\frac{1}{2}^{\circ}$ west. During the next century it diminished, reaching a minimum in 1800 of $1\frac{1}{2}^{\circ}$ west, since which time it has been increasing, and is now, January, 1887, at the Philadelphia State House, lat. $39^{\circ} 56' 54''$, long. $75^{\circ} 09'$, $6^{\circ} 50'$, with an annual increase of $5'$.

243. Mr. Charles A. Schott, late chief of the computing division of the U. S. C. & G. S., tabulated the declinations observed at various stations, and deduced from them formulas by which the magnetic declination at various places may be computed.*

The places are arranged geographically as far as practicable, and are given by latitude and longitude (west of Greenwich). The epoch to which the formulas refer is 1850, or $m = t - 1850$.

* U. S. C. & G. S., 1882. App. 12.

FORMULAS EXPRESSING THE MAGNETIC DECLINATION AT VARIOUS PLACES
 IN THE UNITED STATES, AND FOR ANY TIME WITHIN THE LIMITS OF
 OBSERVATION.

| NAME OF STATION AND LOCATION. | LATI- TUDE. | LONGI- TUDE. | EXPRESSION FOR MAGNETIC DECLINATION. |
|----------------------------------|----------------|-----------------|---|
| Portland, Me. | 43° 38.8' | 70° 16.6' | $D = +10.72 + 2.68 \sin(1.33 m + 24.1)$ |
| Burlington, Vt. | 44° 28.2' | 73° 12.3' | $D = +10.81 + 3.65 \sin(1.30 m - 20.5)$ $+ 0.18 \sin(7.0 m + 132)$ |
| Rutland, Vt. | 43° 36.5' | 72° 55.5' | $D = +10.03 + 3.82 \sin(1.5 m - 24.3)$ |
| Portsmouth, N.H. | 43° 04.8' | 70° 43.0' | $D = +10.63 + 3.17 \sin(1.44 m - 4.7)$ |
| Newburyport, Mass. | 42° 48.4' | 70° 49.0' | $D = +10.07 + 3.10 \sin(1.4 m + 1.9)$ |
| Salem, Mass. | 42° 31.9' | 70° 52.5' | $D = + 9.80 + 3.61 \sin(1.50 m - 1.0)$ |
| Boston, Mass. | 42° 21.5' | 71° 03.8' | $D = + 9.52 + 2.93 \sin(1.30 m + 5.0)$ |
| Cambridge, Mass. | 42° 22.9' | 71° 07.7' | $D = + 9.58 + 2.69 \sin(1.3 m + 7.0)$ $+ 0.18 \sin(3.2 m + 44)$ |
| Nantucket, Mass. | 41° 17.0' | 70° 06.0' | $D = + 9.29 + 2.78 \sin(1.35 m + 5.5)$ |
| Providence, R.I. | 41° 49.5' | 71° 24.1' | $D = + 9.10 + 2.99 \sin(1.45 m - 3.4)$ $+ 0.19 \sin(7.2 m + 116)$ |
| Hartford, Conn. | 41° 45.9' | 72° 40.4' | $D = + 8.06 + 2.90 \sin(1.25 m - 26.4)$ |
| New Haven, Conn. | 41° 18.5' | 72° 55.7' | $D = + 7.78 + 3.11 \sin(1.40 m - 22.1)$ |
| Albany, N.Y. | 42° 39.2' | 73° 45.8' | $D = + 8.17 + 3.02 \sin(1.44 m - 8.3)$ |
| Oxford, N.Y. | 42° 26.5' | 75° 40.5' | $D = + 6.19 + 3.24 \sin(1.35 m - 18.9)$ |
| Buffalo, N.Y. | 42° 52.8' | 78° 53.5' | $D = + 3.66 + 3.47 \sin(1.4 m - 27.8)$ |
| Toronto, Can. | 43° 39.4' | 79° 23.4' | $D = + 3.60 + 2.82 \sin(1.4 m - 44.7)$ $+ 0.09 \sin(9.3 m + 136)$ $+ 0.08 \sin(19 m + 247)$ |
| Erie, Pa. | 42° 07.8' | 80° 05.4' | $D = + 2.26 + 2.71 \sin(1.55 m - 29.7)$ |
| Marietta, Ohio | 39° 25.0' | 81° 28.0' | $D = + 0.02 + 2.89 \sin(1.4 m - 40.5)$ |
| Cleveland, Ohio | 41° 30.3' | 81° 42.0' | $D = + 0.10 + 2.07 \sin(1.40 m - 6.2)$ |
| Detroit, Mich. | 42° 20.0' | 83° 03.0' | $D = - 0.97 + 2.21 \sin(1.50 m - 15.3)$ |
| Sault de St. Marie, Mich. | 46° 29.9' | 84° 20.1' | $D = + 1.54 + 2.70 \sin(1.45 m - 58.5)$ |
| Cincinnati, Ohio | 39° 08.6' | 84° 25.3' | $D = - 2.40 + 2.62 \sin(1.42 m - 39.8)$ |
| St. Louis, Mo. | 38° 38.0' | 90° 12.2' | $D = - 7.15 + 2.33 \sin(1.4 m - 20.1)$ |
| New York, N.Y. | 40° 42.7' | 74° 00.4' | $D = + 6.40 + 2.29 \sin(1.6 m - 5.5)$ $+ 0.14 \sin(6.3 m + 6.4)$ |
| Hatborough, Pa. | 40° 12.0' | 75° 07.0' | $D = + 5.23 + 3.28 \sin(1.54 m - 13.2)$ $+ 0.22 \sin(4.1 m + 157)$ |
| Philadelphia, Pa. | 39° 56.9' | 75° 09.0' | $D = + 5.38 + 3.29 \sin(1.55 m - 23.9)$ $+ 0.39 \sin(4.0 m + 161)$ |
| Harrisburg, Pa. | 40° 15.9' | 76° 52.9' | $D = + 2.93 + 2.98 \sin(1.50 m + 0.2)$ |
| Baltimore, Md. | 39° 17.8' | 76° 37.0' | $D = + 3.20 + 2.57 \sin(1.45 m - 21.2)$ |
| Washington, D.C. | + 38° 53.3' | + 77° 00.6' | $D = + 2.47 + 2.52 \sin(1.40 m - 14.6)$ |
| Cape Henry, Va. | + 36° 55.5' | + 76° 00.5' | $D = + 2.54 + 2.41 \sin(1.50 m - 35.4)$ |
| Charleston, S.C. | 32° 46.6' | 79° 55.8' | $D = - 2.14 + 2.74 \sin(1.35 m - 1.3)$ |
| Savannah, Ga. | 32° 04.9' | 81° 05.5' | $D = - 2.54 + 2.32 \sin(1.5 m - 28.6)$ |
| Key West, Fla. | 24° 33.5' | 81° 48.5' | $D = - 3.90 + 2.93 \sin(1.4 m - 33.5)$ |
| Havann, Cuba. | 23° 09.3' | 82° 21.5' | $D = - 4.52 + 2.00 \sin(1.3 m - 26.7)$ |

FORMULAS EXPRESSING THE MAGNETIC DECLINATION.— *Continued.*

| NAME OF STATION AND LOCATION. | LATI- TUDE. | LONGI- TUDE. | EXPRESSION FOR MAGNETIC DECLINATION. |
|---|----------------|-----------------|---|
| Kingston, Jamaica . . . | 17° 55.9' | 76° 50.6' | $D = - 4.64 + 2.04 \sin(1.2 m + 15.9)$ |
| Panama, New Granada . | 8° 57.1' | 79° 32.2' | $D = - 6.80 + 1.82 \sin(0.9 m + 10.4)$ |
| Florence, Ala. | 34° 47.2' | 87° 41.5' | $D = - 4.25 + 2.33 \sin(1.3 m - 52.8)$ |
| Mobile, Ala. | 30° 41.4' | 88° 02.5' | $D = - 4.40 + 2.69 \sin(1.45 m - 76.4)$ |
| New Orleans, La. . . . | 29° 57.2' | 90° 03.9' | $D = - 5.61 + 2.57 \sin(1.4 m - 61.9)$ |
| Vera Cruz, Mexico . . . | 19° 11.9' | 96° 08.8' | $D = - 4.38 + 5.04 \sin(1.10 m - 65.0)$ |
| Mexico, Mexico | 19° 25.9' | 99° 06.0' | $D = - 4.34 + 4.44 \sin(1.0 m - 79.2)$ |
| Acapulco, Mexico . . . | 16° 50.5' | 99° 52.3' | $D = - 4.13 + 4.82 \sin(1.0 m - 81.1)$ |
| San Blas, Mexico . . . | 21° 32.6' | 105° 15.7' | $D = - 6.51 + 2.74 \sin(0.9 m - 106.8)$ |
| Magdalena Bay, L. Cal. . | 24° 38.4' | 112° 08.9' | $D = - 7.52 + 3.27 \sin(1.25 m - 140.6)$ |
| San Diego, Cal. | 32° 42.1' | 117° 14.3' | $D = -12.52 + 1.60 \sin(1.2 m - 179.8)$ |
| Monterey, Cal. | 36° 36.1' | 121° 53.6' | $D = -12.90 + 3.28 \sin(1.0 m - 142.6)$ |
| San Francisco, Cal. . . . | 37° 47.5' | 122° 27.2' | $D = -13.34 + 3.28 \sin(1.00 m - 130.3)$ |
| Cape Disappointm't, W.T. | 46° 16.7' | 124° 02.0' | $D = -20.26 + 2.36 \sin(1.25 m - 180.0)$ |
| Sitka, Alaska | 57° 02.9' | 135° 19.7' | $D = -26.77 + 2.33 \sin(1.4 m - 111.6)$ |
| Unalashka, Alaska . . . | 53° 52.6' | 166° 31.5' | $D = -18.34 + 1.45 \sin(1.4 m - 67.8)$ |
| Tyrone, Pa. | 40° 40.0' | 78° 15.5' | $D = + 3.46 + 0.0550(t - 1875.5)$ |
| Pittsburg, Pa. | 40° 27.6' | 80° 00.8' | $D = + 2.36 + 0.0566(t - 1878.7)$ |
| Chicago, Ill. | 41° 50.0' | 87° 36.7' | $D = - 6.03 + 0.0281(t - 1850)$ $+ 0.00082(t - 1850)^2$ |
| Grand Haven, Mich. . . | 43° 05.2' | 86° 12.6' | $D = - 4.95 + 0.0380(t - 1850)$ $+ 0.00120(t - 1850)^2$ |
| Madison, Wis. | 43° 04.6' | 89° 24.2' | $D = - 6.43 + 0.0655(t - 1880.3)$ |
| Duluth, Minn.; and Supe- rior City, Wis. | 46° 45.5' | 92° 04.5' | $D = -10.17 + 0.0868(t - 1875.8)$ |
| Rio Janeiro, Brazil. . . | -22° 54.8' | 43° 09.5' | $D = + 0.282 + 0.1395(t - 1850)$ $+ 0.00545(t - 1850)^2$ |
| San Antonio, Tex. . . . | +29° 25.4' | 98° 29.3' | $D = -10.14 + 0.0204(t - 1850)$ $+ .000024(t - 1850)^2$ |
| Omaha, Neb.; and Council Bluffs, Iowa. | 41° 15.7' | 95° 56.5' | $D = -11.66 + 0.0439(t - 1850)$ |
| Denver, Col. | 39° 45.3' | 104° 59.5' | $D = -14.79 + 0.0258(t - 1872.9)$ |
| Salt Lake City, Utah. . . | 40° 46.1' | 111° 53.8' | $D = -15.51 - 0.0930(t - 1850)$ $+ 0.00180(t - 1850)^2$ |

To illustrate the use of the table: Suppose it is desired to ascertain the declination of the needle at Harrisburg for the last of September, 1877, or $t = 1877.75$.

Take from the table the expression for the declination at Harrisburg; that is:

$$D = + 2.93 + 2.98 \sin (1.50 m + 0.2).$$

Find $m = 1877.75 - 1850 = 27.75$;

$$1.50m + 0.2 = 41.625 + 0.2 = 41.825,$$

and $2.98 \times \text{natural sin } 41.825 = 2.98 \times .66686 = 1.987.$

$\therefore D = 2.93 + 1.987 = 4.917 = 4^\circ 55'$ *west* (the result being *plus*). The observed declination for the same time was $4^\circ 53' 5''$. The difference between the computed and observed declination is seen to be very small.

In running old lines it may be necessary to determine the declination at a time anterior to 1850 ; then m will be *negative*. Suppose the declination at Washington, D.C., for the year 1841 is desired. The tabular expression is :

$$D = 2.47 + 2.52 \sin (1.4m - 14.6),$$

$$m = 1841 - 1850 = -9,$$

$$(1.4m - 14.6) = -27.2,$$

$$2.52 \sin (-27.2) = -1.15.$$

$\therefore D = 2.47 - 1.15 = 1.32$ *west* (the resulting sign being *plus*), which agrees practically with the observed declination.

244. The following table is taken from U. S. C. & G. S. Report, 1882, App. 12, Mr. Schott's paper on Secular Variation. It exhibits the computed epoch of greatest easterly deflection reached in the secular motion ; *i.e.*, the date when last reached, or the date (in parenthesis) when it is next expected to be in that position ; the amount in degrees and fractions, and direction (+ west, - east) at this, the nearest stationary epoch ; and the computed annual changes in the declination of the magnetic needle for the years 1870, 1880, and 1885, a plus sign indicating north end of needle moving westward, a minus sign indicating north end of needle moving eastward.

| LOCATION. | NEAREST STATIONARY EPOCH OF EASTERLY DIGRESSION. | AMOUNT AT EASTERLY DIGRESSION. | ANNUAL CHANGE. | | |
|---------------------------|---|--------------------------------------|----------------|----------|----------|
| | | | IN 1870. | IN 1880. | IN 1885. |
| Paris, France | 1581 | - 10.6° | - 7.0' | - 6.1' | - 9.5' |
| Halifax, Nova Scotia . | 1728 | + 12.4° | + 1.8' | + 1.0' | + 0.5' |
| Quebec, Canada | 1809 | + 12.1° | + 4.2' | + 1.6' | + 0.5' |
| Montreal, Canada . . . | 1816 | + 7.6° | + 5.1' | + 3.1' | + 2.8' |
| Eastport, Me. | 1760 | + 12.5° | + 3.3' | + 2.7' | + 2.3' |
| Portland, Me. | 1764 | + 8.0° | + 2.4' | + 1.6' | + 1.2' |
| Burlington, Vt. | 1810 | + 7.2° | + 5.0' | + 6.0' | + 5.8' |
| Rutland, Vt. | 1806 | + 6.2° | + 6.0' | + 5.6' | + 5.3' |
| Portsmouth, N.H. . . . | 1791 | + 7.5° | + 4.4' | + 3.7' | + 3.3' |
| Newburyport, Mass. . . | 1784 | + 7.0° | + 3.9' | + 3.3' | + 2.9' |
| Salem, Mass. | 1791 | + 6.2° | + 5.0' | + 4.1' | + 3.5' |
| Boston, Mass. | 1777 | + 6.6° | + 3.4' | + 2.9' | + 2.5' |
| Cambridge, Mass. . . . | 1783 | + 6.9° | + 2.9' | + 2.1' | + 1.8' |
| Nantucket, Mass. . . . | 1779 | + 6.5° | + 3.3' | + 2.7' | + 2.4' |
| Providence, R.I. | 1780 | + 6.1° | + 3.8' | | |
| Hartford, Conn. | 1799 | + 5.2° | + 3.8' | + 3.7' | + 3.6' |
| New Haven, Conn. . . . | 1802 | + 4.7° | + 4.6' | + 4.3' | + 4.1' |
| Albany, N.Y. | 1793 | + 5.2° | + 4.3' | + 3.7' | + 3.4' |
| Oxford, N.Y. | 1797 | + 3.0° | + 4.5' | + 4.3' | + 4.0' |
| Buffalo, N.Y. | 1806 | + 0.2° | + 5.1' | + 5.0' | + 4.8' |
| Toronto, Canada | | | + 4.8' | + 4.5' | + 2.3' |
| Erie, Pa. | 1811 | - 0.5° | + 4.4' | + 4.2' | + 4.0' |
| Marietta, O. | 1815 | - 2.9° | + 4.2' | + 4.2' | + 4.2' |
| Cleveland, O. | 1790 | - 2.0° | + 2.8' | + 2.5' | + 2.2' |
| Detroit, Mich. | 1800 | - 3.2° | + 3.4' | + 3.0' | + 2.8' |
| Sault de St. Marie, Mich. | 1828 | - 1.2° | + 3.6' | + 4.0' | + 4.1' |
| Cincinnati, O. | 1815 | - 5.0° | + 3.8' | + 3.9' | + 3.8' |
| St. Louis, Mo. | 1800 | - 9.5° | + 3.4' | + 3.2' | + 3.0' |
| New York, N.Y. | 1797 | + 4.0° | + 2.4' | + 2.5' | + 2.6' |
| Hatborough, Pa. | 1797 | + 1.8° | + 4.6' | + 4.5' | |
| Philadelphia, Pa. | 1800 | + 1.9° | + 4.9' | + 4.9' | + 5.3' |
| Baltimore, Md. | 1802 | + 0.6° | + 3.9' | + 3.6' | + 3.2' |
| Harrisburg, Pa. | 1790 | 0.0° | + 4.1' | + 3.3' | + 2.8' |
| Washington, D.C. . . . | 1796 | 0.0° | + 3.5' | + 3.2' | + 3.0' |

| LOCATION. | NEAREST STATIONARY EPOCH OF EASTERLY DIPRESSION. | AMOUNT AT EASTERLY DIPRESSION. | ANNUAL CHANGE. | | |
|---------------------------|---|--------------------------------------|----------------|----------|----------|
| | | | IN 1870. | IN 1880. | IN 1885. |
| Cape Henry, Va. | 1814 | + 0.1° | + 3.8' | + 3.7' | + 3.6' |
| Charleston, S.C. | 1784 | - 4.9° | + 3.5' | + 3.0' | + 2.7' |
| Savannah, Ga. | 1809 | - 4.9° | + 3.6' | + 3.5' | + 3.3' |
| Key West, Fla. | 1810 | - 6.8° | + 4.3' | + 4.2' | + 4.1' |
| Havana, Cuba | 1801 | - 6.5° | + 2.7' | + 2.7' | + 2.6' |
| Kingston, Jamaica . . . | 1762 | - 6.7° | + 2.0' | + 1.6' | + 1.4' |
| Panama, New Granada | 1739 | - 8.6° | + 1.5' | + 1.4' | + 1.3' |
| Florence, Ala. | 1821 | - 6.6° | + 2.8' | + 3.1' | + 3.2' |
| Mobile, Ala. | 1841 | - 7.1° | + 2.8' | + 3.4' | + 3.7' |
| New Orleans, La. | 1830 | - 8.2° | + 3.1' | + 3.5' | + 3.7' |
| Vera Cruz, Mexico . . | 1827 | - 9.4° | + 4.2' | + 4.9' | + 5.2' |
| Mexico, Mexico | 1839 | - 8.8° | + 2.4' | + 3.0' | + 3.3' |
| Acapulco, Mexico . . . | 1841 | - 9.0° | + 2.4' | + 3.2' | + 3.5' |
| San Blas, Mexico . . . | 1868 | - 9.3° | + 0.1' | + 0.5' | + 0.7' |
| Magdalena Bay, L.Cal. | (1890) | - 10.8° | - 1.8' | - 1.0' | - 0.5' |
| San Diego, Cal. | (1925) | - 14.1° | - 1.8' | - 1.6' | - 1.5' |
| Monterey, Cal. | (1903) | - 16.2° | - 1.8' | - 1.3' | - 1.0' |
| San Francisco, Cal. . . | (1890) | - 16.6° | - 1.0' | - 0.5' | - 0.3' |
| C. Disappointm't, W.T. | (1922) | - 22.6° | - 2.8' | - 2.5' | - 2.2' |
| Sitka, Alaska | 1865 | - 29.1° | + 0.4' | + 1.2' | + 1.6' |
| Unalashka, Alaska . . . | 1834 | - 19.8° | + 1.6' | + 1.9' | + 2.0' |
| Tyrone, Pa. | | | | + 3.3' | |
| Pittsburg, Pa. | | | | + 3.4' | |
| Chicago, Ill. | 1833 | - 6.3° | | + 4.6' | + 5.1' |
| Grand Haven, Mich. . . | 1834 | - 5.3° | | + 6.6' | + 7.3' |
| Madison, Wis. | | | | + 3.9' | |
| Duluth, Wis. } | | | | + 5.2' | |
| Superior City, Wis. . } | | | | | |
| Rio Janeiro, Brazil . . | | | | + 10.3' | + 10.7' |
| San Antonio, Tex. . . . | | | | + 2.1' | + 2.2' |
| Omaha, Neb. } | | | | + 2.6' | |
| Council Bluffs, Ia. . . } | | | | | |
| Denver, Col. | | | | + 1.6' | |
| Salt Lake City, Utah . | 1876 | - 16.7° | | + 0.9' | + 2.0' |

The variability of the change will be noticed. For example, take New York, Philadelphia, and Harrisburg, places comparatively near together.

At New York the change in 1870 was only one-half that at Philadelphia; but, both increasing, this ratio was maintained throughout the 15 years. At Harrisburg, on the contrary, the annual change in 1870 was nearly six-sevenths that at Philadelphia, but the change constantly increasing at the latter place while diminishing rapidly at the former, the annual variation at Harrisburg in 1885 was only a little more than one-half that at Philadelphia.*

245. Effects of the Secular Change. It is evident that if a surveyor should ignore this change, in attempting to establish the corners and to trace the boundary lines of a farm from their description in an old deed, it would be possible for him to return to his place of beginning, but probably none of his other corners would coincide with the true corners.

A line in the vicinity of Philadelphia, which 12 years ago had a bearing N. 19° E., would now bear N. 20° E., and in the same locality a bearing which at that time was recorded N. 19° W. would now be N. 18° W. A variation which, if not corrected, would indicate the end of a line 15 chains long over 26 links from its true position,

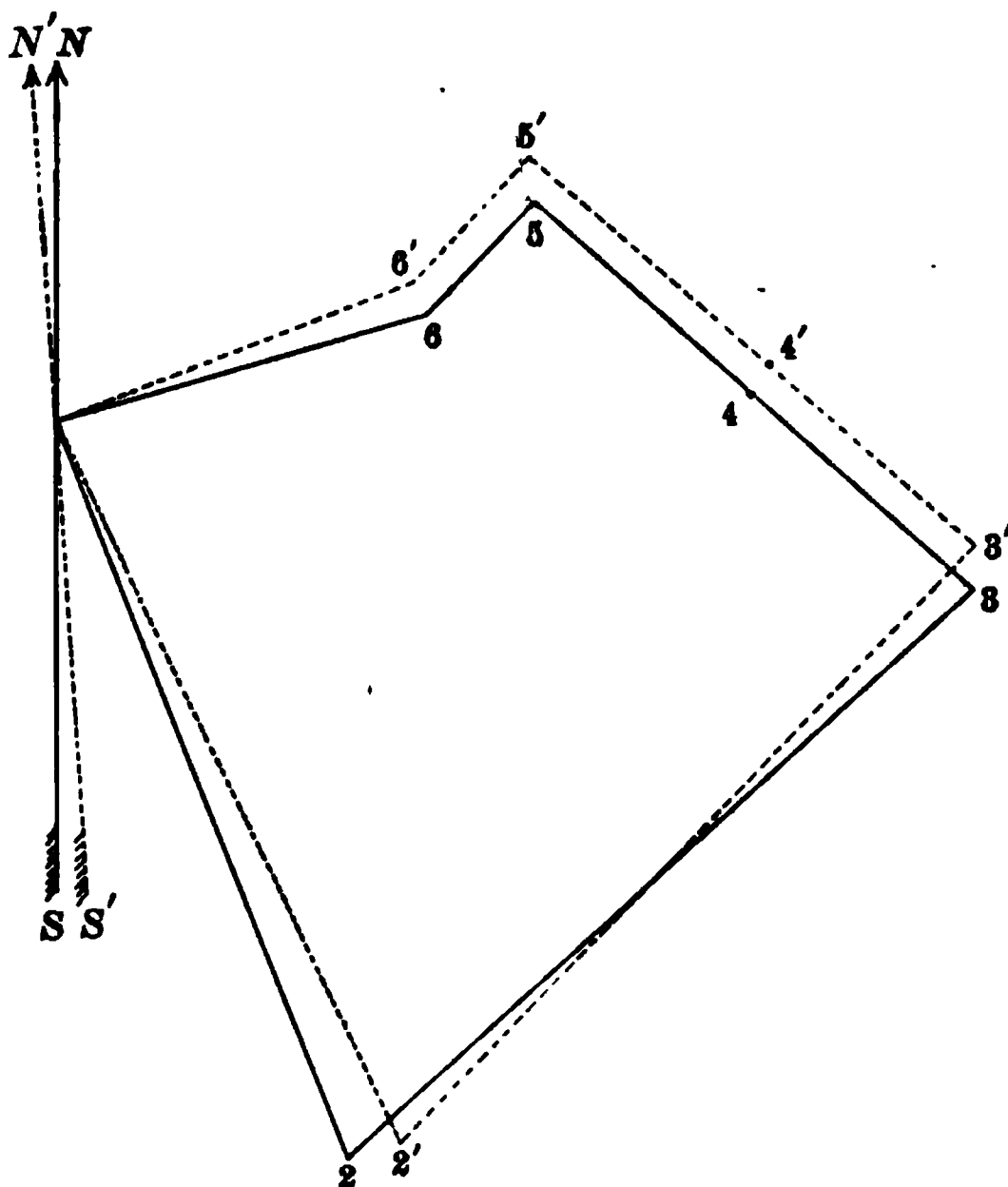
Take, for example, the notes given in Article 208, page 161, and suppose an interval has elapsed sufficient to make the variation two degrees. The accompanying figure shows the true lines and corners; also those corresponding to a survey made without taking the variation into account.

The bearings and distances are as follows:

- | | | |
|-----|------------------------|---------------|
| (1) | S. $20^{\circ} 53'$ E. | 13.11 chains; |
| (2) | N. $48^{\circ} 10'$ E. | 13.62 “ |
| (3) | N. $43^{\circ} 40'$ W. | 4.73 “ |

* For extended investigations on magnetic declination, see U. S. C. & G. S. Reports, 1879, 1881, and 1882.

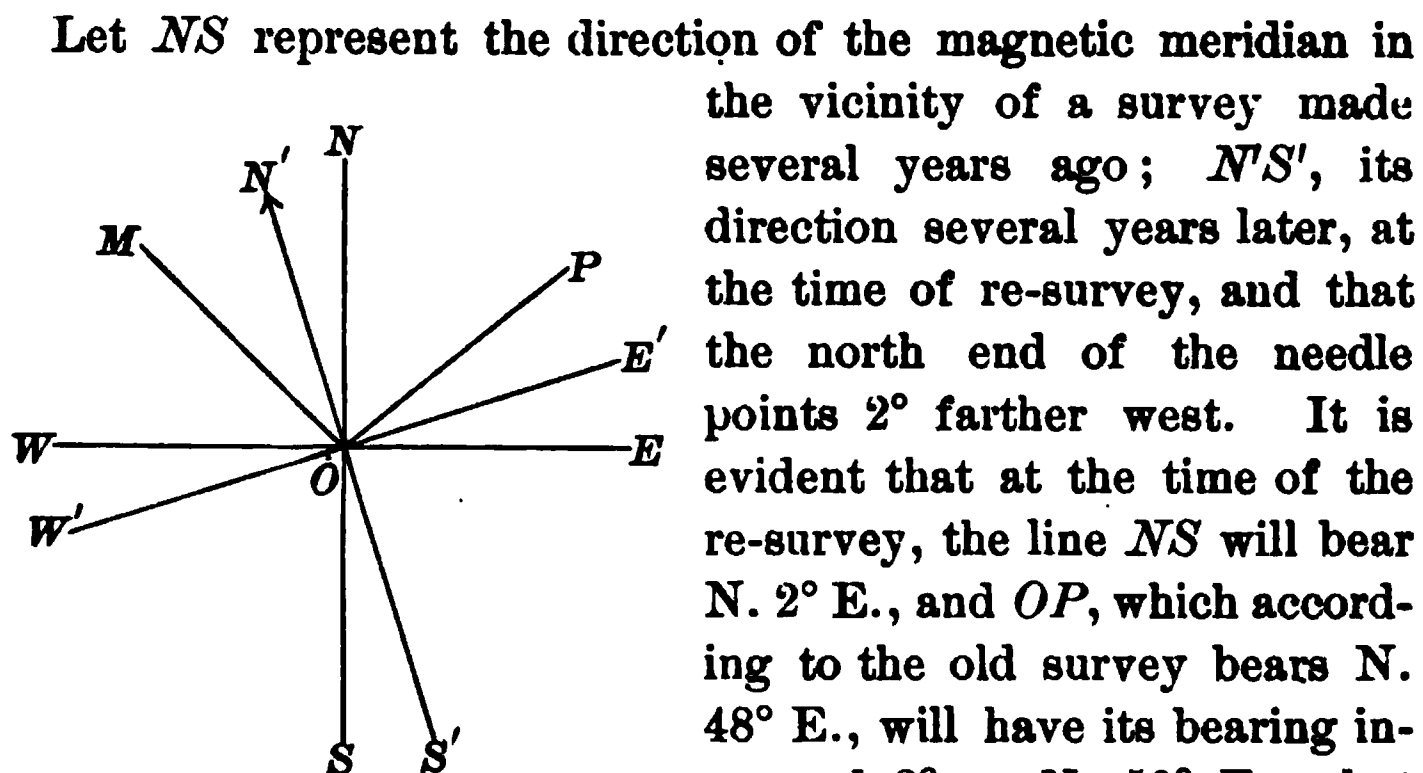
- (4) N. $45^{\circ} 08'$ W. 4.75 chains ;
- (5) S. $51^{\circ} 30'$ W. 2.58 “
- (6) S. $72^{\circ} 30'$ W. 6.56 “



To allow for a variation of two degrees, we should have the following bearings :

- (1) S. $18^{\circ} 53'$ E. ;
- (2) N. $50^{\circ} 10'$ E. ;
- (3) N. $41^{\circ} 40'$ W. ;
- (4) N. $43^{\circ} 08'$ W. ;
- (5) S. $53^{\circ} 30'$ W. ;
- (6) S. $74^{\circ} 30'$ W.

246. To deduce a general rule for obtaining the magnetic bearings of old lines when the variation is known.



Let NS represent the direction of the magnetic meridian in the vicinity of a survey made several years ago; $N'S'$, its direction several years later, at the time of re-survey, and that the north end of the needle points 2° farther west. It is evident that at the time of the re-survey, the line NS will bear $N. 2^\circ E.$, and OP , which according to the old survey bears $N. 48^\circ E.$, will have its bearing increased 2° or $N. 50^\circ E.$; but the line OM , the bearing of which was $N. 42^\circ W.$, will now bear $N. 40^\circ W.$ A line recorded as east will be traced by a course $S. 88^\circ E.$, and so on.

Hence the rule: Increase by the change the bearings which are northeasterly or southwesterly, and diminish by the same amount the bearings which are northwesterly or southeasterly. The foregoing rule is directly applicable *now* in the United States, except on the Pacific coast, because the variation is *west*. That is, the north end of the needle is moving west, thereby increasing the readings of bearings in the $N. E.$ and $S. W.$ quarters, and diminishing the readings of those in the $N. W.$ and $S. E.$ quarters. When it becomes *east*, the words "increase" and "diminish" should be interchanged to make it correct. If a vernier compass is used, the variation may be set off and the lines traced by the old bearings.

247. Change Determined by Old Lines. If the bearing and date of survey of a line are known, and its extremities visible from each other, setting the instrument on one end and sighting the other will give, by comparison with the recorded bearing, the variation.

NOTE. — Care must be taken by the surveyor, when called upon to run out old lines, the corners not being definitely marked, that the time of the former survey be known; the date of the deed does not indicate that of the survey. The description of the lines may have been copied, as they frequently are, from an older deed.

The variation to be applied to correct magnetic bearings is frequently determined in this way.

If the boundaries of a tract of land are to be traced, whether the date of the previous survey be known or not, the surveyor seeks to find, if possible, two consecutive marked corners; then, taking the bearing of these and comparing with the record, he obtains the change sought.

This change, properly applied to each side, should indicate its direction.

It frequently, and in large tracts generally, happens that though the corners at the end of a line may be established, they cannot be observed from each other. In such case run a line as nearly as possible from one corner towards the other by the bearing given in the deed, or make first an allowance which may seem proper from the data at hand; measure from the end of the line thus run the distance to the true corner, and by the 57.3, rule, Article 177; or, by the tangent method, same article, find the angle to be added or subtracted, as the case may require, to correct the bearing with which to run the line. The difference between the bearing given in the deed and the corrected bearing will be the change in the declination since the survey recorded in the deed.

EXAMPLES.

1. A line, said to have been surveyed in 1860; recorded N. 18° 30' E., 24.40 chains, was run in 1885 with a bearing N. 19° 45' E.,—the variation being about 3' to the west per year in its locality, — and the corner was 7 links to the right (farther easterly) of the end of the line run. The corrected magnetic bearing and variation are required.

$$1^{\circ} 15' + \frac{57.3 \times 7}{24.40} = 1^{\circ} 15' + 10' = 1^{\circ} 25' = \text{variation.}$$

Adding the variation to the bearing of the line run, since the true corner was farther to the east, there results N. 19° 55' E. as the corrected magnetic bearing of the line.

2. If in Example 1 the corner had been found 7 links to the *left*, what would be the correct bearing of the line?

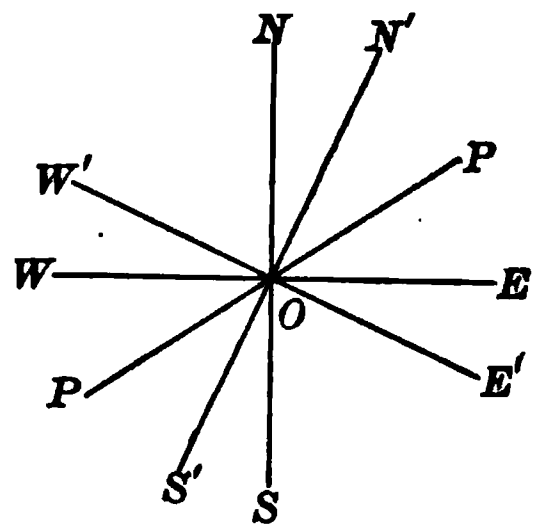
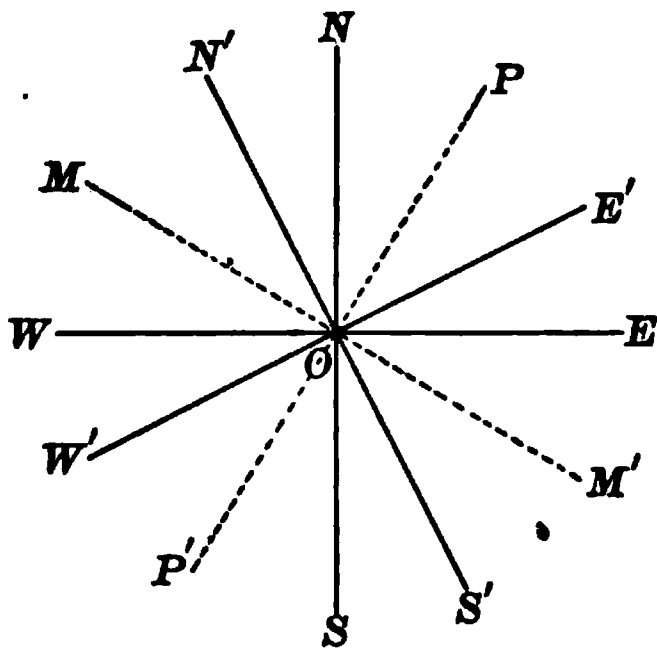
3. A line which in 1862 ran S. $34^{\circ} 15'$ W. 18.56 chains, in 1886 bore S. $35^{\circ} 35'$ W. What was the average change in the declination per year?

4. Give the corrected magnetic bearing for 1886 of a line in the same locality as that in Example 3, which in 1868 ran due east.

5. In 1876 a line had a bearing S. $89^{\circ} 45'$ W. 16.80 chains; in 1886, running by the same bearing, the true corner was 20 links to the north. Give the average annual change, and correct the bearing.

6. If a line 60.00 chains in length were surveyed in the early part of the day, where the needle deviates 5 minutes east of the mean magnetic meridian, and the same line surveyed soon after mid-day, the needle then pointing 5 minutes *west* of the mean magnetic meridian, how far apart would the lines be at their ends, and what the area included between them?

248. To Obtain the True Bearing of a Line, that is, the bearing with respect to the geographical meridian, when the



declination is *west*. Assume NS and $N'S'$ (left-hand figure) to represent respectively the true and magnetic meridian. Then it is evident that the bearing of any line between the north and

east, or south and west, as OP or OP' , will be *less* referred to NS than when referred to $N'S'$ by the amount of the angle $NON' = SOS' =$ the declination.

A line running between north and west, as OM , or south and east, as OM' , will evidently have its bearing increased by the amount of the change.

The reverse is true where the declination is *east*, as may be perceived by reference to the right-hand figure.

Hence, to get the true bearing from the magnetic for all places east of the line of no declination, *i.e.* where the declination is *west*, *subtract* the declination from a bearing which is northeasterly or southwesterly, and *add* the declination to a bearing which is northwesterly or southeasterly. Where the declination is *east*, as at all places west of the line of no declination, *add* the declination to a bearing which is northeasterly or southwesterly, and *subtract* the declination from a bearing which is northwesterly or southeasterly. Where the declination is *west*, a bearing that reads north, when reduced to the true bearing, will evidently be west of north the amount of the declination; if the declination is 3° , the bearing will be N. 3° W., and supposing the same declination, a line running due east magnetically will be truly N. 87° E.

The reverse of the last paragraph is true where the declination is *east*.

REMARK. If, when applying the rule, a negative result is obtained, care must be exercised in the interpretation of it. For example, if the declination is 3° West, and the needle indicates the bearing of a line N. 1° E., there results, by the rule, -2° . This shows simply that the true bearing is to the *west* of north, or N. 2° W. If the bearing is S. 89° E., adding the declination, as the rule requires, gives evidently the reading N. 88° E.

Reduce to their true bearings the following, the declination being $2^\circ 55'$ W.:

N. $2^\circ 15'$ E., East, S. 45° E., South; S. $87^\circ 30'$ W., N. $88^\circ 15'$ W., North.

Also the following, the declination being $3^\circ 40'$ E.:

N. 88° E., East, S. 2° E., South; S. $88^\circ 30'$ W., N. 40° W., North.

249. To Ascertain the Declination.* If a geographical meridian were traced on the earth convenient to the operations of the surveyor, he would have the means always at hand by which to determine the declination. He could simply set up his instrument at a point on the meridian, take the bearing of another point in it, and the reading would be the declination. So the problem resolves itself into the determination of a geographic or true meridian.

250. By Polaris. If there was a celestial object precisely at the point where the prolongation of the earth's axis pierces the celestial sphere, the direction of the meridian could be obtained by simply sighting to the object. This, however, is not the case, but Polaris, or Alpha Ursæ Minoris, is a star whose polar distance is, January, 1887, $1^{\circ} 17' 38''$,† and which apparently revolves about the north pole in 23 hours 56 minutes. It therefore culminates twice daily, and twice it attains its greatest distance directly east and west of the pole, called respectively its eastern and western elongation. If, therefore, the Pole Star could be observed at the instant of its culmination, the line of sight would be in the meridian plane; but since in general the local time of transit is not precisely known, and since the star is then moving at right angles to the plane of the meridian respecting which its motion is at that time a maximum, and consequently a small difference in time would introduce a considerable error in arc, this method is not as reliable as that by means of Polaris at its eastern or western elongation, as then the star for a few minutes appears to move in the direction of the vertical wire, or compass-slit, thus affording a favorable

* For other methods, see Chapter II. Section I., Solar Attachment; and Chapter VI., Art. Solar Compass.

† Its polar distance is diminishing at the rate of $20''$ ($19.06''$) per year. This diminution will continue until the star is within half a degree of the pole, when it will recede.

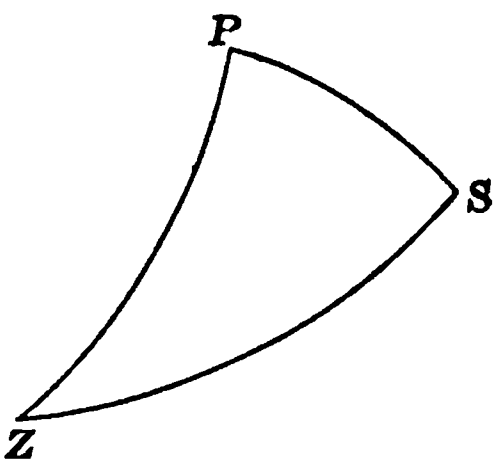
In 1890 its polar distance will be $1^{\circ} 16' 42''$.

In 1900 its polar distance will be $1^{\circ} 13' 38''$.

In 1910 its polar distance will be $1^{\circ} 10' 26''$.

opportunity for observing it, and the precise time of observation need not be known.

Conceive a spherical triangle, the vertices of which are, Z , the zenith of the observer; P , the north pole; and S , Polaris. This triangle, when the star is at an elongation, will be right-angled at the star. In this right-angled spherical triangle are known the co-latitude of the observer's station, and the co-declination or polar distance of the star, to find the azimuth* and hour angle.† Using natural functions, the formula for the hour angle is $\cos P = \tan PS \cot PZ$, and for the azimuth,



$$\sin Z = \frac{\sin PS}{\sin PZ} = \frac{\sin PS}{\cos \text{lat}}. \dagger$$

It may be well to remark, though it has only a theoretical significance, that these formulas are not applicable to *all* north latitudes. In other words, there will be no hour angle shown by the first formula, nor azimuthal angle by the second, on that parallel of latitude which agrees in arc distance with Polaris from the equator, and for any point between that parallel and the pole the formulas fail.

This remark is in general applicable to any circumpolar star.

QUERIES. Is Polaris a longer time passing from eastern to western elongation, than from western to eastern, to an observer whose latitude is 40° ? What is the difference in time to an observer whose latitude is 60° ? 80° ? Where would this difference be a minimum? Where a maximum?

* The azimuth of a star is the angle between the meridian plane and the vertical plane through the star.

† The angle SPZ included between the meridian plane PZ and the plane PS passing through the star.

‡ The azimuth of Polaris at elongation varies with the latitude and with the year, as may be seen by the table on page 217

251. Table of mean local time astronomical (from noon) of the elongations and culminations of Polaris for 1885, latitude 40° , and longitude 6 hours west of Greenwich.

| FIRST DAY OF | E. E. | U. C. | W. E. | L. C. |
|---------------------|--------------|--------------|--------------|--------------|
| | <i>h. m.</i> | <i>h. m.</i> | <i>h. m.</i> | <i>h. m.</i> |
| January | 0 35.3 | 6 29.9 | 12 24.6 | 18 28.0 |
| February | 22 29.0 | 4 27.6 | 10 22.2 | 16 25.6 |
| March | 20 38.5 | 2 37.1 | 8 31.8 | 14 35.1 |
| April | 18 36.4 | 0 35.0 | 6 29.7 | 12 33.1 |
| May | 16 38.6 | 22 33.3 | 4 31.8 | 10 35.2 |
| June | 14 37.0 | 20 31.7 | 2 30.3 | 8 33.7 |
| July | 12 39.5 | 18 34.2 | 0 32.8 | 6 36.2 |
| August | 10 38.1 | 16 32.8 | 22 27.5 | 4 34.8 |
| September | 8 36.6 | 14 31.3 | 20 26.0 | 2 33.3 |
| October | 6 38.9 | 12 33.6 | 18 28.2 | 0 35.5 |
| November | 4 37.0 | 10 31.7 | 16 26.4 | 22 29.7 |
| December | 2 38.9 | 8 33.5 | 14 28.2 | 20 31.6 |

To correct the tabular times so as to apply to any year *subsequent* to 1885, *add* 0.35 minutes for every year. For any year *previous* to that date, *subtract* 0.35 minutes for every year.

For days not given in the table, interpolate, or allow 3.94 minutes for each day, the times varying by this amount.

To allow for difference of latitude between the limits of 30° and 50° , *add* 0.14 for every degree south of 40° ; *subtract* 0.18 for every degree north of 40° .

To refer the tabular times to any year in a quadriennium, observe —

For the first year after a leap year the table is perfect; for the second year after a leap year *add* 1 minute; for the third year after a leap year, *add* 2 minutes; for a leap year, and before March 1, *add* 3 minutes; and for the remainder of the year *subtract* 1 minute.

It will be noticed that there occur two eastern elongations on Jan. 9, and two western elongations on July 9.

AZIMUTH (FROM THE NORTH) OF POLARIS, WHEN AT ELONGATION, BETWEEN THE YEARS 1887-1895, FOR DIFFERENT LATITUDES BETWEEN + 25° AND + 50°.

| Lat. | 1887. | 1888. | 1889. | 1890. | 1891. | 1892. | 1893. | 1894. | 1895. |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| + 25° | 1°25.7' | 1°25.3' | 1°25.0' | 1°24.6' | 1°24.3' | 1°23.9' | 1°23.6' | 1°23.2' | 1°22.9' |
| 26 | 26.4 | 26.0 | 25.7 | 25.3 | 25.0 | 24.6 | 24.3 | 23.9 | 23.6 |
| 27 | 27.1 | 26.8 | 26.4 | 26.0 | 25.7 | 25.4 | 25.1 | 24.7 | 24.3 |
| 28 | 27.9 | 27.6 | 27.2 | 26.8 | 26.5 | 26.2 | 25.8 | 25.4 | 25.1 |
| 29 | 28.8 | 28.4 | 28.0 | 27.6 | 27.3 | 27.0 | 26.6 | 26.3 | 25.9 |
| 30 | 29.6 | 29.3 | 28.9 | 28.5 | 28.2 | 27.8 | 27.5 | 27.1 | 26.8 |
| 31 | 30.5 | 30.2 | 29.8 | 29.4 | 29.1 | 28.8 | 28.4 | 28.0 | 27.6 |
| 32 | 31.5 | 31.2 | 30.8 | 30.4 | 30.1 | 29.7 | 29.3 | 29.0 | 28.6 |
| 33 | 32.6 | 32.2 | 31.8 | 31.4 | 31.1 | 30.7 | 30.3 | 30.0 | 29.6 |
| 34 | 33.6 | 33.3 | 32.9 | 32.5 | 32.1 | 31.8 | 31.4 | 31.0 | 30.6 |
| 35 | 34.8 | 34.4 | 34.0 | 33.6 | 33.2 | 32.9 | 32.5 | 32.1 | 31.7 |
| 36 | 36.0 | 35.6 | 35.2 | 34.9 | 34.4 | 34.0 | 33.6 | 33.2 | 32.9 |
| 37 | 37.2 | 36.8 | 36.4 | 36.0 | 35.6 | 35.2 | 34.8 | 34.5 | 34.1 |
| 38 | 38.5 | 38.1 | 37.7 | 37.3 | 36.9 | 36.5 | 36.1 | 35.7 | 35.3 |
| 39 | 39.9 | 39.5 | 39.1 | 38.7 | 38.3 | 37.9 | 37.5 | 37.1 | 36.7 |
| 40 | 41.4 | 41.0 | 40.5 | 40.1 | 39.7 | 39.3 | 38.9 | 38.5 | 38.1 |
| 41 | 42.9 | 42.5 | 42.0 | 41.6 | 41.2 | 40.8 | 40.4 | 40.0 | 39.6 |
| 42 | 44.5 | 44.1 | 43.6 | 43.2 | 42.8 | 42.4 | 42.0 | 41.5 | 41.1 |
| 43 | 46.1 | 45.7 | 45.3 | 44.9 | 44.4 | 44.0 | 43.6 | 43.2 | 42.7 |
| 44 | 47.9 | 47.5 | 47.1 | 46.6 | 46.2 | 45.8 | 45.3 | 44.9 | 44.4 |
| 45 | 49.8 | 49.4 | 48.9 | 48.5 | 48.1 | 47.6 | 47.1 | 46.7 | 46.2 |
| 46 | 51.8 | 51.3 | 50.9 | 50.4 | 50.0 | 49.5 | 49.0 | 48.6 | 48.2 |
| 47 | 53.8 | 53.4 | 52.9 | 52.5 | 52.0 | 51.5 | 51.0 | 50.6 | 50.2 |
| 48 | 56.0 | 55.6 | 55.1 | 54.6 | 54.2 | 53.7 | 53.5 | 52.8 | 52.3 |
| 49 | 58.3 | 57.9 | 57.4 | 56.9 | 56.5 | 56.0 | 55.5 | 55.0 | 54.5 |
| + 50° | 2°00.8' | 2°00.3' | 1°59.8' | 1°59.3' | 1°58.8' | 1°58.4' | 1°57.9' | 1°57.4' | 1°56.9' |

252. To Establish a True Meridian with a Transit.* See that the instrument is in good adjustment. Allow sufficient time before an elongation of the star to "set up" the transit in a desirable position.† See that it is planted firmly, levelled carefully, and that the cross-wires are illuminated ‡ and properly focused. For convenience, set the vernier at zero, and unclamp the lower plate.

Observe the star a few minutes before its elongation, and keep the vertical wire on it by clamping the lower plate and using the slow-motion screws attached to it. When it has attained its greatest elongation, it will appear for a few moments to coincide with the vertical wire, and then retrograde. Unclamp the vernier plate, and turn off with it the amount of the azimuth § corresponding to the time and place as given in the table of the preceding article. The telescope will then point in the direction of the true meridian, and a mark should be set at as long range as practicable. If preferred, a stake may be set in line of sight at elongation, leaving the turning off of azimuth, and setting mark in meridian until the next day. It would be a little more accurate to take the mean of several observations — direct and reverse — at eastern and western elongations.

* See Solar Attachment, Chapter II. Section I.; also Solar Compass, Chapter VI.

† Twenty to thirty minutes usually, depending upon the observer.

‡ Perforated silvered reflectors, for this purpose, can be obtained of instrument makers. Or, cover with white paper a board 12 or 15 inches square, make a perforation through it of 2 or 3 inches' diameter, and nail on a piece of board to hold a candle. This reflector may be attached to a staff, that it can slide up and down, and adjusted to the height of the telescope. It should be placed about a foot from the object-glass, so that the reflection from the paper will render the cross-wires visible, and at such a height that the star can be observed through the opening.

§ The meridian will lie to the west or east of the direction of the telescope when elongation was observed, according as the elongation was east or west. The azimuth must be turned off accordingly. Since the direction of the line from the observer's station to the star at elongation is known, the declination may be ascertained even before the meridian is established.

253. The Direction of the Meridian may be found, though less accurately, by means of a compass-sight and plumb-line.

Take a smooth plank about 3 feet in length, and fix it firmly level, and nearly east and west, on supports about 2 feet high. Attach a compass-sight to a board 6 or 8 inches square. At 15 or 20 feet north of the plank suspend a plumb-line by artificial supports, from some projecting point on a building or at the end of a staff projecting from a high window.

At fifteen or twenty minutes before the time of elongation of the star let an assistant hold a light in such position that the plumb-line may be distinctly seen through the compass-sight when placed on the plank. Move the sight until the plumb-line covers the star. Continue to keep the star and line in that relative position until the star begins to retrograde. The direction of the line of sight then corresponds to that observed by the transit as indicated in the preceding article; and applying the azimuth therein directed, the meridian may be set out.*

254. To Obtain approximately the Meridian. In old works on surveying it is stated that the north star (Polaris) is very nearly the meridian when it and Alioth † are in the same vertical plane or line. Others add the time that must elapse after one is vertically above the other before the north star makes its transit, and then by sighting the north star at that instant the meridian may be found.

This interval is, January, 1887, nearly half an hour. Other stars are now used, being more suitable. Zeta, or Mizar, the star next to Alioth in the tail of the Great Bear, comes to the meridian now almost simultaneously with Polaris and at a convenient time in the autumn and early winter to make the obser-

* If possible, a night should be chosen when there is no wind. The slightest disturbance in the air causes considerable vibration of the plumb-line. Using a heavy "bob," and allowing it to vibrate in a vessel of water, will tend to the accuracy of the result.

† Alioth, or Epsilon: the star in the tail of the Great Bear nearest the quadrilateral.

vation. Delta Cassiopeiæ, which is on the same side of the pole as Polaris, makes its transit also about the same time with it, and may be used in the spring and early summer when it is not practicable to make use of Zeta. To make either of these observations, use a transit, or a plumb-line and compass-sight, as explained in the preceding articles; watch the movements of the stars until they coincide with the plumb-line. The direction of the line of sight then will indicate quite closely the meridian.*

* The vertical plane including Zeta and Polaris is slowly moving eastward at about the rate of two minutes in six years. At the present time (1887) Polaris is on the meridian about two minutes before Zeta of the Great Bear, but in six years their respective upper and lower transits will coincide. The vertical plane, including Delta Cassiopeiæ and Polaris, is moving westward at about the same rate. Polaris now comes to the meridian about one minute before this star.

CHAPTER IV.

LAYING OUT AND DIVIDING LAND.

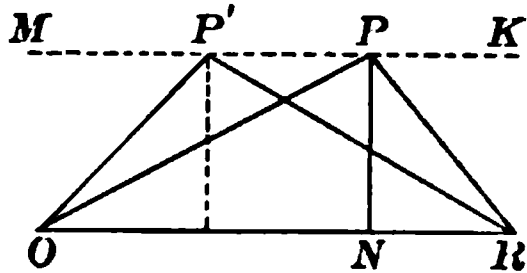
SECTION I.

LAYING OUT LAND.

A. TRIANGLES.

255. *To lay out a given quantity of land in the form of a triangle when the length of the base is given.*

Denote the given area in square chains or square rods by A ,* the length of the base (referred to the same unit) by b , and the unknown altitude by x . Then $\frac{bx}{2} = A$, or $x = \frac{2A}{b}$. Measure the base, and at any point in it erect a perpendicular equal to $\frac{2A}{b}$. Join the ex-



tremity of the perpendicular with the extremities of the base, and a triangle fulfilling the conditions of the question will be exhibited.

256. *When the area is given and the base and altitude in a given ratio.*

* Why not let A denote the number of acres?

NOTE. The locus of the vertices of the triangles answering the conditions is a line parallel to the given base and at a distance therefrom $= \frac{2A}{b}$.

Designate, as before, the area by A , the base and altitude respectively by x and y , and $\frac{x}{y} = \frac{m}{n}$ the ratio; then

$$y = \sqrt{\frac{2An}{m}},$$

$$x = \sqrt{\frac{2Am}{n}}.$$

Or, let $mx = \text{base}$ and $nx = \text{altitude}$; then

$$mnx^2 = 2A;$$

whence

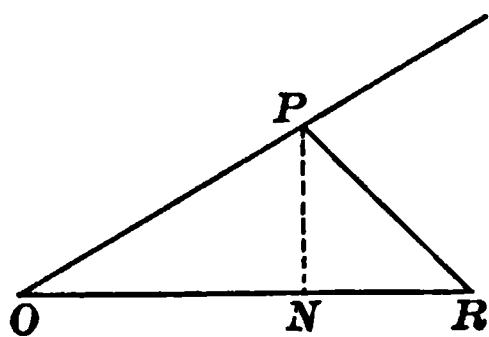
$$x = \sqrt{\frac{2A}{mn}},$$

and

$$mx = \sqrt{\frac{2Am}{n}},$$

$$nx = \sqrt{\frac{2An}{m}}.$$

257. *Given area, base, and one side, to make a given angle with the base.*



Denote the base and area as above; then, since

$$PN = OP \sin O,$$

and

$$A = \frac{b \times OP \sin O}{2},$$

$$OP = \frac{2A}{b \sin O}.$$

EXAMPLES.

1. Lay out an isosceles triangle to contain 6 acres, making the base $\frac{5}{8}$ the altitude. Locate the altitude and find its length.
2. Lay out a right triangle containing 4 acres, having the base $\frac{4}{5}$ the altitude.
3. It is required to lay out 2 acres in the form of a triangle, the base to be 7.50 chains. Find the length of a side of this triangle which shall make an angle of 40° with the base.

258. *To lay out an equilateral triangle to contain a given area.*

Let x = the side, and A = the area; then, since

$$\frac{x^2}{4} \sqrt{3} = A,$$

$$x = 2 \sqrt{\frac{A}{\sqrt{3}}} = \sqrt{\frac{A}{.433}}.$$

259. *Given the area and the two sides, to lay out the triangle.*

Denote the given sides by b and c , the area by A , and the unknown angle by a ; then, since

$$\frac{bc}{2} \sin a = A,$$

$$\sin a = \frac{2A}{bc}.$$

EXAMPLES.

1. Find the side of an equilateral triangle containing one acre.

2. What is the altitude of the triangle in Example 1? How far is it from the foot of the perpendicular to the centre of the figure? How far from either angle to the centre?

3. Lay out a triangle containing 2 acres, two sides to be 8 chains and 6 chains. What must be the included angle?

B. QUADRILATERALS.

SQUARES.

260. *To lay out a given quantity of land in the form of a square.*

Denote the required area in square chains or square rods by A , and one of the sides by x ; then $x = \sqrt{A}$.

Measure a distance equal to the \sqrt{A} ; at each extremity of this line erect a perpendicular of the same length; connect the extremities of the perpendiculars; the figure will be a square.

RECTANGLES.

261. *To lay out a given quantity of land in the form of a rectangle, one side being given.*

Denote, as before, the area by A , the given side by b , and by x the unknown side ; then

$$x = \frac{A}{b}.$$

262. *Given the area, and the length to the breadth in a given ratio.*

Denote the area as above ; the length and breadth respectively by x and y ; m and n their ratio, so that

$$\frac{x}{y} = \frac{m}{n}.$$

Then, since $xy = A$, there results, by substitution,

$$x = \sqrt{\frac{Am}{n}},$$

$$y = \sqrt{\frac{An}{m}}.$$

Or, let $mx =$ the length, and $nx =$ the breadth ; then

$$mnx^2 = A ;$$

whence
$$mx = \sqrt{\frac{Am}{n}},$$

and
$$nx = \sqrt{\frac{An}{m}}.$$

263. *Given the area and the sum of the length and breadth.*

Denote the sum of the sides by S ; the other notation as above ; then

$$xy = A,$$

and
$$x + y = S ;$$

whence
$$x = \frac{S + \sqrt{S^2 - 4A}}{2},$$

and
$$y = \frac{S - \sqrt{S^2 - 4A}}{2}.$$

264. *Given the area and the difference of the length and breadth.*

Denote the difference of the sides by d ; the other notation as before; then

$$xy = A,$$

$$x - y = d;$$

whence

$$x = \frac{\sqrt{d^2 + 4A} + d}{2};$$

and

$$y = \frac{\sqrt{d^2 + 4A} - d}{2}.$$

EXAMPLES.

1. How many rods in each side of a square lot which contains 1 acre? How many chains? How many yards?
2. Lay out 6 acres in the form of a rectangle, the length of one side to be 10 chains. Find the adjacent side.
3. Find the sides of a rectangle which shall contain 15 acres, and the length $\frac{3}{2}$ the breadth.
4. It is required to lay out a rectangle containing 12 acres, so that the sum of two adjacent sides shall equal 26 chains. What must be the length and breadth?
5. Find the sides of a rectangle which shall contain 640 square rods, and the difference of whose sides is 10 rods.

PARALLELOGRAMS.

265. *To lay out a given quantity of land in the form of a parallelogram, the base being given.*

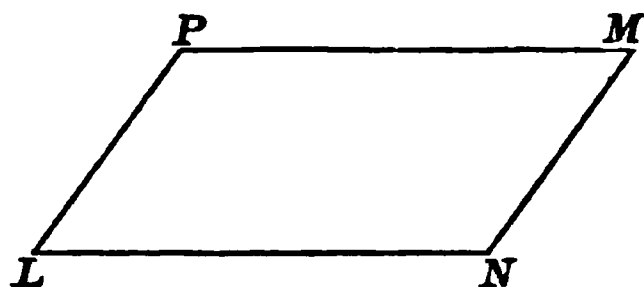
Denote the area and base, as above, and the altitude by x ; then

$$x = \frac{A}{b}.$$

From any point in the base erect a perpendicular equal to $A \div b$, and through the extremity of the perpendicular run a line parallel and equal to the base: a parallelogram will thus be formed, fulfilling the conditions of the question.

266. *Given the area, one side, and adjacent angle.*

Denote the area by A , the base by b , the given angle by α , and by x the side adjacent; then



$$bx \sin \alpha = A;$$

whence
$$x = \frac{A}{b \sin \alpha}.$$

Turn off at L and N , the given angle, measure the distances LP and NM , equal x , and connect M and P for the desired figure.

267. *Given the area and two adjacent sides, to find the included angle.*

Denote the sides by b and c , their included angle by α , and the area as above; then

$$bc \sin \alpha = A;$$

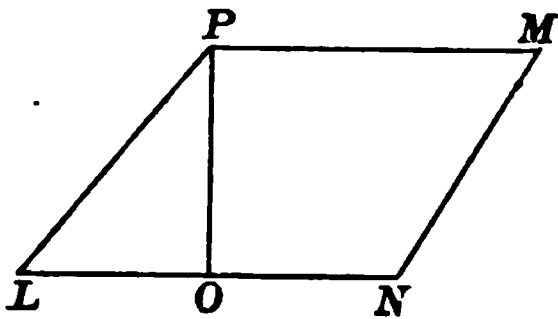
whence
$$\sin \alpha = \frac{A}{bc}.$$

QUERIES. What will the figure become when $bc = A$? When $b = c$? May the product of bc be less than A ? Can an expression for the sine be obtained for each case?

EXAMPLES.

1. It is required to lay out a parallelogram to contain 200 square rods, having a base of 20 rods. What must be the altitude?

2. If in Example 1 it is required that the perpendicular shall be erected at the middle of the base, and terminate at the angle P , as per figure, what length must be given LP , and what the magnitude of the angle L ?



3. It is required to lay out a parallelogram to contain 48 square chains, one side to be 8 chains, and the adjacent angle 70° . What must be the length of the adjacent side?

4. It is required to lay out a parallelogram to contain 2.4 acres, the base and adjacent side to be respectively 6 and 5 chains. Determine the altitude and tell how to lay out the land.

5. It is required to lay out a rhombus to contain 32 square chains, each side to be 6 chains. Compute the altitude, and state how to set out the tract; that is, to establish every corner.

C. POLYGONS.

268. *To lay out a given quantity of land in the form of a regular polygon of any number of sides.*

Denote the area by A , the number of the sides by n , and the length of one of the sides, as PN in the figure, by x , and ON , the radius of the circumscribed circle, by y ; then

$$n \times OL \times LN = A.$$

But the angle $LON = \frac{180^\circ}{n}$, $OL = \frac{x}{2} \cot \frac{180^\circ}{n}$,

and

$$LN = \frac{x}{2}.$$

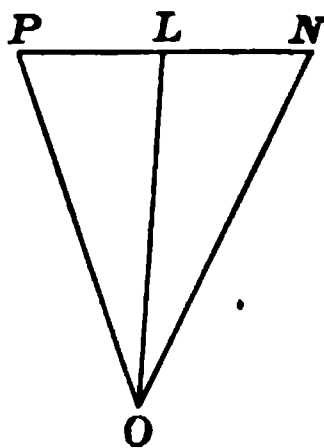
$$\therefore n \times \frac{x^2}{4} \times \cot \frac{180^\circ}{n} = A.$$

Whence

$$x = 2 \sqrt{\frac{A \tan \frac{180^\circ}{n}}{n}},$$

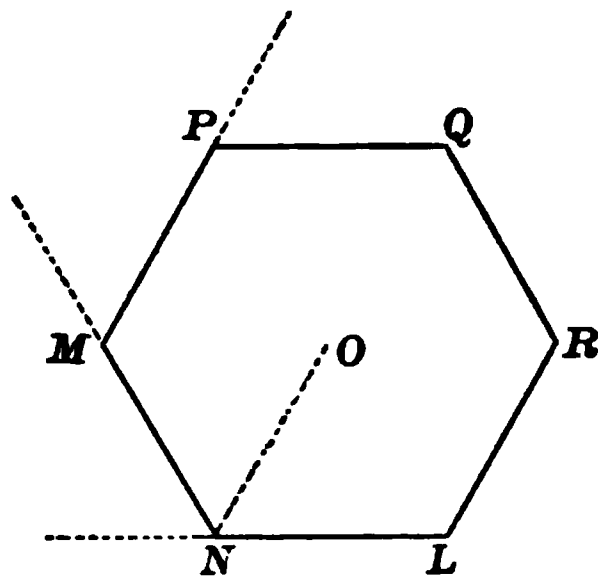
and

$$y = \frac{x}{2 \sin \frac{180^\circ}{n}} = \frac{x}{2} \operatorname{cosec} \frac{180^\circ}{n}.$$



To lay out the tract, find by the above formula the length of one side, as LN , and stake it out. Then with an instrument for measuring angles (transit) set up at one end, as N , sight L , plunge the telescope, deflect $\frac{360^\circ}{n}$ to M . Measure $NM = NL$. Remove the instrument to M , deflect from the prolongation of MN , as before, $\frac{360^\circ}{n}$, measure MP , and so continue around, locating PQ ,

and finally returning to L . The figure will be the polygon required.



In a small polygon, if the centre is fixed, it will be better to set up on it and measure therefrom a distance y to N , turn off an angle (the instrument still at the centre) $= \frac{360^\circ}{n}$, and measure the same distance to M , again turning off an angle equal to the last, measure the same distance to P , and so on. A stake planted at each extremity of the radial lines will indicate the angular points of the tract.*

EXAMPLES.

1. Show how to lay out 1210 square yards in the form of an octagon. The same for a pentagon; decagon.
2. Show how by Article 57 the length of a side of a polygon of a given area and any number of sides, within the limits of the table, may be found.

D. CIRCLES AND ELLIPSES.

CIRCLES.*

269. *To lay out a given quantity of land in the form of a circle.*

Denote the area by A , and the radius by x ; then, since $\pi x^2 = A$,

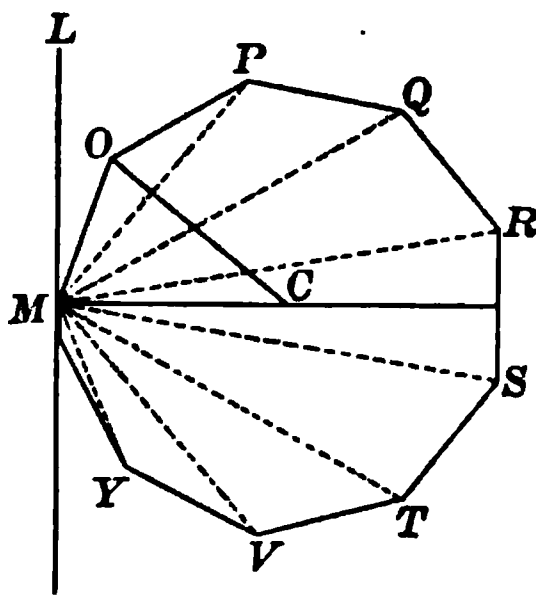
$$x = \sqrt{\frac{A}{\pi}}.$$

* A small lot, when great accuracy is not required, may be laid out by fastening one end of a tape at O , and with a length ON mark out a circumference by means of a pin. Then, beginning at any point in the circumference, measure off the distance x , and continue round the curve, driving a stake at the extremity of each side.

When great accuracy is not required, and small circles generally may be laid out by fastening one end of a tape at the centre, and with a common marking-pin held firmly and perpendicularly along it at x distance, describe and mark out the circumference.

270. Or, fix the extremities of two diameters run out perpendicular to each other, connect these with chords, and the versed sine of 45° to the known radius will give at once the perpendicular distance from the centre of each chord to the circumference. If necessary, the points thus located may be connected and others found in a similar manner. Or the perpendicular distance from any given point in a chord, of known length, to the circumference may be found by simple geometrical truths deduced from the right triangle.

271. If the circle is too large to be laid out as above, it may be accomplished by means of deflection angles as follows: With the known radius find the angle at the centre C , which is subtended by a chord OM of any length, say 100 feet; then with the instrument at M , deflect from the tangent ML to O an angle $LMO = \text{one-half the central angle } OCM$, and measure the distance $MO = 100$ feet. O is a point in the curve.* Again deflect an angle $OMP = \text{one-half the central angle}$, and measure $OP = 100$ feet to locate P , another point in curve,* and so on to locate the others. If there is a fractional part of the deflection angle at the closing point, the corresponding fractional part of 100 feet may be used.



* The angle formed by the tangent and chord drawn to the point of contact is measured by one-half the intercepted arc. An inscribed angle has the same measure.

ELLIPSES.

272. *To lay out a given quantity of land in the form of an ellipse, the greater and lesser diameters to be in a given ratio.*

Denote the area by A , the greater and less diameter (axes) respectively by mx and nx , in which m and n express the given ratio; then

$$\frac{\pi}{4} mx^2 = A; *$$

whence

$$x = \sqrt{\frac{4A}{\pi mn}},$$

$$mx = 2\sqrt{\frac{Am}{\pi n}},$$

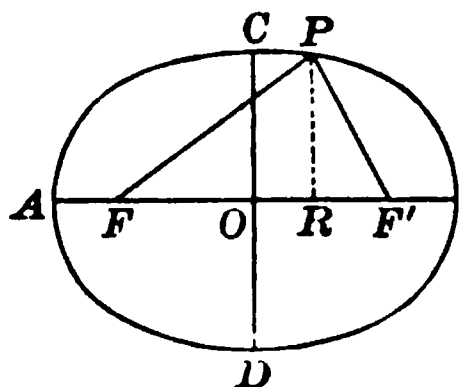
and

$$nx = 2\sqrt{\frac{An}{\pi m}}.$$

273. *Given the area and one of the diameters, to find the other diameter.*

Denote the given diameter by d , the unknown by x , and the area as before; then, since

$$\frac{\pi}{4} dx = A,$$



we have

$$x = \frac{4A}{\pi d}.$$

An ellipse of small size may be laid out as follows:

Measure AB equal to the greater diameter (transverse axis), and from the centre O lay off $OF = OF'$, each equal to the square root of the difference of the squares of the semi-diameters OA , OC . Fix the ends of a steel wire or ribbon of the length AB at F and F' , and with a continuous motion of a marking-pin P , held perpendicularly, keeping the wire taut, the required curve will be traced.

* See any work on General Geometry or Conic Sections for the area of an ellipse.

Or, having found the axis as above, P being any point in the curve, and PR perpendicular to AB at R , by setting off any number of points on AB , we may find from the proportion

$$\overline{PR}^2 : RB \times AR = \overline{OC}^2 : \overline{OA}^2,$$

the corresponding values of PR .

EXAMPLES.

1. Find the radius of a circle containing 1 acre.
2. Find the radius of a sector containing 20 square rods, the angle at the centre being 72° .
3. The area of an ellipse is 1 acre, its diameters in the ratio of 3 : 2 ; find their length.
4. An ellipse contains 80 square rods, its greater diameter 12 rods ; find the lesser diameter.
5. The greater diameter of an elliptical plot of ground enclosed by a wall 1 foot thick is 240 links, and the lesser 160 links, inside measurements. What is the area of the plot, and how much land is occupied by the wall?

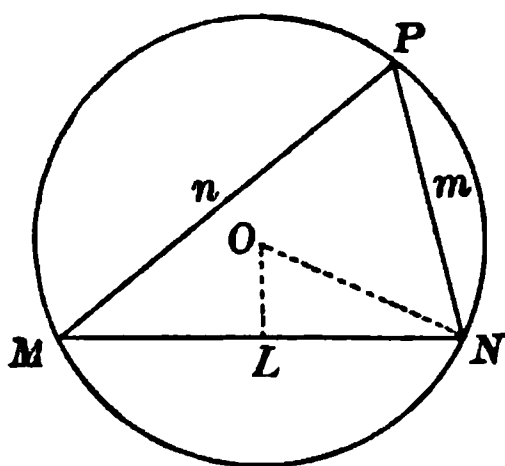
274. Let it be required to lay out a circle circumscribing a triangle, the sides of which are m , n , and p .

Let O be the centre of the circle, R the radius, OL a perpendicular to MN , $p = MN$, and the other sides as indicated in the figure.

Now $NL = \frac{p}{2}$, and angle $NOL = P$.

$$\therefore \frac{p}{2} = R \sin P,$$

$$\text{or} \quad R = \frac{\frac{p}{2}}{\sin P} = \frac{p}{2 \sin P}.$$



To find an expression for R in terms of the three sides, substitute for $\sin P$ its value

$$2 \sin \frac{1}{2} P \cos \frac{1}{2} P = 2 \frac{\sqrt{\frac{1}{2}s(\frac{1}{2}s - m)(\frac{1}{2}s - n)(\frac{1}{2}s - p)}}{mn};$$

$$\text{whence } R = \frac{mnp}{4\sqrt{\frac{1}{2}s(\frac{1}{2}s - m)(\frac{1}{2}s - n)(\frac{1}{2}s - p)}},$$

in which s represents the sum of the sides of the triangle.

ADDITIONAL EXAMPLES.

1. Circumscribe a circle about a triangle the sides of which are 10, 15, and 20 chains.

2. Find an expression for the radius with which to inscribe a circle in a triangle the sides of which are m , n , and p .

Ans. Twice the area of the triangle, divided by the sum of the sides.

3. Describe a circle in a triangle the sides of which are 30, 40, and 50 rods.

5. A circular walk, 6 feet wide, is to be made inside of a square which contains $\frac{1}{2}$ an acre; required the area of the walk.

5. The area of a square is 1 acre, and a circular walk is required to be made in it, touching each side at a point, of such a width that it will take up $\frac{1}{8}$ the area of the square. Find the width of the walk and the length of its centre line.

6. The area of a circular sector of d° is m rods; find an expression for the radius. If $d = 60$ and $m = 300$, find R .

SECTION II.

DIVIDING LAND.

A. TRIANGLES.

275. *To divide a given triangle into two parts in the ratio of $m : n$ by a line parallel to one side.*

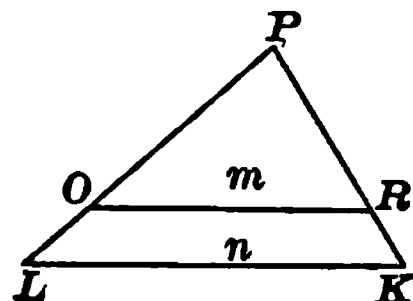
To solve the problem fully, and furnish a check on the work, requires the location of the point O or R , and the length of OR . Denote OR by x , OP by y , and by p and k the sides respectively opposite the angles P and K ; then

$$p^2 : x^2 = m + n : m,$$

or
$$x = p \sqrt{\frac{m}{m+n}}.$$

Again,
$$k^2 : y^2 = m + n : m;$$

whence
$$y = k \sqrt{\frac{m}{m+n}}.$$



If the triangle is to be equally divided, then $m = n$, and there results

$$x = \frac{p}{2} \sqrt{2}, \quad \text{and} \quad y = \frac{k}{2} \sqrt{2}.$$

QUERIES. Is it necessary that LK be known to find either PO or PR ? Must LK be given to find OR ?

EXAMPLES.

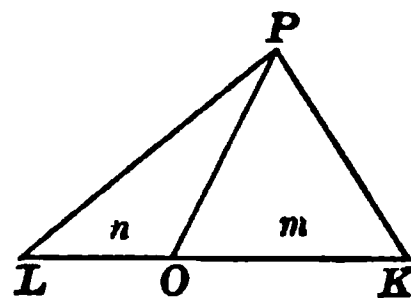
1. Find a general expression for the distance RK (last figure).
2. Show how to divide the triangle LKP into four equivalent parts by lines parallel to the base.

276. To divide a given triangle into two parts in the ratio of $m : n$ by a line from a vertex to the opposite side.

Let PO be the line, $x = LO$, and p as above. Then, since triangles having the same altitude are to each other as their bases, we have

$$p : x = m + n : n;$$

whence
$$x = \frac{pn'}{m+n}.$$



EXAMPLES.

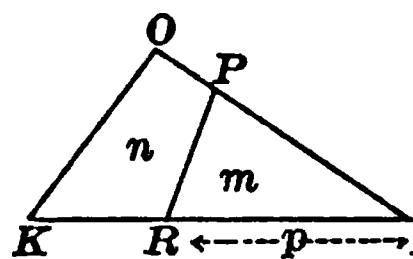
1. Locate O on the supposition that the triangle is to be divided into two equivalent parts.

2. Find where the lines from P will meet the base dividing the triangle into three equivalent parts.

3. The same for any number n parts.

277. To divide a given triangle into two parts in the ratio of $m : n$ by a line through a given point in one of the sides.

Denoting PL by x , and the other sides in the usual manner, we have



$$m + n : m = ko : px;$$

whence $x = \frac{mko}{p(m+n)}.$

If the parts are to be equivalent, $m = n$, and there results

$$x = \frac{ko}{2p}.$$

EXAMPLE.

Show how the given triangle LKO may be divided into three equivalent parts by lines radiating from a given point R .

NOTE. The lines may or may not fall on the same side. Examine both cases.

278. The same conditions as in the last case, except the triangle is to be isosceles.

Using the same notation and figure as in that case, we have the following equality of ratios :

$$m + n : m = ko : x^2;$$

whence $x = \sqrt{\frac{mko}{m+n}}.$

If the parts are to be equivalent, $m = n$, and we have

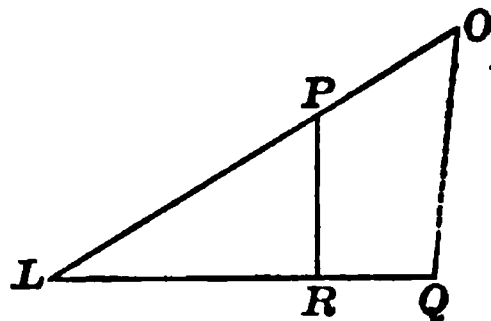
$$x = \sqrt{\frac{ko}{2}} = \frac{1}{2}\sqrt{2ko}.$$

EXAMPLE.

Show how to cut off a given area, in the form of an isosceles triangle, from the corner of a field, only the angle being given.

279. *The bearings of two sides of a field being given, to cut off a triangle having a given area by a line running in a given direction and intersecting the given sides.*

a. Suppose the division line is to make a right angle with either side. Let LO and LQ be the sides, the bearings of which are known, and PR the division line perpendicular to LQ . The angle L becomes known through the bearings of the sides which include it, and there follows



$$p \tan L = PR = l.$$

But
hence

$$\frac{1}{2}pl = \text{area} = A;$$

$$\frac{1}{2}p^2 \tan L = A,$$

and

$$p = \sqrt{\frac{2A}{\tan L}}.$$

b. Suppose the angle at R is oblique. Denote LR by x , and LP by y , and find from the bearings the angles at P and R . Then from the two equations,

$$\frac{1}{2}xy \sin L = A$$

and

$$\frac{x}{y} = \frac{\sin P}{\sin R}$$

may be deduced

$$x = \sqrt{\frac{2A \sin P}{\sin L \sin R}}$$

and

$$y = \sqrt{\frac{2A \sin R}{\sin L \sin P}}.$$

QUERY. Is it necessary that the bearings of LO and LQ be given if the field is triangular and the length of the sides given?

EXAMPLES.

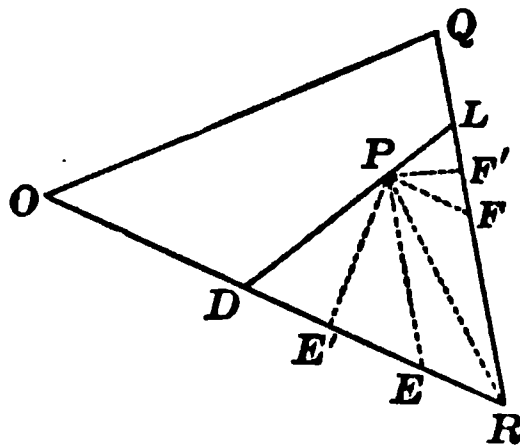
1. The bearing of LO (last figure) is N. 50° E., and LQ S. 82° E. It is required to find the lengths of LR and PR perpendicular thereto, so that 3 acres may be contained in the triangle PLR .

2. Suppose $LO = 10$, $LQ = 8$, and $OQ = 6$ chains. Find the position and length of the division line PR , which, with an angle $PRL = 84^\circ$, will cut off a triangle PRL containing 2.5 acres.

3. Show that if three lines be drawn connecting the middle points of the three sides of a triangle, the four triangles thus formed will be equal.

280. To divide in a given ratio a given triangle by a line passing through a given point within it.

Let OQR represent the given triangle, and P the point within; DL the required division line, and $DRL : LDOQ = m : n$.



The point P may be located by coordinates as PF and PE , lines parallel respectively to OR and QR ; or by its bearing and distance from one of the corners, as R ; or by perpendicular distances PF' , PE' from the sides.

The distances PF and PE may be calculated if the direction and distance PR be known. Denote PF by d , PE by b , DR by x , and RL by y ; then

$$x : y = d : y - b, \quad \text{or } xy = bx + dy;$$

and $xy : qo = m : m + n$, or $xy = \frac{mqo}{m + n};$

hence $bx + dy = \frac{mqo}{m + n}.$

Or, substituting the value of $y = \frac{mqo}{(m + n)x}$ from equation above, we obtain

$$bx + \frac{dmqo}{(m + n)x} = \frac{mqo}{m + n};$$

whence, by reducing and completing the square, there results

$$x = \frac{mqo \pm \sqrt{m^2 q^2 o^2 - 4 b d m q o (m + n)}}{2 b (m + n)},$$

$$y = \frac{2bmgo}{mgo \pm \sqrt{m^2q^2o^2 - 4bdmgo(m+n)}}.$$

If the question were to cut off from a corner of a tract of land a given area, by a line passing through a given point within, we might proceed more simply, as follows :

Denote the area to be cut off by A , and the other notation as above ; then

$$xy \sin R = 2A,$$

and

$$x : y = d : y - b ;$$

whence there results

$$x = \frac{A \pm \sqrt{A^2 - 2Abd \sin R}}{b \sin R},$$

$$y = \frac{2Ab}{A \pm \sqrt{A^2 - 2Abd \sin R}}.$$

In each of the two preceding problems there are in general two division lines, as indicated by the double sign, fulfilling the conditions of the question. The student will point out when, if ever, one of these results will not practically answer the first case. Would either result answer practically the second? When, if ever, would the result be imaginary? Why?

If P were located by its distance PR , and the angle PRL or PRD , the lines PF and PE could be calculated, as before remarked, and the solution above given made applicable ; or we may proceed as follows :

Denote PR by d , $d \sin PRD$ by b , $d \sin PRL$ by c , and the other notation as above ; then

$$xy \sin DRL = 2A,$$

$$bx + cy = 2A.$$

Substituting the value of y from the first equation in the second, and reducing, there results,

$$x^2 - \frac{2Ax}{b} = -\frac{2cA}{b \sin R};$$

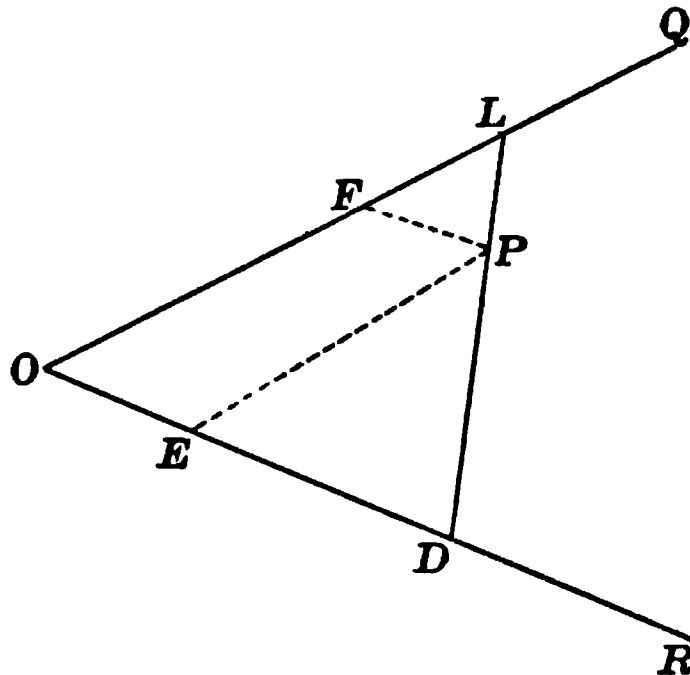
whence
$$x = \frac{A}{b} \pm \sqrt{\frac{A^2}{b^2} - \frac{2cA}{b \sin R}},$$

$$y = \frac{2A}{\left[\frac{A}{b} \pm \sqrt{\frac{A^2}{b^2} - \frac{2cA}{b \sin R}} \right] \sin R};$$

or,
$$y = \frac{2Ab}{A \sin R \pm \sqrt{A^2 \sin^2 R - 2bcA \sin R}}.$$

EXAMPLES.

1. Given the three sides of a triangular tract of land (see last figure), $QR = 17$, $OQ = 19$, and $OR = 22$ chains, to divide it into two equivalent parts by a line passing through a point P , within the field. PF and $PE =$ respectively 4 and 9.50 chains. The location and length of the division line are required.



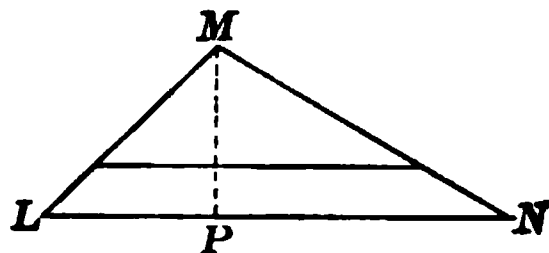
2. It is required to cut off from the angle O , which is 60° , a triangular field to contain 10 acres, by a line DL passing through a point P . The distances PF and PE being 4 and 12 chains respectively, the location and length of the division line are required.

3. Given the angle $ORQ = 56^\circ$ (see last figure but one), $PRL = 20^\circ$, and $PR = 12$ chains. It is required to cut off a

triangle DRL , containing 8 acres, by a line DL passing through the point P . The location and length of the division line are required.

4. Divide a triangular piece of land into three equal parts by lines radiating from a point within.

SUGGESTION. The locus of the vertices of all triangles having the base LN and one-third the area of LMN is a line parallel to LN and at $\frac{1}{3}$ the height PM . Similarly for any other side. Find point of intersection.



5. Apply the principle employed in Example 4 to divide a triangle into three parts, in the ratio of 1, 2, and 3, by lines radiating from a point within.

6. Given two sides of a triangle 6 and 8 chains; it is required to locate a division line which shall cut off from the vertex an isosceles triangle whose area shall be to the area of the given triangle as 3 : 4.

7. Given the sides of a triangle 8, 10, and 12 chains; it is required to divide it into a triangle and a trapezium, the ratio of the former to the latter as 2 : 3, by a line extending from the middle of the longest side to some point on the medium side.

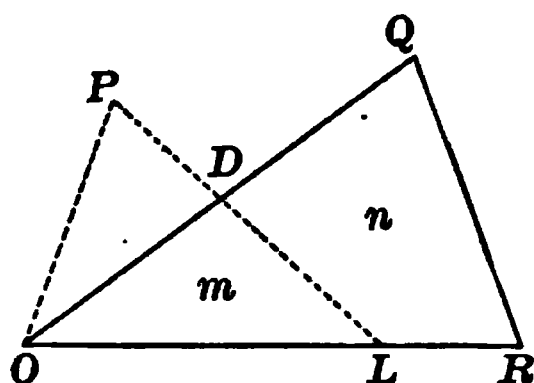
The location of this point and the length of the division line are required.

8. Divide the triangle given in Example 7 into three equivalent parts by lines radiating from the middle of the longest side. Locate the extremities of the division lines.

9. An angle QOP of a field $= 42^\circ 30'$; it is required to cut off from some point D , in the line OP , by a line DL , making an angle $LDO = 78^\circ 30'$, a triangle containing 2 acres. Locate the division line, and determine its length.

10. The sides of a triangle are 16, 18, and 24 chains; it is required to divide it into two parts in the ratio of 2 : 3 by a line perpendicular to the longest side. Locate the division line, and determine its length.

281. To divide a given triangle in a given ratio by a line passing through a given point without it.



Let ORQ represent the triangle, P the point given by the angle POQ and distance OP , DL the line which shall divide the triangle, so that

$$ODL : DLRQ = m : n.$$

Denote OP by b , OL by x , OD by y , the angle DOL by O , the angle POD by O' , and the $\frac{m}{m+n}$ part of the area by A ; then

$$\frac{1}{2}xy \sin O = A; \quad (1)$$

$$\text{also} \quad \frac{1}{2}by \sin O' = \text{area } POD, \quad (2)$$

$$\text{and} \quad \frac{1}{2}bx \sin (O + O') = \text{area } POL. \quad (3)$$

$$\therefore \frac{1}{2}bx \sin (O + O') - \frac{1}{2}by \sin O' = A. \quad (4)$$

Substituting in the last equation the value of y taken from (1) and reducing, there results,

$$bx \sin (O + O') - \frac{2Ab \sin O'}{x \sin O} = 2A;$$

$$\text{or,} \quad x^2 - \frac{2Ax}{b \sin (O + O')} = \frac{2A \sin O'}{\sin O \sin (O + O')};$$

$$\text{whence} \quad x = \frac{A}{b \sin (O + O')} \pm \sqrt{\frac{2A \sin O'}{\sin O \sin (O + O')} + \frac{A^2}{b^2 \sin^2 (O + O')}}.$$

y may be found by substituting the value thus obtained for x , and thence the length of the division line DL .

EXAMPLES.

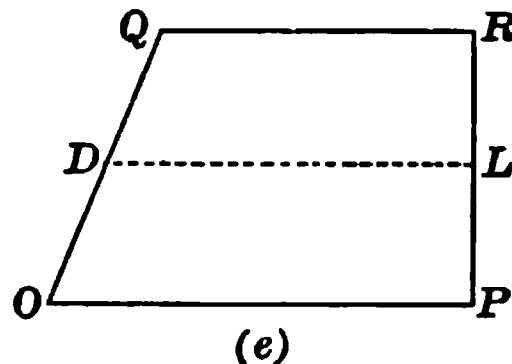
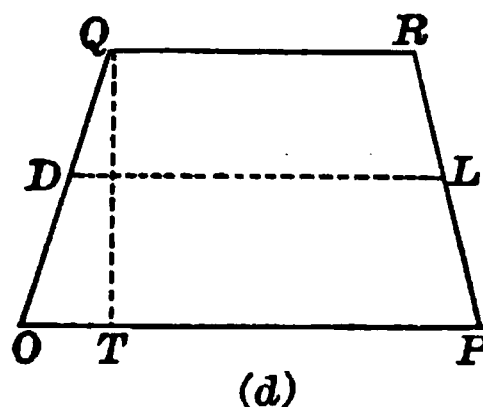
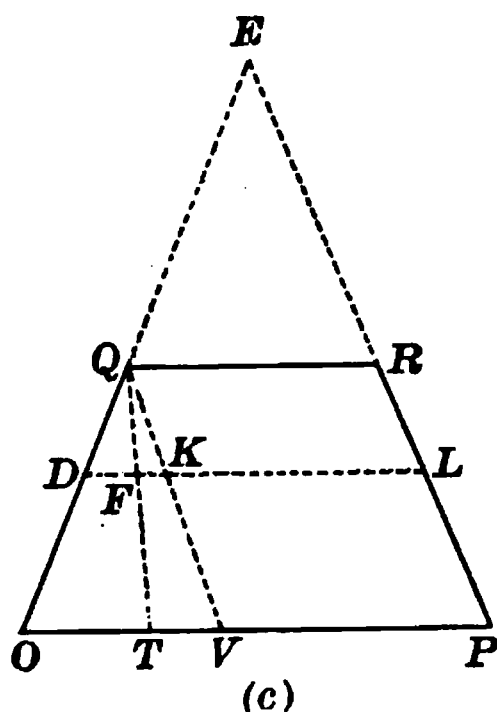
Given, in the triangle OQR , $OR = 18.40$ chains, $RQ = 10.20$ chains, $QO = 20.60$ chains, $OP = 9.50$ chains, and the angle

$POQ = 28^\circ 30'$, to divide the triangle into two parts so that $OLD : DLRQ = 3 : 4$. The position and length of the division line DL are required.

B. QUADRILATERALS.

TRAPEZOIDS.

282. *Given the parallel sides of a trapezoid and the perpendicular distance between them, to divide it by a line parallel to these sides into two parts having a given ratio.*



Let $OPQR$ (Fig. c) be the trapezoid, the sides OP , OQ , and the perpendicular distance QT between the bases being given. It is required to divide it by a line DL , so that $OPLD : DLRQ = m : n$; that is, practically to locate and determine the length of the division line DL .

Denote the lower base by b , the upper base by b' , the perpendicular distance between the bases by h , the perpendicular distance between the upper base and division line by x , the length of the division line by y , and the area $OPQR$ by A . Draw QV parallel to RP ; then the similar triangles give

$$OV : DK = QT : QF.$$

Or, $OP - QR : DL - QR = QT : QF.$

Or, substituting proper values,

$$b - b' : y - b' = h : x;$$

whence $x = \frac{(y - b') h}{b - b'}. \quad (1)$

But the area of $DLQR = (y + b') \frac{x}{2} = \frac{An}{m + n}.$

Representing for convenience the right-hand member of the last equation by A' , we may write

$$xy + b'x = 2A',$$

and $y = \frac{2A'}{x} - b'. \quad (2)$

Substituting the value of x from (1) in (2) and reducing, there results

$$y = \sqrt{\frac{2A'(b - b')}{h} + b'^2},$$

and $x = \frac{-b'h \pm \sqrt{2A'h(b - b') + b'^2h^2}}{b - b'}.$

Restoring the value of A' , we obtain,

$$y = \sqrt{\frac{2An}{h(m + n)}(b - b') + b'^2},$$

$$x = \frac{-b'h \pm \sqrt{\frac{n}{m + n} 2Ah(b - b') + b'^2h^2}}{b - b'}.$$

The student may indicate how he would trace out on the field the division line thus found.

283. If instead of the perpendicular distance there be given one of the sloping sides, as OQ (Fig. c).

Denote OQ by d , OD by x , and the other notation as above. Produce the sides until they meet in some point E ; then

$$OPE : QRE = b^2 : b'^2,$$

$$DLE : QRE = y^2 : b'^2;$$

or, by division, $OPQR : QRE = b^2 - b'^2 : b'^2,$

and $DLQR : QRE = y^2 - b'^2 : b'^2;$

whence $OPQR : DLQR = b^2 - b'^2 : y^2 - b'^2.$

By division $OPLD : DLQR = b^2 - y^2 : y^2 - b'^2;$

inserting values, $m : n = b^2 - y^2 : y^2 - b'^2;$

whence
$$y = \sqrt{\frac{b^2n + b'^2m}{m + n}}.$$

The similar triangles OVQ and QDK give

$$b - b' : y - b' = d : d - x.$$

$$\therefore x = \frac{d(b - y)}{b - b'};$$

or,
$$x = \frac{d}{b - b'} \left[b - \sqrt{\frac{b^2n + b'^2m}{m + n}} \right].$$

In Figure *d*, the unknown sides are symmetrical with respect to a line joining the centres of the parallel sides; in Figure *e*, PR is perpendicular to the parallel sides. The student will show what modification, if any, may be made in the formulas of the two preceding cases for either of these.

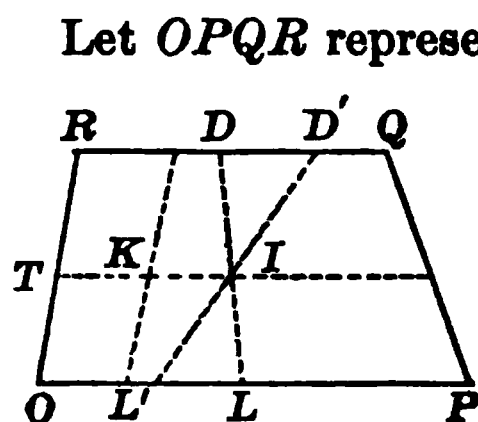
EXAMPLES.

1. Given $OP = 20$ chains, $QR = 15$ chains, $QT = 18$ chains, to find the length of the division line DL , so that QLD shall contain two-thirds as much land as $OPLD$.

2. In Figure *d*, whose sides are equally inclined to the bases, $OP = 24$ chains, $QR = 16$ chains, and the perpendicular distance $QT = 20$ chains; it is required to locate the extremities of the division line DL , and determine its length, so that it shall divide the tract into two equivalent parts.

3. In Figure *e*, suppose $QR : OP : PR = 3 : 4 : 5$, and that the area = 1750 rods; locate and find the length of the division line DL that shall divide the tract, making $OPLD : QRLD = 3 : 4$.

284. To divide a given trapezoid into two parts having a given ratio, by a line intersecting the parallel sides.



Let $OPQR$ represent the trapezoid, and let it be required to divide it into two equal parts. It is evident if the bases be bisected, and a line, as DL , be drawn connecting the points of division, it will be the division line required.

Similarly, if the ratio is $m : n$; denote OP by b , and RQ by b' ; then take $OL = \frac{mb}{m+n}$, $RD = \frac{mb'}{m+n}$, and join DL for the line required.

The student will give the reason.

If the division line is to pass through a given point D' , obtain DL as above directed, then measure from D to D' , and lay off this distance from L to L' . Join $D'L'$ for the division line required. Why?

To divide a trapezoid by a line perpendicular to the bases, or parallel to one of the non-parallel sides, divide the line joining the middle points of the non-parallel sides into two parts in the given ratio, and through the point of division run the required line. If $m : n$ is the ratio, and the bases b and b' , the distance TK in the last figure = $\frac{m(b+b')}{2(m+n)}$.

The student will give the reason.

EXAMPLES.

1. Divide a given trapezoid into three equivalent parts by lines intersecting the parallel sides.

2. Divide a given trapezoid into three parts in the ratio of $m : n : p$, by lines intersecting the parallel sides.

3. The bases of a trapezoid are, $OP = 20$ chains, and $QR = 15$ chains. It is required to divide it into two parts in the ratio of 2 : 3. $OL' = 8.50$ chains; locate D' .

4. Show that I , being the centre of the line connecting the middle of the bases of a trapezoid, is the point through which, if any straight line be drawn meeting the parallel sides, it will divide the trapezoid into two equivalent parts.

285. *Given one side and the adjacent angles of a tract of land, to cut off a trapezoid of a given area by a line parallel to the given side.*

Let PO be the given base, P and O the known angles indicating the direction of the sides PQ and OR . Denote the area $OPLD$, to be cut off by A ; the given side OP by s , PD by y , OL by x , DL by z , and suppose

$$(O + P) < 180^\circ.$$

Produce OR and PQ until they meet in V .

Then area $OPV - \text{area } LDV = A$;

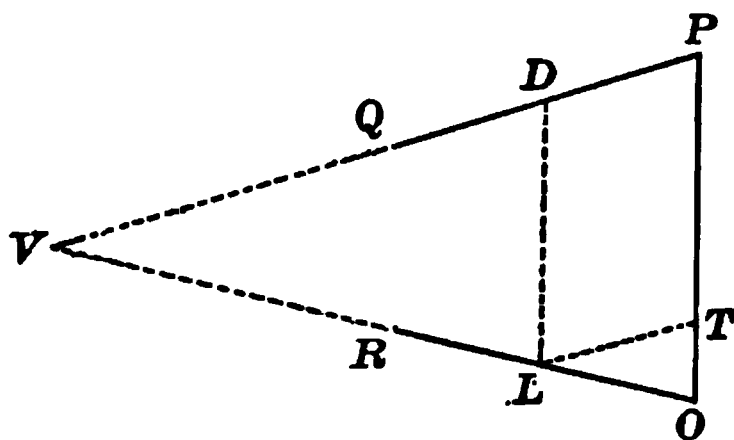
or,
$$\frac{s^2 \sin O \sin P}{\sin V} - \frac{z^2 \sin O \sin P}{\sin V} = 2A;$$

whence
$$z = \sqrt{s^2 - \frac{2A \sin(O + P)}{\sin O \sin P}}.$$

When $(O + P) > 180^\circ$, the produced lines meet in a point on the other side of OP , the $\sin(O + P)$ is also negative, and therefore the fraction under the radical becomes positive. Draw LT parallel to VP ; then in the triangle LOT , by sine proportion, $\sin L (= \sin V) : \sin T (= \sin P) = s - z : x$;

whence
$$x = \frac{(s - z) \sin P}{\sin V}.$$

Similarly,
$$y = \frac{(s - z) \sin O}{\sin V}.$$



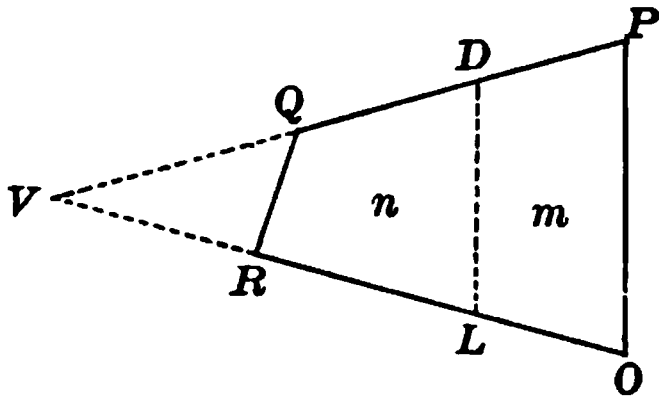
REMARK. When great accuracy is not required, and especially if the tract is small and the sides nearly parallel, an approximate perpendicular distance between the bases OP and DL may be obtained by dividing the area to be cut off by the given side OP ; then measure the perpendicular and a line through its extremity parallel to the base for an approximate division line. Calculate the area thus cut off, divide the difference between it and the required area by the approximate division line for a new perpendicular, and thence obtain more nearly the division line sought.

EXAMPLES.

1. Deduce an expression for DL by another method.
2. Show by other methods how OL or PD may be determined.
3. Given OP , N. $16^\circ 30'$ W., 8.40 chains; PQ , S. $62^\circ 15'$ W.; and OR , S. 82° W., to cut off a trapezoid containing 4 acres, by a line DL parallel to OP . The position and length of the division line are required.
4. Given a side of a tract of land 20 chains, and the adjacent angles 105° and 130° , to cut off 36 acres by a line parallel to the given side. Required the position and length of the division line.

TRAPEZIUMS.

286. Given the area of a trapezium, one of its sides and adjacent angles, to divide it by a line parallel to the given side into two parts having the ratio $m : n$.



Produce the sides PQ and OR to meet in V . Let $OP = s$, $OL = x$, $PD = y$, $DL = z$.

Calculate the area of

$$OPV = A' = \frac{s'^2 \sin O \sin P}{2 \sin V};$$

then $A' - A = \text{area } DLV,$

and the formula $\frac{z^2 \sin O \sin P}{\sin V} = 2(A' - A)$

gives $z = \sqrt{\frac{2 \sin V (A' - A)}{\sin O \sin P}}.$

Having found z , x and y may be deduced as in the foregoing case.

$$x = \frac{(s - z) \sin P}{\sin V},$$

$$y = \frac{(s - z) \sin O}{\sin V}.$$

REMARK. This problem may be solved by Article 285, taking for the given area to be cut off $\frac{m}{m + n}.$

EXAMPLE.

The boundaries of a trapezium are as follows :

- (1) N. 2° E. 8.00 chains ;
- (2) N. $58\frac{1}{2}^\circ$ E. 13.85 “
- (3) S. $31\frac{1}{2}^\circ$ E. 14.80 “
- (4) S. $82\frac{1}{2}^\circ$ W. 20.00 “

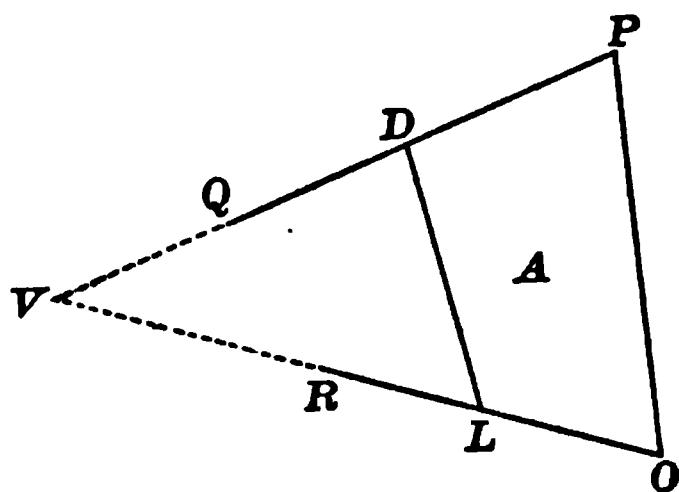
It is required to divide it into two equivalent parts by a line parallel to the third side. Locate it, and determine its length.

287. *Given the bearings of three adjacent sides of a tract of land and the length of the middle one, to cut off a trapezium having a given area, by a line running in a given direction.*

Produce the sides PQ and OR till they meet at V . As before, denote OP by s , OL by x , PD by y , and LD by z . Obtain the angles from the bearings, calculate the area of

$$POV = A' = \frac{s^2 \sin O \sin P}{2 \sin V},$$

and find area $DLV = A' - A$.



Whence the division line $DL = z$ may be found from the formula

$$\frac{z^2 \sin D \sin L}{2 \sin V} = A' - A,$$

or

$$z = \sqrt{\frac{2 \sin V (A' - A)}{\sin D \sin L}}.$$

By the sine proportion

$$VO = \frac{s \sin P}{\sin V},$$

and

$$VL = \frac{z \sin D}{\sin V};$$

whence

$$VO - VL = LO = x = \frac{s \sin P - z \sin D}{\sin V},$$

and

$$y = \frac{s \sin O - z \sin L}{\sin V}.$$

REMARK. If $(O + P) > 180^\circ$, $A' - A$ in the equation for z will become $A' + A$, and in the formulas for x and y the signs in the numerators will be interchanged, or

$$x = \frac{z \sin D - s \sin P}{\sin V},$$

and

$$y = \frac{z \sin L - s \sin O}{\sin V}.$$

EXAMPLE.

Given LO , S. 76° E. ; OP , N. 8° W. 12.40 chains ; PD , S. 72° W. ; it is required to cut off 7 acres by a line bearing N. 23° W. The length of the division line and the distances OL and DP are to be computed.

288. *Given a trapezium, to divide it into two parts having a given ratio, by a line extending from a given point in one of the sides.*

Let $OPQR$ represent the trapezium the area of which is A , m and n the given ratio. Prolong the sides PQ and OR till they meet in V . Let $OR = v$, the division line $DL = z$, RL the given distance to the point $L = d$, and $QD = y$. Calculate the area of $QRV = A'$, and add it to $\frac{n}{m+n} A$, thereby obtaining area of DLV .

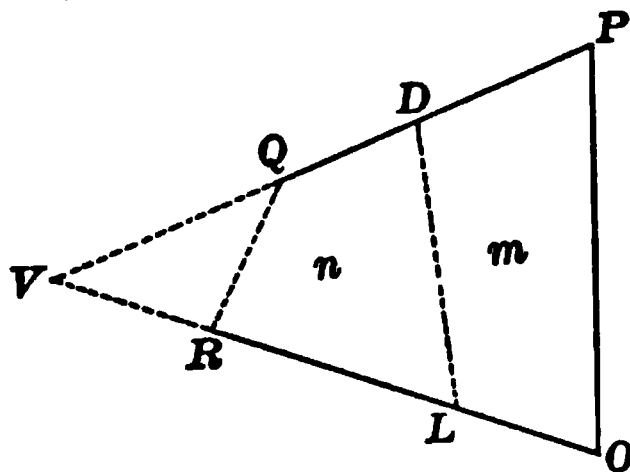
Find by the sine proportion VR , and add it to RL , thus obtaining VL .

Putting $VD = x$, and $VL = b$,

$$bx \sin V = 2 \left(\frac{n}{m+n} A + A' \right).$$

Whence $x = VD$ may be found.

Finally, with the two sides VD and VL and the included angle V , compute the angle L , and the direction and length of the division line DL ; y may be calculated by a preceding method to check the work.



EXAMPLES.

1. Given in a trapezium $MNOP$ (no figure) :

MN , 13.00 chains ;

NO , 7.30 “

OP , 10.40 “

PM , 11.10 “

and diagonal PN , 13.70 “

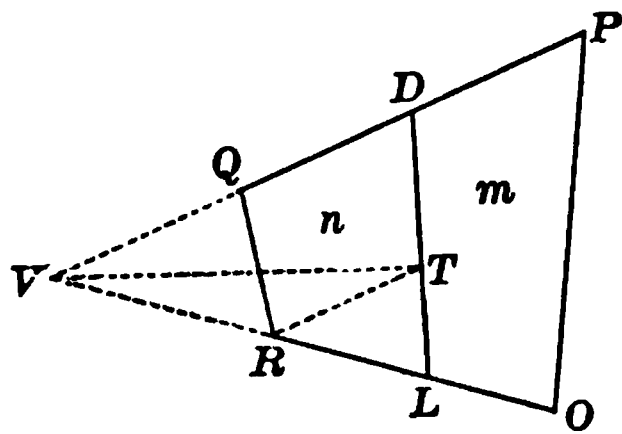
It is required to divide it into two equivalent parts by a line running from a point in the side MN , 6 chains from M . Find

the length of the division line and locate the other extremity of it.

2. Divide the tract described in Example 1 into two parts, in the ratio of 3 : 4, by a line DL running from some point in MN , and falling perpendicularly upon PO . The part $PMDL$ is to be the greater. Locate the line required, and determine its length.

289. *Given a trapezium, to divide it into two parts having a given ratio, by a line passing through a given point within the tract.*

Let $OPQR$ represent the given trapezium T , the point within it, given by its bearing and distance from some angle, as R . Produce the sides OR and PQ to meet in V . Denote the ratio by m and n , the area $OPQR$ by A , QR by v , DL by z , VL by x , and VO by y . Find by the sine proportion



$$VR = \frac{v \sin Q}{\sin V}, \quad VQ = \frac{v \sin R}{\sin V},$$

and thence the area $VQR = A'$. Then in the triangle VRT , having two sides and the included angle, compute VT , which call b , and the angle $TVR = a$. Putting $V - a = \beta$, and $\frac{n}{m+n} A + A' = A''$, the following equations may be written :

$$xy \sin V = 2 A'', \quad (1)$$

$$\text{and} \quad bx \sin a + by \sin \beta = 2 A''. \quad (2)$$

Substituting in (2) the value of y from (1), and reducing, there results,

$$x = \frac{A''}{b \sin a} \pm \sqrt{\frac{A''^2}{b^2 \sin^2 a} - \frac{2 A'' \sin \beta}{\sin a \sin V}},$$

$$\text{and } RL = x - VR = \frac{A''}{b \sin a} \pm \sqrt{\frac{A''^2}{b^2 \sin^2 a} - \frac{2 A'' \sin \beta}{\sin a \sin V}} - \frac{v \sin Q}{\sin V}.$$

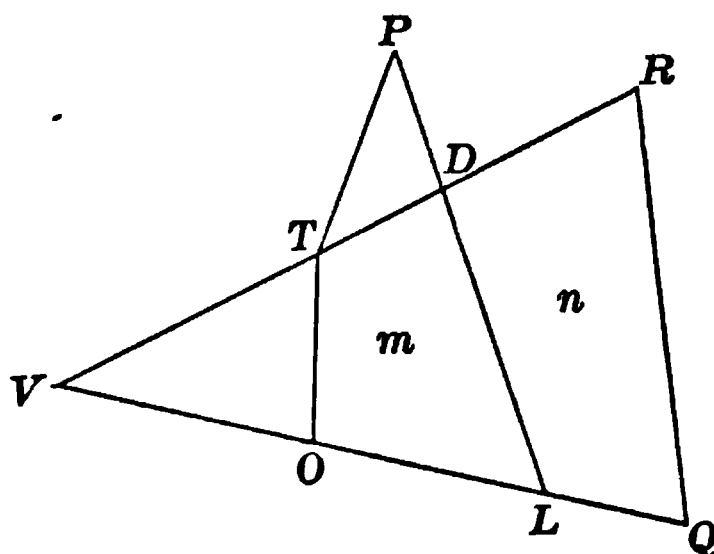
EXAMPLE.

Given the boundaries of a trapezium as follows :

- (1) N. $16\frac{1}{4}^\circ$ W. 24.63 chains ;
- (2) S. 79° W. 27.00 “
- (3) S. $\frac{1}{4}^\circ$ W. 34.28 “
- (4) N. 65° E. 37.20 “

To divide it into two equivalent parts by a line extending from the first to the third side, and passing through a point 20 chains distant from the first and second corners. Locate the line and find its length.

290. *Given a trapezium, to divide it into two parts having a given ratio, by a line passing through a given point without the tract.*



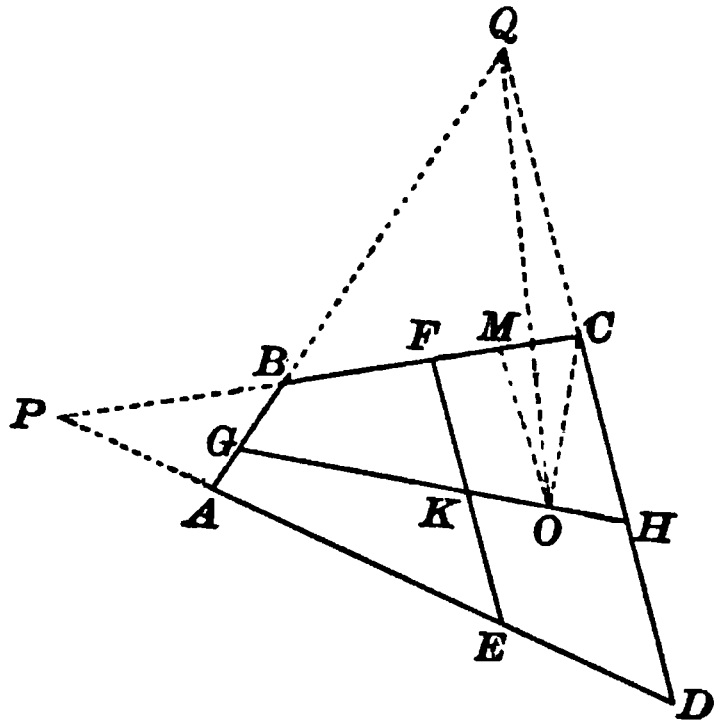
Let $OQRT$ represent the trapezium given by the bearings and distances of its sides, P the point without, located by its bearing and distance from T , the ratio $m : n$. Extend the sides RT and QO until they meet in V . Then the problem may be solved in a similar manner to that in Article 281.

291. *Given a trapezium, to divide it into four equivalent parts, by two lines intersecting opposite sides, one of the division lines being parallel to one of the given sides of the tract.*

Let $ABCD$ represent the given trapezium, FE the division line parallel to DC , and GH the other division line. It is re-

quired to locate both division lines. Prolong the sides AD and BC to meet in P ; also DC and AB to Q . Find AE and EF by methods already given.

Now, any line cutting the parallel sides of a trapezoid and dividing it into two equivalent parts must pass through a point O (the middle of the middle line between the bases). See Article 284. Hence MO becomes known $= \frac{1}{2}(CD + EF)$, and



also $MC = \frac{1}{2}FC$. In the triangle OMC , compute the angle MCO and the line OC ; add $\angle MCO$ to $\angle MCQ$, and having previously calculated QC , find in the triangle QCO the angle CQO and the side QO . Subtract $\angle CQO$ from $\angle CQB$ and obtain $\angle OQB$. Then putting the side $QO = a$, $QH = x$, and $QG = y$, we may write the following equations:

$$xy \sin HQG = 2 \text{ area } HQG,$$

$$ax \sin CQO + ay \sin OQG = 2 \text{ area } HQG.$$

From these equations obtain y . Subtract it from AQ , found by sine proportion, and the distance from the corner A to the extremity of the division line GH at G will be the result.

Then in the triangle QGH find QH ; whence the length and bearing of GH may be computed.

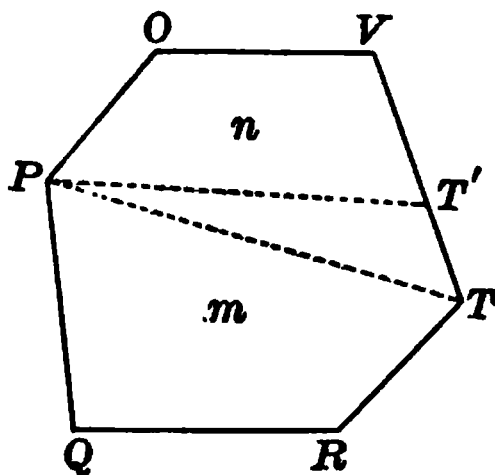
EXAMPLE.

It is required to divide the farm described in 288 (Example 1) into four equivalent parts by two lines intersecting opposite sides; one of the division lines is to be parallel to the first side. Locate the division lines, and determine their lengths.

C. POLYGONS.

292. *Given a polygon, to divide it into two parts having a given ratio, or to cut off a given area, by a line through a given point.*

Let $OPQRTV$ represent the polygon given by its bearings and distances, or angles and sides, and suppose the line be required to run from P , either an angle or any given point in a side. Calculate the area of the polygon, and take the $\frac{m}{m+n}$ part of it as the area to be cut off to the right of the line extending from P .



Run a *trial line** from P as PT , calculate the area of $PQRT$, and determine whether the area thus cut off is too small or too large, and how much. Suppose it is too small; then the extremity T of the division line PT must be moved towards V to some point T' . To find this point, denote TT' by x , the angle $T'TP$ by T , the distance PT by b , and the area of the triangle PTT' by a ; then

$$\frac{1}{2} bx \sin T = a,$$

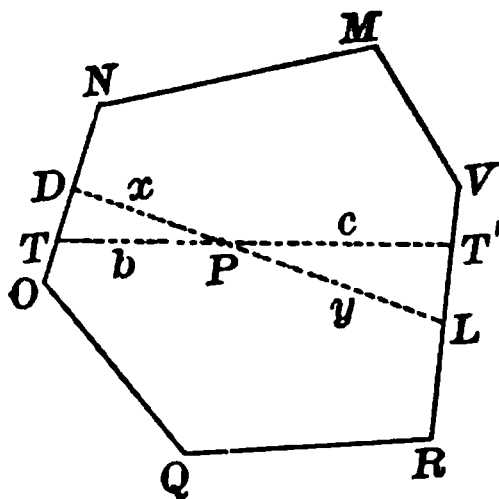
from which we find
$$x = \frac{2a}{b \sin T}.$$

* The bearing and distance of PT may be calculated from the data given—without a trial line—as in supplying omissions. If, however, this is done, the surveyor should not omit to measure the division line to verify his work. In fact, it is the best practice, no matter what method is adopted to obtain the division line, to always test the computation by measurement.

This distance measured from T to T' will locate T' , a point which connected with P will give the division line sought.

293. *Given a polygon, to divide it into two parts in a given ratio, or to cut off a given area, by a line through a given point within the tract.*

Let the marginal figure represent the tract, P the given point.



If the area to be cut off is not directly given, calculate the contents of the tract, and then by the ratio determine the quantity to be cut off, and denote it by A . Run a trial line TT' through P , dividing the polygon as nearly as may be judged in the required manner. Measure $TP = b$, $PT' = c$, and the angles T and T' . Calculate the area of either part of the polygon, and thus ascertain whether T should approach or recede from O . Suppose the area $TNMVT'$ is calculated and found too small by a quantity a , and that DL represents the division line. Put $DP = x$, $PL = y$, the angle $PTD = T$, $PT'L = T'$, and the angle at the point $P = P$, which is required, since that will indicate the direction of the division line.

$$\text{Then} \quad \frac{1}{2} cy \sin P - \frac{1}{2} bx \sin P = a \quad (1)$$

$$x = \frac{b \sin T}{\sin (T + P)}, \quad (2)$$

$$y = \frac{c \sin T'}{\sin (T' + P)}. \quad (3)$$

Substituting the values of x and y from (2) and (3) in (1), there results

$$\frac{c^2 \sin T' \sin P}{\sin (T' + P)} - \frac{b^2 \sin T \sin P}{\sin (T + P)} = 2a. \quad (4)$$

Expanding the denominators, dividing each fraction, numerator and denominator, by its numerator, and writing for $\frac{\cos}{\sin}$ the cot, there results

$$\frac{c^2}{\cot P + \cot T'} - \frac{b^2}{\cot P + \cot T} = 2a. \quad (5)$$

Putting $\cot P = p$, $\cot T = t$, and $\cot T' = t'$, we may write more simply :

$$\frac{c^2}{p + t'} - \frac{b^2}{p + t} = 2a,$$

or
$$p^2 + t + t' - \frac{c^2 - b^2}{2a} = \frac{tc^2 - b^2t'}{2a} - tt';$$

whence
$$p = -\frac{1}{2}\left(t + t' - \frac{c^2 - b^2}{2a}\right) \pm \sqrt{\frac{c^2t - b^2t'}{2a} - tt' + \left[\frac{1}{2}\left(t + t' - \frac{c^2 - b^2}{2a}\right)\right]^2}.$$

Restoring values, we have

$$\cot P = -\frac{1}{2}\left(\cot T + \cot T' - \frac{c^2 - b^2}{2a}\right) \pm \sqrt{\frac{c^2 \cot T - b^2 \cot T'}{2a} - \cot T \cot T' + \frac{1}{4}\left(\cot T + \cot T' - \frac{c^2 - b^2}{2a}\right)^2}.$$

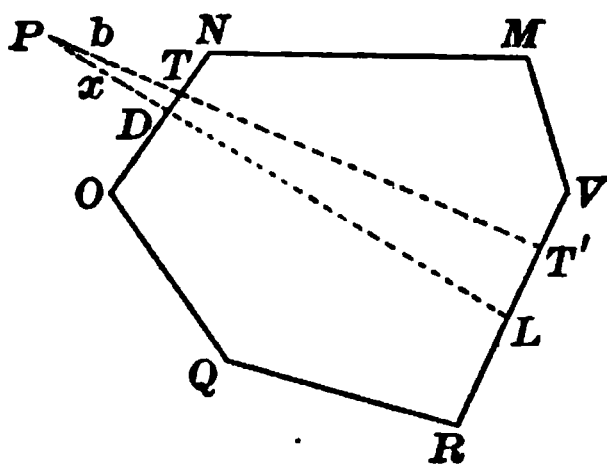
The problem may be simplified when it is practicable to run the trial line at right angles to one of the sides of the polygon. In the tract given, suppose TT' to be run perpendicularly to RV ; then $\cot T' = 0$, and Equation (5) may be written

$$\frac{c^2}{\cot P} - \frac{b^2}{\cot P + \cot T} = 2a,$$

and
$$\cot P = -\frac{1}{2}\left(\cot T - \frac{c^2 - b^2}{2a}\right) \pm \sqrt{\frac{c^2 \cot T}{2a} + \frac{1}{4}\left(\cot T - \frac{c^2 - b^2}{2a}\right)^2}.$$

294. *Given a polygon, to cut off a given area by a line passing through a given point without the tract.*

Let the marginal figure represent the case.



As in the preceding article, run a trial line PT' from P , and suppose it is made perpendicular to RV . Calculate, as before, the content of $TNMVT'$, and ascertain the amount to be added to make the required area. Denote, as before, this area by a , $PT=b$, $PT'=c$, $PD=x$, $PL=y$; the angles at P , T , etc., by P , T , etc., and DL the division line; then

$$\frac{1}{2}cy \sin P - \frac{1}{2}bx \sin P = a, \quad (1)$$

$$x = \frac{b \sin T}{\sin (P + T)}, \quad (2)$$

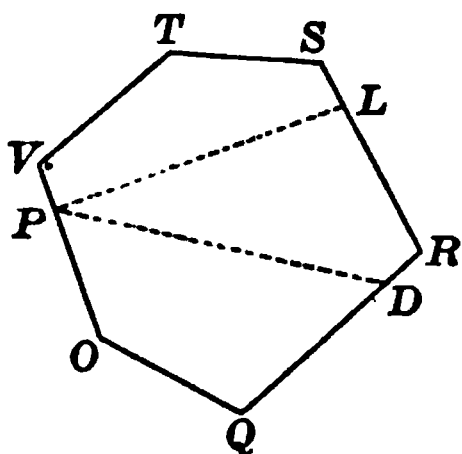
$$y = \frac{c}{\cos P}. \quad (3)$$

Substituting the values of x and y from (2) and (3) in (1), and reducing as in the preceding problem, there results

$$\cot P = -\frac{1}{4a} \left(2a \cot T + b^2 - c^2 \right) \pm \sqrt{\frac{c^2 \cot T}{2a} + \left[\frac{1}{4a} \left(2a \cot T + b^2 - c^2 \right) \right]^2}.$$

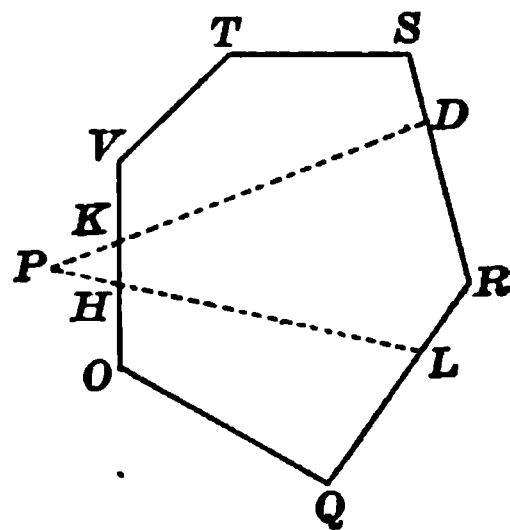
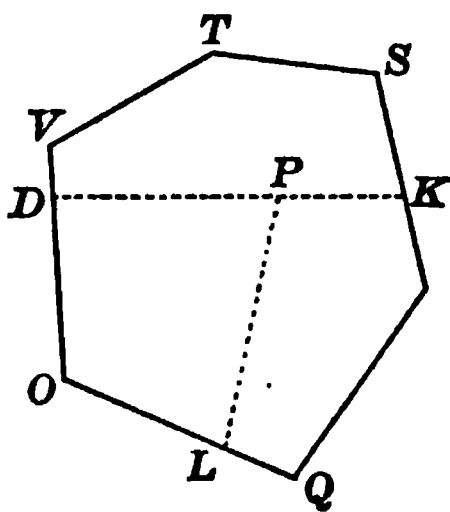
The student may verify the value found.

295. *Given a polygon, to divide it into three parts having a given ratio, by lines radiating from a given point.*



a. Let the figure represent the polygon, and suppose the point is in one side at *P*. Calculate the area of the whole tract and ascertain how much each division is to contain ; then, by Article 292, cut off the required areas *PVTSL* and *POQD*, and the problem is solved.

b. If the point is within the tract,* cut off, by Article 293, one required portion $DVTSKD$ by a line DK through P , and by the preceding article divide the remainder by the line PL as required. If PL cuts off a quadrilateral on either side, Article 288 may be used.



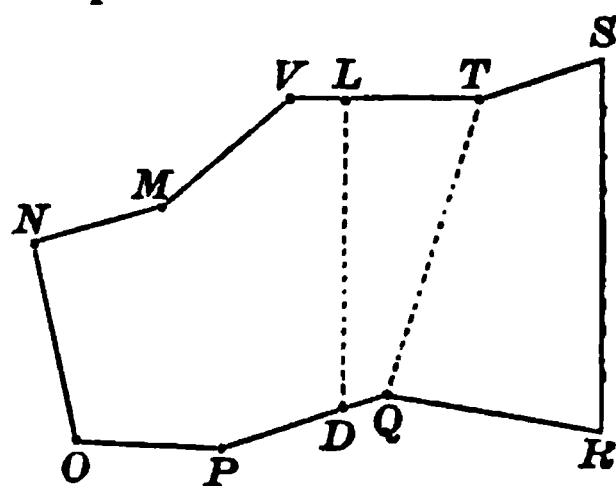
c. If the point is without,* proceed, as in Article 294, to cut off the required portion $KVTSDK$ and $HOQLH$; the remainder $HLRDKH$ will be the third portion.

It is evident that this principle may be extended to any number of parts.

296. To cut off from a given polygon a given area by a line running in a given direction.

Let the figure represent a tract which it is required to divide into two equivalent parts by a line DL parallel to RS .

Join QT , calculate its length and bearing, and also the content of $QRST$. Subtract said content from one-half the area of the whole tract, thereby obtaining the area $DLTQ$. Then, by Article 287, the length and position of the division line may be determined. It is evident that this principle may be extended to any number of subdivisions.



* Calculate the area, and ascertain, by the ratio, how much each division is to contain.

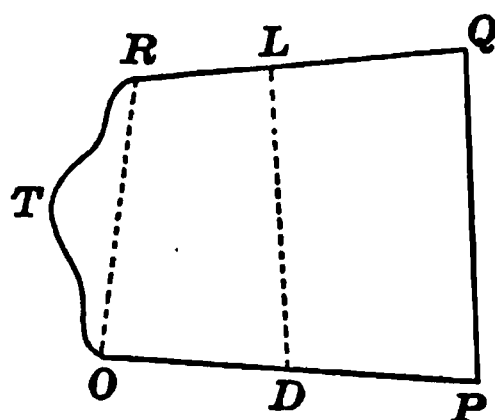
EXAMPLES.

1. The student may indicate how he would divide $DQRSTL$ into two equivalent parts by a line perpendicular to DL .

2. Show how to divide $DPONMVL$ into two equivalent parts by a line extending from the middle of DL .

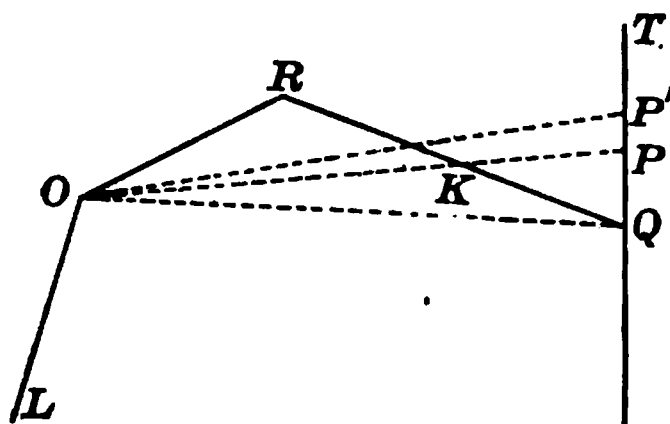
3. Divide the farm described in Article 234, Example 4, into two equivalent parts by a line running due east.

297. *From a tract of land of which one or more of the boundary lines is irregular, to cut off a given area.*



Let $OPQRT$ represent the tract which it is required to divide into two equal parts by a line DL parallel to PQ . Survey the land, taking offsets along RO , and calculate the area. Then the problem may be solved by Article 285.

298. To Straighten Boundary Lines. It is sometimes required to substitute a straight line for an irregular or crooked one between farms, and to leave the same quantity of land as before in each tract. Let ORQ be the line which it is required



to straighten by a line extending from O , the bearings and distances OR , RQ , and the bearing of QT being known. Run a trial line OP , noting the distances RK , OK , KP , and PQ , and calculate the areas of the triangle ROK and PQK . If it

happens that the triangle ROK is equivalent to PQK , then OP will represent the line sought. If, as is generally the case, their areas are not equal, take the difference, and suppose in this case PQK the less. The problem, then, is simply this: Given one side, OP , of a triangle, and the direction of another,

PT , to cut off a given area by a line OP' , to find the distance PP' . The solution is given in Article 257.

Otherwise, with the given bearings and distances calculate the area of the triangle ORQ and the length and bearing of the closing line OQ . Then, as before, having one side of a triangle, the direction of another, and the area, find QP' and the bearing and distance of OP' . The work should be verified by actual measurement of angle and distance.

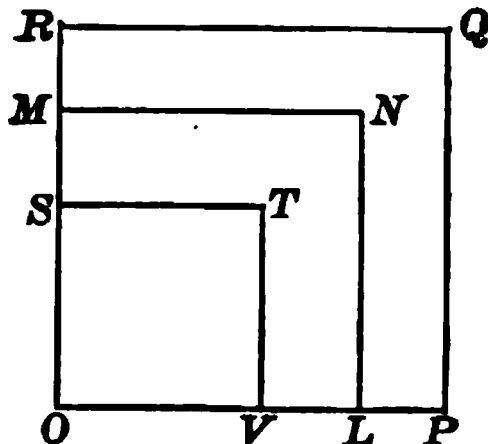
EXAMPLE.

Given OR , N. $59^\circ 30'$ E. 10.60 chains; RQ , S. $70^\circ 15'$ E. 19.32 chains; QT , N. 12° W., to find QP' and the bearing and distance of a line OP' which will straighten the boundary.

MISCELLANEOUS EXAMPLES.

1. It is required to lay out a lot to contain one acre, and having an equal frontage on two streets which intersect at an angle of $84^\circ 40'$. Locate the corners of the property.

2. From a square tract of land $OPQR$, which originally contained 160 acres, the southwest quarter was sold. It is required to find the uniform width of a strip $MNLVTS$ which shall contain 40 acres. How many rods of fencing will the tract require?



3. A rectangular tract of land 16.20 chains long, and 8.60 chains wide, valued at \$200 per acre, is to be divided among three persons so that the first shall have \$1,000 worth of it; the second, \$900; and the third, the remainder. Locate the points of division on the long side.

4. The bearings of two sides of a triangle are OM , N 60° E., and ON , S. 40° E. It is required to cut off from the corner O an isosceles triangle containing 16 acres. Locate and find the length of the division line.

5. There is a farm in the form of a trapezium the area of which is given as 87.78 acres. The description of its boundaries is very much effaced; all that is legible is as follows:

Beginning at the northwest corner, thence (1) S. 76° E. (distance effaced); (2) S. 10° E., distance 25 chains; (3) S. 62° W. (distance effaced); (4) N. 6° W. (distance effaced).

It is required to perfect the description.

SUGGESTION. Prolong the second and fourth sides until they meet, and calculate the area of the triangle exterior to the tract. Add it to the given area, whence the length of the first side may be readily computed; the second and fourth sides may be found easily by either of two methods.

6. Required the length of a chord which will cut off one-third part of a circle whose radius is 100 feet.

SUGGESTION. Let 2θ denote the central angle, and r the radius, for convenience. Then $\frac{\pi r^2 \theta}{180} - \frac{r^2}{2} \sin 2\theta = \frac{\pi r^2}{3}$. Whence θ may be obtained, and hence the chord. The angle will be the same, no matter what the radius may be.

7. A trapezoidal field, the two parallel sides of which are 16 and 10 chains, and the perpendicular distance between them, 12 chains, is to be divided into two equivalent parts by a line parallel to the given sides. It is required to determine the length of the division line and locate its extremities, the sides being equally inclined to the bases.

8. Given the sides of a triangle OR , 280 yards; RQ , 200 yards; OQ , 300 yards; the distance from O to a point P outside the tract, 220 yards; and the angle POQ , 20° . It is required to run the centre line of a straight road through P and across the field, so as to divide the tract into two equal parts. Locate the points where the road will cross the triangle.

9. Given the sides of an irregular pentagon, and the perpendicular distance to each from a point within. Show how to divide the tracts into their equivalent parts. Also into three parts, having the ratio $m : n : p$.

10. Given in a trapezoid $MNOP$ (no figure), $PM = 38.50$ chains; MN , one of the parallel sides, 64.80 chains; NO , 41 chains; the angle M , $85^\circ 30'$; and N , $75^\circ 40'$. It is required to divide the tract into two parts in the ratio of 2 : 3, by a line DL parallel to the parallel sides. The part $MNDL$ is to be the greater. Find the length and location of the division line.

11. Given one side of a triangular field, 120 yards; the angle opposite, 20° ; and the ratio of the other two sides, 7 : 10. Find the area.

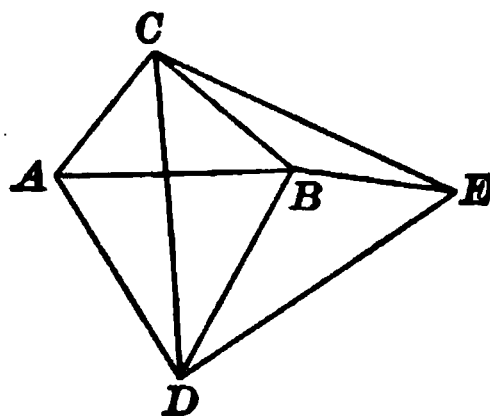
12. Show that the area of a trapezium is equal to one-half the product of its diagonals, by the sine of the angle of their intersection.

13. From a point within a triangular field, the sides of which were equal, I measured the distances to the three angles, and found them 12.5, 10, and 7.5 chains respectively; required the area.

Ans. 12 A. 1 R. 23 P.

The above problem is given in Gummere's Surveying, and by some surveyors it is considered difficult. The following is an outline of a solution; the student will supply what is wanting:

With the given distances form the triangle ABC . On AB describe an equilateral triangle ABD ; join CD by a right line, and on it describe an equilateral triangle CDE . CDE is the triangle in question, and B the point within. For BC and BD are evidently two of the measured distances, and BE , it will be perceived, is the other, through the similarity and equality of the triangles ADC and BDE . To find the area of CDE , compute the angle BAC , whence the angle CAD becomes known; now with the two sides AC , AD , and the included angle CAD , CD is easily determined, and hence the required area of the triangle CDE .



CHAPTER V.

PLANE-TABLE SURVEYING.

299. The Plane-Table, as its name indicates, is a table or board which, being covered with paper, and having certain appliances for levelling and sighting, enables the surveyor to determine points and lines, and to delineate them on the paper in their relative position.

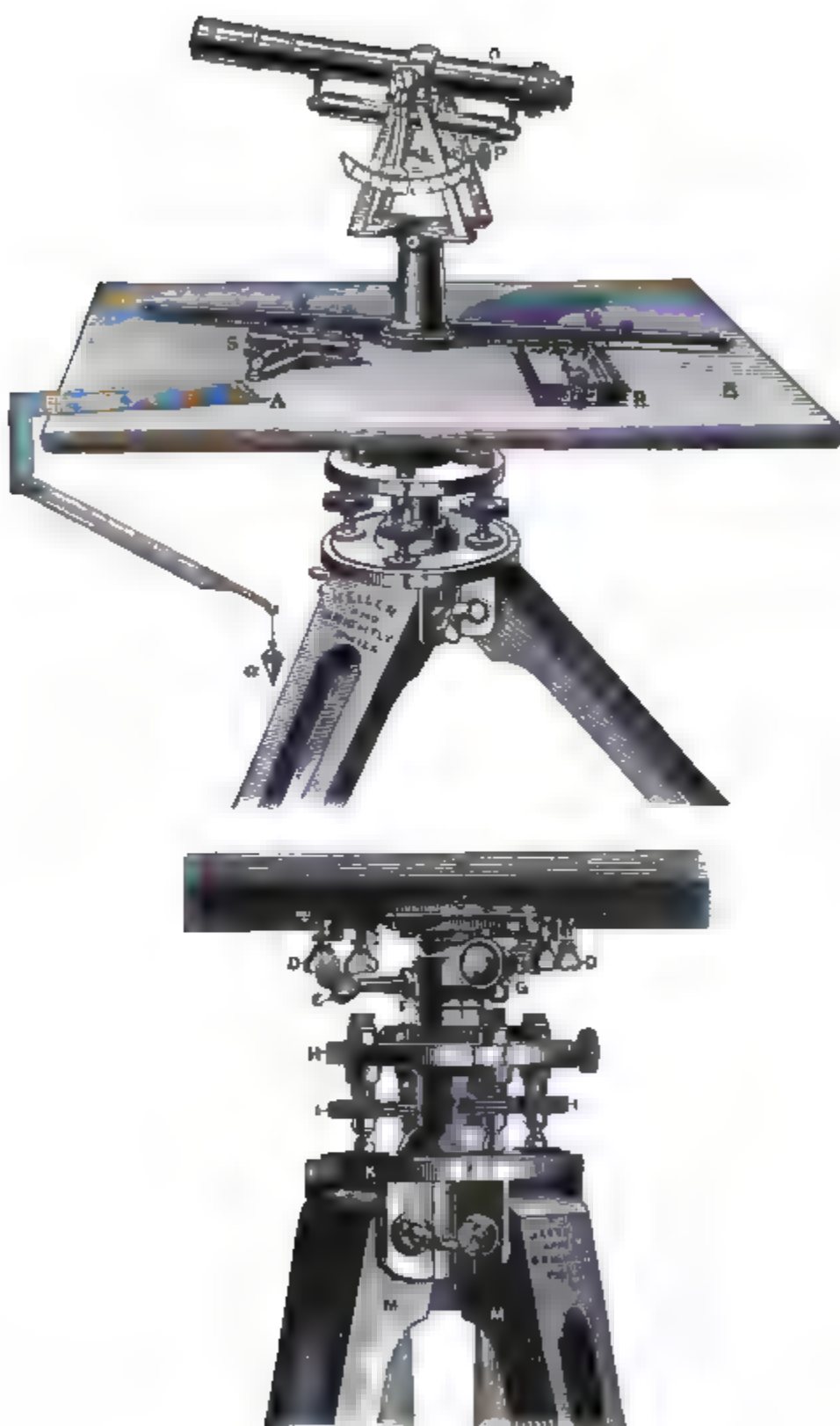
It is used in "filling in" the details of topographical work, and generally for the location of points where great accuracy is not required, on account of the rapidity with which surveys by it may be effected.

300. The Board, which is rectangular in shape, usually 24 by 30 inches, is made of pieces of well-seasoned wood joined advantageously together to prevent warping, and is furnished with rollers or clamps, by means of which the paper is kept securely stretched upon it.

301. The Plumbing-Arm, which is pointed at one end, and from the other a plummet is suspended, is used to determine the point on the ground immediately under its representative on the board, or *vice versa*. The lower part of it moves upon an axis which has an index at its extremity, by means of which it may be ascertained when the bob and point upon the table are in the same vertical line.

302. The Tripod and its Head are similar to those of the ordinary transit, though heavier.

A metallic plate, screwed fast to the table and having a solid conical spindle projecting from its centre, affords the means of attaching the head to the table.



PLANE-TABLE,
AS MADE BY HELLEN & BRIGHTLY, PHILADELPHIA, PA.

The tripod-head admits of a slight lateral motion to the board, and is provided with levelling-clamp and tangent-screws similar to the common transit.

303. Of Alidades there are several kinds. One of the best, however, for ordinary purposes is indicated in the figure. It consists of a brass ruler or straight edge about 22 inches long and two inches wide, from which rises a column surmounted by a telescope. The power of the telescope at least equals that of the common transit, and it is provided with stadia wires, has an attached level, vertical arc, with the necessary adjusting movements. It is set on the column so that the line of collimation is in or near the same vertical plane with the bevelled edge of the ruler.

A parallel ruler allowing a very slight deviation from this plane is sometimes used, and the work is thereby facilitated. A small level is placed on the top of the column, which serves to indicate any unequal settling of the instrument. Two spirit levels at right angles to each other are placed upon the table to indicate when by the levelling-screws it is made horizontal; or, the levels are attached to the ruler of the alidade, one in the longitudinal direction of the ruler, the other perpendicular to it.

304. The Declinator is simply a box containing a magnetic needle which has a range of 12 or 15 degrees on each side of the zero. It is used in *orienting* the table; that is, to place a given point on the table over that on the ground which it represents, and to cause a line of the paper to lie in the same vertical plane, or parallel thereto, with its counterpart on the ground.

Before the table is removed from its first position, or at the time of drawing the first line of the survey, the declinator may be placed upon it, and the needle allowed to rest at zero; then a pencil drawn alongside the box will trace a north and south line, since the sides of the box are made parallel to the line of

zeros.* When the table is oriented at any other station, the declinator will give the same reading if placed along the same line.

ADJUSTMENTS.

305. From the nature of the service in some sections of the country, the plane-table is often necessarily subjected to rough usage, and there is a constant liability to a disturbance of the adjustments; still, in careful hands, a well-made instrument may be used under very unfavorable conditions for a long time without being perceptibly affected. One should not fail, however, to make occasional examinations, and while at work, if any difficulty be encountered which cannot otherwise be accounted for, it should lead directly to a scrutiny of the adjustments.

306. The Fiducial Edge of the Ruler. This should be a true, straight edge. Place the ruler upon a smooth surface, and draw a line along the edge, marking also the lines at the ends of the ruler. Reverse the ruler, and place the opposite ends upon the marked points, and again draw the line. If the two lines coincide, no adjustment is necessary; if not, the edge must be made true.

There is one deviation from a straight line which, by a very rare possibility, the edge of the ruler might assume, and yet not be shown by the above test; it is when a part is convex and a part similarly situated at the other end concave in exactly the same degree and proportion. In this case, on reversal, a line drawn along the edge of the ruler would be coincident with the other, though not a true right line; this can be tested by an exact straight edge.

307. The Level Attached to the Ruler. Place the instrument in the middle of the table, and bring the bubble to the centre by means of the levelling-screws of the table; draw lines

* Any other bearing which may be read will answer the purpose.

along the edge and ends of the ruler upon the board to show its exact position, then reverse 180° . If the bubble remain central, it is in adjustment; if not, correct it one-half by means of the levelling-screws of the table, and the other half by the adjusting-screws attached to the level. This should be repeated until the bubble keeps its central position, whichever way the ruler may be placed upon the table. This presupposes the plane of the board to be true. If two levels are on the rulers, they are examined and adjusted in a like manner.

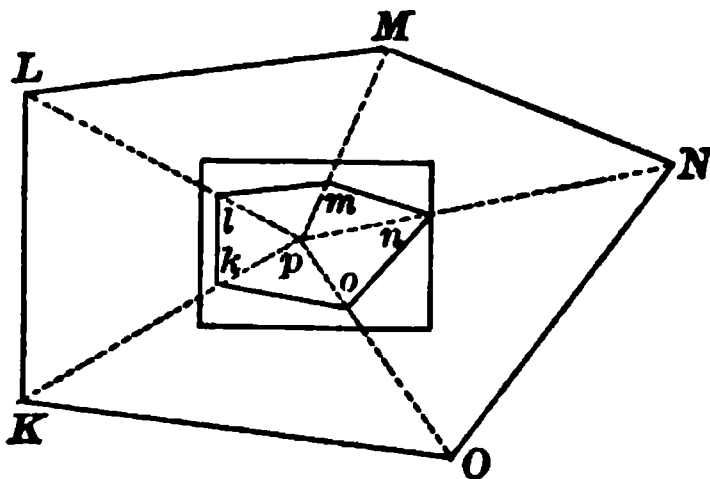
Great care should be exercised in manipulation, lest the table be disturbed.

308. Cause the line of sight to revolve in a vertical plane, make the bubble of the level attached to the telescope read zero when the line of sight is horizontal, and test the vernier arc for index error, each as in the transit.

METHODS EMPLOYED IN PLANE-TABLE SURVEYING.

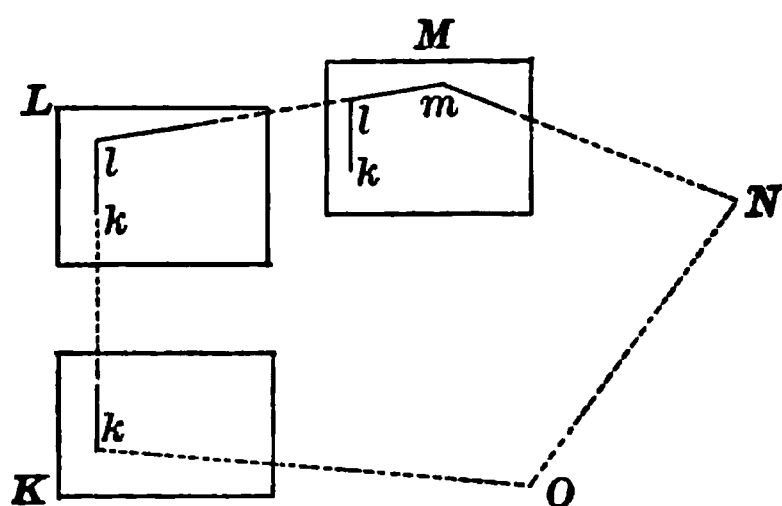
309. Points may be located with respect to one another by either of four methods. In actual practice, however, a combination of some of them is frequently employed.

310. By Radiation. Suppose it is required to make a plot of a field *KLMNO*, all the corners of which can be seen from a point *P* within it. Place the instrument at *P*, level and clamp it. Find a point *p* on the paper, directly over *P* on the ground, and, keeping the bevelled edge of the ruler on *p*, point the telescope to any corner of the tract, as *K*. By means of the stadia wires, or chain, obtain the distance *PK*, and lay it off to any desired scale in the direction of the point sighted,



thus plotting pk . In a similar manner, locate the other corners. Join by straight lines the points thus determined; and the resulting figure $klmno$ will represent the tract surveyed. It is obvious that the position of objects such as buildings, trees, etc., if visible, may be determined by this method, and that it is immaterial whether the instrument be set up in the field or at one of the angles, providing all the stations can be seen from the point selected.

311. By Progression. This method requires the instrument to be set up at every station of the tract to be surveyed. Let



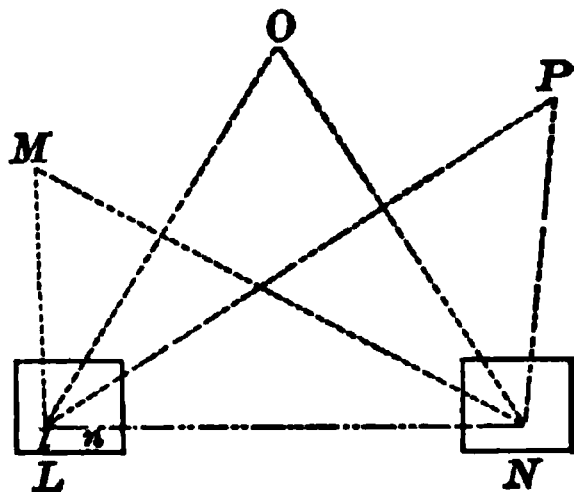
$KLMNO$ represent, as before, the field, and suppose the instrument is first placed at K , and that k on the paper designates this point. With the alidade directed towards L , draw along it an indefinite line. Obtain by stadia or chain the distance

KL , and lay it off to a desired scale, thus locating l . Remove the instrument to L , orient it, and locate m . Continue in the same manner to locate n and o .

When the table is oriented at any station, as M , the line ML should lie in the vertical plane, with its representative ml on the plot, and, having gone round the tract, the last line should close with the first station k .

This method, in conjunction with the preceding, may be employed advantageously in the survey of a road, stream, etc. The centre line of the road or bank of the stream may be traversed by the instrument, placing it at each angle or bend, as in the survey of a field by progression, and determine by the method of radiation the position of prominent objects, such as buildings, bridges, trees, etc. If there be added to the above a sketch of the general features of the ground, a complete map will be had of the belt of country traversed.

312. By Intersections. Let it be required to plot the stations M , O , P . Measure carefully the base line LN , and draw to a convenient scale ln on the paper to represent it. At the extremities of this base line orient and point the instrument to the several stations. The intersections of the pairs of lines drawn from the base line to these stations will indicate their position on the plot. Their distances from the base line, if desired, may be obtained by applying the scale used in the construction of ln .



If a field or closed tract of land is to be surveyed, a portion or all of one side may be used as a base line, or a base may be chosen outside the tract.

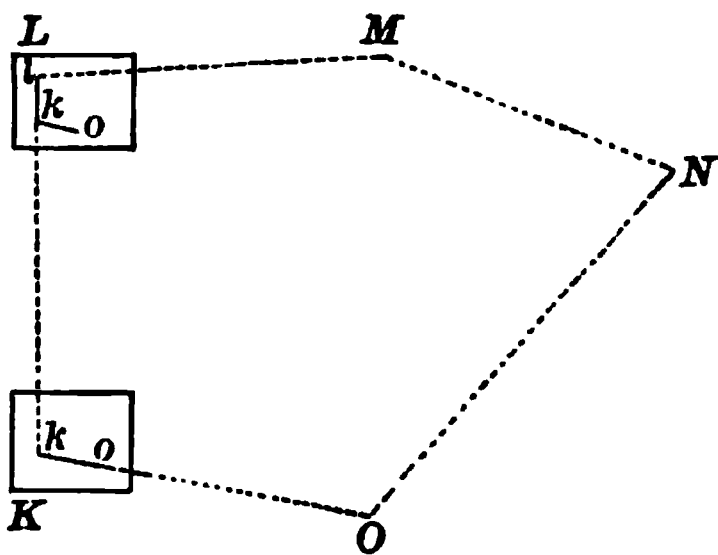
This method is obviously well adapted to the mapping of harbors, shore lines, and generally to inaccessible points.

Of course in this, as in all triangulations, well-conditioned triangles give more satisfactory results; that is to say, avoid, if possible, angles less than 30° or greater than 150° .

313. By Resection. This method requires the measurement of one line and the accessibility of all the stations.

Let $KLMNO$ represent the points to be plotted.

Obtain the distance between two of them, as OK , lay it off on the table to a suitable scale, and let ok represent it. Orient the table at k , point the alidade to L , and draw along its fiducial edge an indefinite line. Remove the instrument to L , and orient it. Then with the alidade centring on o , point it in the direction of O , and draw a line along its



edge: this line will intersect kL in some point l , which will locate L on the plot. Through l draw a line towards M , remove the instrument to M , and proceed as before. Objects on either side of the lines may be determined by radiation or by intersection, and further details, if desired, sketched in as the work proceeds.

314. Determination of Position by Resection on Three Known Points. In this problem three stations, L , N , O , are plotted, as l , n , o , on the table, and the instrument being set up over a fourth point P , it is required to find the position of this point on the map. This is the three-point problem of which geometrical constructions and analytical solutions are given in Chapter II. Section IV. It may be solved thus: Fasten a sheet of tracing-paper on the board, fix a point p to represent the station at which the instrument is set; with the alidade centring on p , direct the telescope successively to L , O , and N , and draw lines of indefinite length along the ruler's edge towards these stations. Then if the tracing-paper be shifted until the three lines thus drawn coincide with the points l , o , and n , the point p will indicate the position of P .

The position of this point may now be transferred, by pricking, to the map, the tracing-paper removed, and the table oriented.

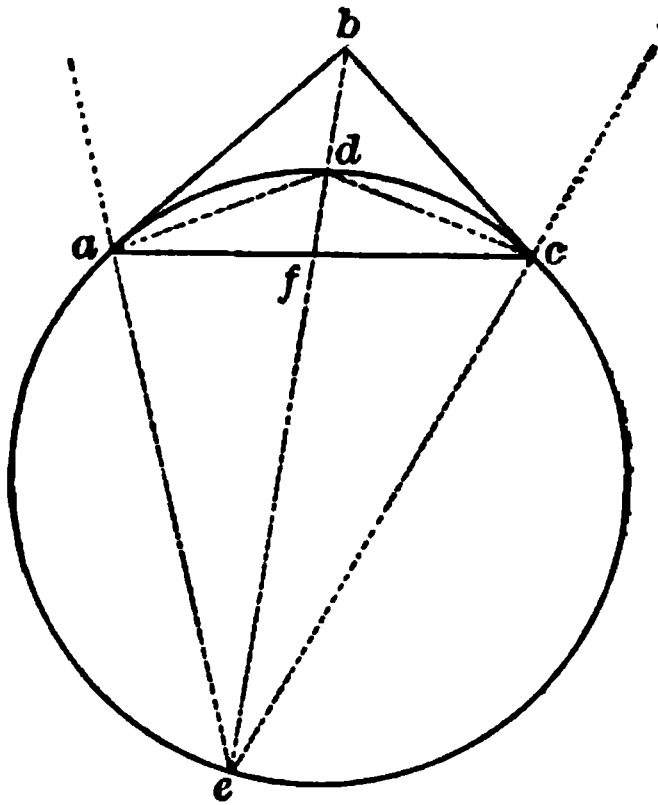
315.* Bessel's Method by Inscribed Quadrilateral. A quadrilateral is constructed with all the angles in the circumference of a circle, one diagonal of which passes through the middle one of the three fixed points and the point sought. On this line the alidade is set, the telescope directed to the middle point, and the table is *in position*. Resection upon the extreme points intersects in this line and determines the position of the point sought.

Let a , b , c , be the points on the sheet representing the signals A , B , C , in the ground.

The table is set up at the point to be determined (d) and

* Articles 315 and 316 are from the U. S. C. & G. S. Report for 1880.

levelled. The alidade is set upon the line ca , and a directed, by revolving the table, to its corresponding signal A , and the table clamped; then, with the alidade centring on c , the middle signal B is sighted with the telescope, and the line ce drawn along the edge of the ruler. The alidade is then set upon the line ac , and the telescope directed to the signal C , by revolving the table, and the table clamped. Then, with the alidade centring on a , the telescope is directed to the middle signal B , and the line ae is drawn along the edge of the ruler. The point e (the intersection of these two lines) will be in the line



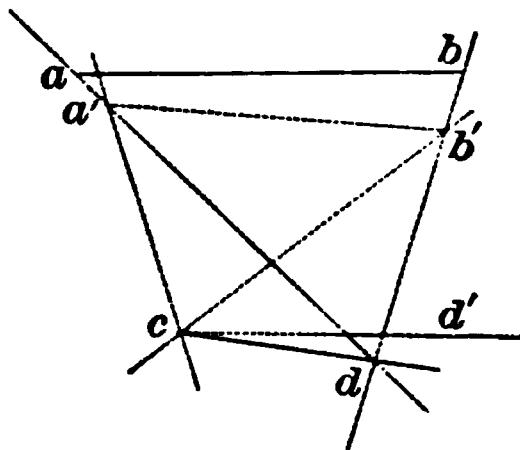
passing through the middle point and the point sought. Set the alidade upon the line be , direct b to the signal B by revolving the table, and the table will be in position. Clamp the table, centre the alidade upon a , direct the telescope to the signal A , and draw along the ruler the line ad . This will intersect the line be at the point sought. Resection upon C , centring the alidade on c in the same manner as upon A , will verify its position.

The opposite angles of the quadrilateral $adce$ being supplementary, angle ace and angle ade are subtended by the same chord ae and cae and cde are subtended by the same chord ce ; consequently, the intersection of ae and ce at e must fall on the line db ; or, the segments of two intersecting chords in a circle being reciprocally proportional, the triangles adf and cef are similar, and the triangles cdf and $ae f$ are similar, and d , f , and e must be in a right line passing through b .

316. Determination of Position by Resection on Two Known Points. This is called the *two-point problem*, there being given

by their projections a , b , two points A and B , to put the plane-table in position at a third point C . (The capital letters refer to points on the ground, and the small ones to their corresponding projections.)

Select a fourth point D , such that the intersections from C and D upon A and B make sufficiently large angles for good determinations. Put the table approximately in position at D , by estimation or by compass, and draw the lines Aa , Bb , intersecting in d ; through d draw a line to C . Then set up at C , and assuming the point c on the line dC at an estimated distance from d , and putting the table in a position parallel to that which is occupied at D , by means of the line cd , draw the lines from c to A , and from c to B . These will intersect the lines dA , dB , at points a' and b' , which form with c and d a quadrilateral *similar* to the true one, but erroneous in size and position.



The angles which the lines ab and $a'b'$ make with each other is the error in position. By constructing now through c a line cd' , making the same angle with cd as that which ab makes with $a'b'$, and directing this line cd' to D , the table will be brought into position, and the true point c can be found by the intersections of aA and bB .

Instead of transferring the angle of error by construction, we may conveniently proceed as follows, observing that the angle which the line $a'b'$ makes with ab is the error in the position of the table. As the table now stands, $a'b'$ is parallel with AB , but we want to turn it so that ab shall be parallel to the same. If, therefore, we place the alidade on $a'b'$, and set up a mark

in that direction, then place the alidade on ab , and turn the table until it again points to the mark, then ab will be parallel to AB , and the table is in position.

317. Practical Suggestions in using the Plane-Table.* The board should be placed so low as to be readily reached, even at the most remote corner, and yet high enough to enable the observer to take sight with comfort. This will bring it a little below the elbow.

Care must be taken that no part of the body touch or rest against the edge of the board. In using the alidade, steady the standard with the left hand, while the right swings the rear end of the ruler in the proper direction.

Thumb-tacks and rollers for holding down the sheet are both found objectionable, especially in high winds. The edges may be pasted underneath, or spring clamps may be used to advantage. A scale graduated upon the fiducial edge of the alidade is inconvenient, and in some positions impracticable and wasteful of time. A detached triangular boxwood or metal scale is greatly to be preferred. Umbrellas or shades, whilst a great relief to the eyes, are cumbersome and troublesome, and by blowing over on the table may cause damage or derangement. Colored glasses screening the eyes will be better, and by using tinted paper, as manilla, instead of white, still more relief is given, and the sheet can be kept cleaner.

Before leaving the station, and at any intervals not otherwise employed, the "check" shots should be tested to determine any displacement of the board.

Use as hard a pencil, and make as few lines, as possible. In locating points of contours, plot the distance at once along the edge of ruler by detached scale, making only a dot at the point which should receive the number of the contour.

Objects on a straight line may be quickly located by plotting the ends and determining the intermediate points by intersecting shots.

* From *The Topographer*, by L. M. Haupt, C.E., Philadelphia.

EXERCISES WITH THE PLANE-TABLE.

1. Make a plane-table survey of a field, using one side as a base line.
2. Make a survey embracing 200 or 300 rods of a road or stream, locating prominent objects on either side.
3. Locate several points on the table by *intersections*, and check the work by *resection* from these points.
4. Locate a non-plotted point by resection on three known points — first method ; check by Bessel's method.

CHAPTER VI.

THE SURVEY OF THE PUBLIC LANDS OF THE UNITED STATES.

THE SOLAR COMPASS.

318. A description of the Solar Compass, the instrument that is extensively used in the survey of the public lands, its adjustment and use, will be given before describing the method employed by the government in these surveys.

This instrument, so ingeniously contrived for readily determining a true meridian or north and south line, was invented by William A. Burt, of Michigan, and patented by him in 1836.

It has since come into general use in the surveys of United States public lands, the principal lines of which are required to be run with reference to the true meridian.

The arrangement of its sockets and plates is similar to that of the Surveyor's Transit, as shown in Chapter II. Section I., except that the sight-vanes are attached to the under plate or limb, and this revolves around the upper or vernier plate on which the solar apparatus is placed.

The limb is divided to half-degrees, is figured in two rows, as usual, and reads by the two opposite verniers to single minutes.

THE SOLAR APPARATUS.

319. The Solar Apparatus is seen in the place of the needle, and in fact operates as its substitute in the field.

It consists mainly of three arcs of circles, by which can be set off the latitude of a place, the declination of the sun, and the hour of the day.

These arcs, designated in the cut by the letters *a*, *b*, and *c*, are therefore termed the latitude, the declination, and the hour arcs respectively.

320. The Latitude Arc *a* has its centre of motion in two pivots, one of which is seen at *d*; the other is concealed in the cut.

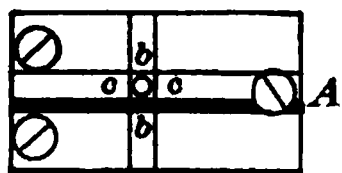
It is moved either up or down within a hollow arc, seen in the cut, by a tangent-screw at *f*, and is securely fastened in any position by a clamp-screw.

The latitude arc is graduated to quarter-degrees, and reads by its vernier *e* to single minutes; it has a range of about 35 degrees, so as to be adjustable to the latitude of any place in the United States.

321. The Declination Arc *b* is also graduated to quarter-degrees, and has a range of about 28 degrees.

Its vernier *v*, reading to single minutes, is fixed to a movable arm *h*, having its centre of motion at the end of the declination arc at *g*; the arm is moved over the surface of the declination arc, and its vernier set to any reading by turning the head of the tangent-screw *k*. It is also securely clamped in any position by a screw, concealed in the engraving.

322. Solar Lenses and Lines. At each end of the arm *h* is a rectangular block of brass, in which is set a small convex lens, having its focus on the surface of a little silver plate *A* (marginal figure), fastened by screws to the inside of the opposite block.



On the surface of the plate are marked two sets of lines intersecting each other at right angles; of these *bb* are termed the hour lines, and *cc* the equatorial lines, as having reference respectively to the hour of the day and the position of the sun in relation to the equator. In the cut the equatorial lines are those on the lower block, parallel to the surface of the



SOLAR COMPASS.

A description, etc., of Solar Attachments to Transit is given in Chapter II. pages 78-88.

hour arc c ; the hour lines are of course those at right angles to the first.

323. Equatorial Sights. On the top of each of the rectangular blocks is seen a little sighting-piece, termed the equatorial sight, fastened to the block by a small, milled head-screw, so as to be detached at pleasure.

They are used, as will be explained hereafter, in adjusting the different parts of the solar apparatus.

324. The Hour Arc c is supported by the two pivots of the latitude arc already spoken of, and is also connected with that arc by a curved arm, as shown in the figure.

The hour arc has a range of about 120° , is divided to half-degrees, and figured in two series, designating both the hours and the degrees, the middle division being marked 12 and 90 on either side of the graduated lines.

325. The Polar Axis. Through the centre of the hour arc passes a hollow socket p containing the spindle of the declination arc, by means of which this arc can be moved from side to side over the surface of the hour arc, or turned completely round, as may be required.

The hour arc is read by the lower edge of the graduated side of the declination arc.

The axis of the declination arc, or indeed the whole socket p , is appropriately termed the polar axis.

326. The Adjuster. Besides the parts shown in the cut, there is also an arm used in the adjustment of the instrument as described hereafter, but laid aside in the box when that is effected.

The parts just described constitute properly the solar apparatus.

Besides these, however, are seen the needle-box n with its arc and tangent screw t , and the spirit levels, for bringing the whole instrument to a horizontal position.

327. The Needle-Box *n* has an arc of about 36 degrees in extent, divided to half-degrees, and figured from the centre or zero mark on either side.

The needle, which is made as in other instruments, except that the arms are of unequal lengths, is raised or lowered by a lever shown in the cut.

The needle-box is attached by a projecting arm to a tangent-screw *t*, by which it is moved about its centre, and its needle set to any variation.

This variation is also read off by the vernier on the end of the projecting arm, reading to three minutes a graduated arc, attached to the plate of the compass.

328. The Levels seen with the solar apparatus have ground-glass vials, and are adjustable at their ends like those of other instruments.

The edge of the circular plate on which the solar work is placed is divided and figured at intervals of 10 degrees, and numbered, as shown, from 0 to 90 on each side of the line of sight.

These graduations are used in connection with a little brass pin, seen in the centre of the plate, to obtain approximate bearings of lines, which are not important enough to require a close observation.

329. Lines of Refraction. The inside faces of the sights are also graduated and figured, to indicate the amount of refraction to be allowed when the sun is near the horizon.

PRINCIPLES OF THE SOLAR COMPASS.

330. The interval between two equatorial lines *cc*, in figure on page 276, as well as between the hour lines *bb*, is just sufficient to include the circular image of the sun, as formed by the solar lens on the opposite end of the revolving arm *h*, figure on page 277.

When, therefore, the instrument is made perfectly horizontal, the equatorial lines and the opposite lenses being accurately adjusted to each other by a previous operation, and the sun's image brought within the equatorial lines, his position in the heavens, with reference to the horizon, will be defined with precision.

Suppose the observation to be made at the time of one of the equinoxes; the arm h , set at zero on the declination arc b ; and the polar axis p , placed exactly parallel to the axis of the earth.

Then the motion of the arm h , if revolved on the spindle of the declination arc around the hour circle c , will exactly correspond with the motion of the sun in the heavens, on the given day and at the place of observation; so that if the sun's image was brought between the lines cc in the morning, it would continue in the same position, passing neither above nor below the lines, as the arm was made to revolve in imitation of the motion of the sun about the earth.

In the morning, as the sun rises from the horizon, the arm h will be in a position nearly at right angles to that shown in the cut, the lens being turned towards the sun, and the silver plate on which his image is thrown directly opposite.

As the sun ascends, the arm must be moved around, until when he has reached the meridian, the graduated side of the declination arc will indicate 12 on the hour circle, and the arm h , the declination arc b , and the latitude arc a will be in the same plane.

As the sun declines from the meridian, the arm h must be moved in the same direction, until at sunset its position will be the exact reverse of that it occupied in the morning.

331. Allowance for Declination. Let us now suppose the observation made when the sun has passed the equinoctial point, and when his position is affected by declination.

By referring to the almanac, and setting off on the arc his declination for the given day and hour, we are still able to

determine his position with the same certainty as if he remained on the equator.

When the sun's declination is south, that is, from the 22d of September to the 20th of March in each year, the arc *b* is turned towards the plates of the compass, as shown in the engraving, and the solar lens *o*, with the silver plate opposite, are made use of in the surveys.

The remainder of the year the arc is turned from the plates, and the other lens and plate employed.

When the solar compass is accurately adjusted, and its plates made perfectly horizontal, the latitude of the place, and the declination of the sun for the given day and hour, being also set off on the respective arcs, *the image of the sun cannot be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth.* The slightest deviation from this position will cause the image to pass above or below the lines, and thus discover the error.

We thus, from the position of the sun in the solar system, obtain a certain direction absolutely unchangeable, from which to run our lines and measure the horizontal angles required.

This simple principle is not only the basis of the construction of the solar compass, but the sole cause of its superiority to the ordinary or magnetic instrument. For in a needle instrument the accuracy of the horizontal angles indicated, and therefore of all the observations made, depends upon "the delicacy of the needle, and the constancy with which it assumes a certain direction, termed the magnetic meridian."

The principal causes of error in the needle, briefly stated, are the dulling of the pivot, the loss of polarity in the needle, the influence of local attraction, and the effect of the sun's rays, producing the diurnal variation.

From all these imperfections the solar instrument is free.

The sights and the graduated limb being adjusted to the solar apparatus, and the latitude of the place and the declination of the sun also set off upon the respective arcs, we are able not only

to run the true meridian, or a due east and west course, but also to set off the horizontal angles with minuteness and accuracy from a direction which never changes, and is unaffected by attraction of any kind.

TO ADJUST THE SOLAR COMPASS.

The adjustments of this instrument, with which the surveyor will have to do, are simple and few in number, and will now be given in order.

332. To Adjust the Levels. Proceed precisely as directed in the account of the other instruments we have described, by bringing the bubbles into the centre of the tubes by the leveling-screws of the tripod, and then reversing the instrument upon its spindle, and raising or lowering the ends of the tubes, until the bubbles will remain in the centre during a complete revolution of the instrument.

333. To Adjust the Equatorial Lines and Solar Lenses. First detach the arm *h* from the declination arc by withdrawing the screws shown in the cut from the ends of the posts of the tangent-screw *k*, and also the clamp-screw, and the conical pivot with its small screws by which the arm and declination arc are connected.

The arm *h* being thus removed, attach the adjuster in its place by replacing the conical pivot and screws, and insert the clamp-screw so as to clamp the adjuster at any point on the declination arc.

Now level the instrument, place the arm *h* on the adjuster, with the same side resting against the surface of the declination arc as before it was detached. Turn the instrument on its spindle so as to bring the solar lens to be adjusted in the direction of the sun, and raise or lower the adjuster on the declination arc, until it can be clamped in such a position as to bring the sun's image as near as may be between the equatorial lines on the opposite silver plate, and bring the image precisely into

position by the tangent of the latitude arc or the levelling-screws of the tripod. Then carefully turn the arm half-way over, until it rests upon the adjuster by the opposite faces of the rectangular blocks, and again observe the position of the sun's image.

If it remains between the lines as before, the lens and plate are in adjustment; if not, loosen the three screws which confine the plate to the block, and move the plate under their heads, until one-half the error in the position of the sun's image is removed.

Again bring the image between the lines, and repeat the operation until it will remain in the same situation, in both positions of the arm, when the adjustment will be completed.

To adjust the other lens and plate, reverse the arm end for end on the adjuster, and proceed precisely as in the former case, until the same result is attained.

In tightening the screws over the silver plate, care must be taken not to move the plate.

This adjustment now being complete, the adjuster should be removed, and the arm *h* with its attachments replaced as before.

334. To Adjust the Vernier of the Declination Arc. Having levelled the instrument, and turned its lens in the direction of the sun, clamp to the spindle, and set the vernier *v* of the declination arc at zero, by means of the tangent-screw at *k*, and clamp to the arc.

See that the spindle moves easily and yet truly in the socket, or polar axis, and raise or lower the latitude arc by turning the tangent-screw *f*, until the sun's image is brought between the equatorial lines on one of the plates. Clamp the latitude arc by the screw, and bring the image precisely into position by the levelling-screws of the tripod or socket, and without disturbing the instrument, carefully revolve the arm *h*, until the opposite lens and plate are brought in the direction of the sun, and note if the sun's image comes between the lines as before.

If it does, there is no index error of the declination arc; if not, with the tangent-screw *k*, move the arm until the sun's image passes over half the error; again bring the image between the lines, and repeat the operation as before, until the image will occupy the same position on both the plates.

We shall now find, however, that the zero marks on the arc and the vernier do not correspond, and to remedy this error, the little flat-head screws above the vernier must be loosened until it can be moved so as to make the zeros coincide, when the operation will be completed.

335. To Adjust the Solar Apparatus to the Compass Sights. First level the instrument, and with the clamp and tangent screws set the main plate at 90° by the verniers and horizontal limb. Then remove the clamp-screw, and raise the latitude arc until the polar axis is by estimation very nearly horizontal, and if necessary, tighten the screws on the pivots of the arc, so as to retain it in this position.

Fix the vernier of the declination arc at zero, and direct the equatorial sights to some distant and well-marked object, and observe the same through the compass sights. If the same object is seen through both, and the verniers read to 90° on the limb, the adjustment is complete; if not, the correction must be made by moving the sights or changing the position of the verniers.

TO USE THE SOLAR COMPASS.

336. Before this instrument can be used at any given place, it is necessary to set off upon its arcs both the declination of the sun as affected by its refraction for the given day and hour, and the latitude of the place where the observation is made.

337. To Set off the Declination. The declination of the sun, given in the ephemeris of the Nautical Almanac from year to year, is calculated for apparent noon at Greenwich, England, or Washington, D.C.

To determine it for any other hour at a place in the United

States, reference must be had, not only to the difference of time arising from the longitude, but also to the change of declination from day to day.

By the use of standard time, which is now quite general throughout the United States, it is very easy to obtain the declination required at any place.

For those using 75th meridian time, a difference of five hours must be allowed for the difference in declination between the place of observation and Greenwich.

The time-piece referred to the 75th meridian as standard indicating 7 A.M. when it is noon at Greenwich.

Where the 90th meridian is used as standard, six hours must be allowed, etc.

To obtain the declination for the other hours of the day, take from the almanac the declination for apparent noon of the given day, and, as the declination is increasing or decreasing, add to or subtract from the declination of the first hour the difference for one hour as given in the ephemeris, which will give, when affected by the refraction, the declination for the succeeding hour; and proceed thus in making a table of the declination for every hour of the day.

338. Refraction. By reason of the increasing density of the atmosphere from its upper regions to the earth's surface, the rays of light from the sun are bent out of their course, so as to make his altitude appear greater than is actually the case.

The amount of refraction varies according to the altitude of the body observed; being 0 when it is in the zenith, about one minute when midway from the horizon to the zenith, and almost 34' when in the horizon.

339. Effect of Incidental Refraction. It will be seen by referring to the instrument, that the effect of the ordinary refraction upon the position of the sun's image with reference to the equatorial lines, which, in fact, are the only ones to be regarded in running lines with the solar compass, is continually

changing, not only with the change of latitude, but also with that of the sun's declination from hour to hour, and the motion of the revolving arm as it follows the sun in its daily revolution.

If the equatorial lines were always in the same vertical plane with the sun, as would be the case at the equator at the time of the equinoxes, it is evident that refraction would have no effect upon the position of the image between these lines, and therefore would not be of any importance to the surveyor.

But as we proceed further north, and as the sun's declination to the south increases, the refraction also increases, and must now be taken into account.

Again, the angle which the equatorial lines make with the horizon is continually changing as the arm is made to follow the motion of the sun during the course of a day.

Thus, in the morning and evening they are more or less inclined to the horizon, while at noon they are exactly parallel to it.

And thus it follows that the excess of refraction at morning and evening is in some measure balanced by the fact that the position of the sun's image with reference to the equatorial lines is then less affected by it, on account of the greater inclination of the lines to the horizon.

340. Allowance for Refraction. The proper allowance to be made for refraction in setting off the declination of the sun upon the solar compass for any hour of any day of the year is given in the following table :

A TABLE OF MEAN REFRACTIONS IN DECLINATION.

To apply on the declination arc of Solar Attachment of either Compass or Transits.*

| HOUR ANGLE. | DECLINATIONS. | | | | | | | | |
|-----------------------|-------------------|-------|-------|------|------|------|--------|--------|--------|
| | FOR LATITUDE 30°. | | | | | | | | |
| | + 20° | + 15° | + 10° | + 5° | 0° | — 5° | — 10° | — 15° | — 20° |
| 0 h. | 10'' | 15'' | 21'' | 27'' | 33'' | 40'' | 48'' | 57'' | 1'08'' |
| 2 | 14 | 19 | 25 | 31 | 38 | 46 | 54 | 1'05 | 1 18 |
| 3 | 20 | 26 | 32 | 39 | 47 | 55 | 1'06 | 1 19 | 1 36 |
| 4 | 32 | 39 | 46 | 52 | 1'06 | 1'19 | 1 35 | 1 57 | 2 29 |
| 5 | 1'00 | 1'10 | 1'24 | 1'52 | 2 07 | 2 44 | 3 46 | 5 43 | 13 06 |
| FOR LATITUDE 32° 30'. | | | | | | | | | |
| 0 h. | 13'' | 18'' | 24'' | 30'' | 36'' | 44'' | 52'' | 1'02'' | 1'14'' |
| 2 | 17 | 22 | 28 | 35 | 42 | 50 | 1'00 | 1 11 | 1 26 |
| 3 | 23 | 29 | 35 | 43 | 51 | 1'01 | 1 13 | 1 28 | 1 47 |
| 4 | 35 | 43 | 51 | 1'01 | 1'13 | 1 27 | 1 46 | 2 13 | 2 54 |
| 5 | 1'03 | 1'15 | 1'31 | 1 53 | 2 20 | 3 05 | 4 25 | 7 36 | |
| FOR LATITUDE 35°. | | | | | | | | | |
| 0 h. | 15'' | 21'' | 27'' | 33'' | 40'' | 48'' | 57'' | 1'08'' | 1'21'' |
| 2 | 20 | 25 | 32 | 38 | 46 | 55 | 1'05 | 1 18 | 1 35 |
| 3 | 26 | 33 | 39 | 47 | 56 | 1'07 | 1 21 | 1 38 | 2 00 |
| 4 | 39 | 47 | 56 | 1'07 | 1'20 | 1 36 | 1 59 | 2 32 | 3 25 |
| 5 | 1'07 | 1'20 | 1'38 | 2 00 | 2 34 | 3 29 | 5 14 | 10 16 | |
| FOR LATITUDE 37° 30'. | | | | | | | | | |
| 0 h. | 18'' | 24'' | 30'' | 36'' | 44'' | 52'' | 1'02'' | 1'14'' | 1'29'' |
| 2 | 22 | 28 | 35 | 42 | 50 | 1'00 | 1 12 | 1 26 | 1 45 |
| 3 | 29 | 36 | 43 | 52 | 1'02 | 1 14 | 1 29 | 1 49 | 2 16 |
| 4 | 43 | 51 | 1'01 | 1'13 | 1 27 | 1 49 | 2 14 | 2 54 | 4 05 |
| 5 | 1'11 | 1'26 | 1 45 | 2 10 | 2 49 | 3 55 | 6 15 | 14 58 | |

* Computed by Edward W. Arms, C.E., for W. and L. E. Gurley, Troy, N.Y.

| Hour Angle. | DECLINATIONS. | | | | | | | | |
|-----------------------|-------------------|-------|-------|------|--------|--------|--------|--------|--------|
| | FOR LATITUDE 40°. | | | | | | | | |
| | + 20° | + 15° | + 10° | + 5° | 0° | − 5° | − 10° | − 15° | − 20° |
| 0 h. | 21'' | 27'' | 33'' | 40'' | 48'' | 57'' | 1'08'' | 1'21'' | 1'39'' |
| 2 | 25 | 32 | 39 | 46 | 52 | 1'06 | 1 19 | 1 35 | 1 57 |
| 3 | 33 | 40 | 48 | 57 | 1'08 | 1 21 | 1 38 | 2 02 | 2 36 |
| 4 | 47 | 55 | 1'06 | 1'19 | 1 36 | 1 58 | 2 30 | 3 21 | 4 59 |
| 5 | 1'15 | 1'31 | 1 51 | 2 20 | 3 05 | 4 25 | 7 34 | 25 18 | |
| FOR LATITUDE 42° 30'. | | | | | | | | | |
| 0 h. | 24'' | 30'' | 36'' | 44'' | 52'' | 1'02'' | 1'14'' | 1'29'' | 1'49'' |
| 2 | 28 | 35 | 39 | 50 | 1'00 | 1 12 | 1 26 | 1 45 | 2 11 |
| 3 | 36 | 43 | 52 | 1'02 | 1 13 | 1 29 | 1 49 | 2 17 | 2 59 |
| 4 | 50 | 1'00 | 1'11 | 1 26 | 1 44 | 2 10 | 2 49 | 3 55 | 6 16 |
| 5 | 1'16 | 1 36 | 1 58 | 2 30 | 3 22 | 5 00 | 9 24 | | |
| FOR LATITUDE 45°. | | | | | | | | | |
| 0 h. | 27'' | 33'' | 40'' | 48'' | 57'' | 1'08'' | 1'21'' | 1'39'' | 2'02'' |
| 2 | 32 | 39 | 46 | 52 | 1'06 | 1 19 | 1 35 | 1 57 | 2 29 |
| 3 | 40 | 47 | 56 | 1'07 | 1 21 | 1 38 | 2 00 | 2 34 | 3 29 |
| 4 | 54 | 1'04 | 1'16 | 1 33 | 1 54 | 2 24 | 3 11 | 4 38 | 8 15 |
| 5 | 1'23 | 1 41 | 2 05 | 2 41 | 3 40 | 5 40 | 12 02 | | |
| FOR LATITUDE 47° 30'. | | | | | | | | | |
| 0 h. | 30'' | 36'' | 44'' | 52'' | 1'02'' | 1'14'' | 1'29'' | 1'49'' | 2'18'' |
| 2 | 35 | 42 | 50 | 1'00 | 1 12 | 1 26 | 1 45 | 2 01 | 2 51 |
| 3 | 43 | 51 | 1'01 | 1 13 | 1 28 | 1 47 | 2 15 | 2 56 | 4 08 |
| 4 | 56 | 1'09 | 1 23 | 1 40 | 2 05 | 2 40 | 3 39 | 5 37 | 11 18 |
| 5 | 1'27 | 1 46 | 2 12 | 2 52 | 4 01 | 6 30 | 16 19 | | |
| FOR LATITUDE 50°. | | | | | | | | | |
| 0 h. | 33'' | 40'' | 48'' | 57'' | 1'08'' | 1'21'' | 1'39'' | 2'02'' | 2'36'' |
| 2 | 38 | 46 | 55 | 1'06 | 1 18 | 1 35 | 1 57 | 2 28 | 3 19 |
| 3 | 47 | 56 | 1'06 | 1 19 | 1 36 | 2 29 | 2 31 | 3 23 | 5 02 |
| 4 | 1'02 | 1'14 | 1 29 | 1 48 | 2 16 | 2 58 | 4 18 | 6 59 | 19 47 |
| 5 | 1 30 | 1 51 | 2 19 | 3 04 | 4 22 | 7 28 | 24 10 | | |

EXPLANATION OF THE TABLE OF REFRACTIONS.*

The table is calculated for latitudes between 30° and 50° at intervals of $2\frac{1}{2}^{\circ}$, that being as near as is required.

The declination ranges from 0 to 20° , both north and south, the + declinations being north, and — south, and is given for every 5 degrees, that being sufficiently near for all practical purposes.

The hour angle in the first column indicates the distance of the sun from the meridian in hours, the refraction given for 0 hours being that which affects the observed declination of the sun when on the meridian, commonly known as meridional refraction; the refraction for the hours just before and after noon is so nearly that of the meridian, that it may be called and allowed as the same.

When the table is used, it must be borne in mind that when the declination is north or + in the table, the refraction is to be added; when the declination is south or — the refraction must be subtracted.

It will be noticed that the refraction in south or — declination increases very rapidly as the sun nears the horizon, showing that observations should not be taken with the sun when south of the equator, less than one hour from the horizon.

Thus, suppose it be required to obtain the declination for any hour in the day, April 16, 1887, at Pittsburg, Pa., where 75th meridian time is used.

The difference in time is 5 hours, so that the declination given in the ephemeris for apparent noon of that day at Greenwich would be that of 7 A.M. at Pittsburg. Proceed as follows:

Declination at Greenwich, mean noon, April 16, 1887,

N. $10^{\circ} 6' 29''$

Add 1' 51'' = refract'n for 5 hrs. [lat. Pittsburg $40^{\circ} 28'$].

Or, N. $10^{\circ} 8' 20''$ = dec. 7 A.M. at Pittsburg.

* See also Refraction Table, page 92.

To get the declination for 8 o'clock, same day and place, add 53'', the difference for one hour — because the declination is increasing — to the declination taken from the almanac, and this increased by the refraction corresponding to 4 hours from noon will give $10^{\circ} 8' 28''$ for the required declination.

Again, suppose it be desired to obtain the corrected declination for 8 A.M. Oct. 15, 1887, same place.

The declination being now south, the refraction is to be subtracted, but the hourly difference is to be added because the declination is increasing, as in the first example; thus:

Declination at Greenwich, mean noon, Oct. 15, 1887,

S. $8^{\circ} 30' 20''$

Add $56'' = \text{dec. for 1 hr., and increasing.}$

S. $8^{\circ} 31' 16''$

Subtract $2' 23'' = \text{refr. 4 hrs. from noon.}$

Or, S. $8^{\circ} 28' 53'' = \text{dec. at 8 A.M. ;}$

and so on for any hour in the day, obtaining from the declination at Greenwich, by the proper application of the hourly motion, the declination corresponding to the hour required, and correcting this for refraction due to altitude.

To facilitate operations, the calculation of the declination for the different hours of the day should be made and noted before the surveyor commences his work.

341. To Set off the Latitude. Find the declination of the sun for the given day at noon, at the place of observation as just described, and with the tangent-screw set it off upon the declination arc, and clamp the arm firmly to the arc.

Observe in the almanac the equation of time for the given day, in order to know about the time the sun will reach the meridian.

Then, about fifteen or twenty minutes before this time, set up the instrument, level it carefully, fix the divided surface of the declination arc at 12 on the hour circle, and turn the instru-

ment upon its spindle until the solar lens is brought into the direction of the sun.

Loosen the clamp-screw of the latitude arc, and with the tangent-screw raise or lower this arc until the image of the sun is brought precisely between the equatorial lines, and turn the instrument from time to time so as to keep the image also between the hour lines on the plate.

As the sun ascends, its image will move below the lines, and the arc must be moved to follow it. Continue thus, keeping it between the two sets of lines until its image begins to pass above the equatorial lines, which is also the moment of its passing the meridian.

Now read off the vernier of the arc, and we have the latitude of the place, which is always to be set off on the arc when the compass is used at the given place.

It is the practice of surveyors using the solar compass to set off, in the manner just described, the latitude of the point where the survey begins, and to repeat the observation and correction of the latitude arc every day when the weather is favorable, there being also an hour at mid-day when the sun is so near the meridian as not to give the direction of lines with the certainty required.

342. To Run Lines with the Solar Compass. Having set off in the manner just given the latitude and declination upon their respective arcs, the instrument being also in adjustment, the surveyor is ready to run lines by the sun.

To do this, the instrument is set over the station and carefully levelled, the plates clamped at zero on the horizontal limb, and the sights directed north and south, the direction being given, when unknown, approximately by the needle.

The solar lens is then turned to the sun, and with one hand on the instrument, and the other on the revolving arm, both are moved from side to side, until the sun's image is made to appear on the silver plate; when, by carefully continuing the operation, it may be brought precisely between the equatorial lines.

Allowance being now made for refraction, the line of sights will indicate the true meridian; the observation may now be made, and the flag-man put in position.

When a due east and west line is to be run, the verniers of the horizontal limb are set at 90° , and the sun's image kept between the lines as before.

The solar compass being so constructed that when the sun's image is in position the limb must be clamped at 0 in order to run a true meridian line, it will be evident that the bearing of any line from the meridian may be read by the verniers of the limb precisely as in the ordinary magnetic compass: the bearings of lines are read from the ends of the needle.

343. Use of the Needle. In running lines, the magnetic needle is always kept with the sun; that is, the point of the needle is made to indicate 0 on the arc of the compass-box by turning the tangent-screw connected with its arm on the opposite side of the plate. By this means the lines can be run by the needle alone in case of the temporary disappearance of the sun; but, of course, in such cases the surveyor must be sure that no local attraction is exerted.

The variation of the needle, which is noted at every station, is read off in degrees and minutes on the arc, by the edge of which the vernier of the needle-box moves.

344. Allowance for the Earth's Curvature. When long lines are run by the solar compass, either by the true meridian, or due east and west, allowance must be made for the curvature of the earth.

Thus, in running north or south, the latitude changes about one minute for every distance of 92 chains 30 links, and the side of a township requires a change on the latitude arc of $5' 12''$, the township, of course, being six miles square.

This allowance is of constant use where the surveyor fails to get an observation on the sun at noon, and is a very close approximation to the truth.

In running due east and west, as in tracing the standard parallels of latitude, the sights are set at 90° on the limb, and the line is run at right angles to the meridian.

If no allowance were made for the earth's curvature, these lines would, if sufficiently produced, reach the equator, to which they are constantly tending.

Of course, in running short lines either east or west, the variation from the parallel would be so small as to be of no practical importance; but when long sights are taken, the correction should be made by taking fore and back sights at every station, noticing the error on the back-sight, and setting off one-half of it on the fore-sight on the side towards the pole.

345. Time of Day by the Sun. The time of day is best ascertained by the solar compass when the sun is on the meridian, as at the time of making the observation for latitude.

The time thus given is that of apparent noon, and can be reduced to mean time, by merely applying the equation of time as directed in the almanac, and adding or subtracting as the sun is slow or fast.

The time, of course, can also be taken before or after noon, by bringing the sun's image between the hour lines, and noticing the position of the divided edge of the revolving arm, with reference to the graduations of the hour circle, allowing four minutes of time for each degree of the arc, and thus obtaining apparent time, which must be corrected by the equation of time as just described.

346. Caution as to the False Image. In using the compass upon the sun, if the revolving arm be turned a little one side of its proper position, a false or reflected image of the sun will appear on the silver plate in nearly the same place as that occupied by the true one. It is caused by the reflection of the true image from the surface of the arm, and is a fruitful source of error to the inexperienced surveyor. It can, however, be

readily distinguished from the real image by being much less bright, and not so clearly defined.

347. Approximate Bearings. When the bearings of lines, such as the course of a stream, or the boundaries of a forest, are not desired with the certainty given by the verniers and horizontal limb, a rough approximation of the angle they make with the true meridian is obtained by the divisions on the outside of the circular plate.

In this operation, a pencil, or thin straight edge of any sort, is held perpendicularly against the circular edge of the plate, and moved around until it is in range with the eye, the brass centre-pin, and the object observed.

The bearing of the line is then read off at the point where the pencil is placed.

348. Time for Using the Solar Compass. The solar compass, like the ordinary instrument, can be used at all seasons of the year, the most favorable time being, of course, in the summer, when the declination is north, and the days are long, and more generally fair.*

ORIGIN OF THE SYSTEM FOR THE SURVEY OF THE PUBLIC LANDS.†

349. The present system of survey of the public lands was inaugurated by a committee appointed by the Continental Congress, of which Thomas Jefferson was chairman. This committee, on May 7, 1784, reported an ordinance requiring public lands to be divided into "hundreds" of ten geographical miles square, and these again subdivided into lots of one mile square, each to be numbered from 1 to 100, commencing in the *northwestern* corner and continuing from west to east and from

* See Article 147.

† The following pages regarding the government surveys are from "Instructions of the General Land Office to the Surveyors-General of the United States relative to the Survey of the Public Lands."

east to west consecutively. By subsequent amendment, April 26, 1785, the ordinance required the surveyors "to divide the said territory into townships of 7 miles square, by lines running due north and south, and others crossing these at right angles. The plots of the townships, respectively, shall be marked by subdivisions into sections of 1 mile square, or 640 acres in the same direction as the external lines, and numbered from 1 to 49, and these sections shall be subdivided into lots of 320 acres." This is the first record of the use of the terms "township" and "section."

This ordinance was subsequently still further amended, and as finally passed on the 20th of May, 1785, provided for townships 6 miles square, containing 36 sections of 1 mile square. The first public surveys were made under this ordinance by the direction of the Geographer of the United States.

| | | | | | |
|----|----|----|----|----|----|
| 6 | 5 | 4 | 3 | 2 | 1 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 17 | 16 | 15 | 14 | 13 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 30 | 29 | 28 | 27 | 26 | 25 |
| 31 | 32 | 33 | 34 | 35 | 36 |

The act of Congress, approved May 18, 1796, provided for the appointment of a surveyor-general, and directed the survey of lands northwest of the Ohio River, and above the mouth of the Kentucky River, "in which the titles of the Indian tribes have been extinguished," and among other provisions, that the "sections shall be numbered respectively,

beginning with the number one in the northeast section and proceeding west and east, alternately, through the township, with progressive numbers till the thirty-sixth be completed." This method of numbering sections, as shown by the preceding diagram, is still in use.

The act of Congress, approved Feb. 11, 1805, directs the subdivisions of the public lands into quarter-sections. The act of April 24, 1820, provides for the sale of the public lands in half-quarter-sections, and that in every case of the division of a quarter-section, the division line shall run north and south. April 5, 1832, Congress directed the subdivision of the public lands into quarter-quarters, and requiring the division line to run east and west.

350. A surveyor-general for each surveying district is appointed by the President, by and with the advice of the Senate. He is required, while in the discharge of the duties of his office, to reside in the district for which he is appointed. His term of office is four years, and he must give bonds, with sufficient security for the penal sum of \$30,000, for the faithful disbursement of all public money placed in his hands, and for the faithful performance of the duties of his office. Among other duties prescribed by law and set forth in the manual, the surveyor-general is required to engage a sufficient number of skilful surveyors as his deputies, and shall cause to be surveyed, measured, and marked, without delay, all base and meridian lines through such points, and perpetuated by such monuments, and such other correction parallels and meridians, as may be prescribed by law, or by instructions from the General Land Office, in respect to the public lands within his surveying district to which the Indian title has been or may be extinguished.

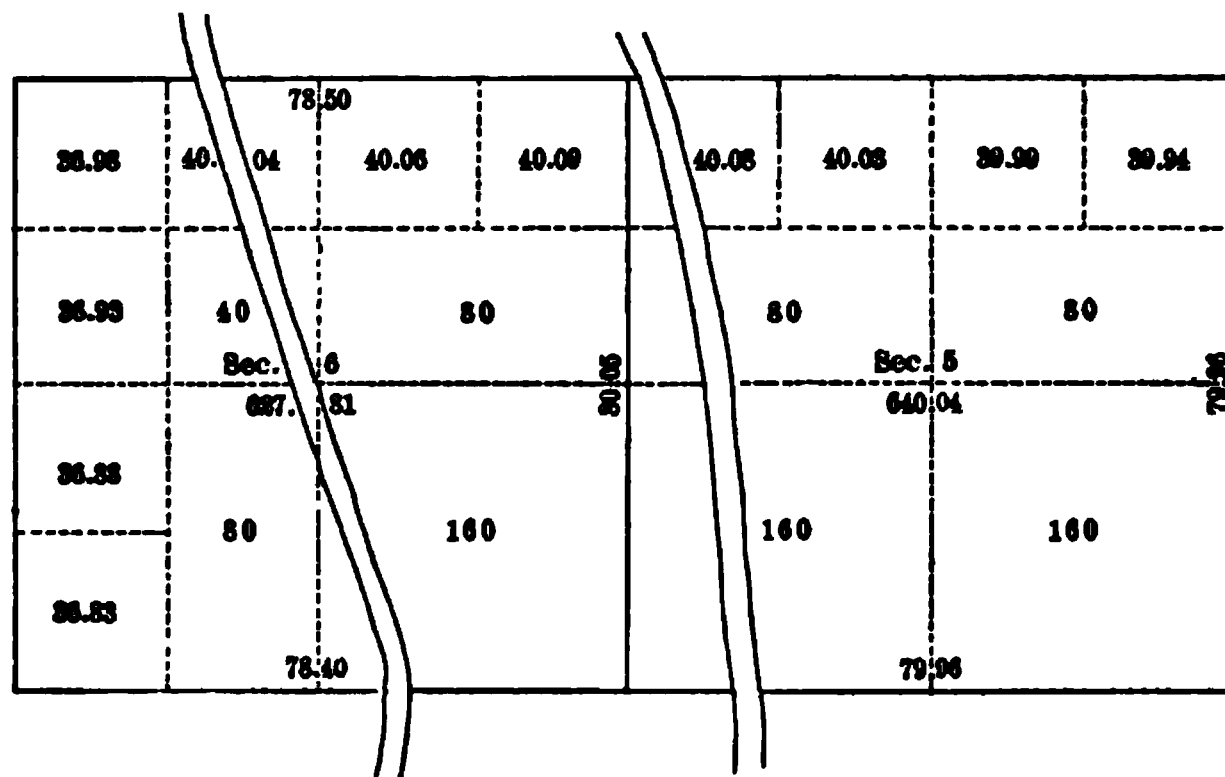
351. System of Rectangular Surveying. The public lands of the United States are ordinarily surveyed into rectangular tracts, bounded by lines conforming to the cardinal points.

The public lands shall be laid off, in the first place, into bodies of land 24 miles square, as near as may be. This shall be done by the extension of standard lines from the principal meridian every 24 miles, and by the extension from the base and standard lines, of auxiliary meridians every 24 miles. Thereafter they shall be laid off into bodies of land 6 miles square, as near as may be, called *townships*, containing, as near as may be, 23,040 acres. The townships shall be subdivided into 36 tracts, called *sections*, each containing, as near as may be, 640 acres. Any number or series of contiguous townships, situate north or south of each other, constitute a *range*.

(a) The law requires that the lines of the public surveys shall be governed by the true meridian, and that the township shall be *six miles square*, — two things involving in connection a mathematical impossibility. For strictly to conform to the meridian necessarily throws the township out of square, by reason of the convergency of meridians, and hence by adhering to the true meridian results the necessity of departing from the strict requirements of law, as respects the precise area of townships and the subdivisional parts thereof; the township assuming something of a trapezoidal form, which inequality develops itself more and more as such, the higher the latitude of the surveys. It is doubtless in view of these circumstances that the law provides (see Section 2 of the act of May 18, 1796) that the section of a mile square shall contain the quantity of 640 acres, *as nearly as may be*; and, moreover, provides (see Section 3 of the act of May 10, 1800) in the following words: “And in all cases where the exterior lines of the townships thus to be subdivided into sections or half-sections shall exceed, or shall not extend, 6 miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west or from south to north; the sections and half-sections bounded on the northern and western lines of such townships

shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity."

Sections 5 and 6 of Township No. 6, North, Range No. 34, east, of the principal meridian, Montana, are exhibited below :



(b) The section lines are surveyed from *south* to north on true meridians, and from *east* to west, in order to throw the excesses or deficiencies in measurements on the north and west sides of the township, as required by law. In a case where a township has been partially surveyed, and it is necessary to complete the survey of the same, or where the character of the land is such that only the north or west portions of the township can be surveyed, this rule cannot be strictly adhered to; but in such cases must be departed from only so far as is absolutely necessary. It will also be necessary to depart from this rule where surveys close upon State or Territorial boundaries, or upon surveys extending from different meridians.

(c) The townships are to bear numbers in respect to the base line, either north or south of it; and the tiers of townships called "ranges" will bear numbers in respect to the meridian line, according to their relative position to it, either on the east or west.

(d) The 36 sections into which a township is subdivided are numbered, commencing with number *one* at the *northeast* angle of the township and proceeding west to number 6, and thence proceeding east to number 12, and so on, alternately until the number 36 is in the southeast angle. In all cases of surveys of fractional townships, the sections should bear the same numbers as they would if the township were full.

(e) Standard parallels shall be established at intervals of every 24 miles, north and south of the base line, and auxiliary meridians at intervals of every 24 miles, east and west of the principal meridian; the object being to confine the errors resulting from convergence of meridians and inaccuracies in measurements, within the tracts of land bounded by the lines so established.

(f) The survey of all principal base and meridian standard parallels, and auxiliary meridian and township lines must be made with an instrument operating independently of the magnetic needle. Burt's *improved solar compass*, or other instrument of equal utility, must be used of necessity in such cases; and it is deemed best that such instrument should be used under all circumstances. Where the needle can be relied on, however, the ordinary compass may be used in subdividing and meandering. Whenever deputies use instruments with magnetic apparatus only, they must test the accuracy of their work and the condition of their instruments by at least three observations upon a circumpolar star, upon different days, between the commencement and close of surveying operations in any given township. Deputies using instruments with solar apparatus are not required to make observations of the star Polaris, but they must test their instruments by taking the latitude daily, weather permitting, in running base, standard, meridian, and range lines, and upon three different days, during the execution of subdivisional surveys in each township. They must make complete records in their field notes, under proper dates, of the making of all observations in compliance with these instructions, showing the style and condition of the instrument in use, and

the angle formed by comparing the line run with the meridian as determined by observations.

(g) The construction and adjustments of all surveying instruments used in the surveying of the public lands of the United States must be tested at least once a year, and oftener if necessary, by comparison with the true meridian, established under the direction of the surveyor-general of the district; and the instruments must be so modified in construction, or in such a way corrected, as may be necessary to produce the closest possible approximation to accuracy and uniformity in the operation of all such instruments. A record will be made of such examinations, showing the number and style of the instrument, name of the maker, the quantity of instrumental error discovered by comparison, in either solar or magnetic apparatus, or both, and means taken for correction. The surveyor-general will allow no surveys to be made until the instruments to be used therefor have been approved by him.

(h) The township lines and the subdivision lines will usually be measured by a two-pole chain of 33.03 feet in length, consisting of 50 links, and each link being 7.92 inches long. On uniform and level ground, however, the four-pole chain may be used. The measurements will, however, always be represented according to the four-pole chain of 100 links. The four-pole chains must be adjusted to lengths of 66.06 feet. The object in adding six-hundredths of a foot to the 66 feet of a four-pole chain is to assure thereby that 66 feet will be set off upon the earth's surface without the application of a greater strain than about 20 pounds by the chainmen, thus providing for loss by vertical curvature of the chain, and at the same time avoiding the uncertain results attending the application of strains taxing its elasticity. The deputy surveyor must provide himself with a measure of the standard chain kept at the office of the surveyor-general, to be used by him as a field standard. The chain in use must be compared and adjusted with this field standard each working day; and such field standard must be returned to the surveyor-general's office for examination when his work is completed.

352. Of Tally-Pins. You will use 11 tally-pins made of steel, not exceeding 14 inches in length, weighty enough toward the point to make them drop perpendicularly, and having a ring at the top, in which is to be fixed a piece of red cloth, or something else of conspicuous color, to make them readily seen when stuck in the ground.

353. Process of Chaining. In measuring lines with a two-pole chain, every *five* chains are called a *tally*; and in measuring lines with a four-pole chain, every *ten* chains are called a *tally*, because at that distance the last of the 10 tally-pins with which the forward chainman set out will have been stuck. He then cries "tally"; which cry is repeated by the other chainman, and each registers the distance by slipping a thimble, button, or ring of leather, or something of the kind, on a belt worn for that purpose, or by some other convenient method. The hind chainman then comes up, and having counted in the presence of his fellow the tally-pins which he has taken up, so that both may be assured that none of the pins have been lost, he then takes the forward end of the chain, and proceeds to set the pins. Thus the chainmen alternately change places, each setting the pins that he has taken up, so that one is forward in all the odd, and the other in all the even, tallies. Such procedure, it is believed, tends to insure accuracy in measurement, facilitates the recollection of the distances to objects on the line, and renders a mis-tally almost impossible.

354. Levelling the Chain and Plumbing the Pins. The length of every line you run is to be ascertained by precise horizontal measurement, as nearly approximating to an air line as is possible in practice on the earth's surface. This all-important object can only be attained by a rigid adherence to the three following observances:

Ever keeping the chain *stretched* to its utmost degree of tension on even ground.

On uneven ground, keeping the chain not only stretched as

aforesaid, but horizontally *levelled*. And when ascending or descending steep ground, hills, or mountains, the chain will have to be *shortened* to one-half its length (and sometimes more), in order accurately to obtain the true horizontal measurement.

The careful plumbing of the tally-pins, so as to attain precisely the *spot* where they should be stuck. The more uneven the surface, the greater the caution needed to set the pins.

355. Marking Lines. All lines on which are to be established the legal corner boundaries are to be marked after this method, viz.: Those trees which may intercept the line must have two chops or notches on each side of them, without any other marks whatever. These are called *sight trees* or *line trees*. A sufficient number of other trees standing within 50 links of the line, on either side of it, are to be blazed on two sides diagonally, or quartering toward the line, in order to render the line conspicuous, and readily to be traced, the blazes to be opposite each other, coinciding in direction with the line where the trees stand very near it, and to approach nearer each other the farther the line passes from the blazed trees.

Where trees two inches or more in diameter are found, the required blazes must not be omitted.

Bushes on or near the line should be bent at right angles therewith, and receive a blow of the axe at about the usual height of blazes from the ground sufficient to leave them in a bent position, but not to prevent their growth.

356. On Trial or Random Lines the trees are not to be blazed, unless occasionally, from indispensable necessity, and then it must be done so guardedly as to prevent the possibility of confounding the marks of the trial line with the *true*. But bushes and limbs of trees may be lopped, and *stakes set* on the trial or random line, at every *ten* chains, to enable the surveyor on his return to follow and correct the trial line, and establish therefrom the *true line*. To prevent confusion, the temporary stakes set on the trial or random lines must be *pulled up* when the surveyor returns to establish the true line.

357. Insuperable Objects on Line; Witness Points. Under circumstances where your course is obstructed by impassable obstacles, such as ponds, swamps, marshes, lakes, rivers, creeks, etc., you will prolong the line across such obstacles by means of right-angle offsets; or, if such be inconvenient, by a traverse or trigonometrical operation, until you regain the line on the opposite side. And in case a north and south, or a true east and west, line is regained in advance of any such obstacle, you will prolong and mark the line back to the obstacle so passed, and state all the particulars in relation thereto in your field-book. And at the intersection of lines with both margins of impassable obstacles you will establish a *witness point* (for the purpose of perpetuating the intersections therewith), by setting a post, and giving in your field-book the course and distance therefrom to two trees on opposite sides of the line, each of which trees you will mark with a blaze and notch facing the post; but on the margins of navigable watercourses or navigable lakes you will mark the trees with the proper number of the fractional section, township, and range.

358. The Best Marking-Tools adapted to the purpose must be provided for marking neatly and *distinctly* all the letters and figures required to be made at corners, *Arabic* figures being used exclusively; and the deputy is always to have at hand the necessary implements for keeping his marking-tools in order.

359. Establishing Corners. To procure the faithful execution of this portion of a surveyor's duty is a matter of the utmost importance. After a true coursing and most exact measurement, the establishment of corners is the consummation of the work. If, therefore, the corners be not perpetuated in a permanent and workmanlike manner, the *great aim* of the surveying service will not have been attained.

The following are the different points for perpetuating corners, viz.:

(a) For township boundaries, at intervals of every 6 miles.

(b) For section boundaries, at intervals of every mile, or 80 chains.

(c) For quarter-section boundaries, at intervals of every half-mile, or 40 chains. Exceptions, however, occur, as fully set forth hereafter in that portion of the manual showing the manner of running township lines and method of subdividing.

(d) Meander corners are established at all those points where the lines of the public surveys intersect the banks of such rivers, bayous, lakes, or islands, as are by law directed to be meandered.

360. Miscellaneous. When a rock in place is established for a corner, its dimensions above ground must be given, and a cross (×) marked at exact corner point.

Where mounds of earth are raised "alongside" of corners on N. and S. lines, they must be placed on the W., and on the E. and W. lines on the N. side of corner. In case the character of the land is such that this cannot be done, the deputy will state in his notes instead of "alongside" "S." (on E.).

In case where pits are practicable, the deputy prefers raising a mound of stone, or stone covered with earth, as more likely to perpetuate the corner; he will use the form given for mound of stone, omitting the words "pits impracticable," and adding "covered with earth," when so established.

Where the requisite number of trees can be found within 300 links of the corner point, three (3) bearing trees should be established for every *standard* or *closing corner*, four (4) for every *corner* common to four *townships* or *sections*, and two (2) for every *quarter-section corner* or *meander corner*. In case the requisite number cannot be found within limits, the deputy must state in his field notes, after describing those established, "no other trees within limits," and "dug pits in secs. — & —," or "raised a mound of stone alongside."

Stones 18 inches and less long must be set two-thirds, and over 18 inches long, three-fourths, of their length in the ground. No stones containing less than 504 cubic inches must be used

for corners. Particular attention is called to the "summary of objects and data required to be noted," on pages — and — of these instructions, and it is expected that the deputy will thoroughly comply with the same in his work and field notes.

No mountains, swamp lands, or lands not classed as surveyable, are to be meandered, and all lines approaching such lands must be discontinued at the section or quarter-section corner.

Where, by reason of impassable objects, the south boundary of a township cannot be established, an east and west line should be run through the township, first random, and then corrected, from one range line to the other, and as far south as possible, and from such line the section lines will be extended in the usual manner, except over any fractions south of said line, which may be surveyed in the opposite direction from the section corners on the auxiliary base thus established.

When no part of the east or west boundaries can be run, both north and south boundaries will be established as true lines. Allowance for the convergency of meridians must be made whenever necessary.*

All letters and figures cut in posts or trees must be marked over with red chalk to make them still more plain and durable. Township corners common to four townships, and section corners common to four sections, are to be set diagonally in the earth, with the angles in the direction of the lines. All other corners are to be set square, with the sides facing the direction of the lines. The sizes of wooden posts, mounds, and pits, noted in foregoing descriptions of corners, are to be regarded as *minimum*, and whenever practicable to increase their dimensions, it is desirable to do so. In establishing corners, stones should be used whenever practicable; then posts; and lastly, mounds, with stake in pit.

It is expected that deputy surveyors will carefully read and familiarize themselves with these instructions, and all others

* See Table of Convergency of Meridians at end of chapter, and explanation of same.

contained in this volume, and will instruct their assistants as to their duties before commencing work. Extra copies will be furnished the deputies for the use of their assistants.

361. Standard Quarter-Section Corners on standard lines must be established in all respects like other quarter-section corners, with the addition of the letters S.C.; and if bearing trees are established for such corners, each tree must be marked S.C. $\frac{1}{4}$ S.B.T. When a pit is dug at a meander corner, it must be 8 links from the corner on the side opposite the river or lake meandered.

The letters M.C., for "meander corner," must be marked on the side facing the river or lake meandered.

362. A Witness Corner, in addition to the marks that would be placed upon the corner for which it is a witness, must have the letters W.C., and be established in all respects like such corner.

If bearing trees are established for a witness corner, each tree must be marked W.C., in addition to the usual marks.

363. Meandering. Both banks of navigable rivers are to be meandered by taking the general courses and distances of their sinuosities.

At those points, when either the township or section lines intersect the banks of a navigable stream, corners are to be established at the time of running these lines. These are called *meander corners*; and in meandering, you are to commence at one of these corners, coursing the banks, and measuring the distance of each course from your commencing corner to the next *meander corner*. By the same method, you are to meander the opposite bank of the same river. The crossing distance *between meander corners* on same line is to be ascertained by triangulation, that the river may be accurately protracted. Rivers not classed under the statute as *navigable*, but which are well-defined natural arteries of internal communication, will only be meandered *on one bank*.

All lakes, bayous, and deep ponds which may serve as public highways of commerce must be meandered.

364. Surveying. Initial points, from which the lines of the public surveys are to be extended, must be established whenever necessary under special instructions, as may be prescribed in each case by the Commissioner of the General Land Office. The locus of such initial points must be selected with great care and due consideration for their prominence and easy identification, and must be established astronomically.

The initial point having been established, the lines of the public surveys are to be extended therefrom as follows :

365. Base Line. The base line shall be extended east and west from the initial point by the use of solar instruments or transits, as may be directed by the surveyor-general in his special written instructions. Where solar instruments are used, the deputy must test said instruments in every 12 miles of line run, by taking the latitude, or by observation on the polar star ; and in all cases where he has reason to suppose that said instrument is in error, he must take an observation on the polar star ; and if error be found, must make the necessary corrections before proceeding with his survey. The proper corners shall be established at each 40 and 80 chains, and at the intersection of the line with rivers, lakes, or bayous that should be meandered, in accordance with the instructions for the establishment of corners. In order to check errors in measurement, two sets of chainmen, operating independently of each other, must be employed.

Where transits are used, the line will be run by setting off at the point of departure on the principal meridians a tangent to the parallel of latitude, which will be a line falling at right angles to the said meridian. The survey will be continued on this line for twelve (12) miles, but the corners will be established at the proper points by offsets northerly from said line, at the end of each half-mile. In order to offset correctly from

the tangent to the parallel, the deputy will be guided by the table of offsets and azimuths contained in the Manual of Instructions.

As the azimuth of the tangent is shown, the angle thence to the true meridian at each mile is readily found, thus indicating the direction of the offset line. The computations are made for a distance of 12 miles, at the end of which observations on the polar star must be taken for the projection of a new tangent. The computations are also upon even degrees of latitude; offsets for intervening parallels can be readily determined by interpolation. Where offset distances quarter-section corners exceed 50 links, their direction to the parallel can be determined in like manner by interpolation for azimuth. When said distances are less than 50 links, interpolation for determining the distances will not be required.

366. Principal Meridian. The principal meridian shall be extended north and south from the initial point, by the use of solar instruments or transits, as may be directed by the surveyor-general in his special written instructions.

Where solar instruments are used, the line will be run in the same manner as prescribed for running the base line by solar instruments. Where transits are used, observations upon the polar star must be taken within each 12 miles of line run. In addition to the above general instructions, it is required that in all cases where the establishment of a new principal meridian seems to be necessary to the surveyor-general, he shall submit the matter, together with his reasons therefor, to the Commissioner of the General Land Office, and the survey of such principal meridian shall not be commenced until written authority, together with such special instructions as he may deem necessary, shall have been received from the Commissioner.

367. Standard Parallels. Standard parallels, which are also called correction lines, shall be extended east and west from the principal meridian, at intervals of every 24 miles north and south of the base line, in the same manner as prescribed for running the base line.

Auxiliary Meridians. Auxiliary meridians shall be extended north and south from the base line, at intervals of every 24 miles east and west from the principal meridian, in the same manner as prescribed for running the principal meridian.

It is contemplated that these base, principal meridian, standard, and auxiliary meridian lines shall first be extended over the territory to be surveyed, and that afterwards township and section lines shall be run, where needed, within these tracts of 24 miles square, formed by the extension of these principal lines; and each surveyor-general will therefore cause said principal lines to be extended as rapidly as practicable.

368. Exterior, or Township Lines. The east and west boundaries of townships are always to be run from south to north on a true meridian line; and the north and south boundaries are to be run from east to west, or from west to east (according to the relation of the township to be surveyed with reference to prior surveys), on a *random* or trial line, and corrected back on a true line. The distance north or south of the township corner to be closed upon, from the point of intersection of these random lines with the east or west boundary of the township, must be carefully measured and noted. Should it happen, however, that such random line should fall short, or overrun in length, or intersect the east or west boundary more than *three chains'* distance from the township corner thereon, as compared with the corresponding boundary on the south (due allowance being made for convergency) the line, and if necessary the entire exterior boundaries of the township, must be retraced, so as to discover and correct the error. In running random lines, temporary corners are to be set at each 40 and 80 chains, and permanent corners established upon the true line as corrected back, in accordance with instructions, throwing the excess or deficiency on the west half-mile, as prescribed by law. Permanent corners are to be established, in accordance with instructions, on the east and west township boundaries at the time they are to be run. Whenever practicable,

the township lines within these tracts of 24 miles square, must be surveyed in regular order from *south* to *north*; *i.e.*, the exterior boundaries of the township, in any one range lying immediately north of the south boundary of such tract of 24 miles square, must first be surveyed, and the exteriors of the other three townships in said range extended therefrom, in regular order, from *south* to *north*; and it is preferable to survey first the entire range of townships in such tract adjoining the east boundary, or adjoining the west boundary, and the other three ranges in regular sequence. In cases, however, where the character of the land is such that this rule cannot be complied with, the following will be observed. In extending the *south* or *north* boundaries of a township to the *west*, where the *southwest* or *northeast* corners cannot be established in the regular way by running a north and south line, such boundaries will be run *west on a true* line, allowing for convergency on the west half-mile; and from the township corner established at the end of such boundary, the west boundary will be run *north* or *south*, as the case may be. In extending *south* or *north* of a township to the *east*, where the *southeast* or *northeast* corner cannot be established in the regular way, the same rule will be observed, except that such boundaries will be run *east on a true* line, and the *east* boundary run *north* or *south*, as the case may be. One set of chainmen only is required in running township lines.

369. Method of Subdividing. The first mile, both on the south and east boundaries of each township you are required to subdivide, is to be carefully traced and measured before you enter upon the subdivision thereof. This will enable you to observe any change that may have taken place in the magnetic variation as it existed at the time of running the township lines, and will also enable you to compare your chaining with that upon the township lines.

Any discrepancy arising either from a change in the magnetic variation or a difference in measure is to be carefully noted in the field notes.

After adjusting your compass to a variation which you have just found will retrace the eastern boundary of the township, you will commence at the corner to Sections 35 and 36, on the south boundary, and run a line parallel to the range line, 40 chains, to the quarter-section corner, which you are to establish between Sections 35 and 36; continuing on said course 40 chains farther, you will establish the corner to Sections 25, 26, 35, and 36.

From the section corner last named, run a *random* line, without blazing, *due east*, for the corner of sections 25 and 36, on east boundary, and at 40 chains from the starting-point set a post for *temporary* quarter-section corner. If you intersect exactly at the corner, you will blaze your random line back, and establish it as the *true* line; but if your random line intersects the said east boundary either north or south of said corner, you will measure the distance of such intersection, from which you will calculate a course that will run a *true* line back to the corner from which your random started. You will establish the *permanent* quarter-section corner at a point equidistant from the two terminations of the *true* line.

From the corner of Sections 25, 26, 35, and 36, run due north between Sections 25 and 26, setting the quarter-section post, as before, at 40 chains, and at 80 chains establishing the corner of Sections 23, 24, 25, and 26. Then run a random *due east* for the corner of Sections 24 and 25 on east boundary; setting temporary quarter-section post at 40 chains; correcting back, and establishing *permanent* quarter-section corner at the equidistant point on the *true* line, in the manner directed on the line between Sections 25 and 36.

In this manner you will proceed with the survey of each successive section in the first tier until you arrive at the north boundary of the township, which you will reach in running up a random line between Sections 1 and 2. If this random line should not intersect at the corner established for Sections 1, 2, 35, and 36, upon the township line, you will note the distance that you fall east or west of the same, from which distance you

will calculate a course that will run a true line south to the corner from which your random started. If the north boundary of a township is a base or standard line, the line between Sections 1 and 2 is to be run north as a *true* line, and the closing corner established at the point of intersection with such base or standard line; and in such case, the distance from said closing corner to the nearest section or quarter-section corner on such base or standard line must be carefully measured and noted as a "connection line."

In like manner proceed with the survey of each successive tier of sections until you arrive at the fifth tier; and from each section corner which you establish upon this tier you are to run random lines to the corresponding corners established upon the range line forming the western boundary of the township; setting as you proceed each *temporary* quarter-section corner at 40 chains from the interior section corner, so as to throw the excess or deficiency of measurement on the extreme tier of quarter-sections contiguous to the township boundary; and on returning establish the *true* line, and establish thereon the *permanent* quarter-section corner.

It is not required that the deputy shall complete the survey of the first tier of sections from north to south before commencing the survey of the second or any subsequent tier, but the corner on which the random line closes must have been previously established by running the line north on which it is established, except as follows: where it is impracticable to establish such section corner in the regular manner, it may be established by running the east and west line *east* or *west*, as the case may be, *on a true line*, setting the quarter-section corner at 40 chains and the section corner at 80 chains.

Quarter-section corners, both upon north and south and upon east and west lines, are to be established at a point "equidistant" from the corresponding section corners, *except* upon the lines crossing on the north and west boundaries of the township, and in those situations the quarter-section corners will always be established at precisely 40 *chains* to the north or

west, as the case may be, of the respective section corners from which those lines respectively *start*, by which procedure the excess or deficiency in the measurements will be thrown, according to law, on the extreme tier of quarter-sections.

370. Prescribed Limits for Closing, and Length of Lines in Certain Cases. Every north-and-south section line, except those terminating in the north boundary of the township, must be 80 *chains* in length.

The east-and-west *section lines*, except those terminating in the west boundary of the township, are to be within 80 *links* of the actual distance established on the south boundary line of the township for the width of said tier of sections, and must close within 80 links north or south of the section corner.

The north boundary and south boundary of any one section, except in the extreme western tier, are to be within 80 *links* of equal length.

The meanders within each fractional section, or between two meander posts, or of an island in the interior of a section, must close within 1 chain and 50 links.

In running *random* township exteriors, if such random lines fall short or overrun in length or intersect the eastern or western boundary, as the case may be, of the township at more than 3 *chains* north or south of the true corner, the lines must be *retraced*, even if found necessary to measure the meridional boundaries of the township. One set of chainmen only is required in subdividing.

371. Subdivision of Sections. Under the provisions of the act of Congress approved Feb. 11, 1805, the course to be pursued in the subdivision of sections is to run straight lines from the established quarter-section corners — United States surveys — to the opposite corresponding corners, and the point of intersection of the lines so run will be the corner common to the several quarter-sections; or, in other words, the legal centre of the section.

In the subdivision of fractional quarter-sections where no opposite corresponding sections have been or can be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east, or west lines, as the case may be, to the watercourse, Indian boundary line, or other external boundary of such fractional section. The law presupposes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines, but in actual experience this is not always the case; hence, in order to carry out the spirit of the law, it will be necessary in running the subdivisional lines through fractional sections to adopt mean courses where the section lines are not due lines, or to run the subdivision line parallel to the section line where there is no opposite section line.

Upon the lines closing on the north and west boundaries of a township the quarter-section corners are established by the United States deputy surveyors at precisely 40 chains to the north or west of the last interior section corners, and the excess or deficiency in the measurement is thrown on the outer tier of lots, as per act of Congress approved May 10, 1800. In the subdivision of quarter-sections, the quarter-quarter corners are to be placed at points equidistant between the section and quarter-section corners, and between the quarter corners and the common centre of the section, *except* on the last half-mile of the lines closing on the north or west boundaries of a township, where they should be placed at 20 chains, proportionate measurement, to the north or west of the quarter-section corner.

The subdivisional lines of fractional quarter-sections should be run from points on the section lines intermediate between the section and quarter-section corners due north, south, east, or west, to the lake, watercourse, or reservation which renders such tracts fractional.

When there are double sets of section corners on township and range lines, the quarter corners for the sections south of the

township lines and east of the range lines are not established in the field by the United States surveyors, but in subdividing such sections said quarter corners should be so placed as to suit the *calculations of the areas of the quarter-sections adjoining the township boundaries* as expressed upon the official plot, adopting proportionate measurements where the present measurements of the north or west boundaries of the sections differ from the original measurements.

372. Re-establishment of Lost Corners. The original corners, when they can be found, must stand as the true corners they were intended to represent, even though not exactly where strict professional care might have placed them in the first instance.

As has been observed, no existing original corner can be disturbed, and it will be plain that any excess or deficiency in measurements between existing corners cannot in any degree affect the distances beyond said existing corners, but must be added or subtracted proportionately to or from the intervals embraced between the corners which are still standing.

373. Summary of Objects and Data required to be Noted. The precise length of every line run, noting all necessary offsets therefrom, with the reason and mode thereof.

The kind and diameter of all *bearing trees*, with the course and distance of the same from their respective corners, and the precise relative position of *witness corners* to the *true corners*.

The kind of materials of which corners are constructed.

Trees on line. The name, diameter, and distance on line to all trees which it intersects.

Intersections by line of *land objects*. The distance at which the line first intersects and then leaves every *settler's claim and improvements*; prairie, river, creek, or other "bottom"; or swamp, marsh, grove, and windfall, with the course of the same at both points of intersection; also the distances at which you begin to ascend, arrive at the top, begin to descend, and

reach the foot of all remarkable hills and ridges, with their courses, and *estimated* height, in feet, above the level land of the surrounding country, or above the bottom lands, ravines, or waters near which they are situated.

Intersection by line of *water objects*.

All rivers, creeks, and smaller streams of water which the line crosses ; the distances on line at the points of intersection ; and their *widths on line*. In cases of *navigable* streams, their width will be ascertained between the *meander corners*, as set forth under the proper head.

The land's *surface* — whether level, rolling, broken, or hilly.

The *soil* — whether first, second, third, or fourth rate.

Timber — the several kinds of timber and undergrowth, in the order in which they predominate.

Bottom lands — to be described as wet or dry ; and if subject to inundation, state to what depth.

Springs of water — whether fresh, saline, or mineral, with the course of the stream flowing from them.

Lakes and ponds — describing their banks and giving their height, and also depth of water, and whether it be pure or stagnant.

Improvements — towns and villages ; houses or cabins ; fields, or other improvements ; sugar-tree groves, sugar camps, mill seats, forges, and factories.

Coal bank or beds ; *peat* or turf grounds ; *minerals* and ores, with particular description of the same as to quality and extent, and all *diggings* therefor ; also *salt* springs and licks. All reliable information you can obtain respecting these objects, whether they be on your immediate line or not, is to appear on the general description to be given at the end of the notes.

Roads and trails, with their directions whence and whither.

Rapids, cataracts, cascades, or falls of water, with the estimated height of their fall in feet.

Precipices, caves, sink holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

Natural curiosities, interesting fossils, petrifications, organic.

| | |
|-------|--|
| 80.00 | <p>Set a post, $4\frac{1}{2}$ ft. long, 4 ins. square, with marked stone, 12 ins. in the ground, for standard cor. to secs. 32 and 33, marked:</p> <p>S.C., T. 13 N., R. 21 E., on N. ;</p> <p>S. 33, on E. ; and</p> <p>S. 32, on W. faces, with 4 notches on E. and 2 notches on W. faces, and raised a mound of stone 2 ft. high, $4\frac{1}{2}$ ft. base, around post.</p> <p>Land, high and mountainous.</p> <p>Soil, sandy, gravelly, and rocky ; 4th rate.</p> <p>Timber, pine, and fir, 80 chs. ; mostly dead and fallen ; some thick undergrowth, same.</p> |
|-------|--|

375. Specimen Field Notes of the survey of Township No. 6 north, Range No. 34 east, of the principal base and meridian of Montana Territory.

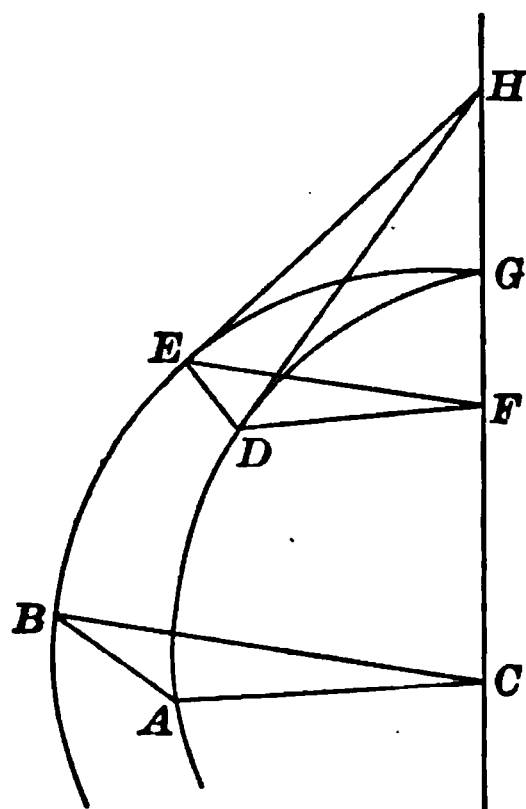
| | |
|---------|---|
| chains. | <p>East, on random line, bet. secs. 5 and 8.</p> <p>Va. $18^{\circ} 45'$ E.</p> <p>Over rolling ground.</p> |
| 16.40 | Road to Williamsburg, course S. |
| 40.00 | Set temporary $\frac{1}{4}$ sec. cor. |
| 79.96 | <p>Intersected N. and S. line 6 lks. N. of cor. to secs. 4, 5, 8, and 9.</p> <p>Thence I run</p> <p>N. $89^{\circ} 56'$ W. on true line, bet. secs. 5 and 8, with same Va.</p> |
| 39.98 | <p>Set a post 3 ft. long, 3 ins. square, with marked stone, 12 ins. in the ground, for $\frac{1}{4}$ sec. cor. marked $\frac{1}{4}$ S. on N. face ; dug pits, $18 \times 18 \times 12$ ins. E. and W. of post $5\frac{1}{2}$ ft. dist., and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ ft. base, around post.</p> |
| 79.96 | <p>The cor. to secs. 5, 6, 7, and 8.</p> <p>Land, rolling.</p> <p>Soil, sandy ; 2d rate.</p> <p>No timber.</p> |

| | |
|-------|---|
| | West, on random line, between secs. 6 and 7. Over rolling ground. |
| 27.15 | Road to Williamsburg, course S. |
| 40.00 | Set temporary $\frac{1}{4}$ sec. cor. |
| 78.40 | Intersect west boundary of township 15 lks. S. of cor. to secs. 1, 6, 7, and 12, which is a post, 4 ft. long, 4 ins. square, marked : T. 6 N.S. 6 on N.E. R. 34 E.S. 7 on S.E. R. 33 E.S. 12 on S.W., and S. 1 on N.W. faces, with pits, $18 \times 18 \times 12$ ins. in each sec., $5\frac{1}{2}$ ft. dist., and mound of earth, 2 ft. high, $4\frac{1}{2}$ ft. base, around post. Thence I run S. $89^{\circ} 54'$ E. on a true line, bet. secs. 6 and 7, with same Va. |
| 38.40 | Set a sandstone, $18 \times 14 \times 3$ ins., 12 ins. in the ground, for $\frac{1}{4}$ sec. cor., marked $\frac{1}{4}$ on N. side; dug pits $18 \times 18 \times 12$ ins. E. and W. of stone $5\frac{1}{2}$ ft. distant, and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ base, alongside. |
| 78.40 | The cor. to secs. 5, 6, 7, and 8. Land, rolling. Soil, sandy; 2d rate. No timber. |
| | North, on a random line, bet. secs. 5 and 6. Va. $18^{\circ} 45'$ E. Over rolling ground. |
| 40.00 | Set temporary $\frac{1}{4}$ sec. cor. Intersect N. boundary of township 20 lks. E. of cor. to secs. 5, 6, 31, and 32, which is a sandstone $30 \times 12 \times 6$ ins., marked with 5 notches on E. and one notch on W. edges, and mound of stone, 2 ft. high, $4\frac{1}{2}$ ft. base, alongside. Thence I run |

| | |
|-------|--|
| | S. $0^{\circ} 09' E.$ on a true line bet. secs. 5 and 6, with same Va. |
| 40.05 | Set a sandstone, $16 \times 12 \times 3$ ins. 11 ins. in the ground, for $\frac{1}{4}$ sec. cor. marked $\frac{1}{2}$ on W. face; dug pits, $18 \times 18 \times 12$ ins., N. and S. of stone, $5\frac{1}{2}$ ft. dist., and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ ft. base, alongside. |
| 80.05 | The cor. to secs. 5, 6, 7, and 8. Land, rolling. Soil, sandy; 2d rate. No timber. |

INCLINATION OF THE MERIDIAN.*

376. In projecting arcs of a great circle it is of the utmost importance that the surveyor be able to tell the inclination of the meridians for any latitude, and for any distance of eastings or westings.



In the following figure, let the two arcs AG and BG be two arcs of a quadrant of the meridian 1° of longitude apart. Let AB = the arc of 1° of longitude on the equator = 69.16 miles.

Let DE be an arc of longitude on any parallel of latitude. Also, let EH and DH be the tangents of those meridians meeting in the earth's axis produced, and corresponding to the parallel of latitude DE .

Then the line $EF = DF = \cos L = \cos AD$ or BE . Also, the angle $DFE = 1^{\circ}$, and the angle DHE = the inclination of

* These articles on the inclination and convergency of meridians, and the table calculated in accordance therewith, are substantially those given in the 1886 catalogue of engineers' and surveyors' instruments, by Buff and Berger, Boston, Mass.

the meridians, which is the angle we wish to find, and which we will represent by X° . And because the two triangles FDE and DHE are on the same base ED , and isosceles, their vertical angles vary inversely as their sides; and we have the equation,

$$1^\circ \times EF = X^\circ \times EH.$$

But $EF = \cos L$, and $EH = \cot L$;

hence $X^\circ \cot L = 1^\circ \cos L$,

or $X^\circ = \cos L \div \cot L = \sin L. \quad (a)$

That is to say,

The inclination of the meridians for any difference of longitude varies as the sine of the latitude.

Since the sine of the latitude is the inclination in decimals of a degree, for one degree of longitude, if we multiply by 3600" we shall have the inclination in seconds of arc. Then, if we divide this by the number of miles in one degree of longitude on that latitude, we shall have the inclination due to one mile on that parallel. Thus, for

| | |
|---|---------------------|
| Latitude 43° | log. sin = 9.833783 |
| Multiply by 3600" | " " = 3.556303 |
| | <hr/> 3.390086 |
| Divide by 50.66 m. = 1° long. on that L , log. = | 1.704682 |
| 48.46" = inclination for one mile of long. | <hr/> 1.685404 |

The use of the inclination, as found by the preceding article, is to show the surveyor how much he must deflect a line of survey from the due east or west, to have it meet the parallel at a given distance from the initial point of the survey; for it will be remembered that a parallel of latitude is a curve having the cosine of the latitude for its radius. And the line due east or west is the tangent of the curve.

Thus, on latitude 43° , it is desired to project a six-mile line west, for the southerly line of a township.

Remembering that in an isosceles triangle the angle at the base is less than a right angle by *half the angle at the vertex*, deflect a line *towards the pole* by the inclination due to *three*

miles, — or in this case $48.46'' \times 3 = 2'.25''$; i.e., *deflection* = $\frac{1}{2}$ *inclination*.

The table on next page, which was computed from the formula (a) above, gives the *inclination* for one mile, and for six miles on any parallel, from 10° to 60° of latitude; also the *convergency* for six miles, on any latitude.

377. The Convergency of the Meridian is readily found for any given distance from the corresponding inclination, by multiplying the *sine* of the inclination by the given *distance*.

Thus, for latitude 43° , the inclination for one mile is $48.46''$; the sine of which is 0.000235. This, multiplied by the number of links in a mile, which = 8,000, we have the convergency for one mile, = 1.88 links.

Multiplying this by the number of miles in a township, = 36, and we have the convergency for a township, = 67.68 links. In this manner were the convergencies of the Table computed.

378. Deflection of Range-Lines from Meridian. The second column of the table shows the surveyor how much he must deflect the range lines between the several sections of a township from the meridian, in order to make the consecutive ranges of sections in a township of uniform width, for the purpose of throwing the effects of convergency into the most westerly range of quarter-sections, agreeably to law.

Thus, say between 45° and 55° of latitude, the inclination is practically 1' for every mile of easting or westing. Then, bearing in mind that in the United States the surveys are regarded as projected from the east and south to the west and north, the surveyor must project the *first range-line* between the sections of a township in those latitudes 1' to the left of the meridian.

The second, 2'; the third, 3'; and so on to the fifth, which must be 5' to the left of the meridian on the east side of the township.

By this means all the convergency of the township is thrown into the *sixth*, or westerly range of sections, as the law directs.

The fourth column of the table below shows the amount of this convergency. This column is also useful in subdividing a block of territory embraced by two standard parallels and two guide meridians into townships. Thus, starting a meridian from a standard parallel on latitude 43° N., for the western boundary of a range of township, — say the first one west from the guide meridian, — and running north, say four townships, the surveyor must make a point that is *east* of the six-mile point on the northern standard parallel, $4 \times 67.7 \text{ links} = 270.8 \text{ links}$. The second meridian should fall $8 \times 67.7 \text{ links}$ to the *right* of the twelve-mile point.

TABLE OF INCLINATION AND CONVERGENCY OF THE MERIDIANS.

| Latitude. | Inclination for one mile. | Inclination for six miles. | Convergency for one township of 36 miles. | Latitude. | Inclination for one mile. | Inclination for six miles. | Convergency for one township of 36 miles. | Latitude. | Inclination for one mile. | Inclination for six miles. | Convergency for one township of 36 miles. |
|-----------|---------------------------|----------------------------|---|-----------|---------------------------|----------------------------|---|-----------|---------------------------|----------------------------|---|
| ° | // | / // | LINKS. | ° | // | / // | LINKS. | ° | / // | / // | LINKS. |
| 10 | 9.18 | 55 | 13.0 | 27 | 26.52 | 2 39 | 36.9 | 44 | 50.19 | 5 01 | 70.1 |
| 11 | 10.13 | 1 01 | 14.2 | 28 | 27.66 | 2 46 | 38.6 | 45 | 52.00 | 5 12 | 72.6 |
| 12 | 11.07 | 1 06 | 15.5 | 29 | 28.85 | 2 53 | 40.2 | 46 | 53.83 | 5 23 | 75.2 |
| 13 | 12.02 | 1 12 | 16.8 | 30 | 30.03 | 3 03 | 41.9 | 47 | 55.67 | 5 34 | 77.8 |
| 14 | 12.98 | 1 18 | 18.1 | 31 | 31.26 | 3 07 | 43.6 | 48 | 57.67 | 5 46 | 80.6 |
| 15 | 13.96 | 1 24 | 19.4 | 32 | 32.49 | 3 15 | 45.4 | 49 | 59.83 | 5 59 | 83.5 |
| 16 | 14.93 | 1 30 | 20.7 | 33 | 33.83 | 3 23 | 47.2 | 50 | 1 02.00 | 6 12 | 86.5 |
| 17 | 15.92 | 1 36 | 22.0 | 34 | 35.17 | 3 31 | 49.1 | 51 | 1 04.17 | 6 25 | 89.7 |
| 18 | 16.91 | 1 41 | 23.4 | 35 | 36.50 | 3 39 | 50.9 | 52 | 1 06.67 | 6 40 | 93.0 |
| 19 | 17.93 | 1 47 | 24.9 | 36 | 37.83 | 3 46 | 52.7 | 53 | 1 09.17 | 6 55 | 96.4 |
| 20 | 18.94 | 1 54 | 26.5 | 37 | 39.17 | 3 55 | 54.7 | 54 | 1 16.67 | 7 10 | 100.0 |
| 21 | 19.98 | 2 00 | 27.8 | 38 | 40.67 | 4 04 | 56.8 | 55 | 1 14.33 | 7 26 | 103.7 |
| 22 | 21.02 | 2 06 | 29.3 | 39 | 42.17 | 4 13 | 58.8 | 56 | 1 17.17 | 7 43 | 107.6 |
| 23 | 22.10 | 2 13 | 30.8 | 40 | 43.67 | 4 22 | 60.9 | 57 | 1 20.00 | 8 00 | 111.8 |
| 24 | 23.17 | 2 19 | 32.3 | 41 | 45.17 | 4 31 | 63.1 | 58 | 1 22.00 | 8 19 | 116.2 |
| 25 | 24.30 | 2 26 | 33.8 | 42 | 46.85 | 4 41 | 65.4 | 59 | 1 26.66 | 8 40 | 120.9 |
| 26 | 25.38 | 2 32 | 35.4 | 43 | 48.52 | 4 51 | 67.7 | 60 | 1 30.00 | 9 00 | 125.7 |

For details of instruction in United States Government Surveying, see Hawes' System of "Rectangular Surveying," Burt's "Key to Solar Compass," and Clevenger's "Government Surveying."

CHAPTER VII.

CITY SURVEYING.

INTRODUCTION.

379. In the broadest sense, the duties of a city engineer in a large city are many and varied. His knowledge and judgment are required in the location of the city, the laying out of streets, and the fixing of suitable grades therefor, the establishment of a proper water supply, the designing of a suitable system of sewers, the improvement of the waterways, and the planning of necessary bridges and buildings. Following his judicial functions as a designer are his ministerial functions as a constructor. The field which is thus opened before him, in carrying into execution the plans for the various public works, is a very wide one.

As the borough grows and expands into the metropolis, its needs in the directions mentioned increase until a division of labor and responsibility becomes expedient and necessary. In securing the best results in engineering practice, as in other work, the tendency is towards specialties; so that in many cities, in order to secure the services of the best men, and also the best results, the numerous and important duties connected with city engineering have been separated. The province of this work, which is not a treatise on engineering, but on land surveying, makes it proper to treat in this chapter, as thoroughly as the intention and limits of the work allow, only what may be classed under the head of surveying, whether it be performed as the special work of the city or town surveyor, or as among the duties of the city engineer,—the qualifications of the

former by no means fitting a man to perform the varied duties of the latter.

Although this work is intended for the instruction of the student, not of the experienced surveyor, and hence in many things may go into details which to the latter may seem unimportant, it is impossible in the limits of a chapter to impart a thorough knowledge of the duties of a city or town surveyor, — indeed, even to mention all his duties and the many operations and methods which only a long and varied practice can impart. General methods will be given and discussed, but any surveyor of a practical turn of mind will have his own methods of performing much of the routine work pertaining to his situation.

It is not in harmony with the plan of this work to go into the statement in this chapter of any elaborate theories regarding surveying and the instruments used therein, but to endeavor to give some methods which are found to be applicable in practice and to give good practical results. A thorough knowledge of any one good method of performing a certain work is of much more value to the student than a misty idea of numerous methods.

Under the two leading heads of this chapter, field instruments and work and office instruments and work, theoretical discussions will not be entered into; not because they do not possess much value, but because we conceive that they are not adapted to the student's present needs and most rapid advancement. Under the former head, in the light of the work which is likely to engage the greater part of the surveyor's time, field instruments and methods of using them will be described. Under the latter, the nature of office plans and records will be described, the instruments and methods used in the work of producing the plans having been described in other chapters.

In dividing land and locating the boundaries between parties it is evident that the greater the value or the prospective value of said lands, the more delicate should be the instruments, and the more exact the methods used in the work. The methods and instruments which would for all practical purposes be sufficiently exact for the location of a line fence in the country,

where land might be purchased for \$100 per acre, would not at all meet the requirements in locating in a city a line between two parties on land worth \$100 per front foot. This fact becomes the more evident when we consider that the structures placed upon party lines in a city are so much more substantial and permanent in their nature than those thus located in the country. To meet these considerations we shall find that while some of the methods of land surveying previously described in this work, and the instruments used therein, are applicable to the purposes of city surveying, many of the methods will be more exact, and the instruments more numerous and delicate.

Following the plan heretofore pursued in this work, we will, before discussing the work of the city surveyor, describe the instruments (not described in previous chapters) of most general use in his work, and explain their adjustments and the general methods of using them. These instruments are the transit and rods, steel tapes, measuring-rods, pocket-thermometer, hand-level, spring-balance, plummet, Y-level, levelling-rods, and rod-levels.

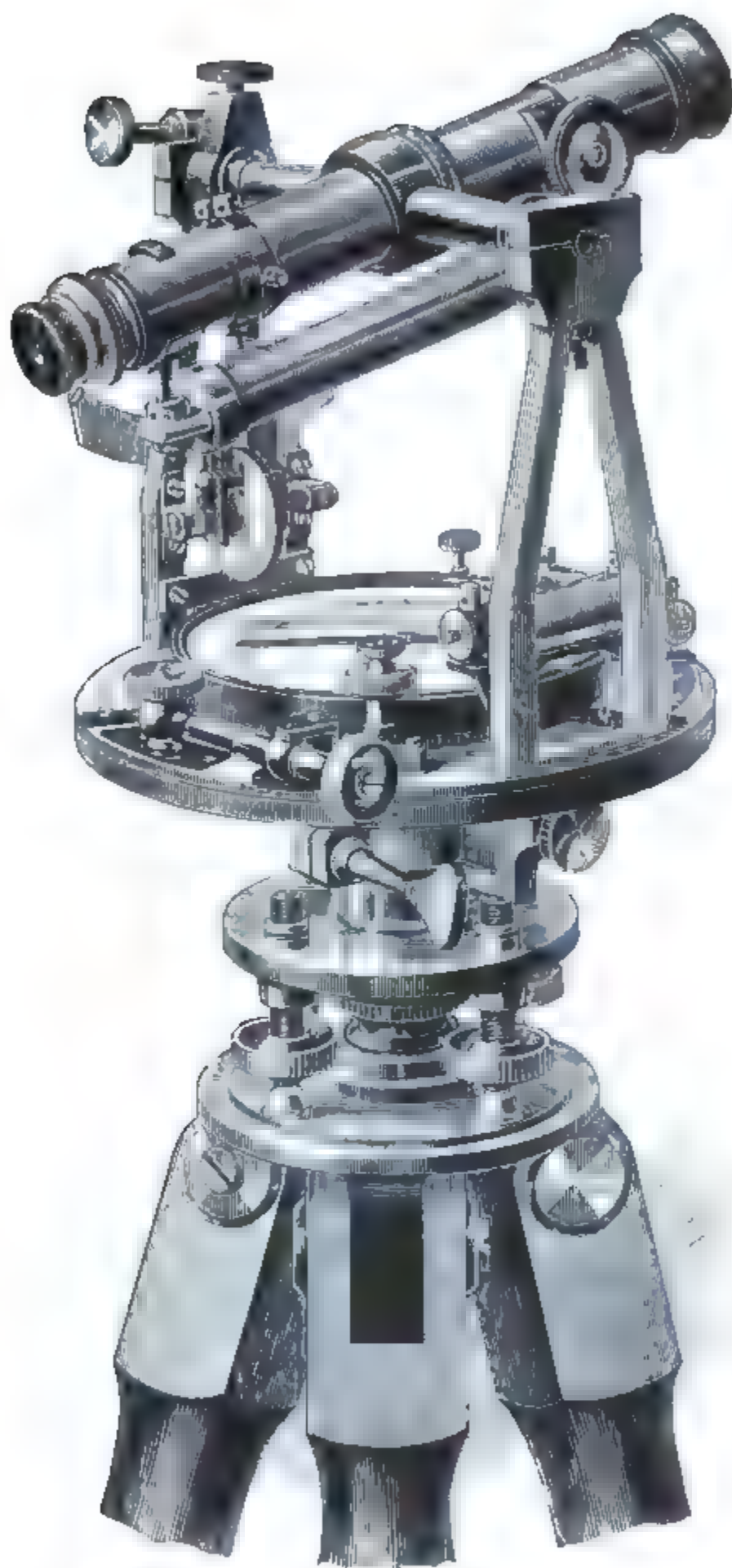
SECTION I.

INSTRUMENTS, THEIR ADJUSTMENTS AND GENERAL USES.

A. FIELD INSTRUMENTS.

380. The Transit. Full description of the transit, its adjustment and uses, may be found in Chapter II.

381. As precision is the distinguishing feature of city and town surveying, the magnetic needle, which is usually found upon the transits, is in this work of but little use. Angles in carefully made surveys are now taken on the horizontal graduated circle of the transit. The instructions already given in this work regarding the magnetic needle are sufficient reason for the



TRANSIT,
WITH GRADIENTER, LEVEL TO TELESCOPE, AND VERTICAL ARC, AS MADE BY
YOUNG & SONS, PHILADELPHIA, PA.

above. It is, however, desirable that in each city and town the true meridian should be determined and permanently marked. Besides being useful in many other ways which will suggest themselves, it will be of great use as an aid in determining the situation of lines described by their bearings in old deeds, the date of the old survey being known.

382. The stadia-hairs * and vertical circle for stadia-measurements are useful attachments, and the telescope should by all means have a long level-tube attached, as this is of much use in city and town work in running grade lines and in levelling for short distances. After the level and the manner of using it have been described, the operation of running a grade line will be explained.

383. Rods. Besides the usual iron-pointed wooden rods, very convenient rods, or pickets, for use with the transit, may be made of gas-pipe about three-quarters of an inch in diameter drawn out on one end to a point, and painted in alternate sections of red and white, — red preferred to black because against red the cross-hairs can be seen.

384. It is by no means as easy a matter to run a straight line with a transit as at first thought it may seem to the student. After the selection of suitable weather, reversing at every extension, care in handling the instrument, and with a corresponding degree of care on the part of assistants, the results are not always what the most careful would desire.

385. In marking a line with stakes, it is convenient to have stake-wood which, in cross-section, has one dimension greater than the other. If, in setting the stake, it always be placed with its broader side towards the instrument, its position will afterwards tell one at a glance in which direction the line was run. This is important when several stakes are set on different

* See Articles 148 to 152, Stadia Measurements.

lines near their intersection, as it will often be the means of avoiding confusion and the resulting errors.

386. Steel Tapes, etc. Before making any important measurements for a city or town, it is necessary, in order to avoid subsequent confusion, that a standard of measurement should be adopted. In many parts of an old city or town the introduction of a new standard would bring inextricable confusion. If there be a standard, even though it has not been carefully preserved, it should, if possible, be ascertained and regarded. When, however, it is at the option of the surveyor to select his standard, the United States standard should, as tending to uniformity, be adopted in this country. Standard rods may be procured of the government. With these rods tape lines and other instruments used for a line purpose should be compared, and the variation noted. It is desirable, also, for purposes of comparison, that a standard, 50 feet or 100 feet, at a known temperature, should be carefully laid down with these rods in the corridor of some building, or in some other convenient place.

Very accurate measuring may be done with graduated wooden rods properly shod with metal ends. These rods are necessarily of but moderate length; hence, work with them is correspondingly slow. For city work, steel tapes are now in very general use; and, when properly handled, give very satisfactory results. They are of different lengths and of different widths. For measuring full hundreds over tolerably level ground the narrow tape, $\frac{3}{8}$ inch wide and 200 feet long, is very convenient. For general city use the 100-foot tape, $\frac{3}{8}$ inch in width, is most convenient.

387. As a rule measurements will be made with the tape in a horizontal position. If not so held, the measurements will afterwards be reduced to the horizontal. In order to determine the horizontal, a hand-level is used to ascertain the difference in elevation of the ground at the two ends of the tape. A cut and description of this convenient little instrument is given below.

Locke's Hand-Level consists of a brass tube about 6 inches long, having, as shown in the figure, a small level on top and near the object end, there being also an opening in the tube beneath, through which the bubble can be seen, as reflected by a glass prism, immediately under the level. Both ends of the tube are closed by plain glass settings to exclude the dust, and there is at the inner end of the sliding or eye tube a semicircular convex lens, which serves to magnify the level bubble, and cross-wire underneath, while it allows the object to be clearly seen through the open half of the tube.



The cross-wire is fastened to a little frame moving under the level-tube, and adjusted to its place by the small screw shown on the end of the level-case. The level of any object in line with the eye of the observer is determined by sighting upon it through the tube, and bringing the air-bubble of the level into a position where it is bisected by the cross-wire.

A short telescope is sometimes applied in place of the plain glass ends, enabling levels to be taken at greater distances and with increased accuracy.

If one or both ends of the tape be held up, the point on the ground vertically under the end of the tape will be determined by means of the plummet, which here needs no description further than to say that its sides should make such an angle with each other as not to prevent the observer when using it from seeing its point; neither should it be so long as to be unsteady.

In all extended and important measurements regard must be had in using the steel tape to standard, temperature, sag, and wind.

Before using a tape its relation to the standard should be

determined by comparison with the standard, marked as previously described, and the variation noted.

388. All important measurements, no matter at what temperature made, should be reduced to a standard temperature; for if, at a certain temperature, we determined with a steel tape the distance apart of two points, at a higher temperature that distance on the same tape would be less because the tape is longer; or, at a lower temperature, greater, because the tape is shorter. The temperature of the air at the time of measurement is ascertained by means of a small thermometer which can be exposed with the tape, and which is so protected that, when not in use, it can be safely carried in the pocket. The standard temperature to which all measurements should be reduced may be taken at pleasure. The correction for expansion and contraction of the steel tape by heat and cold is 0.000006 per unit per degree F.

389. When the tape is held suspended, it will always sag in a vertical direction. Hence the horizontal distance between the extreme graduations will be less than if there were no sag. For this reason, when used to measure the distance between two points, it will, without correction, give a result too great; when used without correction to lay down a given distance, it will give it too small. While a formula may be derived by which to make a correction for sag, it will be found quite as satisfactory to determine it by actual trial. The amount of sag will of course depend upon the tension, or pull. This may be regulated by using at one end of the tape a small spring-balance. It is, however, very desirable that on important work the same men at the same ends of the tape should make all measurements. The experience gained in working together will be a most important factor in securing uniform results.

The effect of wind is in the same direction as that of sag. While much of the work of the surveyor, particularly that involving short measurements, must be done regardless of wind,

no good results in long and important measurements can be secured in windy weather. The best correction for wind is to wait for a calm. In windy weather a narrow tape, as it exposes less surface to the wind, is useful.

390. To illustrate what has been said in regard to the corrections to be applied to measurements made with the steel tape, let us suppose two examples.

First. With a steel tape 100 feet long ($\frac{3}{8}$ inch wide) suspended each length at one or both ends, the temperature of the air being 79° F., the distance on the tape between two points is found to be 550 feet $6\frac{7}{8}$ inches. If the tape is $\frac{1}{8}$ inch longer than the standard, and parts of its length proportionately longer, the standard temperature, 60° F., and the sag $\frac{1}{4}$ inch in 100 feet, what are the corrections, and what is the actual distance between the points?

On account of differing from the standard, as the tape is too long, the distance obtained is too short; the correction for standard is therefore additive. On account of difference in temperature, the temperature being higher than the standard, as the tape is too long, the distance obtained is too short; the correction for temperature is therefore additive. On account of the sag, as the tape is thereby made too short, the distance obtained is too long; the correction for sag is therefore subtractive.

Correction for standard :

$$\frac{1}{8} \text{ in.} \times 5\frac{1}{2} = \frac{1}{4} \text{ in. additive.}$$

Correction for temperature ($79^{\circ} - 60^{\circ} = 19^{\circ}$) :

$$0.000006 \text{ ft.} \times 550 \times 19 = 0.0627 \text{ ft.}$$

$$0.0627 \text{ ft.} \times 12 = 0.7524 \text{ in.} = \frac{1}{8} \text{ in. additive.}$$

Correction for sag :

$$\frac{1}{4} \text{ in.} \times 5\frac{1}{2} = \frac{7}{8} \text{ in. subtractive.}$$

Total correction :

$$+ \frac{1}{4} \text{ in.} + \frac{1}{8} \text{ in.} - \frac{7}{8} \text{ in.} = + \frac{1}{8} \text{ in. additive.}$$

Actual distance between points :

$$550 \text{ ft. } 6\frac{7}{8} \text{ in. } + \frac{1}{8} \text{ in. } = 550 \text{ ft. } 6\frac{8}{8} \text{ in.}$$

Second. Suppose it be required, — other things being as before, — to locate with the steel tape, when the temperature of the air is 52° F., two points which shall at the standard temperature be 225 feet 4½ inches apart.

What length on the tape must be taken ?

Correction for standard :

$$\frac{1}{8} \text{ in. } \times 2\frac{1}{4} = \frac{9}{82} \text{ in. subtractive.}$$

Correction for temperature (60° — 52° = 8°) :

$$0.000006 \text{ ft. } \times 225 \times 8 = 0.0108 \text{ ft.}$$

$$0.0108 \text{ ft. } \times 12 = 0.1296 \text{ in. } = \frac{4}{82} \text{ in. additive.}$$

Correction for sag :

$$\frac{1}{4} \text{ in. } \times 2\frac{1}{4} = \frac{18}{82} \text{ in. additive.}$$

Total correction :

$$-9 \frac{9}{82} \text{ in. } + \frac{4}{82} \text{ in. } + \frac{18}{82} \text{ in. } = + \frac{13}{82} \text{ in. additive.}$$

Length to be taken on tape :

$$225 \text{ ft. } 4\frac{1}{2} \text{ in. } + \frac{13}{82} \text{ in. } = 225 \text{ ft. } 4\frac{29}{82} \text{ in.}$$

When the tape is not suspended, correction for sag will not be made.

In short and less important measurements the same attention to corrections is not necessary.

In practice, the above method has been found to give satisfactory results.

391. In placing stakes to hold measurements, it is best, and in harmony with the method suggested for placing them on instrument lines, to set them with the greater dimension of cross-section in the direction in which the measurement is being made.

Measuring is a very important part of the work of the surveyor. Even when done with the greatest care, it is difficult to obtain results *entirely* satisfactory.

Measurements which are to be directly compared, or are to be used in connection, as in locating parallel lines, should be made under circumstances as nearly as possible identical. Experience and a correct idea of the importance of the work will enable the surveyor to determine the degree of accuracy therein necessary.

LEVELLING-INSTRUMENTS.

392. The Y-Level. Of the different varieties of the levelling-instrument, that termed the Y-level has been almost universally preferred by American engineers, on account of the facility of its adjustment and superior accuracy.

The engraving represents a twenty-inch Y-level as made by W. and L. E. Gurley, Troy, N.Y.

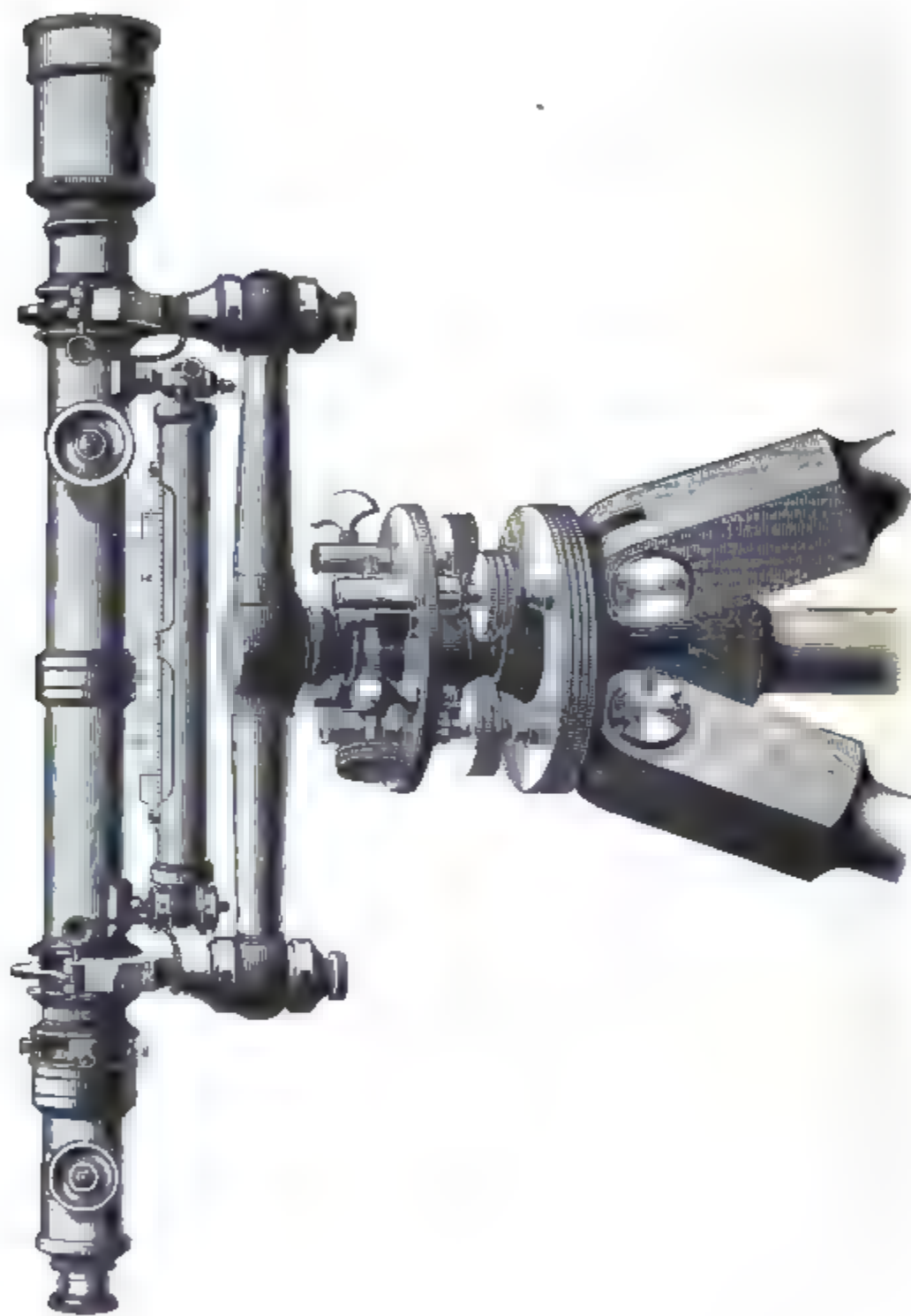
393. The Telescope has at each end a ring of bell-metal, turned very truly, and both of exactly the same diameter; by these it revolves in the wyes, or can be at pleasure clamped in any position when the clips of the wyes are brought down upon the rings, by pushing in the tapering-pins.

394. The Level or ground bubble tube is attached to the under side of the telescope, and furnished at the different ends with the usual movements, in both horizontal and vertical directions.

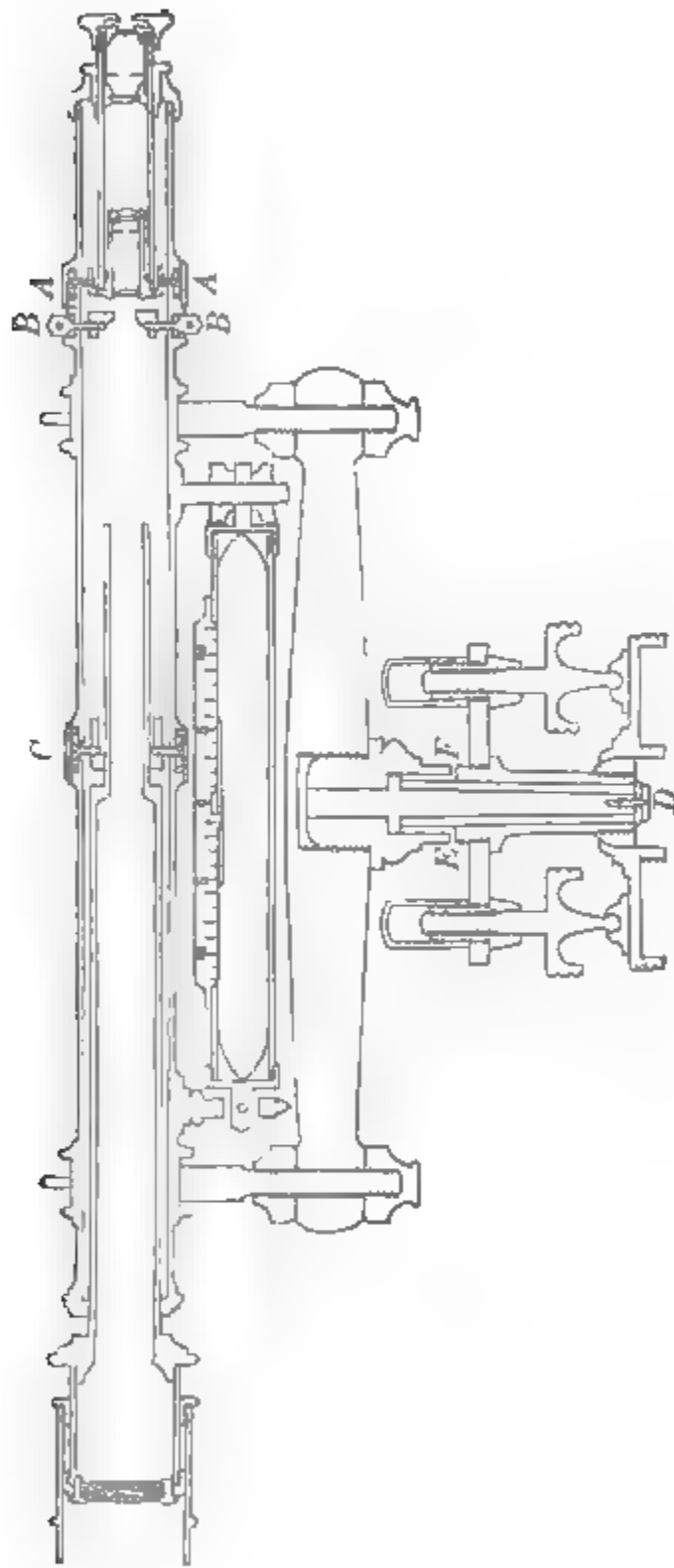
The aperture of the tube, through which the glass vial appears, is about $5\frac{1}{4}$ inches long, being crossed at the centre by a small rib or bridge, which greatly strengthens the tube.

The level-scale which extends over the whole length is graduated into tenths of an inch, and figured at every fifth division, counting from zero at the centre of the bridge; the scale is set close to the glass.

The bubble vial is made of thick glass tube, selected so as to have an even bore from end to end, and finely ground on its upper interior surface, that the run of the air-bubble may be uniform throughout its whole range.



20-INCH Y-LEVEL.



SECTIONAL VIEW OF 20-INCH Y-LEVEL.

395. The **Wyes** are made large and strong, of the best bell-metal, and each has two nuts, both being adjustable with the ordinary steel pin.

The clips are brought down on the rings of the telescope-tube by the Y-pins, which are made tapering, so as to clamp the rings very firmly.

The clip of one of the wyes has a little pin projecting from it, which, entering a recess filed in the edge of the ring, insures the vertical position of the level and cross-wire.

396. The **Level-Bar** is made round, of the best bell-metal, and shaped so as to possess the greatest strength in the parts most subject to sudden strains.

Connected with the level-bar is the head of the tripod-socket.

397. The **Tripod-Socket** is compound; the interior spindle *D*, sectional view, upon which the whole instrument is supported, is made of steel, and nicely ground, so as to turn evenly and firmly in a hollow cylinder of bell-metal; this, again, has its exterior surface fitted and ground to the main socket *EE* of the tripod-head.

The bronze cylinder is held upon the spindle by a washer and screw, the head of the last having a hole in its centre, through which the string of the plumb-bob is passed.

THE ADJUSTMENTS.

398. The three adjustments of the level which the surveyor usually has to attend to are the following:

1. *To adjust the line of collimation*, or, in other words, to bring both wires into the optical axis, so that their point of intersection will remain on any given point during an entire revolution of the telescope.

2. *To bring the level-bubble parallel* with the bearings of the Y-rings, and with the longitudinal axis of the telescope.

3. *To adjust the wyes*, or to bring the bubble into a position at right angles to the vertical axis of the instrument.

399. To Adjust the Line of Collimation, set the tripod firmly, remove the Y-pins from the clips, so as to allow the telescope to turn freely, clamp the instrument to the tripod-head, and, by the levelling and tangent screws, bring either of the wires upon a clearly marked edge of some object, distant from 100 to 500 feet.

Then, with the hand, carefully turn the telescope half-way around, so that the same wire is compared with the object assumed.

Should it be found above or below, bring it half-way back by moving the capstan-head screws at right angles to it, remembering always the inverting property of the eye-piece; now bring the wire again upon the object, and repeat the first operation until it will reverse correctly.

Proceed in the same manner with the other wire until the adjustment is completed.

Should both wires be much out, it will be well to bring them nearly correct before either is entirely adjusted.

When this is effected, unscrew the covering of the eye-piece centring-screws, shown in the sectional view at *AA*, and move each pair in succession with a small screw-driver, until the wires are brought into the centre of the field of view.

The inverting property of the eye-piece does not affect this operation, and the screws are moved direct.

To test the correctness of the centring, revolve the telescope, and observe whether it appears to shift the position of an object.

Should any movement be perceived, the centring is not perfectly effected.

It may here be repeated, that in all telescopes the position and adjustment of the line of collimation depends upon that of the object-glass; and, therefore, that the movement of the eye-piece does not affect the adjustment of the wires in any respect.

When the centring has been once effected, it remains permanent, the cover being screwed on again to conceal and protect it from derangement at the hands of the curious or inexperienced operator.

400. To Adjust the Level-Bubble. Clamp the instrument over either pair of levelling-screws, and bring the bubble into the centre of the tube.

Now turn the telescope in the wyes, so as to bring the level-tube on either side of the centre of the bar. Should the bubble run to the end, it would show that the vertical plane passing through the centre of the bubble was not parallel to that drawn through the axis of the telescope-rings.

To correct the error, bring the bubble entirely back, with the capstan-head screws, which are set in either side of the level-holder, placed usually at the object end of the tube.

Again bring the level-tube over the centre of the bar, and the bubble to the centre; turn the level to either side, and, if necessary, repeat the correction until the bubble will keep its position, when the tube is turned half an inch or more to either side of the centre of the bar.

The necessity for this operation arises from the fact that when the telescope is reversed end for end in the wyes in the other and principal adjustment of the bubble, we are not certain of placing the level-tube in the same vertical plane; and therefore it would be almost impossible to effect the adjustment without a lateral correction.

Having now, in great measure, removed the preparatory difficulties, we proceed to make the level-tube parallel with the bearings of the Y-rings.

To do this, bring the bubble into the centre with the levelling-screws, and then, without jarring the instrument, take the telescope out of the wyes and reverse it end for end. Should the bubble run to either end, lower that end, or, what is equivalent, raise the other by turning the small adjusting-nuts, on one end of the level, until by estimation half the correction is made;

again bring the bubble into the centre, and repeat the whole operation, until the reversion can be made without causing any change in the bubble.

It would be well to test the lateral adjustment, and make such correction as may be necessary in that, before the horizontal adjustment is entirely completed.

401. To Adjust the Wyes. Having effected the previous adjustments, it remains now to describe that of the wyes, or, more precisely, that which brings the level into position at right angles to the vertical axis, so that the bubble will remain in the centre during an entire revolution of the instrument.

To do this, bring the level-tube directly over the centre of the bar, and clamp the telescope firmly in the wyes, placing it, as before, over two of the levelling-screws, unclamp the socket, level the bubble, and turn the instrument half-way around, so that the level-bar may occupy the same position with respect to the levelling-screws beneath.

Should the bubble run to either end, bring it half-way back by the Y-nuts on either end of the bar; now move the telescope over the other set of levelling-screws, bring the bubble again into the centre, and proceed precisely as above described, changing to each pair of screws, successively, until the adjustment is very nearly perfected, when it may be completed over a single pair.

The object of this approximate adjustment is to bring the upper parallel plate of the tripod-head into a position as nearly horizontal as possible, in order that no essential error may arise, in case the level, when reversed, is not brought precisely to its former situation. When the level has been thus completely adjusted, if the instrument is properly made, and the sockets well fitted to each other and the tripod-head, the bubble will reverse over each pair of screws in any position.

Should the surveyor be unable to make it perform correctly, he should examine the outside socket carefully to see that it sets securely in the main socket, and also notice that the clamp does not bear upon the ring which it encircles.

When these are correct, and the error is still manifested, it will probably be in the imperfection of the interior spindle.

After the adjustments of the level have been effected, and the bubble remains in the centre, in any position of the socket, the surveyor should turn the telescope in the wyes until the pin on the clip of the wye will enter the little recess in the ring to which it is fitted, and by which is insured the vertical position of the spirit-level and cross-wire.

When the pin is in its place, the vertical wire may be applied to the edge of a building; and in case it should not be parallel with it, two of the cross-wire screws that are at right angles to each other may be loosened, and by the screws outside, the cross-wire ring turned until the wire is vertical; the line of collimation must then be corrected again and the adjustments of the level will be complete.

402. To Use the Level. Set the legs firmly into the ground. The bubble should then be brought over each pair of levelling-screws successively and levelled in each position, any correction that may appear necessary being made in the adjustments.

Bring the wires precisely in focus and the object distinctly in view, so that all errors of parallax may be avoided.

This error is seen when the eye of an observer is moved to either side of the centre of the eye-piece of a telescope, in which the foci of the object and eye-glasses are not brought precisely upon the cross-wires and object; in such a case the wires will appear to move over the surface, and the observation will be liable to inaccuracy.

In all instances the wires and object should be brought into view so perfectly that the cross-wires will appear to be fastened to the surface, and will remain in that position however the eye is moved.

Care should be exercised during an observation, lest the hand touching the instrument inadvertently, or a foot placed near the leg of the tripod, impair the adjustment.

The weight of a level having a 20-inch telescope, with level-



NEW YORK.



PHILADELPHIA.

LEVELLING-RODS.

ling-head, exclusive of the tripod, is between thirteen and fourteen pounds.

LEVELLING-RODS.

403. The various levelling-rods used by American engineers are made in two or more parts, which slide from each other as they are extended in use.

404. The New York Rod. This rod, which is shown in the engraving as cut in two, so that the ends may be exhibited, is made of maple, in two pieces, but sliding one from the other, the same end being always held on the ground, and the graduations starting from that point.

The graduations are made to tenths and hundredths of a foot, the tenth figures being black, and the feet marked with a large red figure.

The front surface, on which the target moves, reads to $6\frac{1}{2}$ feet; when a greater height is required, the horizontal line of the target is fixed at that point, and the upper half of the rod, carrying the target, is moved out of the lower, the reading being now obtained by a vernier on the graduated side, up to an elevation of 12 feet.

The target is round, made of thick sheet brass, having, to strengthen it still more, a raised rim, which also protects the paint from being defaced.

The target moves easily on the rod, being kept in any position by the friction of the two flat plates of brass which are pressed against two alternate sides, by small spiral springs, working in little thimbles attached to the band which surrounds the rod.

There is also a clamp-screw on the back, by which it may be securely fastened to any part of the rod.

The face of the target is divided into quadrants by horizontal and vertical diameters, which are also the boundaries of the alternate colors with which it is painted.

The colors usually preferred are white and red ; sometimes white and black.

The opening in the face of the target is a little more than a tenth of a foot long, so that in any position a tenth or a foot figure can be seen on the surface of the rod.

The right edge of the opening is chamfered, and divided into ten equal spaces, corresponding with nine-hundredths on the rod ; the divisions start from the horizontal line which separates the colors of the face.

The vernier, like that on the side of the rod, reads to thousandths of a foot.

The clamp, which is screwed fast to the lower end of the upper sliding-piece, has a movable part which can be brought by the clamp-screw firmly against the front surface of the lower half of the rod, and thus the two parts immovably fastened to each other without marring the divided face of the rod.

405. The Philadelphia Rod. This rod is made of two strips of cherry, each about $\frac{3}{4}$ inch thick by $1\frac{1}{2}$ inches wide and 7 feet long, connected by two metal sleeves, the lower one of which has a clamping-screw for fastening the two parts together when the rod is raised for a higher reading than 7 feet.

Both sides of the back strip and one side of the front one are planed out $\frac{1}{8}$ inch below the edges ; these depressed surfaces are painted white, divided into feet, tenths and hundredths of a foot, and the feet and tenths figured.

The front piece reads from the bottom upward to 7 feet, the foot figures being red and an inch long, the tenth figures black and eight-tenths of an inch long. When the rod is extended to full length, the front surface of the rear half reads from 7 to 13 feet, and the whole front of the rod is figured continuously and becomes a self-reading rod 13 feet long.

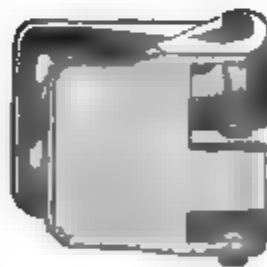
The back surface of the rear half is figured from 7 to 13 feet, reading from the top down ; it has a vernier also by which the rod is read to two-hundredths of a foot as it is extended. The target is round and made of sheet-brass, raised

on the perimeter to increase its strength, and is painted in white and red quadrants; it has also a scale on its chamfered edge, reading to two-hundredths of a foot.

When a level of less than 7 feet is desired, the target is moved up or down the front surface, the rod being closed together and clamped; but when a greater height is required, the target is fixed at 7 feet and the rear half slid out, the scale on the back giving the readings like those of the target to two-hundredths of a foot.

This rod is so graduated that the leveller is enabled to take the reading direct from it, the rodman's duties being simply to hold the rod vertical over the points. It is hence called a *self-reading or speaking rod*.

406. The Rod-Level. The figures below represent a level recently devised, for the more accurate plumbing of levelling-rods.



ROD-LEVEL.



ROD-LEVEL AS APPLIED TO A ROD.

The left-hand figure shows it when folded for convenience in carrying. Its convenience and value commend it to general favor.

407. Levelling is measuring in a vertical direction. In his treatise on levelling, Frederick W. Simms says: "Levelling is the art of tracing a line at the surface of the earth which shall

cut the directions of gravity everywhere at right angles. . . . The direction of gravity invariably tends towards the centre of the earth, and may be considered as represented by a plumb-line when hanging freely, and suspended beyond the sphere of attraction of the surrounding objects. . . . The operation of levelling may be defined as the art of finding how much higher or lower any one point is than another, or, more properly, the difference of their distances from the centre of the earth."

A surface like that of still water may be called a level surface. The curve formed by the intersection with such a surface of a vertical plane is a *line of true level*; a line tangent to the latter is a *line of apparent level*.

Levelling is the art of determining the differences of elevation of two or more points, or of determining how much one point is above or below a line of true level passing through the other point.

408. From the foregoing it is evident that, on account of the curvature of the earth, a horizontal line is not really throughout its length a level line; that of two points in the same level line each will have its own horizon. Hence, in levelling, the effect of the curvature of the earth upon the comparative elevations of different points must be taken into consideration. The effect of the curvature is to make objects appear lower than they really are.

The air nearer the surface of the earth is denser than that farther removed from the surface. This difference in density, causing refraction of light, will affect the elevation of a point as observed through the telescope of a level, so that it also must be taken into consideration. Its effect is to make objects appear higher than they really are. The error caused by refraction is one-seventh as great as that caused by curvature.

Let us first find an expression for the correction due to the curvature of the earth. That is —

409. *To find the deviation from its tangent of a line of true level.*

Let O represent the centre of the earth, PN a line of true level, and PN' its tangent, or a line of apparent level. The distance NN' corresponding to the length of sight PN is required.

From Geometry,

$$\overline{PN'}^2 = NN'(2ON + NN');$$

$$\text{or, } NN' = \frac{\overline{PN'}^2}{2ON + NN'}.$$

For ordinary distances, the length of the arc may be regarded as that of the tangent, and NN' as inconsiderable in comparison with $2ON$, the diameter of the earth. Therefore, calling the length of sight d , the correction c , and the radius of the earth r , we have

$$c = \frac{d^2}{2r},$$

and the correction for refraction

$$= \frac{1}{7}c = \frac{1}{7} \times \frac{d^2}{2r} = \frac{d^2}{14r};$$

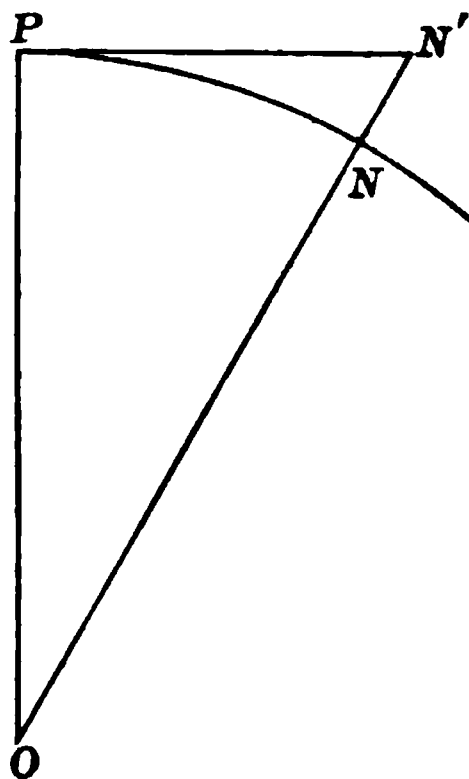
then the correction due to curvature and refraction, which we will call C , is

$$c - \frac{1}{7}c = \frac{d^2}{2r} - \frac{d^2}{14r};$$

$$\text{or, } C = \frac{3d^2}{7r}.$$

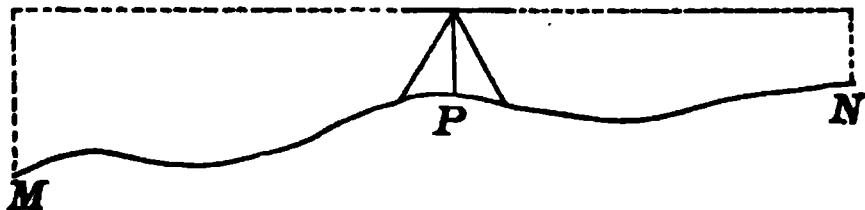
This correction must be added to the height of the object as found by the level.

In practice, the necessity for using the above formula is avoided whenever it is possible to set the level at equal distances from the points whose difference of height is required.

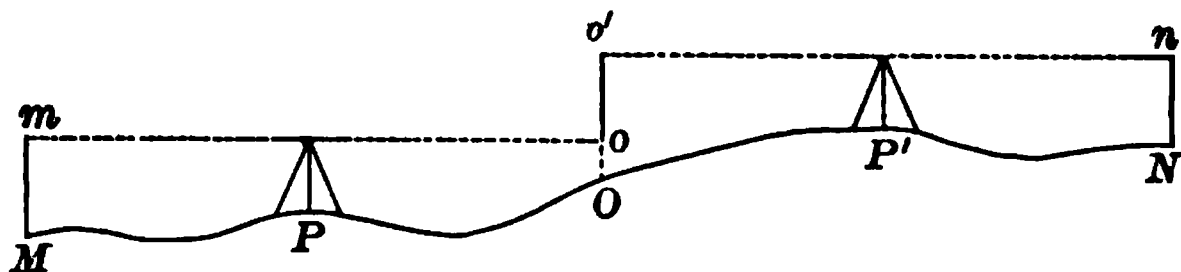


EXERCISES.

1. Assuming the diameter of the earth 7,926 miles, show that for a mile sight c = about 8 inches. Find the value of C for the same distance.
2. What is the correction due to curvature for half a mile?
3. What is the length of sight when C equals one-tenth of a foot?
4. Show that, practically, the correction for curvature in feet is equal to two-thirds the square of the distance in miles.



410. If two points M , N , whose difference of elevation is required, can be observed upon from some point P about equidistant* from them, not necessarily in their line, set up the level at P , and note the reading of a rod held vertically over each point. The difference of the two readings will indicate the difference of level required.



411. If the above method is impracticable, set up the instrument at some point P —either in or out of the line, no matter which — from which a rod may be observed on the first station M , and also on another point O in the direction of N , about equidistant with M from the instrument. Remove the level to a

* Placing the instrument in this position lessens the effects of inaccurate adjustment and renders unnecessary the corrections indicated in Article 409.

new position P' , whence observe again the rod on O , also the rod reading at N .

The difference between the readings of the rod at M and O shows how much higher the latter is than the former, and in like manner the difference of the readings at O and N gives the difference in elevation of these points, and so on, no matter what the number of stations. The difference in height of M and N

$$= Mm - Oo + Oo' - Nn;$$

or,

$$Mm + Oo' - Oo - Nn$$

$$= Mm + Oo' - (Oo + Nn).$$

Calling Mm and Oo' back-sights, and the other two, fore-sights, we perceive that the difference of level of two points is shown by subtracting the sum of the fore-sights from the sum of the back-sights.

412. Again, in levelling, we measure, by means of the rod, how much lower than the line of sight (height of instrument) certain points are. Thus we may determine the relative elevations of the points. Suppose, for example, it be required to determine the difference in elevation of any two points. For reasons already given, set the level equally distant from the points. If this cannot be done, and both observations have to be taken from one of the stations, especially if the distance between them is considerable, correction as previously described must be made. But in this case suppose it is possible; and suppose that when held on one point, the rod reads 7.255; that is, this point may be considered 7.255 below the line of sight, and 4.755 when held on the other; then the first may be considered $7.255 - 4.755$, or 2.500 farther than the second below the line of sight, or lower than the second.

413. Suppose it be required to determine the difference in elevation between two points, of which one is so much higher than the other that the rod is too short to give a reading on both points for one position of the instrument. In such a case

one or more auxiliary points, called turning-points (T.P.), must be used, and their relative elevations determined. Suppose the reading on the first point is 0.824, and on a turning-point is 10.432; the latter is then 9.608 below the former. Now the instrument must be moved and set up so as to obtain a reading on the turning-point; and (we will suppose) on the other of the given points. Suppose that on the former it is 1.302, and on the latter 8.634; the latter is then 7.332 below the turning-point, or $9.608 + 7.332$, or 16.940, below the first of the two given points.

The first sight taken after setting up the level is called a back-sight, or plus sight; those taken after this, and before the instrument is moved, are called fore-sights or minus sights. As the difference of the readings of the rod on two points gives their difference of elevation, the difference of the sum of the plus sights, and the sum of the minus sights on T.P.'s and the last point will give the difference in elevation of the extreme points. In the above example

| | |
|--|--------------|
| 0.824 | 10.432 |
| 1.302 | 8.634 |
| <hr/> 2.126 | <hr/> 19.066 |
| $19.066 - 2.126 = 16.940$, as before. | |

This is used as a check on level-notes.

In extended levelling, permanent elevations fixed during the progress of the work for future reference are called bench marks or benches (B.M.).

414. In levelling, it is customary to refer all elevations to an assumed level plane, called the plane of reference, the datum plane, or simply the datum. Points are then said to be so much above or below the datum. As this plane may be assumed at pleasure, it is generally so taken as to be lower than any point whose elevation is to be determined. In city levelling this plane may be assumed at the height of mean low water.

which elevation may be called zero. Then a point which has the elevation 125.37 will be 125.37 above low water.

If two points have the elevations 125.375 and 105.213 respectively, the former is $125.375 - 105.213$, or 20.162 higher than the latter.

The datum having once been determined, its elevation, or that of a point a known distance above it, should be permanently fixed for future reference and comparison.

415. The levels for profile given under Street Grades, on page 365, show how the field notes in levelling may be kept. The elevation of the bench-mark from which they start is 51.415 above the datum. The first plus sight is 7.030, which, added to 51.415, gives 58.445, the height of the instrument (H.I.) above the datum. The first minus sight, which is on a turning-point (T.P.), is 0.870, which, subtracted from 58.445, gives 57.575, the height of the T.P. above the datum. The instrument is then moved, set up again in a convenient place, and the work proceeds.

At one setting of the instrument, the elevations of any points, besides the turning-point, which are not too high or too low to be reached, may be ascertained. It is evident that if any error be made at a T.P., all the following elevations will thereby be affected; but if made at one of these other points, only the elevation of that point will be affected. Hence the importance of careful observations at T.P.'s.

In the above-mentioned form for the keeping of the field notes, all the observations (Obs.) are set in one column. If desired, plus sights and minus sights may be set in different columns; and of minus sights, those on turning-points may be set in a column by themselves. It will then be easy to apply the check before described. However, the form given is in practice very convenient.

EXERCISE.

Tabulate in both of the above forms, also in the form headed

| STA. | + S. | H. I. | - S. | ELEVATION. | REMARKS. |
|------|------|-------|------|------------|----------|
| | | | | | |

the following level notes :

| | | | | | | | | |
|--|---|---|---|---|---|---|---|----------|
| Height of B.M. | . | . | . | . | . | . | . | 100.000. |
| Obs. on B.M. | . | . | . | . | . | . | . | 5.132. |
| " " Sta. 0 | . | . | . | . | . | . | . | 6.28. |
| " " " 1 | . | . | . | . | . | . | . | 7.12. |
| " " " 2 | . | . | . | . | . | . | . | 8.84. |
| " " T.P. 3 | . | . | . | . | . | . | . | 9.780. |
| From new position of inst. obs. on Sta. 3, | | | | | | | | 2.160. |
| Obs. on Sta. 4 | . | . | . | . | . | . | . | 5.89. |
| " " " 5 | . | . | . | . | . | . | . | 7.92. |
| " " " 6 | . | . | . | . | . | . | . | 10.18. |
| " " T.P. 7 | . | . | . | . | . | . | . | 12.020. |
| Again on " 7 | . | . | . | . | . | . | . | 1.260. |
| Obs. on Sta. 8 | . | . | . | . | . | . | . | 4.23. |
| " " " 9 | . | . | . | . | . | . | . | 5.87. |
| " " " 10 | . | . | . | . | . | . | . | 6.94. |

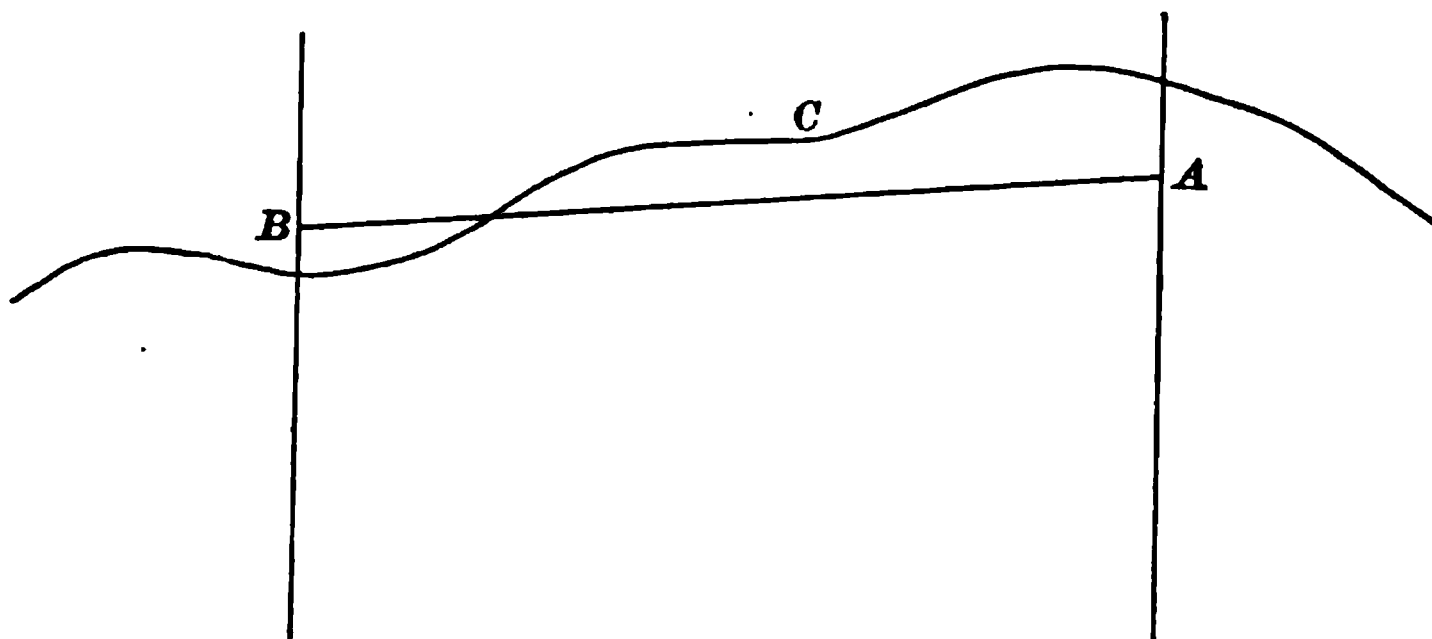
416. Wind and sunshine affect the accuracy of levelling, as of work with the transit. For very good work it is desirable to have a calm day on which the sun is obscured by clouds. In addition to a proper manipulation of the instrument, the sights should not be longer than from 200 to 300 feet, the rod should be held vertical, and the rodman should select for turning-points good and firm points on stones, pegs, etc., on which the rod may be freely turned or spun around.

417. Numerous bench-marks should be located in convenient places. In a city such places are at the intersections of streets, on door-sills of buildings which have become thoroughly settled, on roots of trees, etc. There are many other suitable places which will suggest themselves.

418. In city work, in making a circuit of levels for the establishment of grade elevations and bench-marks, the work should check out with no greater error than 0.01 foot in three miles.

In levelling, as in all other work, regard must be had to the difference between actual mistakes, the results of carelessness, and the degree of accuracy actually obtainable by the observer.

We will now describe a general method of running a grade-line with the transit. In the figure the irregular line represents the profile of the ground, and the straight line the grade-line.



Let it be required to run a grade-line from *A*, elevation 30.29, to *B*, elevation 28.79; elevation of plug or ground at *A* 33.49, at *B* 27.26; therefore cut at *A* 3.20 and fill at *B* 1.53.

Set the transit over *A*; and, using the long level-tube, take the elevation from a convenient bench. Suppose the H.I. is found to be 38.21; then the length of the rod for marking the grade-line (called working height) is $38.21 - 30.29 = 7.92$. The rod will then be taken to *B* and held on the plug. But as the plug is 1.53 below the grade-line at *B*, the target, when the rod is held for grade on that plug, will be set at $7.92 + 1.53 = 9.45$. When thus held, the observer will set the horizontal cross-hair on the middle of the target and clamp the telescope. The line of sight will then be a line parallel with the grade-line and 7.92 above it. Care must be taken to use the rod 7.92, and

not 9.45, as the working height. Measurements may now be made from the line of sight to determine the cut to the grade-line at any intermediate point.

Suppose at *C* the rod read 5.97; then the cut at that point is $7.92 - 5.97 = 1.95$.

How would you proceed if the instrument were set at *B*?

The cuts or fills to grade at any points may be determined by taking the elevations of the ground at those points and calculating the grade elevations at the same points. The difference of elevation will be the cut or fill required.

B. OFFICE INSTRUMENTS.

419. In addition to the various drawing-instruments previously described the student should understand the use of that elegant instrument the polar planimeter. In ascertaining the areas of figures having irregular boundaries it will be found extremely useful. He should also become acquainted with the different methods for the rapid reproduction of drawings.

SECTION II.

WORK.

420. The work of the city surveyor may be divided into two classes: first, public work, or that which he is called upon to perform for the city government; second, private work, or that which he performs for private citizens. The former is generally connected with the streets; the latter, with the property between them.

Again, all of his work may be classed as field work or office work, the former of which we will now consider.

A. FIELD WORK.

421. Public Work. There are many and varied natural features and artificial influences affecting the original location

of a town or city. To the thoughtful student many of these will readily suggest themselves. While in the choice of a site the surveyor may have a voice, it is more than probable that his work will commence upon a site already selected. We will now describe some of his more important duties as performed for the town or city government.

422. Street Lines. The city consists of streets for public use, and of the blocks bounded by them, the land in which is divided and sold to individuals for their private use. Hence we have first to consider the general plan or arrangement of the streets, their widths (the distances between house lines), and their distances apart. There are many general plans which may be adopted, or may be used as the foundation for new ones. When general convenience and the economical division of property are considered, I believe there is none which better meets the requirements than that which is characterized by two systems of parallel streets crossing at right angles. With this general arrangement, and some well-located diagonal avenues, we have the lay-out of a beautiful and convenient city.

The general directions of the streets should be such that the greatest number may during the day be visited by the sunshine. This will be accomplished if one set of parallel streets runs in a northeasterly and southwesterly direction.

Every important street should be at least 60 feet wide, while some of the main streets should be at least 100 feet wide, with avenues even wider. The streets will then admit freely air and sunshine, which latter is too often in narrow streets cut off by tall buildings; while the avenues will be in harmony with their design as elegant thoroughfares.

Another important consideration which affects the width of streets is the expense of paving and of keeping them in order.

The distances of the streets from each other will vary very much, according to the purposes for which the included property is to be used, and how it is to be divided. They may vary

from 300 to 600 feet. The sidewalks will be from one-fifth to one-fourth of the width of the streets.

In small towns an elaborate design will not be attempted; but it is always best to have in view the possibilities of future growth.

423. With the transit, the surveyor will run and extend street lines, and will turn off required horizontal angles on the horizontal graduated circle of that instrument. It is convenient to work upon the centre lines of the streets. Two base lines having been carefully located at right angles with each other, the centre lines of the two sets of streets will, with the most reliable measuring-instruments at the disposal of the surveyor, be carefully located parallel with them respectively. If the land is quite level, a 200-foot steel tape is useful. If it be inclined and irregular, a 100-foot tape is better suited to the purpose. In any case, the hand-level, plummet, thermometer, etc., should be used. The work, like all work of the surveyor, should be carefully checked by a test of the different angles and distances. All this work should be done with the greatest care. It is desirable, in order to guard against future difficulties in regard to measurements by other parties, to make streets and block distances a little full; that is, greater than they are actually required to be — say about one-fourth of an inch in 100 feet. As the work progresses, it will be properly marked with stakes, as before described. After the satisfactory location of the centre lines of the street, the house lines may easily be located therefrom.

424. The work of the surveyor may be not in laying out and regulating a new town, but in connection with one already laid out. The extensions of the old town may be carried on in harmony with the plan already existing, or they may be on a plan altogether different, and after the manner already described for a new town. He will find that the already built-up portions of the town have been previously regulated, or that they have

not been. If they have been, it is advisable in carrying on the work therein to adhere as closely as possible to established lines, elevations, standard of measurement, etc., lest any alterations should lead to expensive and unnecessary legal complications. If the town has never been regulated, the first steps will be to regulate its streets. In doing this a complete survey will be required. Instrument lines will be carefully located with the transit on all streets, and the angles at their intersections determined. These lines will be the basis for the location, by offsets, of all buildings, fences, etc. As the survey goes on, the results will be carefully plotted to a conveniently large scale; and from the completed plot, an advantageous location of the streets may be determined upon. They will then be located upon the ground to correspond. All important measurements will be made, as before described, with the steel tape, with all the corrections carefully attended to. Offsets to fences, etc., need not be made with so much care, and the corrections will, as a rule, be superfluous. During the progress of the work in an old town, as in a new one, all important lines will be carefully marked with stakes, and upon permanent objects, as houses, etc.

425. The streets in any city or town having been satisfactorily located according to the general plan, it is necessary, in order to preserve work already done, and to prevent conflict in future work, that the location of the street lines should be preserved. On account of the perishable nature of wooden stakes, and the fact that they may soon be disturbed, it is necessary to use something more permanent. This is generally found in stones. Mere stones, or monuments used for permanently holding the lines of streets, are differently located and are of different sizes, depending upon their location. Sometimes they are placed in the sidewalks 5 feet from the house lines. Then they need not be more than 4 or 5 inches square and 2 feet in length. The line is determined by a small hole drilled in the top of the stone. Sometimes the top of the stone is placed below the surface of the pavement; sometimes it is placed flush

therewith. Larger stones set in the intersections of the streets, where their centre lines cross, are very conveniently situated for use, and afford a very satisfactory means of marking street lines. On account of their more exposed position, they must be larger than those previously described, and should be set with the greatest care, the materials around them being well packed and rammed. They should be paved about and well protected from danger from traffic. The stones should be square in cross-section about 3 feet long, about 8 inches square on the top, and about 1 foot square on the bottom, the top and bottom being at right angles with the axis of the stone. The line is determined as before by a hole drilled in the top of the stone. From their situation we call these stones centre stones. It is well also to mark substantial buildings standing at the corners of streets with their distances from the house lines of the streets, these distances having been carefully determined by measurements. In general, a line having once been determined upon as satisfactory, every available means should be employed to preserve its location, as any change would obviously be attended with inconvenience and danger.

426. Street Grades. In the selection of a site for a town, and in the location of the streets of a town or city, a topographical map will be of much service. This map will show at a glance the shape of the ground under consideration. If the surface of the earth were cut by horizontal planes 5, 10, 20, or more feet apart, and the curves in which these planes intersect the surface were projected upon a horizontal plane, the resulting lines would be called contour lines or contours. These curves would represent points of the same elevation. Their distances apart would represent relative inclination in the ground, the curves being nearer as the ground is steeper. The determination of these contours is an important feature in topographical surveying. In addition to its other uses, such a map would be of service in locating sewers, also in fixing proper elevations and grades for streets. The field work necessary in the prep-

aration of topographical maps, which we will briefly notice, may be done as follows: Two sets of parallel lines having been located at right angles with each other by means of the transit and tape, the level will be set up, and a number of points at any one elevation above the datum found with the level and the rod, and their locations with reference to the two sets of lines determined. Another set of points as far above or below the former as the planes are apart will in like manner be determined and located, and so on until the entire ground has been gone over. The above method of topographical surveying in determining contours is not a very rapid one. The stadia method is more rapid, and is well adapted to large areas. In addition to the usual horizontal cross-hair in the transit, two others are introduced, one above and one below the former. The instrument has also a vertical circle. The stadia-hairs are so arranged that when the level rod is held at a certain distance from the transit, a certain number of feet on the rod is included between them. The *distance* of any point from the instrument can be determined, as it varies with the number of feet intercepted on the rod. The line of sight must be at right angles to the rod; if it is not, a calculation must be made to determine the distance. By this distance and a horizontal angle the point is located horizontally.* The *elevation* of the point above the station at which the instrument is placed is obtained by observing on the rod a point as much above the ground as the telescope is, and taking the vertical angle. The product of the horizontal distance and the tangent of the angle will give the required difference in elevation. The plane table also has been much used in making topographical surveys.

Street grades themselves will be determined upon in the office, after the necessary data has been obtained in the field.

427: A very convenient method of obtaining the data necessary for the determination of elevations and grades for the streets is to obtain a continuous profile of the ground on the

* See Chapter II., Stadia Measurements, Articles 148-152.

centre line of each street. The work is done in the following manner: The level having been set up, and the height of instrument determined from a convenient bench-mark, an elevation will be taken on a level plug set at the intersection of the centre lines of two streets. Elevations will then be taken at stations, say 50 feet apart, about on the centre line, measurements with the tape being commenced at the intersection before mentioned, and made carefully enough to avoid any error that might affect the work. In addition to the elevations at the stations, elevations should be taken at any intermediate points where the shape of the ground abruptly changes; and the points should be located by measurement. These intermediate points are called pluses. When the next intersection is reached, measurements will be commenced anew, and the levelling continued in the same manner. Elevations on level plugs at intersections, on turning-points, and on benches, which, if not previously established should be established as the work progresses, should be carefully taken with the target. The elevations for the profile should be read without the target to the nearest hundredth. Such circuits should be made in levelling for profiles, and the levelling on the cross-streets should be so carried on as to check the work in every way. The level notes, taken as described for the profile of the centre line of a street, are shown below. They are from actual practice. The datum is mean low water in the ——— River, the elevation of which is taken as zero. The manner of plotting these notes, and of determining grade lines is given under the head Office Work.

428. In order to avoid errors in giving grade lines, the grade elevations at the intersections of streets should be permanently marked. This may be done by placing the centre stones before described so that their tops shall be at the grade elevation. In order to preserve these elevations in case of the removal or disturbance of the stones, bench-marks should be established on convenient door-sills, and in other safe and con-

**LEVELS ON FIFTH AVENUE, SOUTHERLY FROM MARY-
LAND AVENUE.**

FOR PROFILE.

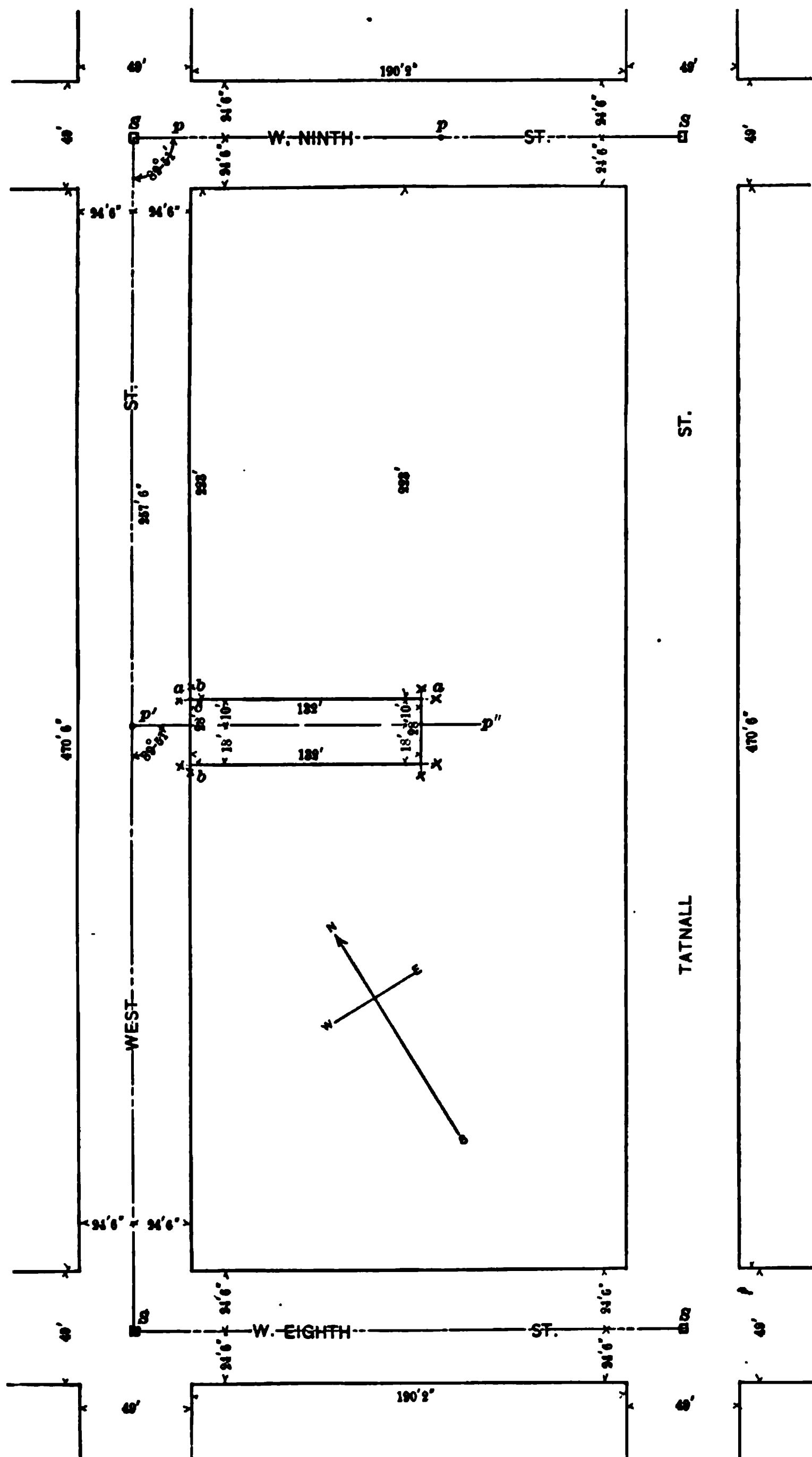
Nov. 21, 1886, A.M.

| STA. | OBS. | H.I. | EL. | | REMARKS. |
|--------------|--------|--------|--------|------|---|
| B.M. | | | 51.415 | | On west end of door-sill, etc. |
| + | 7.030 | 58.445 | | | |
| { P. | 0.870 | | 57.575 | | |
| { + | 10.005 | 67.580 | | | |
| B.M. & { P. | 1.300 | | 66.280 | | { On highest point of red rock, etc. |
| { + | 0.900 | 67.180 | | | |
| Sta. 0. | 0.000 | | 67.180 | | { Plug middle of 5th and Md. Aves. |
| 0 + 25. | 1.55 | | 65.63 | | |
| 0 + 35. | 0.28 | | 66.90 | | |
| 1. | 1.50 | | 65.68 | | { 50-ft. Sta. meas. south from mid. of Md. Ave. |
| 2. | 3.91 | | 63.27 | | |
| 3. | 6.20 | | 60.98 | | |
| 4. | 8.83 | | 58.35 | | |
| 5. | 11.80 | | 55.38 | | |
| 6. | 13.20 | | 53.98 | | |
| 7. | | | | | |
| Plug & { P. | 11.352 | | 55.828 | | { Plug centre 5th Ave. and Anchorage St. |
| { + | 4.365 | 60.193 | | | |
| B.M. | 5.480 | | 54.713 | | { Temporary — on plug near fence, etc. |
| Sta. 1. | 5.13 | | 55.06 | | { 50-ft. sta. meas. south from middle of Anchorage St. |
| 2. | 4.65 | | 55.54 | | |
| 3. | 4.93 | | 55.26 | | |
| 4. | 5.69 | | 54.50 | | |
| 5. | 7.26 | | 52.93 | | |
| 6. | 11.00 | | 49.19 | | |
| Plug 6 + 34. | 12.224 | | 47.969 | | { Plug centre 5th Ave. and Brown St. |

venient places. Besides serving as benches for the stones, these bench-marks will be used in doing very close final leveling, the tops of the stones being too uneven for that purpose.

429. Marking of Lines and Grades. The lines and grades of the streets having been finally determined, and the means of preserving them having been established, the marking of these lines and grades for any public work, as street extension and grading, curb setting, sewer and water-pipe laying, etc., can be readily done. Street lines will be run with the transit; and, in the manner previously described, grade lines will be run with the same instrument. The marking of street lines and grades for the purposes mentioned, the giving of lines and elevations for other public work, and measurements of various kinds, as of earthwork, constitute the principal part of the field work to be done for the town or city government by the city or town surveyor; or, as the officer who does this work may have more extended duties, the principal part of the *surveying* to be done by the city engineer.

430. Private Work. Continuing the description of the field work of the town or city surveyor, we will notice the second general class in which his work is comprised; that is, work for individuals, or private work. In general, — for other duties in this connection will fall to his lot, such as surveying large tracts according to methods already described, etc., — this work will consist in marking property lines and in giving grades and elevations. As a rule, in a town or city more property lines are marked for buildings than for any other purpose. When the surveyor is called upon to locate the lines of a lot, his first inquiry will be as to the data by which to locate them. It is of course understood that in this connection the only power of the surveyor is to locate lines according to given data, not, as many persons seem to think, to establish of his own volition new lines. So we will inquire what is proper data for locating such lines. In general, the



party desiring to have the lines of a lot marked will produce his deed for the property. The young surveyor will be inclined to think that the distances given in deeds are, as to the location of lines, final. This is not always the case. When walls, alleys, stones, and other permanent landmarks are called for, and can be found, they will take precedence of distances in locating lines. When walls, fences, and other holdings prove undisputed possession for a period of years, though they may not be described in the deed, they govern. In such cases it would be superfluous to mark lines. In towns and cities lots are now as a rule located from the streets. Let us take, in marking the lines for a lot, an example from actual practice. The description taken from the deed is definite, and is as follows:

Beginning at the easterly side of West Street, between Eighth and Ninth Streets, at the distance of 223 feet from the southerly side of Ninth Street; thence easterly, parallel with Ninth Street, 132 feet to a corner; thence southerly, parallel with West Street, 28 feet to a corner; thence westerly, parallel with the first-described line and Ninth Street, 132 feet to the aforesaid easterly side of West Street; and thence thereby, northerly, 28 feet to the place of beginning. The lot is located as shown in the sketch. The owner desired to have marked upon the ground, for use in building, the two lines parallel with Ninth Street and the line of the easterly side of West Street. In order that they may not be removed in making excavations for cellars, walls, etc., the nail plugs to mark the lines are set 3 or 4 feet outside of the lot. In the sketch, *S, S, S, S* represent the stone monuments set at the intersections of the centre lines of the streets to mark lines and grade elevations. Each street is 49 feet wide. In marking the lot, points *p, p*, will be taken in the centre line of Ninth Street. From these points (if there are no obstructions that prevent) measurements will be made parallel with West Street. Twenty-four feet six inches, half the width of Ninth Street, and 223 feet, the distance from the southerly side of Ninth

Street to the northerly side of the lot, will be laid down, and nails placed in nail plugs at a , a , to mark the northerly line of the lot. From these the southerly line will be located. In a similar manner the front and back lines will be located. Lines strained from a to a and from b to b will cross at c , giving a corner of the lot, the nail plugs being undisturbed as the work of building progresses.

If, on account of impassable obstacles, as buildings, walls, etc., a measurement cannot be made from Ninth Street to the place for the nail plug a back of the lot, the marking of the side lines will be done as follows: The southeast angle at the intersection of Ninth and West Streets, $89^{\circ} 51'$, if not known, will be taken. In addition to the points taken in the centre line of West Street for use in locating the front and back lines of the lot, an additional point p' will be taken, and at this point the angle $89^{\circ} 51'$ will be thrown in, and the random line $p'p''$ located parallel with Ninth Street. On this random line points for the location of the side lines will be taken. Now, suppose the point p' is found by measurement to be 257 feet and 6 inches from the centre of Ninth Street (all corrections having been made), or 233 feet from the southerly side thereof. Then the northerly side line will be located by measuring northerly from the line $p'p''$ 10 feet, and the southerly side line by measuring southerly from the line $p'p''$ 18 feet. If the surveyor is in possession of an instrument thoroughly reliable for use in angular measurements, the latter method of marking side lines is to be preferred. When one measurement is made along a sidewalk where there are no obstructions, and the other through fences and over various obstructions, it is hardly possible to obtain the degree of accuracy that may be obtained by the angular method. Sometimes it may be necessary to turn off an angle from the random line in order to locate the back line of a lot. The location of lines is often marked by nails in fences, measurements to houses, walls, etc., instead of by nails in plugs.

After the street lines have been located and marked, the

work in each block should be done independently of the other blocks.

In the intervals between routine work it is desirable, in connection with gathering other data, to take and record in a suitable book, for use as described above, the angles at the intersections of the streets, thus saving time in marking the lines of lots.

The location from the deed of the lines of a lot is not always so easy as in the example given. It is frequently the case that the distances given are indefinite; sometimes none are given. In such cases, in the absence of established holdings, or other means of determining the location of property lines, the matter must be settled by an arrangement between adjoining owners.

In some cases a lot is described in whole or part without distances, but as bounded by the property of other owners. In such a case the location of the lines may, if the descriptions in the deeds of these other proprietors are sufficiently definite, be determined by marking the lines of the other lots.

431. The city or town surveyor will frequently be called upon for surveys to locate new lines with reference to the street lines, or for surveys of tracts of land in or adjoining the city or town. In such cases his manner of working will be based upon the methods of land-surveying already described.

Private parties will frequently require, for use in building operations, the marking of grade lines. This will be done in the manner previously described. In marking the grade and height of the building line in front of a lot, it will very often be found convenient to mark the tops of the front line plugs as so much above or below grade elevation.

B. OFFICE WORK.

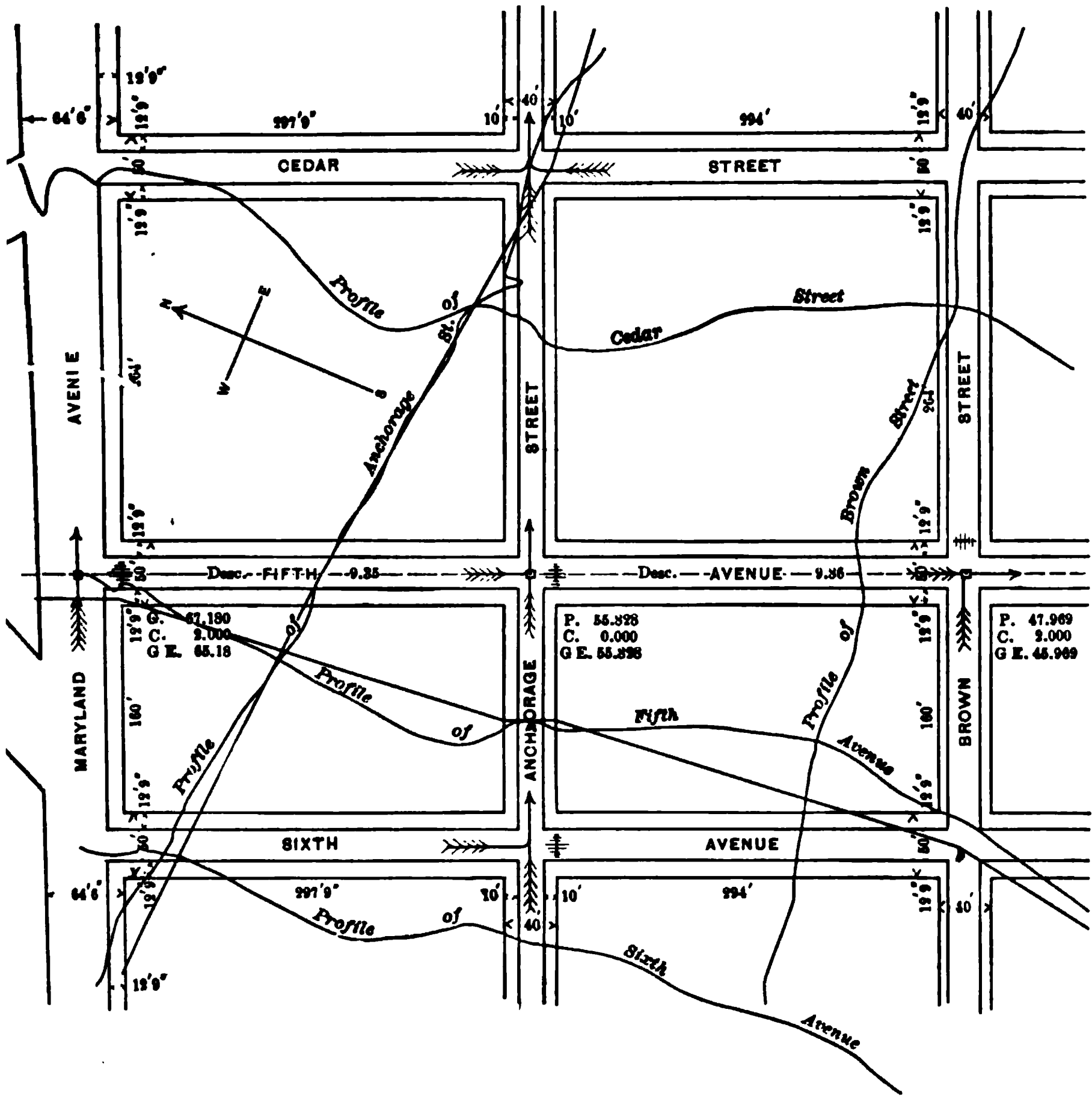
432. Like the field work, the office work of the surveyor may be classified as *Public Work* and *Private Work*.

433. Public Work. All field notes should be sufficiently elaborate to be understood by those who may have occasion to

horizontal distances, if both were plotted to the same scale, or, as we say, if the vertical and horizontal scales were made equal, the differences in elevation will scarcely be apparent. This is remedied by conveniently exaggerating the vertical scale. For example, if the horizontal scale be made 100 feet to the inch, the vertical scale might be made 10 feet to the inch. In the sketch the two scales have this ratio.

EXERCISE. Let the student select scales, and, in the manner described above, prepare a profile from the field notes given on page 356.

437. Having thus plotted the streets and profiles in a large area, we may, by use of the plan thus made, determine suitable grades for the streets. This will involve careful study of the shape of the ground, location of watercourses, probable location of sewers, and effect upon property. The effect of a proposed grade for one street upon those which it crosses must be particularly noticed. To properly perform this work involves that knowledge and judgment which can only be acquired by long experience. The straight lines drawn in connection with the profiles represent the surface grades of the finished streets. In fixing the grade for Fifth Avenue, those of the other streets having been taken into consideration, it was found best to have a cut of 2 feet at Maryland Avenue, no cut or fill at Anchorage Street, and a cut of 2 feet at Brown Street. The elevations of the surface at the intersections of Fifth Avenue with Maryland Avenue, Anchorage Street and Brown Street, are respectively 67.180 on the ground, 55.828 and 47.969 on plugs flush with the ground. The grade line having been fixed, the grade elevations (G.E.) at the centres are respectively 65.180, 55.828, and 45.969, and the descents 9.35 feet and 9.86 feet, as shown in the sketch. The nature of grades will depend much upon local considerations. Grades should always be steep enough to secure proper drainage. The inclination should not be less than 1 in 100. Considering the accumulations of dirt on many of our city streets, from 1 to 1.5 in 100 is to be preferred.



438. In streets in which surface water is carried on the streets, some streets will carry the water in gutters across others. In the sketch such streets are indicated by having arrows drawn in their directions across intersections. In this manner Fifth Avenue carries the water across Brown Street, and Anchorage Street carries it across Fifth Avenue. The water flowing on Fifth Avenue, from Maryland Avenue towards Anchorage Street, will turn into Anchorage Street. The opposite side of Anchorage Street, at the house line, will be a knuckle as high as the centre of the street; and the water will flow from that point towards Brown Street. In fixing grades great care must be taken to so arrange them that one street shall not be overtaxed with water from the others. An outlet for the surface water is formed in the natural watercourses.

If the grade of Anchorage Street were very heavy, so that if continued across it would make one side of Fifth Avenue much higher than the other, it would be desirable to break the grade of Anchorage Street at the curb lines of Fifth Avenue, giving only sufficient fall to carry the water across the Avenue.

439. If the section is sewered, and if the sewers are made large enough to carry the surface water, the gutters across the streets will be dispensed with, and inlets to the sewers placed at the curb corners of the blocks.

440. It is often convenient and useful to have plotted on separate streets the profile and grades of each street.

441. Besides making street and grade plans, it will be a part of the office work of the surveyor to plot, in the usual manner of plotting such work, the surveys made in and about the city or town, for both the city and individuals.

442. In some cities a registry of property is kept. The plotting of lots in suitable record books, and the keeping up of the records, will be a part of the city surveyor's work.

443. Private Work. This includes the preparation of any plans ordered for their own use by parties other than those connected with the city government.

CONCLUSION.

444. The student must bear in mind that he can never, from books, learn to be an accomplished surveyor. The practice is ever in advance of the books. Though he should store his mind with book knowledge upon the subject, he will yet be wanting in the knowledge and readiness regarding actual work which can only be acquired by a long experience. Many operations which can with difficulty be understood from pages of explanation, will, when their actual performance is seen, be comprehended in a short time. Again, there is that which can *never* be learned from books; that is, the judgment which must be constantly exercised in practising the delicate duties of a city surveyor. Among other things, this judgment will teach him to be *very* cautious about giving voluntary advice, and careful in giving even that which is requested; to perform his duties conscientiously, and to keep clear of all entangling alliances. Let him learn everything connected with a complete performance of his work, from the work of the axeman up; that, when he directs, he may do it with the same grace with which he should ever follow the directions of his superiors.

The practice of city surveying is a most excellent drill. If conscientiously performed, it will develop careful and thoughtful habits. However, in practice the student will also have to learn to avoid "fussing" over work, and to proportion to the importance of the work in hand the time and care spent upon a particular work.

BOOKS.

445. Valuable information regarding the matters treated of in this chapter will be found in the following publications:

The manuals and catalogues of instrument-makers.

“A Treatise on the Principles and Practice of Levelling,” by Frederick W. Simms; published by D. Van Nostrand, New York.

“A Descriptive Treatise on Mathematical Drawing-Instruments,” by William F. Stanley; published by E. & F. N. Spon, New York and London.

“A Manual of Drafting Instruments and Operations,” by S. Edward Warren; published by John Wiley & Son, New York.

“The Draughtsman’s Handbook of Plan and Map Drawing,” by George S. André; published by E. & F. N. Spon, New York and London.

The student of surveying who wishes to extend his studies into the field of city engineering will find information upon that subject in the numerous works upon its special branches, and in the current technical periodicals of that class. Much information regarding present American practice in city engineering will be found in the series of papers on **“Municipal Engineering”** now being published in **“Engineering News.”** When completed, these in book form will make a very useful volume.

CHAPTER VIII.

MINE SURVEYING.

446. The survey of underground excavations (mines) to determine their position and extent may be principally for the purpose of projecting the points upon a horizontal plane as in land surveying.

But in strata of high inclination and in cavernous spaces various vertical projections will be needed to complete the graphical representation of the workings; and in fissure veins the elevation may be more important than the plan.

447. Surveys to depict areas underground may be made with surveyors' compass and chain, but generally now the transit or theodolite is used to take the angles, and the steel tape to measure the distances, and in some mines the tape may be with advantage hundreds of feet in length; but generally 50 feet for the chain or 100 feet for the tape are most convenient lengths.

448. The surveyor and each assistant, of course, requires a lamp, and "the sights" are ranged with lamp and plummet, the sight from the instrument being taken upon the flame of the miner's lamp (or candle, it may be) suitably held at the plummet line, which is held to depend from a point fixed or to be fixed in the "roof" or over a point in the "bottom." The plummet string itself may be seen within 300 feet. A chain-pin (arrow) can be used to plumb the light over or under a point. It is advised to display the light at a station *for sight only*, and therefore in moving it, for any reason, other than vertically, in giving the point, it should be hidden from the observer.

The point may be marked by a nail in the timber cap or sill, or be a nail in a peg; the place of the point in smooth roof is to be made conspicuous by a ring of white paint around it, and also as it may be by reference marks at the sides (pillars) of the passage-way.

It is a refinement to use a lamp which is also a plummet, and further to place an extra lamp on the bottom under it; two lights seen in the vertical line making its place more certain, and helping to decide that the sight is ready to be taken.*

It may happen that the line of reflection from standing water can be taken for the line of incidence of a light held under a point, when the roof droops between, the passage being "in swamp" there.

The surveyor's lamp is made entirely of brass or copper, so as not to affect the magnetic needle of the instrument.

For use in low openings the tripod of the instrument must be one of short legs (an extra set of shifting legs will answer the purpose), or have extension legs.

It has been suggested to use two extra tripods, one to set up in advance, for keeping the place of fore-sight and for receiving the instrument alone, carried forward to be mounted there at the same exact spot with facility, while the tripod, left standing at the last place of the instrument, marks the point for back-sight with equal certainty: thus each of three tripods taking its turn in being at a place for fore-sight, remaining there for mounting the instrument upon it, and still remaining for back-sight after the instrument is taken for mounting at next station. There are obvious objections to this in the weight of the luggage, and that only some instruments are made for such ready separate handling.

Some rays of light must be thrown into the telescope at its object end to make visible the cross-hairs therein. This is generally done by the surveyor, while taking a sight, holding his

* Eckley B. Coxe devised the plummet lamp, and also a form of it with wire-gauze covering, like the Davy Safety Lamp, for use where fire-damp may be expected.

lamp in his left hand at the front, but a little to one side of the object-glass. A reflector mounted at the object end is a help. One is a silvered flat ring, standing bias, about 2 inches forward from a collar which is slipped over the object end of the telescope. It reflects light into the instrument as an annular beam. Another one is a diminutive hemisphere which scatters light caught from the lamp into the tube.

The change to, and the equable temperature of, the mine require the trying and favor the making of the ordinary adjustments of the instrument there.

449. Stations are generally made only at the angle points of survey lines, and are therefore not regularly distanced. They may be numbered, lettered, or designated by the total distance from the zero of the measurements of their line. Intermediate points are made on the line where, opposite to lateral openings, other lines of survey or important short connections by measurement merely may start. The corners of chambers along the passage may be noted by distance without making points; the size and position of parts of chambers being afterwards taken and noted by sketch with dimensions relatively marked thereon, there being mostly a parallelism in the rock measures which simplifies the position and shape that chambers take, so that no special survey of directions is regularly required for them.

450. Angles between vertical planes of sight (in azimuth) are noted for obtaining the courses as reduced courses from the initial course of survey, by the successive additions and subtractions to it and from it of the angles as taken, and modified according to the series of 90° in each quadrant of the circle.

The initial course had better be referred to true meridian, and comparison with bearings made with allowance for the variation (declination) of the needle. But it has always been recognized that the course, in degrees and minutes, of a quadrant—and therefore liable to mistakes as to the particular one of four quadrants—would be absolute if the full circle be graduated

around to 90° , 180° , 270° , and 360° , in the successive quadrants. While it is not agreed whether north or south shall be the zero, the direction of graduation with the movement of the hands on the dial of a watch or clock is conventionally fixed. The bearings will be a key to which zero was used in the notes.

451. It is but seldom that in drifts of mines the alignment as well as the grade requires adjustment to the regularity of straight lines and curves similar to surface railroads; for the tram-cars will run around very sharp turns, and for them there is therefore no necessity of expensive improvements in line. But when a locomotive is to be used, or wire-rope haulage is to be introduced, there is apt to be a call for regulation of the line, with regard, especially, to minimum radius of curvature.

Unlike the longer, flat curves of a railroad, — designated according to the American system by the even angular deflections from each other of chords of 100 feet, — these sharper curves will go by assumed even radii (in length not less than ten times the gauge of track), and the deflection angles for running them in by the instrument upon short chords will have to be calculated.

One-half the chord divided by the radius will equal the sine of the angle of deflection from tangent, which is half the angle that two such equal chords will make with each other, and also half the angle at the centre of the circle subtended by the chord. From any point on the circular curve as a position of the instrument, successive deflections of the angle will fix the ends of consecutive chords as measured in. Shorter chords (like those less than 100 feet in a railroad curve) have deflection angles approximately proportional to their lengths.

For ranging the line of direction of a passage that is being opened into the solid, two points for placing lights are given at the start, necessarily near together, until the prolongation of open space allows testing the line by the instrument and giving new points of line. From the three points of a curve line that mark the chords of half the arc, obviously, by simple measure-

ments, a like fourth point may be derived as the face (breast) of the working is advanced. In driving a passage-way describing a semicircle — to save weakening pillar at foot of shaft — a long, curved gas-pipe was used in ranging around. A large-scale working plot showing offsets secures the proper location of curving and branching passages.

Outside, besides the fixing of projected curves by deflection angles as above, the laying off of points of arc intermediate on the chord is by foot-rule measurement of ordinates at right angles.

But without strict regard to data, an expedient way of uniting two intersecting straight lines of track by a circular curve (as an arc starting from the one straight line at any distance short of the apex of the lines and ending on the other line an equal distance from the apex) is to find points by linear measurement merely. Assuming any tangential distance back from apex to P.C. (point of curve), the beginning, and the same to P.T. (point of tangent), the end of curve, we find a third point of the arc, its middle, as a point midway between the middle of the chord of the whole arc and the apex. One-fourth of this *versed sine* will be the versed sine (middle ordinate) to be erected on each chord of half the arc for points of the arc. And any other middle ordinates will be as the squares of their arcs or chords.

This principle applies in rounding off intersecting grades into vertical curves, either convex or concave; by vertical allowances and according to horizontal distances, starting with that at the apex and proceeding similarly to the foregoing as to subdivisions.

The laying off of curves by chords and versed sine so derived does not require knowledge of length of radius or of amplitude of angle. But when the extent of circular arc between two tangents is to be determined by the length of radius, the tangential distance from apex will equal radius multiplied by natural tangent of half the angle of intersection; and between P.C. and P.T. there will be the same measures of chord as there are of chord angles in angle of intersection.

452. In the note-book the left-hand page is used for stations, distances, angles, courses (reduced), and bearings (magnetic), and the opposite right-hand page for offset distances — marked relative to a perpendicular line dividing the page, together with sketches and remarks. The notes should begin at the bottom of the pages and proceed upwards, to appear as on the plan to which their results are to be transferred, in their proper relation of position and observation forward.

The plan of underground work is begun with the plotted network of the lines of survey, then the outline of parts excavated is drawn in detail, and these are shaded, as the places become closed in and abandoned, to distinguish what is open work at any period.

The scale of maps showing the workings, etc., of coal mines is now fixed by law in many of the States at 1:1200 as the least; that is, at not less than 1 inch for 100 feet; the purpose of the maps being to aid the official inspection and regulation of the mines for securing the health and safety of the miners. The plan will generally require to show the relation of the workings to surface openings, watercourses, and bounding lines, and to improvements, such as buildings, roads, and railroads.

The line of outcrops (exposure at the surface of the ground of the mineral beds) within its range will appear on the map, but general topographical detail is reserved for the extended small-scale maps of the surface, which will represent what may be learned of mineral indications also; from which data in advance of the workings may be derived and confirmed by special explorations, as of proof-holes and deep boring. But upon the mine plan such elevations (heights of surface above datum) as seem most essential, such as principal ones along the outcrops, highest points of hills, and lowest of streams should be mapped.

The use of the *pantograph*, for reducing the irregular figures of mine plans with all details from one scale to another, has found much approval; and the *planimeter* is liked for labor-saving and accuracy in determining such areas.

453. In *veins*, the work being deep and narrow, and pursued from *levels* or galleries (horizons of working) generally about 60 feet apart in height, plans of these levels, drawn in different colors to distinguish them, are superimposed on the map of general plan. They show the openings, — the gangways, the cross-cuts, etc., — with the defining lines of the walls of the vein, and may embrace other separations of the mineral. Longitudinal elevation and vertical cross-sections will show the shafts and other connections between the levels, together with the *chambers*, whether open, filled in, or caved.

Ore bodies occurring detached and of the most varying dimensions, though often resembling each other as lenticular in shape, make the workings appear in plan, elevation, and cross-section, as the results of exploration in patches. Shafts in the vein will be parallel to pitch of one wall, and therefore varying from the vertical.

A stratified bed that is to be operated upon, — opened, and won by mining, — may be conceived as a seam of uniform small thickness extending within limits as a plane surface and in relative position defined by the “strike” (the course of all its level lines, which will all be parallel) and its “dip” (the greatest pitch at right angles to the course of the level-line). But upon the large scale the seam occurs of variable thickness, and with lines of level changing in direction and not parallel at different elevations, to the degree that instead of a plane it is a warped surface.

The arrangement of permanent works upon the surface of the ground with reference to the lay of the bed as well as the topography and improvements existing or suited to it, the favorable connection of the lines of haulage and drainage inside, with all to govern outside, present to the mind of the mathematical surveyor applications of the theorems of Descriptive Geometry, as included in adaptation to the ends of practical economy.

454. Location upon the surface of the ground of the plan of inside work, is a repetition of courses and distances outside in the

same vertical planes. Any particular portion of the workings in progress can thus be compared in natural scale upon actual plan of surface of the ground over them.

Overlaid plans with elevations and cross-sections of workings, such as were described for workings in veins, are required to show the development in high pitching beds. The "lifts" or levels in such of coal are 100 yards apart, measured on line of pitch.

Overlaid plans of different parallel seams worked through same shaft are also made, but without systematic elevation and cross-section; the connections (shafts, slopes, or tunnels) between the beds being through barren ground, and limited to the exigencies of hoisting, draining, and ventilating.

455. Following the determination in azimuth by courses and distances of the passages in the mine is the determination of their changes in level by the spirit levelling-instrument and the level-rod (as a separate operation, even if the transit be a combined instrument having a parallel spirit level attached to its telescope), the work being quite similar to such above ground. But the rod must be limited in height to the low spaces where it is to be used, and is preferably marked with red figures for the feet, and white figures for the tenths, upon a black ground. The top of a simple white target is safer to take, however, than the reading from the instrument of the figures themselves. For accuracy, sights, as above ground, should be limited to 300 feet in distance from the instrument.

From the elevations of points taken by levelling, contour lines can be shown on plan as the mineral bed is exploited.

Blue is the conventional color for these contour lines and the figures marking their elevation above the datum, on a mine plan, and *brown* suits for the contrasted surface elevations.

456. Levelling along passage-ways for the purpose of fixing better gradients of hauling-roads, or for fall of water by rectification of undulating bottom to improve drainage, requires sta-

tions especially chained in at regular distances of 50 feet or less; the marks being temporary ones on the sides to serve for taking the levels and to be referred to as to heights in grading, when the *variation* of level of bottom from the *grade* of a station governs the cutting or filling of bottom there, or change of the whole cross-section in height, as it may be. For the adoption of suitable gradients along an extended line, a longitudinal vertical section is drawn, called a profile, which exhibits the relation of ground-line levels, and allows the fixing of grade with assurance. The profile may include the line of top as well as of bottom, with section of rock measures to be affected by "ripping" of the roof and "cutting" of bottom.

457. A Drift or passage along with the measures of a bed will make undulating grade, if course be followed; and if the drainage-rise be allowed to govern, the alignment will be sacrificed.

Tunnelling, however, being arbitrary, across the measures, is mostly upon directed line and grade. *Slopes* are mostly upon directed course; but if within the measures of an inclined bed will mostly be variable in grade. So with an *adit*, driven to give drainage outfall to the surface. For it, shortening of the distance will probably be the governing condition principally.

458. For the workings at high pitch, the determination of horizontal and vertical components of the distances on the sloping lines of top and bottom in a bed, and "hanging wall" and "foot wall" in a vein, will bring the vertical arc of the instrument into requisition, for obtaining the vertical angle, which is always taken as the full angle above the horizontal. Vertical sections, besides such longitudinal ones following broken line of passage within a stratum and showing only adjacent rock measures, may be made of particular places where there is *folding*, or *fault*, of the measures, and for geological or more general purposes they may exhibit the lay and thickness of the various rocks up to the surface, which will as a correct

margin show the outcroppings in profile. Vertical sections may be projections upon planes that traverse the measures according to various conditions, and may be constructed of related points from the map that were not determined for their relevancy to this purpose.

It seems that vertical arcs have had *versed sines* corresponding to radius 1 marked around them for the purpose of telling the allowance upon slope measurement to obtain corresponding horizontal distances, the versed sine being the difference between the hypotenuse as the radius and the horizontal base as the cosine of the vertical right-angled triangle formed; and the slope length for a given horizontal distance would be greater, according to the versed sine of the angle.

Vertical arcs have had *tangents* as rises corresponding to the unit of horizontal distance for the different angles marked upon them.

A method of dividing the arc according to the *sines*, without the intervention of the equal graduation into degrees necessarily, is the subject of a contribution to "Van Nostrand's Engineering Magazine" for July, 1876, and is appended at the end of this chapter.

459. The measurement down deep borings or shafts is best made by special flat steel wire, with suitable plummet heavy enough to insure its making the wire line taut.

The transfer of points down a shaft, as of two to determine a base line for connecting surveys below with those on the surface of the ground, is made by very heavy plummets attached to ordinary wire run off of reels. A portable box to contain the reels, their cranks, and the plummets, is convenient; the best arrangement being that of reels fixed in a frame that stays in the box. The suspended plummets are to be received below each in a bucket of water, or, if hanging from considerable height, in some thicker liquid to settle the wire lines to a steady position for ranged observation by the instrument below. And the observation will be easier upon wire that is whitened there by chalk or paint after being placed.

The plummets in the shaft of the Washington Monument, for showing changes in the verticality of the structure, are steadied in vessels containing a mixture of glycerine and molasses.

460. For taking courses on pitches at high angles an extra telescope on the axis extended to the outside of one of the standards of transit has been used. Another *mining transit* has for the same purpose the sweep of the telescope to the vertical position, made possible by having its standards made inclined to overhang. But the *object-prism* placed before the object-glass, allowing sighting at true right angles in any plane, seems most simply to fulfil the requirements for sighting up or down, as well as sidewise, and is a ready means applicable to the telescope of any ordinary instrument. A transit adapted in any of these ways for taking vertical sights enables the points of base line, as transferred by plummets to the bottom of the shaft, to be tested and compared with the extended line across the pit top, provided the atmosphere be clear in the shaft and obstructions do not intervene. The vertical adjustment of the instrument itself would be tested by this check, the usual test being on high objects, with reversal of standards to opposite sides by turning the horizontal plates.

A heavy, substantial, simple transit, not weighted with "attachments," is the most reliable.

461. The use of the *hanging compass* and of the *hanging clinometer* of the olden time is retained in small and crooked passages of some metalliferous mines. And their subsidiary use in excavations inconvenient of access or footing of the ordinary (the standing instruments) has lately been recommended as of wider application, and they have been introduced into this country. Each of the instruments is to hang by its two hooks, turned opposite ways, to the cord that marks the line. The compass-box levels itself by its gimbals (double trunnions), like a ship's compass, in the frame of which the flat hooks with long bearings in line are a part. The clinometer

hangs as a vertical arc with plummet to give the inclination of the cord from the horizon, while the compass gives the needle course. The cord is stretched from one low stout tripod to another, or in a curving space may be fastened to a gimlet screwed into side timber beyond intersecting point or angle of two cords. The tripod serves as a stool also for the assistant holding cord to the point on it firmly. The distances are accurately measured along the cord by applying a graduated rod to it. The horizontal and vertical components of the measurements have to be calculated for plotting on plan and section. In the old mining regions of Europe the surface surveys were also carried on with the same appliances. With care and patience surprisingly good results in locating connections were attained. The old instruments were graduated in hours and minutes, and the English designations of *dial* and *dialling* for the mine compass and operations with it seem to refer to the same original division of its circle. It seems strange to learn that the plotting was protracted by the same compass (swung there on horizontal plate used for straight edge), reference being had to a meridian line fixed in the office, and the drawing-table being a smooth and level stone slab resting on foundation independent of the office floor.

462. Formerly, when topography was used more for the picturing of the plan of landscape in mapping the features for the information of the tourist or the military commander, than for the projection of the contour accurately to fit the location of artificial ways of the different kinds to the ground, *hachures* were used to indicate character of sloping elevations, and they survive in use upon small-scale maps, to indicate mountain chains. They are intended to be lines of pitch, drawn close together so as to graduate changes naturally, and they should be broken at the intersection of the successive level planes with the surface to make terraces however narrow, and suggest level stages in measure of elevation. Now we have on topographical plans *contour lines* to represent the lines of suc-

cessive levels, say 10 feet apart in rise. They are plotted by connecting all points of elevation that may be determined over the area with regard to the requirements of accuracy in noting the changes; and they may be considered the margins made by a body of water that had successively risen or receded 10 feet in height at a time over the area. They are to be marked by their elevation above the lowest datum plane, preferably over that of mean tide of the ocean. They turn upon themselves where they enclose a peak or a basin — according as the next ones indicate them as higher or lower in the series; they are farther apart in horizontal distance as slopes are flatter, and where two or more coincide for any distance there is a precipice.

These points of even elevations of the ground are determined from the levels run along the survey lines, and the cross-section profiles taken at the stations of the lines — slopes being taken at right angles to the line with *straight edge pole* and *clinometer* or *plummet slope level* applied to it. Each of these angle instruments having a vertical graduated arc, the former with arm hinged at centre of arc and carrying a spirit-level to ascertain the vertical angle included between the levelled arm and the slope of the straight edge under it; the latter, by the departure from the perpendicular of the plummet, showing the equal departure from the horizontal of the straight edge.

From the profile of each slope sketched in the field-book and marked with distances and degrees of rise and fall across the survey line, the successive even 10-foot points can be laid off on plan, regard being had in starting with elevation of station to the partial changes required for the first even 10-foot point each way. A scale of horizontal distances for each degree of the arc, to gain 10 feet rise, is made by the topographer of Bristol-board to lay off the points derived by sloping at the stations, and saves the plotting of the profile of cross-section.

The topographer prefers to draw the contours in the field as taken, using demi-sheets of paper that can be joined at their margins, and upon each of which a portion of the line corresponding to its number is plotted, the line having dots along it, spacing the successive stations intermediate of the angle points

of line, and having the elevations corresponding in pencil alongside. The sheets are held in a box that is carried by a shoulder-strap, and the side of which is used in the field as a drawing-board, the particular sheet in use at the time being tacked on it.

463. The topographer will sketch in the streams, buildings, etc., with reference to measurements however, and will have special lines with small compass, etc., run for him to make contour connections. The operations will rise to the scope of plane-table work, if the drawing-board have a socket with clamps, and be mounted and levelled (by applying a loose hand-level) upon a tripod; the ruler used on it having small compass sights screwed to its ends for sighting to objects and fixing their position on plot by the graphic triangulation of intersected sight-lines from different stations on the survey-line; the station on plot when over its place on ground having a needle stuck upright in it, that has a sealing-wax head for convenient handling, for the purpose of resting the ruler against when sighting. Interpolation, or resection, is the reverse sighting from without the line over the plot to two or three poles on stations of the line or other previously located objects, to attain position, it being understood that the plane table stands with plot in proper relative position always. Secondary triangulation will extend the area of topographic sketching, but this should be checked by connections beyond with surveyed lines and levels.

The Locke level may be used for taking rises by finding all the points in sight that are at a level of the eye, and, in connection with the levelling-rod, the fall of ground may also be determined by this instrument. For gently undulating ground the use of it is better than sloping.

464. Contour lines are drawn 10 feet apart in elevation on most plans of extended land and other surveys that are measured in detail, but it is obvious that cases occur where for large-scale work they are taken closer in elevation or farther for small-scale mapping. In the former case of large-scale work they may be required exactly as elevations directly located by

spirit levelling-instrument, in the latter case as the approximation from altitudes taken in a few places by the barometer.

The scope of their usefulness on plans for projecting improvements it would be difficult to describe exhaustively. They may be for use in locating the drives and walks and terraces, etc., of a park; the shaping of grounds, under-draining, etc., about a residence; the laying out of streets, etc., in a hilly town; the leading of streams of water, large or small, for all purposes in partial or wholly artificial channels, for navigation, water power and supply, irrigation, etc.; the location of roads and railroads with regard to ease of construction and of favorable gradients, as well as the uses in mining directly, and location of all surface erections collateral thereto or elsewhere, collectively known as "the Works."

ANGULAR CROSS-SECTIONING.

By F. Z. SCHELLENBERG, C.E.

Written for "Van Nostrand's Engineering Magazine," July, 1876.

A most direct and expeditious method to get differences in level between points in sight is by the use of a vertical arc graduated to the successive sines 1, 2, 3, . . . 100, in quadrant, for the radius of arc 100.

Multiplying the distance measured in hundreds on the slope by the rate per hundred indicated on the arc gives the difference in level in units. In the higher parts of the arc the corresponding cosines may be marked for deriving horizontal distances.

The applicability of this graduation to such purposes, as described under this caption by R. Bell, C.E., in May number, is obvious, as may also be its use for more extended profiles, for geological cross-sections, for road-grading, or wherever between points obtained by the levelling-instrument its accuracy is not indispensable.

A clinometer thus graduated enables contour lines for topographical work to be most readily determined. The table following gives the 100 points in the quadrant in terms of the common graduation of 90° to the quadrant.

| Vertical Distance for 100 measured on Slope. | Horizontal Distance for 100 measured on Slope. | Angle with Horizon. | Vertical Distance for 100 measured on Slope. | Horizontal Distance for 100 measured on Slope. | Angle with Horizon. |
|---|---|------------------------|---|---|------------------------|
| 0 | | 0° 00' | 51 | | 30° 40' |
| 1 | | 0° 34' | 52 | | 31° 20' |
| 2 | | 1° 09' | 53 | | 32° 00' |
| 3 | | 1° 43' | 54 | | 32° 41' |
| 4 | | 2° 18' | 55 | 83.5 | 33° 22' |
| 5 | 99.9 | 2° 52' | 56 | | 34° 03' |
| 6 | | 3° 26' | 57 | | 34° 45' |
| 7 | | 4° 01' | 58 | | 35° 27' |
| 8 | | 4° 35' | 59 | | 36° 09' |
| 9 | | 5° 10' | 60 | 80.0 | 36° 52' |
| 10 | 99.5 | 5° 44' | 61 | | 37° 35' |
| 11 | | 6° 19' | 62 | | 38° 19' |
| 12 | | 6° 54' | 63 | | 39° 03' |
| 13 | | 7° 28' | 64 | | 39° 48' |
| 14 | | 8° 03' | 65 | 76.0 | 40° 32' |
| 15 | 98.9 | 8° 38' | 66 | | 41° 18' |
| 16 | | 9° 12' | 67 | | 42° 04' |
| 17 | | 9° 47' | 68 | | 42° 51' |
| 18 | | 10° 22' | 69 | | 43° 38' |
| 19 | | 10° 57' | 70 | 71.4 | 44° 26' |
| 20 | 98.0 | 11° 32' | 71 | | 45° 14' |
| 21 | | 12° 07' | 72 | | 46° 03' |
| 22 | | 12° 43' | 73 | | 46° 53' |
| 23 | | 13° 18' | 74 | | 47° 44' |
| 24 | | 13° 53' | 75 | 66.2 | 48° 35' |
| 25 | 96.8 | 14° 29' | 76 | | 49° 28' |
| 26 | | 15° 04' | 77 | | 50° 21' |
| 27 | | 15° 40' | 78 | | 51° 16' |
| 28 | | 16° 16' | 79 | | 52° 11' |
| 29 | | 16° 51' | 80 | 60.0 | 53° 08' |
| 30 | 95.4 | 17° 27' | 81 | | 54° 06' |
| 31 | | 18° 04' | 82 | | 55° 05' |
| 32 | | 18° 40' | 83 | | 56° 06' |
| 33 | | 19° 16' | 84 | | 57° 08' |
| 34 | | 19° 53' | 85 | 52.7 | 58° 13' |
| 35 | 93.7 | 20° 29' | 86 | | 59° 19' |
| 36 | | 21° 06' | 87 | | 60° 28' |
| 37 | | 21° 43' | 88 | | 61° 39' |
| 38 | | 22° 20' | 89 | | 62° 52' |
| 39 | | 22° 57' | 90 | 43.6 | 64° 09' |
| 40 | 91.6 | 23° 35' | 91 | | 65° 30' |
| 41 | | 24° 12' | 92 | | 66° 56' |
| 42 | | 24° 50' | 93 | | 68° 26' |
| 43 | | 25° 28' | 94 | | 70° 03' |
| 44 | | 26° 06' | 95 | 31.2 | 71° 48' |
| 45 | 89.3 | 26° 45' | 96 | | 73° 44' |
| 46 | | 27° 23' | 97 | | 75° 56' |
| 47 | | 28° 02' | 98 | | 78° 31' |
| 48 | | 28° 41' | 99 | | 81° 54' |
| 49 | | 29° 20' | 100 | 00.0 | 90° 00' |
| 50 | 86.6 | 30° 00' | | | |



TRANSIT,

AS FIRST MADE IN 1831 BY THE INVENTOR, WILLIAM J. YOUNG, PHILADELPHIA, PA.

APPENDIX.

THE JUDICIAL FUNCTIONS OF SURVEYORS.*

WHEN a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his actions by facts which lead him away from the results which theoretically he ought to reach.

Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan, all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is, that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed, it is impossible to determine by the record, by the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats were

* By Chief Justice Cooley of the Supreme Court of Michigan.

more accurately surveyed, as indeed they should have been; for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear.

The fact probably is, that town surveys are quite as inaccurate as those made under authority of the general government. It is now upwards of fifty years since a major part of the public surveys, in what is now the State of Michigan, were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold and the work of improvement began. A generation has passed away since they were converted into cultivated farms, and few, if any, of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must have been relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush-fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing was substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary. If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determine the boundary line between the proprietors.

However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though

the effect be to make one half-quarter section 90 acres, and the one adjoining 70 ; for parties buy, or are supposed to buy, in reference to these monuments, and are entitled to what is within their lines, and no more, be it more or less. While the witness trees remain, there can generally be no difficulty in determining the locality of the stakes. When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men, whose theoretical education is thought to make them experts, who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and would have been if placed correctly. This is a serious mistake. The problem is now the same that it was before : To ascertain by the best lights of which the case admits where the original lines were. The mistake above referred to is supposed to have found expression in our legislation ; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed. An act passed in 1869 (Compiled Laws, 593), amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows :

“ *Second.* Extinct interior section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

“ *Third.* Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners ; in all other cases, at its proportionate distance between the nearest original corners on the same line.”

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near. To

estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct.

If the monument was set for an interior section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word "extinct" may be used in this connection: one, the sense of physical disappearance; the other, the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man loses his deed, he shall lose his land altogether. But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and Federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether State or Federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged. The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost, until after he has thoroughly sifted the evidence, and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary, when no other is attainable; and the surveyor should inquire when it originated, how and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or

may not have been trustworthy; but to bring these lines into discredit, when the people concerned do not question them, not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance, and perhaps discredit, since in a legal controversy the law, as well as common sense, must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. *Stewart v. Carleton*, 31 Mich. Reports, 270; *Diehl v. Zanger*, 39 Mich. Reports, 601. And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession must often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees and no monuments to govern, except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these without looking farther.

Now it may perhaps be known in a particular case that a certain monument still remaining was the starting-point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and used for many years, and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot-owners quarrel, and one of them calls in a surveyor that he may be sure that his neighbor

shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was or not, and the first result is, he notifies the lot-owners that there is error in the street line, and that all fences should be moved, say, one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot-owners will generally allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result? It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. *McNamara v. Seaton*, 82 Ill. Reports, 498; *Bunce v. Bidwell*, 43 Mich. Reports, 542. Public policy requires that such lines be not lightly disturbed or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go as an officer of the law is to express his opinion where the monument should be at the same time that he imparts the information

to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer, and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument for any considerable period as strong, if not conclusive, evidence of such settlement. The peace of the community absolutely requires this rule. *Foyce v. Williams*, 26 Mich. Reports, 332. It is not long since that in one of the leading cities of the State an attempt was made to move houses two or three rods into a street, on the ground that a survey, under which the street had been located for many years, had been found on a more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor, where boundaries are in dispute, must be varied by the circumstances. (1) He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties express or implied have rendered them unimportant. By monuments in the case of government surveys we mean, of course, the corner and quarter stakes; blazed lines or marked trees on the lines are not monuments; they are merely guides or finger-posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. (2) If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any State statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts, giving due prominence to the acts of parties concerned, and always keeping in mind, first, that neither his opinion nor his survey can be conclusive upon parties concerned; and, second, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and same rules that will govern theirs. On town plats if a surplus or

deficiency appears in a block when the actual boundaries are compared with the original figures, and there is no evidence to fix the exact location of the stakes which marked the division into lots, the rule of common sense and the law is that the surplus or deficiency is to be apportioned between the lots on an assumption that the error extended alike to all parts of the block. *O'Brien v. McGrane*, 29 Wis. Reports, 446 ; *Quinnin v. Reixers*, 46 Mich. Reports, 605.

It is always possible when corners are extinct that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so ; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course it is desirable that all such agreements be reduced to writing ; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines. The subject to which allusion will now be made is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless, it is often found that surveyors misapprehend them, or err in their application ; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that meander lines are boundary lines, and all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river ; and while some courts have held

that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the State while conceding to the owners of the bank all riparian rights. The practical difference is not very important. In this State the rule that the centre line is the boundary line is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation in some cases with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the centre line of the river. Of course the original survey supposes that each purchaser of land on the stream has a water-front of the length shown by the field notes ; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely ; and in some cases, if it were continued in the same direction to the centre line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation or other valuable use of the water, while the other might have a water-front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless, it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule. Extend the line of division between the two parcels from

the meander line to the centre line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water-front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water-lots which appear as such on town plats. *Bay City Gas Light Co. v. The Industrial Works*, 28 Mich. Reports, 182. It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out. When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties, and may constitute a sufficient guide when peculiar cases arise. Each riparian lot-owner ought to have a line on the legal boundary, namely, the centre line of the stream, proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream, when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the

original admeasurement. Wherever that was the case the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of government sale afterwards, for the reason that the rights of the bank owners are fixed by their purchase: when making that they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it cannot afterwards be taken from them. It is believed, however, that the Federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the Great Lakes.

On all the little lakes of the State, which are mere expansions near their mouths of the rivers passing through them, — such as the Muskegon, Pere Marquette, and Manistee, — the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. *Rice v. Ruddiman*, 10 Mich. 125. If such a lake were circular, the lines would converge to the centre; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for some considerable distance, in order to determine where one may lawfully moor vessels or rafts for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. *Lorman v. Benson*, 8 Mich. 18; *People's Ice Co. v. Steamer Excelsior*, recently decided.

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is, that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them. As regards the lakes and ponds of the State, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not: (1) To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them? (2) What public rights exist in them? (3) If there are islands in them which were not surveyed out and sold by the United States, can this be done now? Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the Great Lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the Great Lakes would belong to the United States until sold, and might be surveyed and measured at any time. The right to take fish in the lakes or to cut ice is public, like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore-owners; but, so far as these public rights can be the subject of ownership, they belong to the State, not the United States; and so, it is believed, does the bed of a lake also. *Pollard v. Hagan*, 3 Howard's U. S. Reports. But such rights are not generally considered proper subjects of sale, but, like the right to make use of the public highways, they are held by the State in trust for all the people. What is said of the large lakes may, perhaps, be said also of many of the interior lakes of the State; such, for example, as Houghton,

Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size, — say, even a mile across, — there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon, before. But the general rules for the extension of shore lines which have already been laid down should govern such cases, or at least should serve as guides in their settlement.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the States may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation. If the State should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as a trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the State could not drain, even for this purpose, against the will of the shore-owners, unless their rights were appropriated and paid for. Upon many questions that might arise between the State as owner of the

bed of a little lake and the shore-owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not, and cannot be, judicial officers, but in a great many cases they act in a *quasi* judicial capacity, with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

TABLES.

TABLE I.



THE

COMMON OR BRIGGS LOGARITHMS

OF THE

NATURAL NUMBERS

From 1 to 10000.



1—100

| N | log | N | log | N | log | N | log | N | log |
|----|----------|----|----------|----|----------|----|----------|-----|----------|
| 1 | 0.00 000 | 21 | 1.32 222 | 41 | 1.61 278 | 61 | 1.78 533 | 81 | 1.90 849 |
| 2 | 0.30 103 | 22 | 1.34 242 | 42 | 1.62 325 | 62 | 1.79 239 | 82 | 1.91 381 |
| 3 | 0.47 712 | 23 | 1.36 173 | 43 | 1.63 347 | 63 | 1.79 934 | 83 | 1.91 908 |
| 4 | 0.60 206 | 24 | 1.38 021 | 44 | 1.64 345 | 64 | 1.80 618 | 84 | 1.92 428 |
| 5 | 0.69 897 | 25 | 1.39 794 | 45 | 1.65 321 | 65 | 1.81 291 | 85 | 1.92 942 |
| 6 | 0.77 815 | 26 | 1.41 497 | 46 | 1.66 276 | 66 | 1.81 954 | 86 | 1.93 450 |
| 7 | 0.84 510 | 27 | 1.43 136 | 47 | 1.67 210 | 67 | 1.82 607 | 87 | 1.93 952 |
| 8 | 0.90 309 | 28 | 1.44 716 | 48 | 1.68 124 | 68 | 1.83 251 | 88 | 1.94 448 |
| 9 | 0.95 424 | 29 | 1.46 240 | 49 | 1.69 020 | 69 | 1.83 885 | 89 | 1.94 939 |
| 10 | 1.00 000 | 30 | 1.47 712 | 50 | 1.69 897 | 70 | 1.84 510 | 90 | 1.95 424 |
| 11 | 1.04 139 | 31 | 1.49 136 | 51 | 1.70 757 | 71 | 1.85 126 | 91 | 1.95 904 |
| 12 | 1.07 918 | 32 | 1.50 515 | 52 | 1.71 600 | 72 | 1.85 733 | 92 | 1.96 379 |
| 13 | 1.11 394 | 33 | 1.51 851 | 53 | 1.72 428 | 73 | 1.86 332 | 93 | 1.96 848 |
| 14 | 1.14 613 | 34 | 1.53 148 | 54 | 1.73 239 | 74 | 1.86 923 | 94 | 1.97 313 |
| 15 | 1.17 609 | 35 | 1.54 407 | 55 | 1.74 036 | 75 | 1.87 506 | 95 | 1.97 772 |
| 16 | 1.20 412 | 36 | 1.55 630 | 56 | 1.74 819 | 76 | 1.88 081 | 96 | 1.98 227 |
| 17 | 1.23 045 | 37 | 1.56 820 | 57 | 1.75 587 | 77 | 1.88 649 | 97 | 1.98 677 |
| 18 | 1.25 527 | 38 | 1.57 978 | 58 | 1.76 343 | 78 | 1.89 209 | 98 | 1.99 123 |
| 19 | 1.27 875 | 39 | 1.59 106 | 59 | 1.77 085 | 79 | 1.89 763 | 99 | 1.99 564 |
| 20 | 1.30 103 | 40 | 1.60 206 | 60 | 1.77 815 | 80 | 1.90 309 | 100 | 2.00 000 |
| N | log | N | log | N | log | N | log | N | log |

1—100

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 100 | 00 000 | 00 043 | 00 087 | 00 130 | 00 173 | 00 217 | 00 260 | 00 303 | 00 346 | 00 389 |
| 101 | 00 432 | 00 475 | 00 518 | 00 561 | 00 604 | 00 647 | 00 689 | 00 732 | 00 775 | 00 817 |
| 102 | 00 860 | 00 903 | 00 945 | 00 988 | 01 030 | 01 072 | 01 115 | 01 157 | 01 199 | 01 241 |
| 103 | 01 284 | 01 326 | 01 368 | 01 410 | 01 452 | 01 494 | 01 536 | 01 578 | 01 620 | 01 662 |
| 104 | 01 703 | 01 745 | 01 787 | 01 828 | 01 870 | 01 912 | 01 953 | 01 995 | 02 036 | 02 077 |
| 105 | 02 119 | 02 160 | 02 202 | 02 243 | 02 284 | 02 325 | 02 366 | 02 407 | 02 449 | 02 490 |
| 106 | 02 531 | 02 572 | 02 612 | 02 653 | 02 694 | 02 735 | 02 776 | 02 816 | 02 857 | 02 898 |
| 107 | 02 938 | 02 979 | 03 019 | 03 060 | 03 100 | 03 141 | 03 181 | 03 222 | 03 262 | 03 302 |
| 108 | 03 342 | 03 383 | 03 423 | 03 463 | 03 503 | 03 543 | 03 583 | 03 623 | 03 663 | 03 703 |
| 109 | 03 743 | 03 782 | 03 822 | 03 862 | 03 902 | 03 941 | 03 981 | 04 021 | 04 060 | 04 100 |
| 110 | 04 139 | 04 179 | 04 218 | 04 258 | 04 297 | 04 336 | 04 376 | 04 415 | 04 454 | 04 495 |
| 111 | 04 532 | 04 571 | 04 610 | 04 650 | 04 689 | 04 727 | 04 766 | 04 805 | 04 844 | 04 883 |
| 112 | 04 922 | 04 961 | 04 999 | 05 038 | 05 077 | 05 115 | 05 154 | 05 192 | 05 231 | 05 269 |
| 113 | 05 308 | 05 346 | 05 385 | 05 423 | 05 461 | 05 500 | 05 538 | 05 576 | 05 614 | 05 652 |
| 114 | 05 690 | 05 729 | 05 767 | 05 805 | 05 843 | 05 881 | 05 918 | 05 956 | 05 994 | 06 032 |
| 115 | 06 070 | 06 108 | 06 145 | 06 183 | 06 221 | 06 258 | 06 296 | 06 333 | 06 371 | 06 408 |
| 116 | 06 446 | 06 483 | 06 521 | 06 558 | 06 595 | 06 633 | 06 670 | 06 707 | 06 744 | 06 781 |
| 117 | 06 819 | 06 856 | 06 893 | 06 930 | 06 967 | 07 004 | 07 041 | 07 078 | 07 115 | 07 151 |
| 118 | 07 188 | 07 225 | 07 262 | 07 298 | 07 335 | 07 372 | 07 408 | 07 445 | 07 482 | 07 518 |
| 119 | 07 555 | 07 591 | 07 628 | 07 664 | 07 700 | 07 737 | 07 773 | 07 809 | 07 846 | 07 882 |
| 120 | 07 918 | 07 954 | 07 990 | 08 027 | 08 063 | 08 099 | 08 135 | 08 171 | 08 207 | 08 243 |
| 121 | 08 279 | 08 314 | 08 350 | 08 386 | 08 422 | 08 458 | 08 493 | 08 529 | 08 565 | 08 600 |
| 122 | 08 636 | 08 672 | 08 707 | 08 743 | 08 778 | 08 814 | 08 849 | 08 884 | 08 920 | 08 955 |
| 123 | 08 991 | 09 026 | 09 061 | 09 096 | 09 132 | 09 167 | 09 202 | 09 237 | 09 272 | 09 307 |
| 124 | 09 342 | 09 377 | 09 412 | 09 447 | 09 482 | 09 517 | 09 552 | 09 587 | 09 621 | 09 656 |
| 125 | 09 691 | 09 726 | 09 760 | 09 795 | 09 830 | 09 864 | 09 899 | 09 934 | 09 968 | 10 003 |
| 126 | 10 037 | 10 072 | 10 106 | 10 140 | 10 175 | 10 209 | 10 243 | 10 278 | 10 312 | 10 346 |
| 127 | 10 380 | 10 415 | 10 449 | 10 483 | 10 517 | 10 551 | 10 585 | 10 619 | 10 653 | 10 687 |
| 128 | 10 721 | 10 755 | 10 789 | 10 823 | 10 857 | 10 890 | 10 924 | 10 958 | 10 992 | 11 025 |
| 129 | 11 059 | 11 093 | 11 126 | 11 160 | 11 193 | 11 227 | 11 261 | 11 294 | 11 327 | 11 361 |
| 130 | 11 394 | 11 428 | 11 461 | 11 494 | 11 528 | 11 561 | 11 594 | 11 628 | 11 661 | 11 694 |
| 131 | 11 727 | 11 760 | 11 793 | 11 826 | 11 860 | 11 893 | 11 926 | 11 959 | 11 992 | 12 024 |
| 132 | 12 057 | 12 090 | 12 123 | 12 156 | 12 189 | 12 222 | 12 254 | 12 287 | 12 320 | 12 352 |
| 133 | 12 385 | 12 418 | 12 450 | 12 483 | 12 516 | 12 548 | 12 581 | 12 613 | 12 646 | 12 678 |
| 134 | 12 710 | 12 743 | 12 775 | 12 808 | 12 840 | 12 872 | 12 905 | 12 937 | 12 969 | 13 001 |
| 135 | 13 033 | 13 066 | 13 098 | 13 130 | 13 162 | 13 194 | 13 226 | 13 258 | 13 290 | 13 322 |
| 136 | 13 354 | 13 386 | 13 418 | 13 450 | 13 481 | 13 513 | 13 545 | 13 577 | 13 609 | 13 640 |
| 137 | 13 672 | 13 704 | 13 735 | 13 767 | 13 799 | 13 830 | 13 862 | 13 893 | 13 925 | 13 956 |
| 138 | 13 988 | 14 019 | 14 051 | 14 082 | 14 114 | 14 145 | 14 176 | 14 208 | 14 239 | 14 270 |
| 139 | 14 301 | 14 333 | 14 364 | 14 395 | 14 426 | 14 457 | 14 489 | 14 520 | 14 551 | 14 582 |
| 140 | 14 613 | 14 644 | 14 675 | 14 706 | 14 737 | 14 768 | 14 799 | 14 829 | 14 860 | 14 891 |
| 141 | 14 922 | 14 953 | 14 983 | 15 014 | 15 045 | 15 076 | 15 106 | 15 137 | 15 168 | 15 198 |
| 142 | 15 229 | 15 259 | 15 290 | 15 320 | 15 351 | 15 381 | 15 412 | 15 442 | 15 473 | 15 503 |
| 143 | 15 534 | 15 564 | 15 594 | 15 625 | 15 655 | 15 685 | 15 715 | 15 746 | 15 776 | 15 806 |
| 144 | 15 836 | 15 866 | 15 897 | 15 927 | 15 957 | 15 987 | 16 017 | 16 047 | 16 077 | 16 107 |
| 145 | 16 137 | 16 167 | 16 197 | 16 227 | 16 256 | 16 286 | 16 316 | 16 346 | 16 376 | 16 406 |
| 146 | 16 435 | 16 465 | 16 495 | 16 524 | 16 554 | 16 584 | 16 613 | 16 643 | 16 673 | 16 702 |
| 147 | 16 732 | 16 761 | 16 791 | 16 820 | 16 850 | 16 879 | 16 909 | 16 938 | 16 967 | 16 997 |
| 148 | 17 026 | 17 056 | 17 085 | 17 114 | 17 143 | 17 173 | 17 202 | 17 231 | 17 260 | 17 289 |
| 149 | 17 319 | 17 348 | 17 377 | 17 406 | 17 435 | 17 464 | 17 493 | 17 522 | 17 551 | 17 580 |
| 150 | 17 609 | 17 638 | 17 667 | 17 696 | 17 725 | 17 754 | 17 782 | 17 811 | 17 840 | 17 869 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 150 | 17 609 | 17 638 | 17 667 | 17 696 | 17 725 | 17 754 | 17 782 | 17 811 | 17 840 | 17 869 |
| 151 | 17 898 | 17 926 | 17 955 | 17 984 | 18 013 | 18 041 | 18 070 | 18 099 | 18 127 | 18 156 |
| 152 | 18 184 | 18 213 | 18 241 | 18 270 | 18 298 | 18 327 | 18 355 | 18 384 | 18 412 | 18 441 |
| 153 | 18 469 | 18 498 | 18 526 | 18 554 | 18 583 | 18 611 | 18 639 | 18 667 | 18 696 | 18 724 |
| 154 | 18 752 | 18 780 | 18 808 | 18 837 | 18 865 | 18 893 | 18 921 | 18 949 | 18 977 | 19 005 |
| 155 | 19 033 | 19 061 | 19 089 | 19 117 | 19 145 | 19 173 | 19 201 | 19 229 | 19 257 | 19 285 |
| 156 | 19 312 | 19 340 | 19 368 | 19 396 | 19 424 | 19 451 | 19 479 | 19 507 | 19 535 | 19 562 |
| 157 | 19 590 | 19 618 | 19 645 | 19 673 | 19 700 | 19 728 | 19 756 | 19 783 | 19 811 | 19 838 |
| 158 | 19 866 | 19 893 | 19 921 | 19 948 | 19 976 | 20 003 | 20 030 | 20 058 | 20 085 | 20 112 |
| 159 | 20 140 | 20 167 | 20 194 | 20 222 | 20 249 | 20 276 | 20 303 | 20 330 | 20 358 | 20 385 |
| 160 | 20 412 | 20 439 | 20 466 | 20 493 | 20 520 | 20 548 | 20 575 | 20 602 | 20 629 | 20 656 |
| 161 | 20 683 | 20 710 | 20 737 | 20 763 | 20 790 | 20 817 | 20 844 | 20 871 | 20 898 | 20 925 |
| 162 | 20 952 | 20 978 | 21 005 | 21 032 | 21 059 | 21 085 | 21 112 | 21 139 | 21 165 | 21 192 |
| 163 | 21 219 | 21 245 | 21 272 | 21 299 | 21 325 | 21 352 | 21 378 | 21 405 | 21 431 | 21 458 |
| 164 | 21 484 | 21 511 | 21 537 | 21 564 | 21 590 | 21 617 | 21 643 | 21 669 | 21 696 | 21 722 |
| 165 | 21 748 | 21 775 | 21 801 | 21 827 | 21 854 | 21 880 | 21 906 | 21 932 | 21 958 | 21 985 |
| 166 | 22 011 | 22 037 | 22 063 | 22 089 | 22 115 | 22 141 | 22 167 | 22 194 | 22 220 | 22 246 |
| 167 | 22 272 | 22 298 | 22 324 | 22 350 | 22 376 | 22 401 | 22 427 | 22 453 | 22 479 | 22 505 |
| 168 | 22 531 | 22 557 | 22 583 | 22 608 | 22 634 | 22 660 | 22 686 | 22 712 | 22 737 | 22 763 |
| 169 | 22 789 | 22 814 | 22 840 | 22 866 | 22 891 | 22 917 | 22 943 | 22 968 | 22 994 | 23 019 |
| 170 | 23 045 | 23 070 | 23 096 | 23 121 | 23 147 | 23 172 | 23 198 | 23 223 | 23 249 | 23 274 |
| 171 | 23 300 | 23 325 | 23 350 | 23 376 | 23 401 | 23 426 | 23 452 | 23 477 | 23 502 | 23 528 |
| 172 | 23 553 | 23 578 | 23 603 | 23 629 | 23 654 | 23 679 | 23 704 | 23 729 | 23 754 | 23 779 |
| 173 | 23 805 | 23 830 | 23 855 | 23 880 | 23 905 | 23 930 | 23 955 | 23 980 | 24 005 | 24 030 |
| 174 | 24 055 | 24 080 | 24 105 | 24 130 | 24 155 | 24 180 | 24 204 | 24 229 | 24 254 | 24 279 |
| 175 | 24 304 | 24 329 | 24 353 | 24 378 | 24 403 | 24 428 | 24 452 | 24 477 | 24 502 | 24 527 |
| 176 | 24 551 | 24 576 | 24 601 | 24 625 | 24 650 | 24 674 | 24 699 | 24 724 | 24 748 | 24 773 |
| 177 | 24 797 | 24 822 | 24 846 | 24 871 | 24 895 | 24 920 | 24 944 | 24 969 | 24 993 | 25 018 |
| 178 | 25 042 | 25 066 | 25 091 | 25 115 | 25 139 | 25 164 | 25 188 | 25 212 | 25 237 | 25 261 |
| 179 | 25 285 | 25 310 | 25 334 | 25 358 | 25 382 | 25 406 | 25 431 | 25 455 | 25 479 | 25 503 |
| 180 | 25 527 | 25 551 | 25 575 | 25 600 | 25 624 | 25 648 | 25 672 | 25 696 | 25 720 | 25 744 |
| 181 | 25 768 | 25 792 | 25 816 | 25 840 | 25 864 | 25 888 | 25 912 | 25 935 | 25 959 | 25 983 |
| 182 | 26 007 | 26 031 | 26 055 | 26 079 | 26 102 | 26 126 | 26 150 | 26 174 | 26 198 | 26 221 |
| 183 | 26 245 | 26 269 | 26 293 | 26 316 | 26 340 | 26 364 | 26 387 | 26 411 | 26 435 | 26 458 |
| 184 | 26 482 | 26 505 | 26 529 | 26 553 | 26 576 | 26 600 | 26 623 | 26 647 | 26 670 | 26 694 |
| 185 | 26 717 | 26 741 | 26 764 | 26 788 | 26 811 | 26 834 | 26 858 | 26 881 | 26 905 | 26 928 |
| 186 | 26 951 | 26 975 | 26 998 | 27 021 | 27 045 | 27 068 | 27 091 | 27 114 | 27 138 | 27 161 |
| 187 | 27 184 | 27 207 | 27 231 | 27 254 | 27 277 | 27 300 | 27 323 | 27 346 | 27 370 | 27 393 |
| 188 | 27 416 | 27 439 | 27 462 | 27 485 | 27 508 | 27 531 | 27 554 | 27 577 | 27 600 | 27 623 |
| 189 | 27 646 | 27 669 | 27 692 | 27 715 | 27 738 | 27 761 | 27 784 | 27 807 | 27 830 | 27 852 |
| 190 | 27 875 | 27 898 | 27 921 | 27 944 | 27 967 | 27 989 | 28 012 | 28 035 | 28 058 | 28 081 |
| 191 | 28 103 | 28 126 | 28 149 | 28 171 | 28 194 | 28 217 | 28 240 | 28 262 | 28 285 | 28 307 |
| 192 | 28 330 | 28 353 | 28 375 | 28 398 | 28 421 | 28 443 | 28 466 | 28 488 | 28 511 | 28 533 |
| 193 | 28 556 | 28 578 | 28 601 | 28 623 | 28 646 | 28 668 | 28 691 | 28 713 | 28 735 | 28 758 |
| 194 | 28 780 | 28 803 | 28 825 | 28 847 | 28 870 | 28 892 | 28 914 | 28 937 | 28 959 | 28 981 |
| 195 | 29 003 | 29 026 | 29 048 | 29 070 | 29 092 | 29 115 | 29 137 | 29 159 | 29 181 | 29 203 |
| 196 | 29 226 | 29 248 | 29 270 | 29 292 | 29 314 | 29 336 | 29 358 | 29 380 | 29 403 | 29 425 |
| 197 | 29 447 | 29 469 | 29 491 | 29 513 | 29 535 | 29 557 | 29 579 | 29 601 | 29 623 | 29 645 |
| 198 | 29 667 | 29 688 | 29 710 | 29 732 | 29 754 | 29 776 | 29 798 | 29 820 | 29 842 | 29 863 |
| 199 | 29 885 | 29 907 | 29 929 | 29 951 | 29 973 | 29 994 | 30 016 | 30 038 | 30 060 | 30 081 |
| 200 | 30 103 | 30 125 | 30 146 | 30 168 | 30 190 | 30 211 | 30 233 | 30 255 | 30 276 | 30 298 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 200 | 30 103 | 30 125 | 30 146 | 30 168 | 30 190 | 30 211 | 30 233 | 30 255 | 30 276 | 30 298 |
| 201 | 30 320 | 30 341 | 30 363 | 30 384 | 30 406 | 30 428 | 30 449 | 30 471 | 30 492 | 30 514 |
| 202 | 30 535 | 30 557 | 30 578 | 30 600 | 30 621 | 30 643 | 30 664 | 30 685 | 30 707 | 30 728 |
| 203 | 30 750 | 30 771 | 30 792 | 30 814 | 30 835 | 30 856 | 30 878 | 30 899 | 30 920 | 30 942 |
| 204 | 30 963 | 30 984 | 31 006 | 31 027 | 31 048 | 31 069 | 31 091 | 31 112 | 31 133 | 31 154 |
| 205 | 31 175 | 31 197 | 31 218 | 31 239 | 31 260 | 31 281 | 31 302 | 31 323 | 31 345 | 31 366 |
| 206 | 31 387 | 31 408 | 31 429 | 31 450 | 31 471 | 31 492 | 31 513 | 31 534 | 31 555 | 31 576 |
| 207 | 31 597 | 31 618 | 31 639 | 31 660 | 31 681 | 31 702 | 31 723 | 31 744 | 31 765 | 31 785 |
| 208 | 31 806 | 31 827 | 31 848 | 31 869 | 31 890 | 31 911 | 31 931 | 31 952 | 31 973 | 31 994 |
| 209 | 32 015 | 32 035 | 32 056 | 32 077 | 32 098 | 32 118 | 32 139 | 32 160 | 32 181 | 32 201 |
| 210 | 32 222 | 32 243 | 32 263 | 32 284 | 32 305 | 32 325 | 32 346 | 32 366 | 32 387 | 32 408 |
| 211 | 32 428 | 32 449 | 32 469 | 32 490 | 32 510 | 32 531 | 32 552 | 32 572 | 32 593 | 32 613 |
| 212 | 32 634 | 32 654 | 32 675 | 32 695 | 32 715 | 32 736 | 32 756 | 32 777 | 32 797 | 32 818 |
| 213 | 32 838 | 32 858 | 32 879 | 32 899 | 32 919 | 32 940 | 32 960 | 32 980 | 33 001 | 33 021 |
| 214 | 33 041 | 33 062 | 33 082 | 33 102 | 33 122 | 33 143 | 33 163 | 33 183 | 33 203 | 33 224 |
| 215 | 33 244 | 33 264 | 33 284 | 33 304 | 33 325 | 33 345 | 33 365 | 33 385 | 33 405 | 33 425 |
| 216 | 33 445 | 33 465 | 33 486 | 33 506 | 33 526 | 33 546 | 33 566 | 33 586 | 33 606 | 33 626 |
| 217 | 33 646 | 33 666 | 33 686 | 33 706 | 33 726 | 33 746 | 33 766 | 33 786 | 33 806 | 33 826 |
| 218 | 33 846 | 33 866 | 33 885 | 33 905 | 33 925 | 33 945 | 33 965 | 33 985 | 34 005 | 34 025 |
| 219 | 34 044 | 34 064 | 34 084 | 34 104 | 34 124 | 34 143 | 34 163 | 34 183 | 34 203 | 34 223 |
| 220 | 34 242 | 34 262 | 34 282 | 34 301 | 34 321 | 34 341 | 34 361 | 34 380 | 34 400 | 34 420 |
| 221 | 34 439 | 34 459 | 34 479 | 34 498 | 34 518 | 34 537 | 34 557 | 34 577 | 34 596 | 34 616 |
| 222 | 34 635 | 34 655 | 34 674 | 34 694 | 34 713 | 34 733 | 34 753 | 34 772 | 34 792 | 34 811 |
| 223 | 34 830 | 34 850 | 34 869 | 34 889 | 34 908 | 34 928 | 34 947 | 34 967 | 34 986 | 35 005 |
| 224 | 35 025 | 35 044 | 35 064 | 35 083 | 35 102 | 35 122 | 35 141 | 35 160 | 35 180 | 35 199 |
| 225 | 35 218 | 35 238 | 35 257 | 35 276 | 35 295 | 35 315 | 35 334 | 35 353 | 35 372 | 35 392 |
| 226 | 35 411 | 35 430 | 35 449 | 35 468 | 35 488 | 35 507 | 35 526 | 35 545 | 35 564 | 35 583 |
| 227 | 35 603 | 35 622 | 35 641 | 35 660 | 35 679 | 35 698 | 35 717 | 35 736 | 35 755 | 35 774 |
| 228 | 35 793 | 35 813 | 35 832 | 35 851 | 35 870 | 35 889 | 35 908 | 35 927 | 35 946 | 35 965 |
| 229 | 35 984 | 36 003 | 36 021 | 36 040 | 36 059 | 36 078 | 36 097 | 36 116 | 36 135 | 36 154 |
| 230 | 36 173 | 36 192 | 36 211 | 36 229 | 36 248 | 36 267 | 36 286 | 36 305 | 36 324 | 36 342 |
| 231 | 36 361 | 36 380 | 36 399 | 36 418 | 36 436 | 36 455 | 36 474 | 36 493 | 36 511 | 36 530 |
| 232 | 36 549 | 36 568 | 36 586 | 36 605 | 36 624 | 36 642 | 36 661 | 36 680 | 36 698 | 36 717 |
| 233 | 36 736 | 36 754 | 36 773 | 36 791 | 36 810 | 36 829 | 36 847 | 36 866 | 36 884 | 36 903 |
| 234 | 36 922 | 36 940 | 36 959 | 36 977 | 36 996 | 37 014 | 37 033 | 37 051 | 37 070 | 37 088 |
| 235 | 37 107 | 37 125 | 37 144 | 37 162 | 37 181 | 37 199 | 37 218 | 37 236 | 37 254 | 37 273 |
| 236 | 37 291 | 37 310 | 37 328 | 37 346 | 37 365 | 37 383 | 37 401 | 37 420 | 37 438 | 37 457 |
| 237 | 37 475 | 37 493 | 37 511 | 37 530 | 37 548 | 37 566 | 37 585 | 37 603 | 37 621 | 37 639 |
| 238 | 37 658 | 37 676 | 37 694 | 37 712 | 37 731 | 37 749 | 37 767 | 37 785 | 37 803 | 37 822 |
| 239 | 37 840 | 37 858 | 37 876 | 37 894 | 37 912 | 37 931 | 37 949 | 37 967 | 37 985 | 38 003 |
| 240 | 38 021 | 38 039 | 38 057 | 38 075 | 38 093 | 38 112 | 38 130 | 38 148 | 38 166 | 38 184 |
| 241 | 38 202 | 38 220 | 38 238 | 38 256 | 38 274 | 38 292 | 38 310 | 38 328 | 38 346 | 38 364 |
| 242 | 38 382 | 38 399 | 38 417 | 38 435 | 38 453 | 38 471 | 38 489 | 38 507 | 38 525 | 38 543 |
| 243 | 38 561 | 38 578 | 38 596 | 38 614 | 38 632 | 38 650 | 38 668 | 38 686 | 38 703 | 38 721 |
| 244 | 38 739 | 38 757 | 38 775 | 38 792 | 38 810 | 38 828 | 38 846 | 38 863 | 38 881 | 38 899 |
| 245 | 38 917 | 38 934 | 38 952 | 38 970 | 38 987 | 39 005 | 39 023 | 39 041 | 39 058 | 39 076 |
| 246 | 39 094 | 39 111 | 39 129 | 39 146 | 39 164 | 39 182 | 39 199 | 39 217 | 39 235 | 39 252 |
| 247 | 39 270 | 39 287 | 39 305 | 39 322 | 39 340 | 39 358 | 39 375 | 39 393 | 39 410 | 39 428 |
| 248 | 39 445 | 39 463 | 39 480 | 39 498 | 39 515 | 39 533 | 39 550 | 39 568 | 39 585 | 39 602 |
| 249 | 39 620 | 39 637 | 39 655 | 39 672 | 39 690 | 39 707 | 39 724 | 39 742 | 39 759 | 39 777 |
| 250 | 39 794 | 39 811 | 39 829 | 39 846 | 39 863 | 39 881 | 39 898 | 39 915 | 39 933 | 39 950 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 250 | 39 794 | 39 811 | 39 829 | 39 846 | 39 863 | 39 881 | 39 898 | 39 915 | 39 933 | 39 950 |
| 251 | 39 967 | 39 985 | 40 002 | 40 019 | 40 037 | 40 054 | 40 071 | 40 088 | 40 106 | 40 123 |
| 252 | 40 140 | 40 157 | 40 175 | 40 192 | 40 209 | 40 226 | 40 243 | 40 261 | 40 278 | 40 295 |
| 253 | 40 312 | 40 329 | 40 346 | 40 364 | 40 381 | 40 398 | 40 415 | 40 432 | 40 449 | 40 466 |
| 254 | 40 483 | 40 500 | 40 518 | 40 535 | 40 552 | 40 569 | 40 586 | 40 603 | 40 620 | 40 637 |
| 255 | 40 654 | 40 671 | 40 688 | 40 705 | 40 722 | 40 739 | 40 756 | 40 773 | 40 790 | 40 807 |
| 256 | 40 824 | 40 841 | 40 858 | 40 875 | 40 892 | 40 909 | 40 926 | 40 943 | 40 960 | 40 976 |
| 257 | 40 993 | 41 010 | 41 027 | 41 044 | 41 061 | 41 078 | 41 095 | 41 111 | 41 128 | 41 145 |
| 258 | 41 162 | 41 179 | 41 196 | 41 212 | 41 229 | 41 246 | 41 263 | 41 280 | 41 296 | 41 313 |
| 259 | 41 330 | 41 347 | 41 363 | 41 380 | 41 397 | 41 414 | 41 430 | 41 447 | 41 464 | 41 481 |
| 260 | 41 497 | 41 514 | 41 531 | 41 547 | 41 564 | 41 581 | 41 597 | 41 614 | 41 631 | 41 647 |
| 261 | 41 664 | 41 681 | 41 697 | 41 714 | 41 731 | 41 747 | 41 764 | 41 780 | 41 797 | 41 814 |
| 262 | 41 830 | 41 847 | 41 863 | 41 880 | 41 896 | 41 913 | 41 929 | 41 946 | 41 963 | 41 979 |
| 263 | 41 996 | 42 012 | 42 029 | 42 045 | 42 062 | 42 078 | 42 095 | 42 111 | 42 127 | 42 144 |
| 264 | 42 160 | 42 177 | 42 193 | 42 210 | 42 226 | 42 243 | 42 259 | 42 275 | 42 292 | 42 308 |
| 265 | 42 325 | 42 341 | 42 357 | 42 374 | 42 390 | 42 406 | 42 423 | 42 439 | 42 455 | 42 472 |
| 266 | 42 488 | 42 504 | 42 521 | 42 537 | 42 553 | 42 570 | 42 586 | 42 602 | 42 619 | 42 635 |
| 267 | 42 651 | 42 667 | 42 684 | 42 700 | 42 716 | 42 732 | 42 749 | 42 765 | 42 781 | 42 797 |
| 268 | 42 813 | 42 830 | 42 846 | 42 862 | 42 878 | 42 894 | 42 911 | 42 927 | 42 943 | 42 959 |
| 269 | 42 975 | 42 991 | 43 008 | 43 024 | 43 040 | 43 056 | 43 072 | 43 088 | 43 104 | 43 120 |
| 270 | 43 136 | 43 152 | 43 169 | 43 185 | 43 201 | 43 217 | 43 233 | 43 249 | 43 265 | 43 281 |
| 271 | 43 297 | 43 313 | 43 329 | 43 345 | 43 361 | 43 377 | 43 393 | 43 409 | 43 425 | 43 441 |
| 272 | 43 457 | 43 473 | 43 489 | 43 505 | 43 521 | 43 537 | 43 553 | 43 569 | 43 584 | 43 600 |
| 273 | 43 616 | 43 632 | 43 648 | 43 664 | 43 680 | 43 696 | 43 712 | 43 727 | 43 743 | 43 759 |
| 274 | 43 775 | 43 791 | 43 807 | 43 823 | 43 838 | 43 854 | 43 870 | 43 886 | 43 902 | 43 917 |
| 275 | 43 933 | 43 949 | 43 965 | 43 981 | 43 996 | 44 012 | 44 028 | 44 044 | 44 059 | 44 075 |
| 276 | 44 091 | 44 107 | 44 122 | 44 138 | 44 154 | 44 170 | 44 185 | 44 201 | 44 217 | 44 232 |
| 277 | 44 248 | 44 264 | 44 279 | 44 295 | 44 311 | 44 326 | 44 342 | 44 358 | 44 373 | 44 389 |
| 278 | 44 404 | 44 420 | 44 436 | 44 451 | 44 467 | 44 483 | 44 498 | 44 514 | 44 529 | 44 545 |
| 279 | 44 560 | 44 576 | 44 592 | 44 607 | 44 623 | 44 638 | 44 654 | 44 669 | 44 685 | 44 700 |
| 280 | 44 716 | 44 731 | 44 747 | 44 762 | 44 778 | 44 793 | 44 809 | 44 824 | 44 840 | 44 855 |
| 281 | 44 871 | 44 886 | 44 902 | 44 917 | 44 932 | 44 948 | 44 963 | 44 979 | 44 994 | 45 010 |
| 282 | 45 025 | 45 040 | 45 056 | 45 071 | 45 086 | 45 102 | 45 117 | 45 133 | 45 148 | 45 163 |
| 283 | 45 179 | 45 194 | 45 209 | 45 225 | 45 240 | 45 255 | 45 271 | 45 286 | 45 301 | 45 317 |
| 284 | 45 332 | 45 347 | 45 362 | 45 378 | 45 393 | 45 408 | 45 423 | 45 439 | 45 454 | 45 469 |
| 285 | 45 484 | 45 500 | 45 515 | 45 530 | 45 545 | 45 561 | 45 576 | 45 591 | 45 606 | 45 621 |
| 286 | 45 637 | 45 652 | 45 667 | 45 682 | 45 697 | 45 712 | 45 728 | 45 743 | 45 758 | 45 773 |
| 287 | 45 788 | 45 803 | 45 818 | 45 834 | 45 849 | 45 864 | 45 879 | 45 894 | 45 909 | 45 924 |
| 288 | 45 939 | 45 954 | 45 969 | 45 984 | 46 000 | 46 015 | 46 030 | 46 045 | 46 060 | 46 075 |
| 289 | 46 090 | 46 105 | 46 120 | 46 135 | 46 150 | 46 165 | 46 180 | 46 195 | 46 210 | 46 225 |
| 290 | 46 240 | 46 255 | 46 270 | 46 285 | 46 300 | 46 315 | 46 330 | 46 345 | 46 359 | 46 374 |
| 291 | 46 389 | 46 404 | 46 419 | 46 434 | 46 449 | 46 464 | 46 479 | 46 494 | 46 509 | 46 523 |
| 292 | 46 538 | 46 553 | 46 568 | 46 583 | 46 598 | 46 613 | 46 627 | 46 642 | 46 657 | 46 672 |
| 293 | 46 687 | 46 702 | 46 716 | 46 731 | 46 746 | 46 761 | 46 776 | 46 790 | 46 805 | 46 820 |
| 294 | 46 835 | 46 850 | 46 864 | 46 879 | 46 894 | 46 909 | 46 923 | 46 938 | 46 953 | 46 967 |
| 295 | 46 982 | 46 997 | 47 012 | 47 026 | 47 041 | 47 056 | 47 070 | 47 085 | 47 100 | 47 114 |
| 296 | 47 129 | 47 144 | 47 159 | 47 173 | 47 188 | 47 202 | 47 217 | 47 232 | 47 246 | 47 261 |
| 297 | 47 276 | 47 290 | 47 305 | 47 319 | 47 334 | 47 349 | 47 363 | 47 378 | 47 392 | 47 407 |
| 298 | 47 422 | 47 436 | 47 451 | 47 465 | 47 480 | 47 494 | 47 509 | 47 524 | 47 538 | 47 553 |
| 299 | 47 567 | 47 582 | 47 596 | 47 611 | 47 625 | 47 640 | 47 654 | 47 669 | 47 683 | 47 698 |
| 300 | 47 712 | 47 727 | 47 741 | 47 756 | 47 770 | 47 784 | 47 799 | 47 813 | 47 828 | 47 842 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 300 | 47 712 | 47 727 | 47 741 | 47 756 | 47 770 | 47 784 | 47 799 | 47 813 | 47 828 | 47 842 |
| 301 | 47 857 | 47 871 | 47 885 | 47 900 | 47 914 | 47 929 | 47 943 | 47 958 | 47 972 | 47 986 |
| 302 | 48 001 | 48 015 | 48 029 | 48 044 | 48 058 | 48 073 | 48 087 | 48 101 | 48 116 | 48 130 |
| 303 | 48 144 | 48 159 | 48 173 | 48 187 | 48 202 | 48 216 | 48 230 | 48 244 | 48 259 | 48 273 |
| 304 | 48 287 | 48 302 | 48 316 | 48 330 | 48 344 | 48 359 | 48 373 | 48 387 | 48 401 | 48 416 |
| 305 | 48 430 | 48 444 | 48 458 | 48 473 | 48 487 | 48 501 | 48 515 | 48 530 | 48 544 | 48 558 |
| 306 | 48 572 | 48 586 | 48 601 | 48 615 | 48 629 | 48 643 | 48 657 | 48 671 | 48 686 | 48 700 |
| 307 | 48 714 | 48 728 | 48 742 | 48 756 | 48 770 | 48 785 | 48 799 | 48 813 | 48 827 | 48 841 |
| 308 | 48 855 | 48 869 | 48 883 | 48 897 | 48 911 | 48 926 | 48 940 | 48 954 | 48 968 | 48 982 |
| 309 | 48 996 | 49 010 | 49 024 | 49 038 | 49 052 | 49 066 | 49 080 | 49 094 | 49 108 | 49 122 |
| 310 | 49 136 | 49 150 | 49 164 | 49 178 | 49 192 | 49 206 | 49 220 | 49 234 | 49 248 | 49 262 |
| 311 | 49 276 | 49 290 | 49 304 | 49 318 | 49 332 | 49 346 | 49 360 | 49 374 | 49 388 | 49 402 |
| 312 | 49 415 | 49 429 | 49 443 | 49 457 | 49 471 | 49 485 | 49 499 | 49 513 | 49 527 | 49 541 |
| 313 | 49 554 | 49 568 | 49 582 | 49 596 | 49 610 | 49 624 | 49 638 | 49 651 | 49 665 | 49 679 |
| 314 | 49 693 | 49 707 | 49 721 | 49 734 | 49 748 | 49 762 | 49 776 | 49 790 | 49 803 | 49 817 |
| 315 | 49 831 | 49 845 | 49 859 | 49 872 | 49 886 | 49 900 | 49 914 | 49 927 | 49 941 | 49 955 |
| 316 | 49 969 | 49 982 | 49 996 | 50 010 | 50 024 | 50 037 | 50 051 | 50 065 | 50 079 | 50 092 |
| 317 | 50 106 | 50 120 | 50 133 | 50 147 | 50 161 | 50 174 | 50 188 | 50 202 | 50 215 | 50 229 |
| 318 | 50 243 | 50 256 | 50 270 | 50 284 | 50 297 | 50 311 | 50 325 | 50 338 | 50 352 | 50 365 |
| 319 | 50 379 | 50 393 | 50 406 | 50 420 | 50 433 | 50 447 | 50 461 | 50 474 | 50 488 | 50 501 |
| 320 | 50 515 | 50 529 | 50 542 | 50 556 | 50 569 | 50 583 | 50 596 | 50 610 | 50 623 | 50 637 |
| 321 | 50 651 | 50 664 | 50 678 | 50 691 | 50 705 | 50 718 | 50 732 | 50 745 | 50 759 | 50 772 |
| 322 | 50 786 | 50 799 | 50 813 | 50 826 | 50 840 | 50 853 | 50 866 | 50 880 | 50 893 | 50 907 |
| 323 | 50 920 | 50 934 | 50 947 | 50 961 | 50 974 | 50 987 | 51 001 | 51 014 | 51 028 | 51 041 |
| 324 | 51 055 | 51 068 | 51 081 | 51 095 | 51 108 | 51 121 | 51 135 | 51 148 | 51 162 | 51 175 |
| 325 | 51 188 | 51 202 | 51 215 | 51 228 | 51 242 | 51 255 | 51 268 | 51 282 | 51 295 | 51 308 |
| 326 | 51 322 | 51 335 | 51 348 | 51 362 | 51 375 | 51 388 | 51 402 | 51 415 | 51 428 | 51 441 |
| 327 | 51 455 | 51 468 | 51 481 | 51 495 | 51 508 | 51 521 | 51 534 | 51 548 | 51 561 | 51 574 |
| 328 | 51 587 | 51 601 | 51 614 | 51 627 | 51 640 | 51 654 | 51 667 | 51 680 | 51 693 | 51 706 |
| 329 | 51 720 | 51 733 | 51 746 | 51 759 | 51 772 | 51 786 | 51 799 | 51 812 | 51 825 | 51 838 |
| 330 | 51 851 | 51 865 | 51 878 | 51 891 | 51 904 | 51 917 | 51 930 | 51 943 | 51 957 | 51 970 |
| 331 | 51 983 | 51 996 | 52 009 | 52 022 | 52 035 | 52 048 | 52 061 | 52 075 | 52 088 | 52 101 |
| 332 | 52 114 | 52 127 | 52 140 | 52 153 | 52 166 | 52 179 | 52 192 | 52 205 | 52 218 | 52 231 |
| 333 | 52 244 | 52 257 | 52 270 | 52 284 | 52 297 | 52 310 | 52 323 | 52 336 | 52 349 | 52 362 |
| 334 | 52 375 | 52 388 | 52 401 | 52 414 | 52 427 | 52 440 | 52 453 | 52 466 | 52 479 | 52 492 |
| 335 | 52 504 | 52 517 | 52 530 | 52 543 | 52 556 | 52 569 | 52 582 | 52 595 | 52 608 | 52 621 |
| 336 | 52 634 | 52 647 | 52 660 | 52 673 | 52 686 | 52 699 | 52 711 | 52 724 | 52 737 | 52 750 |
| 337 | 52 763 | 52 776 | 52 789 | 52 802 | 52 815 | 52 827 | 52 840 | 52 853 | 52 866 | 52 879 |
| 338 | 52 892 | 52 905 | 52 917 | 52 930 | 52 943 | 52 956 | 52 969 | 52 982 | 52 994 | 53 007 |
| 339 | 53 020 | 53 033 | 53 046 | 53 058 | 53 071 | 53 084 | 53 097 | 53 110 | 53 122 | 53 135 |
| 340 | 53 148 | 53 161 | 53 173 | 53 186 | 53 199 | 53 212 | 53 224 | 53 237 | 53 250 | 53 263 |
| 341 | 53 275 | 53 288 | 53 301 | 53 314 | 53 326 | 53 339 | 53 352 | 53 364 | 53 377 | 53 390 |
| 342 | 53 403 | 53 415 | 53 428 | 53 441 | 53 453 | 53 466 | 53 479 | 53 491 | 53 504 | 53 517 |
| 343 | 53 529 | 53 542 | 53 555 | 53 567 | 53 580 | 53 593 | 53 605 | 53 618 | 53 631 | 53 643 |
| 344 | 53 656 | 53 668 | 53 681 | 53 694 | 53 706 | 53 719 | 53 732 | 53 744 | 53 757 | 53 769 |
| 345 | 53 782 | 53 794 | 53 807 | 53 820 | 53 832 | 53 845 | 53 857 | 53 870 | 53 882 | 53 895 |
| 346 | 53 908 | 53 920 | 53 933 | 53 945 | 53 958 | 53 970 | 53 983 | 53 995 | 54 008 | 54 020 |
| 347 | 54 033 | 54 045 | 54 058 | 54 070 | 54 083 | 54 095 | 54 108 | 54 120 | 54 133 | 54 145 |
| 348 | 54 158 | 54 170 | 54 183 | 54 195 | 54 208 | 54 220 | 54 233 | 54 245 | 54 258 | 54 270 |
| 349 | 54 283 | 54 295 | 54 307 | 54 320 | 54 332 | 54 345 | 54 357 | 54 370 | 54 382 | 54 394 |
| 350 | 54 407 | 54 419 | 54 432 | 54 444 | 54 456 | 54 469 | 54 481 | 54 494 | 54 506 | 54 518 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 350 | 54 407 | 54 419 | 54 432 | 54 444 | 54 456 | 54 469 | 54 481 | 54 494 | 54 506 | 54 518 |
| 351 | 54 531 | 54 543 | 54 555 | 54 568 | 54 580 | 54 593 | 54 605 | 54 617 | 54 630 | 54 642 |
| 352 | 54 654 | 54 667 | 54 679 | 54 691 | 54 704 | 54 716 | 54 728 | 54 741 | 54 753 | 54 765 |
| 353 | 54 777 | 54 790 | 54 802 | 54 814 | 54 827 | 54 839 | 54 851 | 54 864 | 54 876 | 54 888 |
| 354 | 54 900 | 54 913 | 54 925 | 54 937 | 54 949 | 54 962 | 54 974 | 54 986 | 54 998 | 55 011 |
| 355 | 55 023 | 55 035 | 55 047 | 55 060 | 55 072 | 55 084 | 55 096 | 55 108 | 55 121 | 55 133 |
| 356 | 55 145 | 55 157 | 55 169 | 55 182 | 55 194 | 55 206 | 55 218 | 55 230 | 55 242 | 55 255 |
| 357 | 55 267 | 55 279 | 55 291 | 55 303 | 55 315 | 55 328 | 55 340 | 55 352 | 55 364 | 55 376 |
| 358 | 55 388 | 55 400 | 55 413 | 55 425 | 55 437 | 55 449 | 55 461 | 55 473 | 55 485 | 55 497 |
| 359 | 55 509 | 55 522 | 55 534 | 55 546 | 55 558 | 55 570 | 55 582 | 55 594 | 55 606 | 55 618 |
| 360 | 55 630 | 55 642 | 55 654 | 55 666 | 55 678 | 55 691 | 55 703 | 55 715 | 55 727 | 55 739 |
| 361 | 55 751 | 55 763 | 55 775 | 55 787 | 55 799 | 55 811 | 55 823 | 55 835 | 55 847 | 55 859 |
| 362 | 55 871 | 55 883 | 55 895 | 55 907 | 55 919 | 55 931 | 55 943 | 55 955 | 55 967 | 55 979 |
| 363 | 55 991 | 56 003 | 56 015 | 56 027 | 56 038 | 56 050 | 56 062 | 56 074 | 56 086 | 56 098 |
| 364 | 56 110 | 56 122 | 56 134 | 56 146 | 56 158 | 56 170 | 56 182 | 56 194 | 56 205 | 56 217 |
| 365 | 56 229 | 56 241 | 56 253 | 56 265 | 56 277 | 56 289 | 56 301 | 56 312 | 56 324 | 56 336 |
| 366 | 56 348 | 56 360 | 56 372 | 56 384 | 56 396 | 56 407 | 56 419 | 56 431 | 56 443 | 56 455 |
| 367 | 56 467 | 56 478 | 56 490 | 56 502 | 56 514 | 56 526 | 56 538 | 56 549 | 56 561 | 56 573 |
| 368 | 56 585 | 56 597 | 56 608 | 56 620 | 56 632 | 56 644 | 56 656 | 56 667 | 56 679 | 56 691 |
| 369 | 56 703 | 56 714 | 56 726 | 56 738 | 56 750 | 56 761 | 56 773 | 56 785 | 56 797 | 56 808 |
| 370 | 56 820 | 56 832 | 56 844 | 56 855 | 56 867 | 56 879 | 56 891 | 56 902 | 56 914 | 56 926 |
| 371 | 56 937 | 56 949 | 56 961 | 56 972 | 56 984 | 56 996 | 57 008 | 57 019 | 57 031 | 57 043 |
| 372 | 57 054 | 57 066 | 57 078 | 57 089 | 57 101 | 57 113 | 57 124 | 57 136 | 57 148 | 57 159 |
| 373 | 57 171 | 57 183 | 57 194 | 57 206 | 57 217 | 57 229 | 57 241 | 57 252 | 57 264 | 57 276 |
| 374 | 57 287 | 57 299 | 57 310 | 57 322 | 57 334 | 57 345 | 57 357 | 57 368 | 57 380 | 57 392 |
| 375 | 57 403 | 57 415 | 57 426 | 57 438 | 57 449 | 57 461 | 57 473 | 57 484 | 57 496 | 57 507 |
| 376 | 57 519 | 57 530 | 57 542 | 57 553 | 57 565 | 57 576 | 57 588 | 57 600 | 57 611 | 57 623 |
| 377 | 57 634 | 57 646 | 57 657 | 57 669 | 57 680 | 57 692 | 57 703 | 57 715 | 57 726 | 57 738 |
| 378 | 57 749 | 57 761 | 57 772 | 57 784 | 57 795 | 57 807 | 57 818 | 57 830 | 57 841 | 57 852 |
| 379 | 57 864 | 57 875 | 57 887 | 57 898 | 57 910 | 57 921 | 57 933 | 57 944 | 57 955 | 57 967 |
| 380 | 57 978 | 57 990 | 58 001 | 58 013 | 58 024 | 58 035 | 58 047 | 58 058 | 58 070 | 58 081 |
| 381 | 58 092 | 58 104 | 58 115 | 58 127 | 58 138 | 58 149 | 58 161 | 58 172 | 58 184 | 58 195 |
| 382 | 58 206 | 58 218 | 58 229 | 58 240 | 58 252 | 58 263 | 58 274 | 58 286 | 58 297 | 58 309 |
| 383 | 58 320 | 58 331 | 58 343 | 58 354 | 58 365 | 58 377 | 58 388 | 58 399 | 58 410 | 58 422 |
| 384 | 58 433 | 58 444 | 58 456 | 58 467 | 58 478 | 58 490 | 58 501 | 58 512 | 58 524 | 58 535 |
| 385 | 58 546 | 58 557 | 58 569 | 58 580 | 58 591 | 58 602 | 58 614 | 58 625 | 58 636 | 58 647 |
| 386 | 58 659 | 58 670 | 58 681 | 58 692 | 58 704 | 58 715 | 58 726 | 58 737 | 58 749 | 58 760 |
| 387 | 58 771 | 58 782 | 58 794 | 58 805 | 58 816 | 58 827 | 58 838 | 58 850 | 58 861 | 58 872 |
| 388 | 58 883 | 58 894 | 58 906 | 58 917 | 58 928 | 58 939 | 58 950 | 58 961 | 58 973 | 58 984 |
| 389 | 58 995 | 59 006 | 59 017 | 59 028 | 59 040 | 59 051 | 59 062 | 59 073 | 59 084 | 59 095 |
| 390 | 59 106 | 59 118 | 59 129 | 59 140 | 59 151 | 59 162 | 59 173 | 59 184 | 59 195 | 59 207 |
| 391 | 59 218 | 59 229 | 59 240 | 59 251 | 59 262 | 59 273 | 59 284 | 59 295 | 59 306 | 59 318 |
| 392 | 59 329 | 59 340 | 59 351 | 59 362 | 59 373 | 59 384 | 59 395 | 59 406 | 59 417 | 59 428 |
| 393 | 59 439 | 59 450 | 59 461 | 59 472 | 59 483 | 59 494 | 59 506 | 59 517 | 59 528 | 59 539 |
| 394 | 59 550 | 59 561 | 59 572 | 59 583 | 59 594 | 59 605 | 59 616 | 59 627 | 59 638 | 59 649 |
| 395 | 59 660 | 59 671 | 59 682 | 59 693 | 59 704 | 59 715 | 59 726 | 59 737 | 59 748 | 59 759 |
| 396 | 59 770 | 59 780 | 59 791 | 59 802 | 59 813 | 59 824 | 59 835 | 59 846 | 59 857 | 59 868 |
| 397 | 59 879 | 59 890 | 59 901 | 59 912 | 59 923 | 59 934 | 59 945 | 59 956 | 59 966 | 59 977 |
| 398 | 59 988 | 59 999 | 60 010 | 60 021 | 60 032 | 60 043 | 60 054 | 60 065 | 60 076 | 60 086 |
| 399 | 60 097 | 60 108 | 60 119 | 60 130 | 60 141 | 60 152 | 60 163 | 60 173 | 60 184 | 60 195 |
| 400 | 60 206 | 60 217 | 60 228 | 60 239 | 60 249 | 60 260 | 60 271 | 60 282 | 60 293 | 60 304 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 400 | 60 206 | 60 217 | 60 228 | 60 239 | 60 249 | 60 260 | 60 271 | 60 282 | 60 293 | 60 304 |
| 401 | 60 314 | 60 325 | 60 336 | 60 347 | 60 358 | 60 369 | 60 379 | 60 390 | 60 401 | 60 412 |
| 402 | 60 423 | 60 433 | 60 444 | 60 455 | 60 466 | 60 477 | 60 487 | 60 498 | 60 509 | 60 520 |
| 403 | 60 531 | 60 541 | 60 552 | 60 563 | 60 574 | 60 584 | 60 595 | 60 606 | 60 617 | 60 627 |
| 404 | 60 638 | 60 649 | 60 660 | 60 670 | 60 681 | 60 691 | 60 703 | 60 713 | 60 724 | 60 735 |
| 405 | 60 746 | 60 756 | 60 767 | 60 778 | 60 788 | 60 799 | 60 810 | 60 821 | 60 831 | 60 842 |
| 406 | 60 853 | 60 863 | 60 874 | 60 885 | 60 895 | 60 906 | 60 917 | 60 927 | 60 938 | 60 949 |
| 407 | 60 959 | 60 970 | 60 981 | 60 991 | 61 002 | 61 013 | 61 023 | 61 034 | 61 045 | 61 055 |
| 408 | 61 066 | 61 077 | 61 087 | 61 098 | 61 109 | 61 119 | 61 130 | 61 140 | 61 151 | 61 162 |
| 409 | 61 172 | 61 183 | 61 194 | 61 204 | 61 215 | 61 225 | 61 236 | 61 247 | 61 257 | 61 268 |
| 410 | 61 278 | 61 289 | 61 300 | 61 310 | 61 321 | 61 331 | 61 342 | 61 352 | 61 363 | 61 374 |
| 411 | 61 384 | 61 395 | 61 405 | 61 416 | 61 426 | 61 437 | 61 448 | 61 458 | 61 469 | 61 479 |
| 412 | 61 490 | 61 500 | 61 511 | 61 521 | 61 532 | 61 542 | 61 553 | 61 563 | 61 574 | 61 584 |
| 413 | 61 595 | 61 606 | 61 616 | 61 627 | 61 637 | 61 648 | 61 658 | 61 669 | 61 679 | 61 690 |
| 414 | 61 700 | 61 711 | 61 721 | 61 731 | 61 742 | 61 752 | 61 763 | 61 773 | 61 784 | 61 794 |
| 415 | 61 805 | 61 815 | 61 826 | 61 836 | 61 847 | 61 857 | 61 868 | 61 878 | 61 888 | 61 899 |
| 416 | 61 909 | 61 920 | 61 930 | 61 941 | 61 951 | 61 962 | 61 972 | 61 982 | 61 993 | 62 003 |
| 417 | 62 014 | 62 024 | 62 034 | 62 045 | 62 055 | 62 066 | 62 076 | 62 086 | 62 097 | 62 107 |
| 418 | 62 118 | 62 128 | 62 138 | 62 149 | 62 159 | 62 170 | 62 180 | 62 190 | 62 201 | 62 211 |
| 419 | 62 221 | 62 232 | 62 242 | 62 252 | 62 263 | 62 273 | 62 284 | 62 294 | 62 304 | 62 315 |
| 420 | 62 325 | 62 335 | 62 346 | 62 356 | 62 366 | 62 377 | 62 387 | 62 397 | 62 408 | 62 418 |
| 421 | 62 428 | 62 439 | 62 449 | 62 459 | 62 469 | 62 480 | 62 490 | 62 500 | 62 511 | 62 521 |
| 422 | 62 531 | 62 542 | 62 552 | 62 562 | 62 572 | 62 583 | 62 593 | 62 603 | 62 613 | 62 624 |
| 423 | 62 634 | 62 644 | 62 655 | 62 665 | 62 675 | 62 685 | 62 696 | 62 706 | 62 716 | 62 726 |
| 424 | 62 737 | 62 747 | 62 757 | 62 767 | 62 778 | 62 788 | 62 798 | 62 808 | 62 818 | 62 829 |
| 425 | 62 839 | 62 849 | 62 859 | 62 870 | 62 880 | 62 890 | 62 900 | 62 910 | 62 921 | 62 931 |
| 426 | 62 941 | 62 951 | 62 961 | 62 972 | 62 982 | 62 992 | 63 002 | 63 012 | 63 022 | 63 033 |
| 427 | 63 043 | 63 053 | 63 063 | 63 073 | 63 083 | 63 094 | 63 104 | 63 114 | 63 124 | 63 134 |
| 428 | 63 144 | 63 155 | 63 165 | 63 175 | 63 185 | 63 195 | 63 205 | 63 215 | 63 225 | 63 236 |
| 429 | 63 246 | 63 256 | 63 266 | 63 276 | 63 286 | 63 296 | 63 306 | 63 317 | 63 327 | 63 337 |
| 430 | 63 347 | 63 357 | 63 367 | 63 377 | 63 387 | 63 397 | 63 407 | 63 417 | 63 428 | 63 438 |
| 431 | 63 448 | 63 458 | 63 468 | 63 478 | 63 488 | 63 498 | 63 508 | 63 518 | 63 528 | 63 538 |
| 432 | 63 548 | 63 558 | 63 568 | 63 579 | 63 589 | 63 599 | 63 609 | 63 619 | 63 629 | 63 639 |
| 433 | 63 649 | 63 659 | 63 669 | 63 679 | 63 689 | 63 699 | 63 709 | 63 719 | 63 729 | 63 739 |
| 434 | 63 749 | 63 759 | 63 769 | 63 779 | 63 789 | 63 799 | 63 809 | 63 819 | 63 829 | 63 839 |
| 435 | 63 849 | 63 859 | 63 869 | 63 879 | 63 889 | 63 899 | 63 909 | 63 919 | 63 929 | 63 939 |
| 436 | 63 949 | 63 959 | 63 969 | 63 979 | 63 988 | 63 998 | 64 008 | 64 018 | 64 028 | 64 038 |
| 437 | 64 048 | 64 058 | 64 068 | 64 078 | 64 088 | 64 098 | 64 108 | 64 118 | 64 128 | 64 137 |
| 438 | 64 147 | 64 157 | 64 167 | 64 177 | 64 187 | 64 197 | 64 207 | 64 217 | 64 227 | 64 237 |
| 439 | 64 246 | 64 256 | 64 266 | 64 276 | 64 286 | 64 296 | 64 306 | 64 316 | 64 326 | 64 335 |
| 440 | 64 345 | 64 355 | 64 365 | 64 375 | 64 385 | 64 395 | 64 404 | 64 414 | 64 424 | 64 434 |
| 441 | 64 444 | 64 454 | 64 464 | 64 473 | 64 483 | 64 493 | 64 503 | 64 513 | 64 523 | 64 532 |
| 442 | 64 542 | 64 552 | 64 562 | 64 572 | 64 582 | 64 591 | 64 601 | 64 611 | 64 621 | 64 631 |
| 443 | 64 640 | 64 650 | 64 660 | 64 670 | 64 680 | 64 689 | 64 699 | 64 709 | 64 719 | 64 729 |
| 444 | 64 738 | 64 748 | 64 758 | 64 768 | 64 777 | 64 787 | 64 797 | 64 807 | 64 816 | 64 826 |
| 445 | 64 836 | 64 846 | 64 856 | 64 865 | 64 875 | 64 885 | 64 895 | 64 904 | 64 914 | 64 924 |
| 446 | 64 933 | 64 943 | 64 953 | 64 963 | 64 972 | 64 982 | 64 992 | 65 002 | 65 011 | 65 021 |
| 447 | 65 031 | 65 040 | 65 050 | 65 060 | 65 070 | 65 079 | 65 089 | 65 099 | 65 108 | 65 118 |
| 448 | 65 128 | 65 137 | 65 147 | 65 157 | 65 167 | 65 176 | 65 186 | 65 196 | 65 205 | 65 215 |
| 449 | 65 225 | 65 234 | 65 244 | 65 254 | 65 263 | 65 273 | 65 283 | 65 292 | 65 302 | 65 312 |
| 450 | 65 321 | 65 331 | 65 341 | 65 350 | 65 360 | 65 369 | 65 379 | 65 389 | 65 398 | 65 408 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 450 | 65 321 | 65 331 | 65 341 | 65 350 | 65 360 | 65 369 | 65 379 | 65 389 | 65 398 | 65 408 |
| 451 | 65 418 | 65 427 | 65 437 | 65 447 | 65 456 | 65 466 | 65 475 | 65 485 | 65 495 | 65 504 |
| 452 | 65 514 | 65 523 | 65 533 | 65 543 | 65 552 | 65 562 | 65 571 | 65 581 | 65 591 | 65 600 |
| 453 | 65 610 | 65 619 | 65 629 | 65 639 | 65 648 | 65 658 | 65 667 | 65 677 | 65 686 | 65 696 |
| 454 | 65 706 | 65 715 | 65 725 | 65 734 | 65 744 | 65 753 | 65 763 | 65 772 | 65 782 | 65 792 |
| 455 | 65 801 | 65 811 | 65 820 | 65 830 | 65 839 | 65 849 | 65 858 | 65 868 | 65 877 | 65 887 |
| 456 | 65 896 | 65 906 | 65 916 | 65 925 | 65 935 | 65 944 | 65 954 | 65 963 | 65 973 | 65 982 |
| 457 | 65 992 | 66 001 | 66 011 | 66 020 | 66 030 | 66 039 | 66 049 | 66 058 | 66 068 | 66 077 |
| 458 | 66 087 | 66 096 | 66 106 | 66 115 | 66 124 | 66 134 | 66 143 | 66 153 | 66 162 | 66 172 |
| 459 | 66 181 | 66 191 | 66 200 | 66 210 | 66 219 | 66 229 | 66 238 | 66 247 | 66 257 | 66 266 |
| 460 | 66 276 | 66 285 | 66 295 | 66 304 | 66 314 | 66 323 | 66 332 | 66 342 | 66 351 | 66 361 |
| 461 | 66 370 | 66 380 | 66 389 | 66 398 | 66 408 | 66 417 | 66 427 | 66 436 | 66 445 | 66 455 |
| 462 | 66 464 | 66 474 | 66 483 | 66 492 | 66 502 | 66 511 | 66 521 | 66 530 | 66 539 | 66 549 |
| 463 | 66 558 | 66 567 | 66 577 | 66 586 | 66 596 | 66 605 | 66 614 | 66 624 | 66 633 | 66 642 |
| 464 | 66 652 | 66 661 | 66 671 | 66 680 | 66 689 | 66 699 | 66 708 | 66 717 | 66 727 | 66 736 |
| 465 | 66 745 | 66 755 | 66 764 | 66 773 | 66 783 | 66 792 | 66 801 | 66 811 | 66 820 | 66 829 |
| 466 | 66 839 | 66 848 | 66 857 | 66 867 | 66 876 | 66 885 | 66 894 | 66 904 | 66 913 | 66 922 |
| 467 | 66 932 | 66 941 | 66 950 | 66 960 | 66 969 | 66 978 | 66 987 | 66 997 | 67 006 | 67 015 |
| 468 | 67 025 | 67 034 | 67 043 | 67 052 | 67 062 | 67 071 | 67 080 | 67 089 | 67 099 | 67 108 |
| 469 | 67 117 | 67 127 | 67 136 | 67 145 | 67 154 | 67 164 | 67 173 | 67 182 | 67 191 | 67 201 |
| 470 | 67 210 | 67 219 | 67 228 | 67 237 | 67 247 | 67 256 | 67 265 | 67 274 | 67 284 | 67 293 |
| 471 | 67 302 | 67 311 | 67 321 | 67 330 | 67 339 | 67 348 | 67 357 | 67 367 | 67 376 | 67 385 |
| 472 | 67 394 | 67 403 | 67 413 | 67 422 | 67 431 | 67 440 | 67 449 | 67 459 | 67 468 | 67 477 |
| 473 | 67 486 | 67 495 | 67 504 | 67 514 | 67 523 | 67 532 | 67 541 | 67 550 | 67 560 | 67 569 |
| 474 | 67 578 | 67 587 | 67 596 | 67 605 | 67 614 | 67 624 | 67 633 | 67 642 | 67 651 | 67 660 |
| 475 | 67 669 | 67 679 | 67 688 | 67 697 | 67 706 | 67 715 | 67 724 | 67 733 | 67 742 | 67 752 |
| 476 | 67 761 | 67 770 | 67 779 | 67 788 | 67 797 | 67 806 | 67 815 | 67 825 | 67 834 | 67 843 |
| 477 | 67 852 | 67 861 | 67 870 | 67 879 | 67 888 | 67 897 | 67 906 | 67 916 | 67 925 | 67 934 |
| 478 | 67 943 | 67 952 | 67 961 | 67 970 | 67 979 | 67 988 | 67 997 | 68 006 | 68 015 | 68 024 |
| 479 | 68 034 | 68 043 | 68 052 | 68 061 | 68 070 | 68 079 | 68 088 | 68 097 | 68 106 | 68 115 |
| 480 | 68 124 | 68 133 | 68 142 | 68 151 | 68 160 | 68 169 | 68 178 | 68 187 | 68 196 | 68 205 |
| 481 | 68 215 | 68 224 | 68 233 | 68 242 | 68 251 | 68 260 | 68 269 | 68 278 | 68 287 | 68 296 |
| 482 | 68 305 | 68 314 | 68 323 | 68 332 | 68 341 | 68 350 | 68 359 | 68 368 | 68 377 | 68 386 |
| 483 | 68 395 | 68 404 | 68 413 | 68 422 | 68 431 | 68 440 | 68 449 | 68 458 | 68 467 | 68 476 |
| 484 | 68 485 | 68 494 | 68 502 | 68 511 | 68 520 | 68 529 | 68 538 | 68 547 | 68 556 | 68 565 |
| 485 | 68 574 | 68 583 | 68 592 | 68 601 | 68 610 | 68 619 | 68 628 | 68 637 | 68 646 | 68 655 |
| 486 | 68 664 | 68 673 | 68 681 | 68 690 | 68 699 | 68 708 | 68 717 | 68 726 | 68 735 | 68 744 |
| 487 | 68 753 | 68 762 | 68 771 | 68 780 | 68 789 | 68 797 | 68 806 | 68 815 | 68 824 | 68 833 |
| 488 | 68 842 | 68 851 | 68 860 | 68 869 | 68 878 | 68 886 | 68 895 | 68 904 | 68 913 | 68 922 |
| 489 | 68 931 | 68 940 | 68 949 | 68 958 | 68 966 | 68 975 | 68 984 | 68 993 | 69 002 | 69 011 |
| 490 | 69 020 | 69 028 | 69 037 | 69 046 | 69 055 | 69 064 | 69 073 | 69 082 | 69 090 | 69 099 |
| 491 | 69 108 | 69 117 | 69 126 | 69 135 | 69 144 | 69 152 | 69 161 | 69 170 | 69 179 | 69 188 |
| 492 | 69 197 | 69 205 | 69 214 | 69 223 | 69 232 | 69 241 | 69 249 | 69 258 | 69 267 | 69 276 |
| 493 | 69 285 | 69 294 | 69 302 | 69 311 | 69 320 | 69 329 | 69 338 | 69 346 | 69 355 | 69 364 |
| 494 | 69 373 | 69 381 | 69 390 | 69 399 | 69 408 | 69 417 | 69 425 | 69 434 | 69 443 | 69 452 |
| 495 | 69 461 | 69 469 | 69 478 | 69 487 | 69 496 | 69 504 | 69 513 | 69 522 | 69 531 | 69 539 |
| 496 | 69 548 | 69 557 | 69 566 | 69 574 | 69 583 | 69 592 | 69 601 | 69 609 | 69 618 | 69 627 |
| 497 | 69 636 | 69 644 | 69 653 | 69 662 | 69 671 | 69 679 | 69 688 | 69 697 | 69 705 | 69 714 |
| 498 | 69 723 | 69 732 | 69 740 | 69 749 | 69 758 | 69 767 | 69 775 | 69 784 | 69 793 | 69 801 |
| 499 | 69 810 | 69 819 | 69 827 | 69 836 | 69 845 | 69 854 | 69 862 | 69 871 | 69 880 | 69 888 |
| 500 | 69 897 | 69 906 | 69 914 | 69 923 | 69 932 | 69 940 | 69 949 | 69 958 | 69 966 | 69 975 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 500 | 69 897 | 69 906 | 69 914 | 69 923 | 69 932 | 69 940 | 69 949 | 69 958 | 69 966 | 69 975 |
| 501 | 69 984 | 69 992 | 70 001 | 70 010 | 70 018 | 70 027 | 70 036 | 70 044 | 70 053 | 70 062 |
| 502 | 70 070 | 70 079 | 70 088 | 70 096 | 70 105 | 70 114 | 70 122 | 70 131 | 70 140 | 70 148 |
| 503 | 70 157 | 70 165 | 70 174 | 70 183 | 70 191 | 70 200 | 70 209 | 70 217 | 70 226 | 70 234 |
| 504 | 70 243 | 70 252 | 70 260 | 70 269 | 70 278 | 70 286 | 70 295 | 70 303 | 70 312 | 70 321 |
| 505 | 70 329 | 70 338 | 70 346 | 70 355 | 70 364 | 70 372 | 70 381 | 70 389 | 70 398 | 70 406 |
| 506 | 70 415 | 70 424 | 70 432 | 70 441 | 70 449 | 70 458 | 70 467 | 70 475 | 70 484 | 70 492 |
| 507 | 70 501 | 70 509 | 70 518 | 70 526 | 70 535 | 70 544 | 70 552 | 70 561 | 70 569 | 70 578 |
| 508 | 70 586 | 70 595 | 70 603 | 70 612 | 70 621 | 70 629 | 70 638 | 70 646 | 70 655 | 70 663 |
| 509 | 70 672 | 70 680 | 70 689 | 70 697 | 70 706 | 70 714 | 70 723 | 70 731 | 70 740 | 70 749 |
| 510 | 70 757 | 70 766 | 70 774 | 70 783 | 70 791 | 70 800 | 70 808 | 70 817 | 70 825 | 70 834 |
| 511 | 70 842 | 70 851 | 70 859 | 70 868 | 70 876 | 70 885 | 70 893 | 70 902 | 70 910 | 70 919 |
| 512 | 70 927 | 70 935 | 70 944 | 70 952 | 70 961 | 70 969 | 70 978 | 70 986 | 70 995 | 71 003 |
| 513 | 71 012 | 71 020 | 71 029 | 71 037 | 71 046 | 71 054 | 71 063 | 71 071 | 71 079 | 71 088 |
| 514 | 71 096 | 71 105 | 71 113 | 71 122 | 71 130 | 71 139 | 71 147 | 71 155 | 71 164 | 71 172 |
| 515 | 71 181 | 71 189 | 71 198 | 71 206 | 71 214 | 71 223 | 71 231 | 71 240 | 71 248 | 71 257 |
| 516 | 71 265 | 71 273 | 71 282 | 71 290 | 71 299 | 71 307 | 71 315 | 71 324 | 71 332 | 71 341 |
| 517 | 71 349 | 71 357 | 71 366 | 71 374 | 71 383 | 71 391 | 71 399 | 71 408 | 71 416 | 71 425 |
| 518 | 71 433 | 71 441 | 71 450 | 71 458 | 71 466 | 71 475 | 71 483 | 71 492 | 71 500 | 71 508 |
| 519 | 71 517 | 71 525 | 71 533 | 71 542 | 71 550 | 71 559 | 71 567 | 71 575 | 71 584 | 71 592 |
| 520 | 71 600 | 71 609 | 71 617 | 71 625 | 71 634 | 71 642 | 71 650 | 71 659 | 71 667 | 71 675 |
| 521 | 71 684 | 71 692 | 71 700 | 71 709 | 71 717 | 71 725 | 71 734 | 71 742 | 71 750 | 71 759 |
| 522 | 71 767 | 71 775 | 71 784 | 71 792 | 71 800 | 71 809 | 71 817 | 71 825 | 71 834 | 71 842 |
| 523 | 71 850 | 71 858 | 71 867 | 71 875 | 71 883 | 71 892 | 71 900 | 71 908 | 71 917 | 71 925 |
| 524 | 71 933 | 71 941 | 71 950 | 71 958 | 71 966 | 71 975 | 71 983 | 71 991 | 71 999 | 72 008 |
| 525 | 72 016 | 72 024 | 72 032 | 72 041 | 72 049 | 72 057 | 72 066 | 72 074 | 72 082 | 72 090 |
| 526 | 72 099 | 72 107 | 72 115 | 72 123 | 72 132 | 72 140 | 72 148 | 72 156 | 72 165 | 72 173 |
| 527 | 72 181 | 72 189 | 72 198 | 72 206 | 72 214 | 72 222 | 72 230 | 72 239 | 72 247 | 72 255 |
| 528 | 72 263 | 72 272 | 72 280 | 72 288 | 72 296 | 72 304 | 72 313 | 72 321 | 72 329 | 72 337 |
| 529 | 72 346 | 72 354 | 72 362 | 72 370 | 72 378 | 72 387 | 72 395 | 72 403 | 72 411 | 72 419 |
| 530 | 72 428 | 72 436 | 72 444 | 72 452 | 72 460 | 72 469 | 72 477 | 72 485 | 72 493 | 72 501 |
| 531 | 72 509 | 72 518 | 72 526 | 72 534 | 72 542 | 72 550 | 72 558 | 72 567 | 72 575 | 72 583 |
| 532 | 72 591 | 72 599 | 72 607 | 72 616 | 72 624 | 72 632 | 72 640 | 72 648 | 72 656 | 72 665 |
| 533 | 72 673 | 72 681 | 72 689 | 72 697 | 72 705 | 72 713 | 72 722 | 72 730 | 72 738 | 72 746 |
| 534 | 72 754 | 72 762 | 72 770 | 72 779 | 72 787 | 72 795 | 72 803 | 72 811 | 72 819 | 72 827 |
| 535 | 72 835 | 72 843 | 72 852 | 72 860 | 72 868 | 72 876 | 72 884 | 72 892 | 72 900 | 72 908 |
| 536 | 72 916 | 72 925 | 72 933 | 72 941 | 72 949 | 72 957 | 72 965 | 72 973 | 72 981 | 72 989 |
| 537 | 72 997 | 73 006 | 73 014 | 73 022 | 73 030 | 73 038 | 73 046 | 73 054 | 73 062 | 73 070 |
| 538 | 73 078 | 73 086 | 73 094 | 73 102 | 73 111 | 73 119 | 73 127 | 73 135 | 73 143 | 73 151 |
| 539 | 73 159 | 73 167 | 73 175 | 73 183 | 73 191 | 73 199 | 73 207 | 73 215 | 73 223 | 73 231 |
| 540 | 73 239 | 73 247 | 73 255 | 73 263 | 73 272 | 73 280 | 73 288 | 73 296 | 73 304 | 73 312 |
| 541 | 73 320 | 73 328 | 73 336 | 73 344 | 73 352 | 73 360 | 73 368 | 73 376 | 73 384 | 73 392 |
| 542 | 73 400 | 73 408 | 73 416 | 73 424 | 73 432 | 73 440 | 73 448 | 73 456 | 73 464 | 73 472 |
| 543 | 73 480 | 73 488 | 73 496 | 73 504 | 73 512 | 73 520 | 73 528 | 73 536 | 73 544 | 73 552 |
| 544 | 73 560 | 73 568 | 73 576 | 73 584 | 73 592 | 73 600 | 73 608 | 73 616 | 73 624 | 73 632 |
| 545 | 73 640 | 73 648 | 73 656 | 73 664 | 73 672 | 73 679 | 73 687 | 73 695 | 73 703 | 73 711 |
| 546 | 73 719 | 73 727 | 73 735 | 73 743 | 73 751 | 73 759 | 73 767 | 73 775 | 73 783 | 73 791 |
| 547 | 73 799 | 73 807 | 73 815 | 73 823 | 73 830 | 73 838 | 73 846 | 73 854 | 73 862 | 73 870 |
| 548 | 73 878 | 73 886 | 73 894 | 73 902 | 73 910 | 73 918 | 73 926 | 73 933 | 73 941 | 73 949 |
| 549 | 73 957 | 73 965 | 73 973 | 73 981 | 73 989 | 73 997 | 74 005 | 74 013 | 74 020 | 74 028 |
| 550 | 74 036 | 74 044 | 74 052 | 74 060 | 74 068 | 74 076 | 74 084 | 74 092 | 74 099 | 74 107 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 550 | 74 036 | 74 044 | 74 052 | 74 060 | 74 068 | 74 076 | 74 084 | 74 092 | 74 099 | 74 107 |
| 551 | 74 115 | 74 123 | 74 131 | 74 139 | 74 147 | 74 155 | 74 162 | 74 170 | 74 178 | 74 186 |
| 552 | 74 194 | 74 202 | 74 210 | 74 218 | 74 225 | 74 233 | 74 241 | 74 249 | 74 257 | 74 265 |
| 553 | 74 273 | 74 280 | 74 288 | 74 296 | 74 304 | 74 312 | 74 320 | 74 327 | 74 335 | 74 343 |
| 554 | 74 351 | 74 359 | 74 367 | 74 374 | 74 382 | 74 390 | 74 398 | 74 406 | 74 414 | 74 421 |
| 555 | 74 429 | 74 437 | 74 445 | 74 453 | 74 461 | 74 468 | 74 476 | 74 484 | 74 492 | 74 500 |
| 556 | 74 507 | 74 515 | 74 523 | 74 531 | 74 539 | 74 547 | 74 554 | 74 562 | 74 570 | 74 578 |
| 557 | 74 586 | 74 593 | 74 601 | 74 609 | 74 617 | 74 624 | 74 632 | 74 640 | 74 648 | 74 656 |
| 558 | 74 663 | 74 671 | 74 679 | 74 687 | 74 695 | 74 702 | 74 710 | 74 718 | 74 726 | 74 733 |
| 559 | 74 741 | 74 749 | 74 757 | 74 764 | 74 772 | 74 780 | 74 788 | 74 796 | 74 803 | 74 811 |
| 560 | 74 819 | 74 827 | 74 834 | 74 842 | 74 850 | 74 858 | 74 865 | 74 873 | 74 881 | 74 889 |
| 561 | 74 896 | 74 904 | 74 912 | 74 920 | 74 927 | 74 935 | 74 943 | 74 950 | 74 958 | 74 966 |
| 562 | 74 974 | 74 981 | 74 989 | 74 997 | 75 005 | 75 012 | 75 020 | 75 028 | 75 035 | 75 043 |
| 563 | 75 051 | 75 059 | 75 066 | 75 074 | 75 082 | 75 089 | 75 097 | 75 105 | 75 113 | 75 120 |
| 564 | 75 128 | 75 136 | 75 143 | 75 151 | 75 159 | 75 166 | 75 174 | 75 182 | 75 189 | 75 197 |
| 565 | 75 205 | 75 213 | 75 220 | 75 228 | 75 236 | 75 243 | 75 251 | 75 259 | 75 266 | 75 274 |
| 566 | 75 282 | 75 289 | 75 297 | 75 305 | 75 312 | 75 320 | 75 328 | 75 335 | 75 343 | 75 351 |
| 567 | 75 358 | 75 366 | 75 374 | 75 381 | 75 389 | 75 397 | 75 404 | 75 412 | 75 420 | 75 427 |
| 568 | 75 435 | 75 442 | 75 450 | 75 458 | 75 465 | 75 473 | 75 481 | 75 488 | 75 496 | 75 504 |
| 569 | 75 511 | 75 519 | 75 526 | 75 534 | 75 542 | 75 549 | 75 557 | 75 565 | 75 572 | 75 580 |
| 570 | 75 587 | 75 595 | 75 603 | 75 610 | 75 618 | 75 626 | 75 633 | 75 641 | 75 648 | 75 656 |
| 571 | 75 664 | 75 671 | 75 679 | 75 686 | 75 694 | 75 702 | 75 709 | 75 717 | 75 724 | 75 732 |
| 572 | 75 740 | 75 747 | 75 755 | 75 762 | 75 770 | 75 778 | 75 785 | 75 793 | 75 800 | 75 808 |
| 573 | 75 815 | 75 823 | 75 831 | 75 838 | 75 846 | 75 853 | 75 861 | 75 868 | 75 876 | 75 884 |
| 574 | 75 891 | 75 899 | 75 906 | 75 914 | 75 921 | 75 929 | 75 937 | 75 944 | 75 952 | 75 959 |
| 575 | 75 967 | 75 974 | 75 982 | 75 989 | 75 997 | 76 005 | 76 012 | 76 020 | 76 027 | 76 035 |
| 576 | 76 042 | 76 050 | 76 057 | 76 065 | 76 072 | 76 080 | 76 087 | 76 095 | 76 103 | 76 110 |
| 577 | 76 118 | 76 125 | 76 133 | 76 140 | 76 148 | 76 155 | 76 163 | 76 170 | 76 178 | 76 185 |
| 578 | 76 193 | 76 200 | 76 208 | 76 215 | 76 223 | 76 230 | 76 238 | 76 245 | 76 253 | 76 260 |
| 579 | 76 268 | 76 275 | 76 283 | 76 290 | 76 298 | 76 305 | 76 313 | 76 320 | 76 328 | 76 335 |
| 580 | 76 343 | 76 350 | 76 358 | 76 365 | 76 373 | 76 380 | 76 388 | 76 395 | 76 403 | 76 410 |
| 581 | 76 418 | 76 425 | 76 433 | 76 440 | 76 448 | 76 455 | 76 462 | 76 470 | 76 477 | 76 485 |
| 582 | 76 492 | 76 500 | 76 507 | 76 515 | 76 522 | 76 530 | 76 537 | 76 545 | 76 552 | 76 559 |
| 583 | 76 567 | 76 574 | 76 582 | 76 589 | 76 597 | 76 604 | 76 612 | 76 619 | 76 626 | 76 634 |
| 584 | 76 641 | 76 649 | 76 656 | 76 664 | 76 671 | 76 678 | 76 686 | 76 693 | 76 701 | 76 708 |
| 585 | 76 716 | 76 723 | 76 730 | 76 738 | 76 745 | 76 753 | 76 760 | 76 768 | 76 775 | 76 782 |
| 586 | 76 790 | 76 797 | 76 805 | 76 812 | 76 819 | 76 827 | 76 834 | 76 842 | 76 849 | 76 856 |
| 587 | 76 864 | 76 871 | 76 879 | 76 886 | 76 893 | 76 901 | 76 908 | 76 916 | 76 923 | 76 930 |
| 588 | 76 938 | 76 945 | 76 953 | 76 960 | 76 967 | 76 975 | 76 982 | 76 989 | 76 997 | 77 004 |
| 589 | 77 012 | 77 019 | 77 026 | 77 034 | 77 041 | 77 048 | 77 056 | 77 063 | 77 070 | 77 078 |
| 590 | 77 085 | 77 093 | 77 100 | 77 107 | 77 115 | 77 122 | 77 129 | 77 137 | 77 144 | 77 151 |
| 591 | 77 159 | 77 166 | 77 173 | 77 181 | 77 188 | 77 195 | 77 203 | 77 210 | 77 217 | 77 225 |
| 592 | 77 232 | 77 240 | 77 247 | 77 254 | 77 262 | 77 269 | 77 276 | 77 283 | 77 291 | 77 298 |
| 593 | 77 305 | 77 313 | 77 320 | 77 327 | 77 335 | 77 342 | 77 349 | 77 357 | 77 364 | 77 371 |
| 594 | 77 379 | 77 386 | 77 393 | 77 401 | 77 408 | 77 415 | 77 422 | 77 430 | 77 437 | 77 444 |
| 595 | 77 452 | 77 459 | 77 466 | 77 474 | 77 481 | 77 488 | 77 495 | 77 503 | 77 510 | 77 517 |
| 596 | 77 525 | 77 532 | 77 539 | 77 546 | 77 554 | 77 561 | 77 568 | 77 576 | 77 583 | 77 590 |
| 597 | 77 597 | 77 605 | 77 612 | 77 619 | 77 627 | 77 634 | 77 641 | 77 648 | 77 656 | 77 663 |
| 598 | 77 670 | 77 677 | 77 685 | 77 692 | 77 699 | 77 706 | 77 714 | 77 721 | 77 728 | 77 735 |
| 599 | 77 743 | 77 750 | 77 757 | 77 764 | 77 772 | 77 779 | 77 786 | 77 793 | 77 801 | 77 808 |
| 600 | 77 815 | 77 822 | 77 830 | 77 837 | 77 844 | 77 851 | 77 859 | 77 866 | 77 873 | 77 880 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 600 | 77 815 | 77 822 | 77 830 | 77 837 | 77 844 | 77 851 | 77 859 | 77 866 | 77 873 | 77 880 |
| 601 | 77 887 | 77 895 | 77 902 | 77 909 | 77 916 | 77 924 | 77 931 | 77 938 | 77 945 | 77 952 |
| 602 | 77 960 | 77 967 | 77 974 | 77 981 | 77 988 | 77 996 | 78 003 | 78 010 | 78 017 | 78 025 |
| 603 | 78 032 | 78 039 | 78 046 | 78 053 | 78 061 | 78 068 | 78 075 | 78 082 | 78 089 | 78 097 |
| 604 | 78 104 | 78 111 | 78 118 | 78 125 | 78 132 | 78 140 | 78 147 | 78 154 | 78 161 | 78 168 |
| 605 | 78 176 | 78 183 | 78 190 | 78 197 | 78 204 | 78 211 | 78 219 | 78 226 | 78 233 | 78 240 |
| 606 | 78 247 | 78 254 | 78 262 | 78 269 | 78 276 | 78 283 | 78 290 | 78 297 | 78 305 | 78 312 |
| 607 | 78 319 | 78 326 | 78 333 | 78 340 | 78 347 | 78 355 | 78 362 | 78 369 | 78 376 | 78 383 |
| 608 | 78 390 | 78 398 | 78 405 | 78 412 | 78 419 | 78 426 | 78 433 | 78 440 | 78 447 | 78 455 |
| 609 | 78 462 | 78 469 | 78 476 | 78 483 | 78 490 | 78 497 | 78 504 | 78 512 | 78 519 | 78 526 |
| 610 | 78 533 | 78 540 | 78 547 | 78 554 | 78 561 | 78 569 | 78 576 | 78 583 | 78 590 | 78 597 |
| 611 | 78 604 | 78 611 | 78 618 | 78 625 | 78 633 | 78 640 | 78 647 | 78 654 | 78 661 | 78 668 |
| 612 | 78 675 | 78 682 | 78 689 | 78 696 | 78 704 | 78 711 | 78 718 | 78 725 | 78 732 | 78 739 |
| 613 | 78 746 | 78 753 | 78 760 | 78 767 | 78 774 | 78 781 | 78 789 | 78 796 | 78 803 | 78 810 |
| 614 | 78 817 | 78 824 | 78 831 | 78 838 | 78 845 | 78 852 | 78 859 | 78 866 | 78 873 | 78 880 |
| 615 | 78 888 | 78 895 | 78 902 | 78 909 | 78 916 | 78 923 | 78 930 | 78 937 | 78 944 | 78 951 |
| 616 | 78 958 | 78 965 | 78 972 | 78 979 | 78 986 | 78 993 | 79 000 | 79 007 | 79 014 | 79 021 |
| 617 | 79 029 | 79 036 | 79 043 | 79 050 | 79 057 | 79 064 | 79 071 | 79 078 | 79 085 | 79 092 |
| 618 | 79 099 | 79 106 | 79 113 | 79 120 | 79 127 | 79 134 | 79 141 | 79 148 | 79 155 | 79 162 |
| 619 | 79 169 | 79 176 | 79 183 | 79 190 | 79 197 | 79 204 | 79 211 | 79 218 | 79 225 | 79 232 |
| 620 | 79 239 | 79 246 | 79 253 | 79 260 | 79 267 | 79 274 | 79 281 | 79 288 | 79 295 | 79 302 |
| 621 | 79 309 | 79 316 | 79 323 | 79 330 | 79 337 | 79 344 | 79 351 | 79 358 | 79 365 | 79 372 |
| 622 | 79 379 | 79 386 | 79 393 | 79 400 | 79 407 | 79 414 | 79 421 | 79 428 | 79 435 | 79 442 |
| 623 | 79 449 | 79 456 | 79 463 | 79 470 | 79 477 | 79 484 | 79 491 | 79 498 | 79 505 | 79 511 |
| 624 | 79 518 | 79 525 | 79 532 | 79 539 | 79 546 | 79 553 | 79 560 | 79 567 | 79 574 | 79 581 |
| 625 | 79 588 | 79 595 | 79 602 | 79 609 | 79 616 | 79 623 | 79 630 | 79 637 | 79 644 | 79 650 |
| 626 | 79 657 | 79 664 | 79 671 | 79 678 | 79 685 | 79 692 | 79 699 | 79 706 | 79 713 | 79 720 |
| 627 | 79 727 | 79 734 | 79 741 | 79 748 | 79 754 | 79 761 | 79 768 | 79 775 | 79 782 | 79 789 |
| 628 | 79 796 | 79 803 | 79 810 | 79 817 | 79 824 | 79 831 | 79 837 | 79 844 | 79 851 | 79 858 |
| 629 | 79 865 | 79 872 | 79 879 | 79 886 | 79 893 | 79 900 | 79 906 | 79 913 | 79 920 | 79 927 |
| 630 | 79 934 | 79 941 | 79 948 | 79 955 | 79 962 | 79 969 | 79 975 | 79 982 | 79 989 | 79 996 |
| 631 | 80 003 | 80 010 | 80 017 | 80 024 | 80 030 | 80 037 | 80 044 | 80 051 | 80 058 | 80 065 |
| 632 | 80 072 | 80 079 | 80 085 | 80 092 | 80 099 | 80 106 | 80 113 | 80 120 | 80 127 | 80 134 |
| 633 | 80 140 | 80 147 | 80 154 | 80 161 | 80 168 | 80 175 | 80 182 | 80 188 | 80 195 | 80 202 |
| 634 | 80 209 | 80 216 | 80 223 | 80 229 | 80 236 | 80 243 | 80 250 | 80 257 | 80 264 | 80 271 |
| 635 | 80 277 | 80 284 | 80 291 | 80 298 | 80 305 | 80 312 | 80 318 | 80 325 | 80 332 | 80 339 |
| 636 | 80 346 | 80 353 | 80 359 | 80 366 | 80 373 | 80 380 | 80 387 | 80 393 | 80 400 | 80 407 |
| 637 | 80 414 | 80 421 | 80 428 | 80 434 | 80 441 | 80 448 | 80 455 | 80 462 | 80 468 | 80 475 |
| 638 | 80 482 | 80 489 | 80 496 | 80 502 | 80 509 | 80 516 | 80 523 | 80 530 | 80 536 | 80 543 |
| 639 | 80 550 | 80 557 | 80 564 | 80 570 | 80 577 | 80 584 | 80 591 | 80 598 | 80 604 | 80 611 |
| 640 | 80 618 | 80 625 | 80 632 | 80 638 | 80 645 | 80 652 | 80 659 | 80 665 | 80 672 | 80 679 |
| 641 | 80 686 | 80 693 | 80 699 | 80 706 | 80 713 | 80 720 | 80 726 | 80 733 | 80 740 | 80 747 |
| 642 | 80 754 | 80 760 | 80 767 | 80 774 | 80 781 | 80 787 | 80 794 | 80 801 | 80 808 | 80 814 |
| 643 | 80 821 | 80 828 | 80 835 | 80 841 | 80 848 | 80 855 | 80 862 | 80 868 | 80 875 | 80 882 |
| 644 | 80 889 | 80 895 | 80 902 | 80 909 | 80 916 | 80 922 | 80 929 | 80 936 | 80 943 | 80 949 |
| 645 | 80 956 | 80 963 | 80 969 | 80 976 | 80 983 | 80 990 | 80 996 | 81 003 | 81 010 | 81 017 |
| 646 | 81 023 | 81 030 | 81 037 | 81 043 | 81 050 | 81 057 | 81 064 | 81 070 | 81 077 | 81 084 |
| 647 | 81 090 | 81 097 | 81 104 | 81 111 | 81 117 | 81 124 | 81 131 | 81 137 | 81 144 | 81 151 |
| 648 | 81 158 | 81 164 | 81 171 | 81 178 | 81 184 | 81 191 | 81 198 | 81 204 | 81 211 | 81 218 |
| 649 | 81 224 | 81 231 | 81 238 | 81 245 | 81 251 | 81 258 | 81 265 | 81 271 | 81 278 | 81 285 |
| 650 | 81 291 | 81 298 | 81 305 | 81 311 | 81 318 | 81 325 | 81 331 | 81 338 | 81 345 | 81 351 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 650 | 81 291 | 81 298 | 81 305 | 81 311 | 81 318 | 81 325 | 81 331 | 81 338 | 81 345 | 81 351 |
| 651 | 81 358 | 81 365 | 81 371 | 81 378 | 81 385 | 81 391 | 81 398 | 81 405 | 81 411 | 81 418 |
| 652 | 81 425 | 81 431 | 81 438 | 81 445 | 81 451 | 81 458 | 81 465 | 81 471 | 81 478 | 81 485 |
| 653 | 81 491 | 81 498 | 81 505 | 81 511 | 81 518 | 81 525 | 81 531 | 81 538 | 81 544 | 81 551 |
| 654 | 81 558 | 81 564 | 81 571 | 81 578 | 81 584 | 81 591 | 81 598 | 81 604 | 81 611 | 81 617 |
| 655 | 81 624 | 81 631 | 81 637 | 81 644 | 81 651 | 81 657 | 81 664 | 81 671 | 81 677 | 81 684 |
| 656 | 81 690 | 81 697 | 81 704 | 81 710 | 81 717 | 81 723 | 81 730 | 81 737 | 81 743 | 81 750 |
| 657 | 81 757 | 81 763 | 81 770 | 81 776 | 81 783 | 81 790 | 81 796 | 81 803 | 81 809 | 81 816 |
| 658 | 81 823 | 81 829 | 81 836 | 81 842 | 81 849 | 81 856 | 81 862 | 81 869 | 81 875 | 81 882 |
| 659 | 81 889 | 81 895 | 81 902 | 81 908 | 81 915 | 81 921 | 81 928 | 81 935 | 81 941 | 81 948 |
| 660 | 81 954 | 81 961 | 81 968 | 81 974 | 81 981 | 81 987 | 81 994 | 82 000 | 82 007 | 82 014 |
| 661 | 82 020 | 82 027 | 82 033 | 82 040 | 82 046 | 82 053 | 82 060 | 82 066 | 82 073 | 82 079 |
| 662 | 82 086 | 82 092 | 82 099 | 82 105 | 82 112 | 82 119 | 82 125 | 82 132 | 82 138 | 82 145 |
| 663 | 82 151 | 82 158 | 82 164 | 82 171 | 82 178 | 82 184 | 82 191 | 82 197 | 82 204 | 82 210 |
| 664 | 82 217 | 82 223 | 82 230 | 82 236 | 82 243 | 82 249 | 82 256 | 82 263 | 82 269 | 82 276 |
| 665 | 82 282 | 82 289 | 82 295 | 82 302 | 82 308 | 82 315 | 82 321 | 82 328 | 82 334 | 82 341 |
| 666 | 82 347 | 82 354 | 82 360 | 82 367 | 82 373 | 82 380 | 82 387 | 82 393 | 82 400 | 82 406 |
| 667 | 82 413 | 82 419 | 82 426 | 82 432 | 82 439 | 82 445 | 82 452 | 82 458 | 82 465 | 82 471 |
| 668 | 82 478 | 82 484 | 82 491 | 82 497 | 82 504 | 82 510 | 82 517 | 82 523 | 82 530 | 82 536 |
| 669 | 82 543 | 82 549 | 82 556 | 82 562 | 82 569 | 82 575 | 82 582 | 82 588 | 82 595 | 82 601 |
| 670 | 82 607 | 82 614 | 82 620 | 82 627 | 82 633 | 82 640 | 82 646 | 82 653 | 82 659 | 82 666 |
| 671 | 82 672 | 82 679 | 82 685 | 82 692 | 82 698 | 82 705 | 82 711 | 82 718 | 82 724 | 82 730 |
| 672 | 82 737 | 82 743 | 82 750 | 82 756 | 82 763 | 82 769 | 82 776 | 82 782 | 82 789 | 82 795 |
| 673 | 82 802 | 82 808 | 82 814 | 82 821 | 82 827 | 82 834 | 82 840 | 82 847 | 82 853 | 82 860 |
| 674 | 82 866 | 82 872 | 82 879 | 82 885 | 82 892 | 82 898 | 82 905 | 82 911 | 82 918 | 82 924 |
| 675 | 82 930 | 82 937 | 82 943 | 82 950 | 82 956 | 82 963 | 82 969 | 82 975 | 82 982 | 82 988 |
| 676 | 82 995 | 83 001 | 83 008 | 83 014 | 83 020 | 83 027 | 83 033 | 83 040 | 83 046 | 83 052 |
| 677 | 83 059 | 83 065 | 83 072 | 83 078 | 83 085 | 83 091 | 83 097 | 83 104 | 83 110 | 83 117 |
| 678 | 83 123 | 83 129 | 83 136 | 83 142 | 83 149 | 83 155 | 83 161 | 83 168 | 83 174 | 83 181 |
| 679 | 83 187 | 83 193 | 83 200 | 83 206 | 83 213 | 83 219 | 83 225 | 83 232 | 83 238 | 83 245 |
| 680 | 83 251 | 83 257 | 83 264 | 83 270 | 83 276 | 83 283 | 83 289 | 83 296 | 83 302 | 83 308 |
| 681 | 83 315 | 83 321 | 83 327 | 83 334 | 83 340 | 83 347 | 83 353 | 83 359 | 83 366 | 83 372 |
| 682 | 83 378 | 83 385 | 83 391 | 83 398 | 83 404 | 83 410 | 83 417 | 83 423 | 83 429 | 83 436 |
| 683 | 83 442 | 83 448 | 83 455 | 83 461 | 83 467 | 83 474 | 83 480 | 83 487 | 83 493 | 83 499 |
| 684 | 83 506 | 83 512 | 83 518 | 83 525 | 83 531 | 83 537 | 83 544 | 83 550 | 83 556 | 83 563 |
| 685 | 83 569 | 83 575 | 83 582 | 83 588 | 83 594 | 83 601 | 83 607 | 83 613 | 83 620 | 83 626 |
| 686 | 83 632 | 83 639 | 83 645 | 83 651 | 83 658 | 83 664 | 83 670 | 83 677 | 83 683 | 83 689 |
| 687 | 83 696 | 83 702 | 83 708 | 83 715 | 83 721 | 83 727 | 83 734 | 83 740 | 83 746 | 83 753 |
| 688 | 83 759 | 83 765 | 83 771 | 83 778 | 83 784 | 83 790 | 83 797 | 83 803 | 83 809 | 83 816 |
| 689 | 83 822 | 83 828 | 83 835 | 83 841 | 83 847 | 83 853 | 83 860 | 83 866 | 83 872 | 83 879 |
| 690 | 83 885 | 83 891 | 83 897 | 83 904 | 83 910 | 83 916 | 83 923 | 83 929 | 83 935 | 83 942 |
| 691 | 83 948 | 83 954 | 83 960 | 83 967 | 83 973 | 83 979 | 83 985 | 83 992 | 83 998 | 84 004 |
| 692 | 84 011 | 84 017 | 84 023 | 84 029 | 84 036 | 84 042 | 84 048 | 84 055 | 84 061 | 84 067 |
| 693 | 84 073 | 84 080 | 84 086 | 84 092 | 84 098 | 84 105 | 84 111 | 84 117 | 84 123 | 84 130 |
| 694 | 84 136 | 84 142 | 84 148 | 84 155 | 84 161 | 84 167 | 84 173 | 84 180 | 84 186 | 84 192 |
| 695 | 84 198 | 84 205 | 84 211 | 84 217 | 84 223 | 84 230 | 84 236 | 84 242 | 84 248 | 84 255 |
| 696 | 84 261 | 84 267 | 84 273 | 84 280 | 84 286 | 84 292 | 84 298 | 84 305 | 84 311 | 84 317 |
| 697 | 84 323 | 84 330 | 84 336 | 84 342 | 84 348 | 84 354 | 84 361 | 84 367 | 84 373 | 84 379 |
| 698 | 84 386 | 84 392 | 84 398 | 84 404 | 84 410 | 84 417 | 84 423 | 84 429 | 84 435 | 84 442 |
| 699 | 84 448 | 84 454 | 84 460 | 84 466 | 84 473 | 84 479 | 84 485 | 84 491 | 84 497 | 84 504 |
| 700 | 84 510 | 84 516 | 84 522 | 84 528 | 84 535 | 84 541 | 84 547 | 84 553 | 84 559 | 84 566 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 700 | 84 510 | 84 516 | 84 522 | 84 528 | 84 535 | 84 541 | 84 547 | 84 553 | 84 559 | 84 566 |
| 701 | 84 572 | 84 578 | 84 584 | 84 590 | 84 597 | 84 603 | 84 609 | 84 615 | 84 621 | 84 628 |
| 702 | 84 634 | 84 640 | 84 646 | 84 652 | 84 658 | 84 665 | 84 671 | 84 677 | 84 683 | 84 689 |
| 703 | 84 696 | 84 702 | 84 708 | 84 714 | 84 720 | 84 726 | 84 733 | 84 739 | 84 745 | 84 751 |
| 704 | 84 757 | 84 763 | 84 770 | 84 776 | 84 782 | 84 788 | 84 794 | 84 800 | 84 807 | 84 813 |
| 705 | 84 819 | 84 825 | 84 831 | 84 837 | 84 844 | 84 850 | 84 856 | 84 862 | 84 868 | 84 874 |
| 706 | 84 880 | 84 887 | 84 893 | 84 899 | 84 905 | 84 911 | 84 917 | 84 924 | 84 930 | 84 936 |
| 707 | 84 942 | 84 948 | 84 954 | 84 960 | 84 967 | 84 973 | 84 979 | 84 985 | 84 991 | 84 997 |
| 708 | 85 003 | 85 009 | 85 016 | 85 022 | 85 028 | 85 034 | 85 040 | 85 046 | 85 052 | 85 058 |
| 709 | 85 065 | 85 071 | 85 077 | 85 083 | 85 089 | 85 095 | 85 101 | 85 107 | 85 114 | 85 120 |
| 710 | 85 126 | 85 132 | 85 138 | 85 144 | 85 150 | 85 156 | 85 163 | 85 169 | 85 175 | 85 181 |
| 711 | 85 187 | 85 193 | 85 199 | 85 205 | 85 211 | 85 217 | 85 224 | 85 230 | 85 236 | 85 242 |
| 712 | 85 248 | 85 254 | 85 260 | 85 266 | 85 272 | 85 278 | 85 285 | 85 291 | 85 297 | 85 303 |
| 713 | 85 309 | 85 315 | 85 321 | 85 327 | 85 333 | 85 339 | 85 345 | 85 352 | 85 358 | 85 364 |
| 714 | 85 370 | 85 376 | 85 382 | 85 388 | 85 394 | 85 400 | 85 406 | 85 412 | 85 418 | 85 425 |
| 715 | 85 431 | 85 437 | 85 443 | 85 449 | 85 455 | 85 461 | 85 467 | 85 473 | 85 479 | 85 485 |
| 716 | 85 491 | 85 497 | 85 503 | 85 509 | 85 516 | 85 522 | 85 528 | 85 534 | 85 540 | 85 546 |
| 717 | 85 552 | 85 558 | 85 564 | 85 570 | 85 576 | 85 582 | 85 588 | 85 594 | 85 600 | 85 606 |
| 718 | 85 612 | 85 618 | 85 625 | 85 631 | 85 637 | 85 643 | 85 649 | 85 655 | 85 661 | 85 667 |
| 719 | 85 673 | 85 679 | 85 685 | 85 691 | 85 697 | 85 703 | 85 709 | 85 715 | 85 721 | 85 727 |
| 720 | 85 733 | 85 739 | 85 745 | 85 751 | 85 757 | 85 763 | 85 769 | 85 775 | 85 781 | 85 788 |
| 721 | 85 794 | 85 800 | 85 806 | 85 812 | 85 818 | 85 824 | 85 830 | 85 836 | 85 842 | 85 848 |
| 722 | 85 854 | 85 860 | 85 866 | 85 872 | 85 878 | 85 884 | 85 890 | 85 896 | 85 902 | 85 908 |
| 723 | 85 914 | 85 920 | 85 926 | 85 932 | 85 938 | 85 944 | 85 950 | 85 956 | 85 962 | 85 968 |
| 724 | 85 974 | 85 980 | 85 986 | 85 992 | 85 998 | 86 004 | 86 010 | 86 016 | 86 022 | 86 028 |
| 725 | 86 034 | 86 040 | 86 046 | 86 052 | 86 058 | 86 064 | 86 070 | 86 076 | 86 082 | 86 088 |
| 726 | 86 094 | 86 100 | 86 106 | 86 112 | 86 118 | 86 124 | 86 130 | 86 136 | 86 141 | 86 147 |
| 727 | 86 153 | 86 159 | 86 165 | 86 171 | 86 177 | 86 183 | 86 189 | 86 195 | 86 201 | 86 207 |
| 728 | 86 213 | 86 219 | 86 225 | 86 231 | 86 237 | 86 243 | 86 249 | 86 255 | 86 261 | 86 267 |
| 729 | 86 273 | 86 279 | 86 285 | 86 291 | 86 297 | 86 303 | 86 308 | 86 314 | 86 320 | 86 326 |
| 730 | 86 332 | 86 338 | 86 344 | 86 350 | 86 356 | 86 362 | 86 368 | 86 374 | 86 380 | 86 386 |
| 731 | 86 392 | 86 398 | 86 404 | 86 410 | 86 415 | 86 421 | 86 427 | 86 433 | 86 439 | 86 445 |
| 732 | 86 451 | 86 457 | 86 463 | 86 469 | 86 475 | 86 481 | 86 487 | 86 493 | 86 499 | 86 504 |
| 733 | 86 510 | 86 516 | 86 522 | 86 528 | 86 534 | 86 540 | 86 546 | 86 552 | 86 558 | 86 564 |
| 734 | 86 570 | 86 576 | 86 581 | 86 587 | 86 593 | 86 599 | 86 605 | 86 611 | 86 617 | 86 623 |
| 735 | 86 629 | 86 635 | 86 641 | 86 646 | 86 652 | 86 658 | 86 664 | 86 670 | 86 676 | 86 682 |
| 736 | 86 688 | 86 694 | 86 700 | 86 705 | 86 711 | 86 717 | 86 723 | 86 729 | 86 735 | 86 741 |
| 737 | 86 747 | 86 753 | 86 759 | 86 764 | 86 770 | 86 776 | 86 782 | 86 788 | 86 794 | 86 800 |
| 738 | 86 806 | 86 812 | 86 817 | 86 823 | 86 829 | 86 835 | 86 841 | 86 847 | 86 853 | 86 859 |
| 739 | 86 864 | 86 870 | 86 876 | 86 882 | 86 888 | 86 894 | 86 900 | 86 906 | 86 911 | 86 917 |
| 740 | 86 923 | 86 929 | 86 935 | 86 941 | 86 947 | 86 953 | 86 958 | 86 964 | 86 970 | 86 976 |
| 741 | 86 982 | 86 988 | 86 994 | 86 999 | 87 005 | 87 011 | 87 017 | 87 023 | 87 029 | 87 035 |
| 742 | 87 040 | 87 046 | 87 052 | 87 058 | 87 064 | 87 070 | 87 075 | 87 081 | 87 087 | 87 093 |
| 743 | 87 099 | 87 105 | 87 111 | 87 116 | 87 122 | 87 128 | 87 134 | 87 140 | 87 146 | 87 151 |
| 744 | 87 157 | 87 163 | 87 169 | 87 175 | 87 181 | 87 186 | 87 192 | 87 198 | 87 204 | 87 210 |
| 745 | 87 216 | 87 221 | 87 227 | 87 233 | 87 239 | 87 245 | 87 251 | 87 256 | 87 262 | 87 268 |
| 746 | 87 274 | 87 280 | 87 286 | 87 291 | 87 297 | 87 303 | 87 309 | 87 315 | 87 320 | 87 326 |
| 747 | 87 332 | 87 338 | 87 344 | 87 349 | 87 355 | 87 361 | 87 367 | 87 373 | 87 379 | 87 384 |
| 748 | 87 390 | 87 396 | 87 402 | 87 408 | 87 413 | 87 419 | 87 425 | 87 431 | 87 437 | 87 442 |
| 749 | 87 448 | 87 454 | 87 460 | 87 466 | 87 471 | 87 477 | 87 483 | 87 489 | 87 495 | 87 500 |
| 750 | 87 506 | 87 512 | 87 518 | 87 523 | 87 529 | 87 535 | 87 541 | 87 547 | 87 552 | 87 558 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 750 | 87 506 | 87 512 | 87 518 | 87 523 | 87 529 | 87 535 | 87 541 | 87 547 | 87 552 | 87 558 |
| 751 | 87 564 | 87 570 | 87 576 | 87 581 | 87 587 | 87 593 | 87 599 | 87 604 | 87 610 | 87 616 |
| 752 | 87 622 | 87 628 | 87 633 | 87 639 | 87 645 | 87 651 | 87 656 | 87 662 | 87 668 | 87 674 |
| 753 | 87 679 | 87 685 | 87 691 | 87 697 | 87 703 | 87 708 | 87 714 | 87 720 | 87 726 | 87 731 |
| 754 | 87 737 | 87 743 | 87 749 | 87 754 | 87 760 | 87 766 | 87 772 | 87 777 | 87 783 | 87 789 |
| 755 | 87 795 | 87 800 | 87 806 | 87 812 | 87 818 | 87 823 | 87 829 | 87 835 | 87 841 | 87 846 |
| 756 | 87 852 | 87 858 | 87 864 | 87 869 | 87 875 | 87 881 | 87 887 | 87 892 | 87 898 | 87 904 |
| 757 | 87 910 | 87 915 | 87 921 | 87 927 | 87 933 | 87 938 | 87 944 | 87 950 | 87 955 | 87 961 |
| 758 | 87 967 | 87 973 | 87 978 | 87 984 | 87 990 | 87 996 | 88 001 | 88 007 | 88 013 | 88 018 |
| 759 | 88 024 | 88 030 | 88 036 | 88 041 | 88 047 | 88 053 | 88 058 | 88 064 | 88 070 | 88 076 |
| 760 | 88 081 | 88 087 | 88 093 | 88 098 | 88 104 | 88 110 | 88 116 | 88 121 | 88 127 | 88 133 |
| 761 | 88 138 | 88 144 | 88 150 | 88 156 | 88 161 | 88 167 | 88 173 | 88 178 | 88 184 | 88 190 |
| 762 | 88 195 | 88 201 | 88 207 | 88 213 | 88 218 | 88 224 | 88 230 | 88 235 | 88 241 | 88 247 |
| 763 | 88 252 | 88 258 | 88 264 | 88 270 | 88 275 | 88 281 | 88 287 | 88 292 | 88 298 | 88 304 |
| 764 | 88 309 | 88 315 | 88 321 | 88 326 | 88 332 | 88 338 | 88 343 | 88 349 | 88 355 | 88 360 |
| 765 | 88 366 | 88 372 | 88 377 | 88 383 | 88 389 | 88 395 | 88 400 | 88 406 | 88 412 | 88 417 |
| 766 | 88 423 | 88 429 | 88 434 | 88 440 | 88 446 | 88 451 | 88 457 | 88 463 | 88 468 | 88 474 |
| 767 | 88 480 | 88 485 | 88 491 | 88 497 | 88 502 | 88 508 | 88 513 | 88 519 | 88 525 | 88 530 |
| 768 | 88 536 | 88 542 | 88 547 | 88 553 | 88 559 | 88 564 | 88 570 | 88 576 | 88 581 | 88 587 |
| 769 | 88 593 | 88 598 | 88 604 | 88 610 | 88 615 | 88 621 | 88 627 | 88 632 | 88 638 | 88 643 |
| 770 | 88 649 | 88 655 | 88 660 | 88 666 | 88 672 | 88 677 | 88 683 | 88 689 | 88 694 | 88 700 |
| 771 | 88 705 | 88 711 | 88 717 | 88 722 | 88 728 | 88 734 | 88 739 | 88 745 | 88 750 | 88 756 |
| 772 | 88 762 | 88 767 | 88 773 | 88 779 | 88 784 | 88 790 | 88 795 | 88 801 | 88 807 | 88 812 |
| 773 | 88 818 | 88 824 | 88 829 | 88 835 | 88 840 | 88 846 | 88 852 | 88 857 | 88 863 | 88 868 |
| 774 | 88 874 | 88 880 | 88 885 | 88 891 | 88 897 | 88 902 | 88 908 | 88 913 | 88 919 | 88 925 |
| 775 | 88 930 | 88 936 | 88 941 | 88 947 | 88 953 | 88 958 | 88 964 | 88 969 | 88 975 | 88 981 |
| 776 | 88 986 | 88 992 | 88 997 | 89 003 | 89 009 | 89 014 | 89 020 | 89 025 | 89 031 | 89 037 |
| 777 | 89 042 | 89 048 | 89 053 | 89 059 | 89 064 | 89 070 | 89 076 | 89 081 | 89 087 | 89 092 |
| 778 | 89 098 | 89 104 | 89 109 | 89 115 | 89 120 | 89 126 | 89 131 | 89 137 | 89 143 | 89 148 |
| 779 | 89 154 | 89 159 | 89 165 | 89 170 | 89 176 | 89 182 | 89 187 | 89 193 | 89 198 | 89 204 |
| 780 | 89 209 | 89 215 | 89 221 | 89 226 | 89 232 | 89 237 | 89 243 | 89 248 | 89 254 | 89 260 |
| 781 | 89 265 | 89 271 | 89 276 | 89 282 | 89 287 | 89 293 | 89 298 | 89 304 | 89 310 | 89 315 |
| 782 | 89 321 | 89 326 | 89 332 | 89 337 | 89 343 | 89 348 | 89 354 | 89 360 | 89 365 | 89 371 |
| 783 | 89 376 | 89 382 | 89 387 | 89 393 | 89 398 | 89 404 | 89 409 | 89 415 | 89 421 | 89 426 |
| 784 | 89 432 | 89 437 | 89 443 | 89 448 | 89 454 | 89 459 | 89 465 | 89 470 | 89 476 | 89 481 |
| 785 | 89 487 | 89 492 | 89 498 | 89 504 | 89 509 | 89 515 | 89 520 | 89 526 | 89 531 | 89 537 |
| 786 | 89 542 | 89 548 | 89 553 | 89 559 | 89 564 | 89 570 | 89 575 | 89 581 | 89 586 | 89 592 |
| 787 | 89 597 | 89 603 | 89 609 | 89 614 | 89 620 | 89 625 | 89 631 | 89 636 | 89 642 | 89 647 |
| 788 | 89 653 | 89 658 | 89 664 | 89 669 | 89 675 | 89 680 | 89 686 | 89 691 | 89 697 | 89 702 |
| 789 | 89 708 | 89 713 | 89 719 | 89 724 | 89 730 | 89 735 | 89 741 | 89 746 | 89 752 | 89 757 |
| 790 | 89 763 | 89 768 | 89 774 | 89 779 | 89 785 | 89 790 | 89 796 | 89 801 | 89 807 | 89 812 |
| 791 | 89 818 | 89 823 | 89 829 | 89 834 | 89 840 | 89 845 | 89 851 | 89 856 | 89 862 | 89 867 |
| 792 | 89 873 | 89 878 | 89 883 | 89 889 | 89 894 | 89 900 | 89 905 | 89 911 | 89 916 | 89 922 |
| 793 | 89 927 | 89 933 | 89 938 | 89 944 | 89 949 | 89 955 | 89 960 | 89 966 | 89 971 | 89 977 |
| 794 | 89 982 | 89 988 | 89 993 | 89 998 | 90 004 | 90 009 | 90 015 | 90 020 | 90 026 | 90 031 |
| 795 | 90 037 | 90 042 | 90 048 | 90 053 | 90 059 | 90 064 | 90 069 | 90 075 | 90 080 | 90 086 |
| 796 | 90 091 | 90 097 | 90 102 | 90 108 | 90 113 | 90 119 | 90 124 | 90 129 | 90 135 | 90 140 |
| 797 | 90 146 | 90 151 | 90 157 | 90 162 | 90 168 | 90 173 | 90 179 | 90 184 | 90 189 | 90 195 |
| 798 | 90 200 | 90 206 | 90 211 | 90 217 | 90 222 | 90 227 | 90 233 | 90 238 | 90 244 | 90 249 |
| 799 | 90 255 | 90 260 | 90 266 | 90 271 | 90 276 | 90 282 | 90 287 | 90 293 | 90 298 | 90 304 |
| 800 | 90 309 | 90 314 | 90 320 | 90 325 | 90 331 | 90 336 | 90 342 | 90 347 | 90 352 | 90 358 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 800 | 90 309 | 90 314 | 90 320 | 90 325 | 90 331 | 90 336 | 90 342 | 90 347 | 90 352 | 90 358 |
| 801 | 90 363 | 90 369 | 90 374 | 90 380 | 90 385 | 90 390 | 90 396 | 90 401 | 90 407 | 90 412 |
| 802 | 90 417 | 90 423 | 90 428 | 90 434 | 90 439 | 90 445 | 90 450 | 90 455 | 90 461 | 90 466 |
| 803 | 90 472 | 90 477 | 90 482 | 90 488 | 90 493 | 90 499 | 90 504 | 90 509 | 90 515 | 90 520 |
| 804 | 90 526 | 90 531 | 90 536 | 80 542 | 90 547 | 90 553 | 90 558 | 90 563 | 90 569 | 90 574 |
| 805 | 90 580 | 90 585 | 90 590 | 90 596 | 90 601 | 90 607 | 90 612 | 90 617 | 90 623 | 90 628 |
| 806 | 90 634 | 90 639 | 90 644 | 90 650 | 90 655 | 90 660 | 90 666 | 90 671 | 90 677 | 90 682 |
| 807 | 90 687 | 90 693 | 90 698 | 90 703 | 90 709 | 90 714 | 90 720 | 90 725 | 90 730 | 90 736 |
| 808 | 90 741 | 90 747 | 90 752 | 90 757 | 90 763 | 90 768 | 90 773 | 90 779 | 90 784 | 90 789 |
| 809 | 90 795 | 90 800 | 90 806 | 90 811 | 90 816 | 90 822 | 90 827 | 90 832 | 90 838 | 90 843 |
| 810 | 90 849 | 90 854 | 90 859 | 90 865 | 90 870 | 90 875 | 90 881 | 90 886 | 90 891 | 90 897 |
| 811 | 90 902 | 90 907 | 90 913 | 90 918 | 90 924 | 90 929 | 90 934 | 90 940 | 90 945 | 90 950 |
| 812 | 90 956 | 90 961 | 90 966 | 90 972 | 90 977 | 90 982 | 90 988 | 90 993 | 90 998 | 91 004 |
| 813 | 91 009 | 91 014 | 91 020 | 91 025 | 91 030 | 91 036 | 91 041 | 91 046 | 91 052 | 91 057 |
| 814 | 91 062 | 91 068 | 91 073 | 91 078 | 91 084 | 91 089 | 91 094 | 91 100 | 91 105 | 91 110 |
| 815 | 91 116 | 91 121 | 91 126 | 91 132 | 91 137 | 91 142 | 91 148 | 91 153 | 91 158 | 91 164 |
| 816 | 91 169 | 91 174 | 91 180 | 91 185 | 91 190 | 91 196 | 91 201 | 91 206 | 91 212 | 91 217 |
| 817 | 91 222 | 91 228 | 91 233 | 91 238 | 91 243 | 91 249 | 91 254 | 91 259 | 91 265 | 91 270 |
| 818 | 91 275 | 91 281 | 91 286 | 91 291 | 91 297 | 91 302 | 91 307 | 91 312 | 91 318 | 91 323 |
| 819 | 91 328 | 91 334 | 91 339 | 91 344 | 91 350 | 91 355 | 91 360 | 91 365 | 91 371 | 91 376 |
| 820 | 91 381 | 91 387 | 91 392 | 91 397 | 91 403 | 91 408 | 91 413 | 91 418 | 91 424 | 91 429 |
| 821 | 91 434 | 91 440 | 91 445 | 91 450 | 91 455 | 91 461 | 91 466 | 91 471 | 91 477 | 91 482 |
| 822 | 91 487 | 91 492 | 91 498 | 91 503 | 91 508 | 91 514 | 91 519 | 91 524 | 91 529 | 91 535 |
| 823 | 91 540 | 91 545 | 91 551 | 91 556 | 91 561 | 91 566 | 91 572 | 91 577 | 91 582 | 91 587 |
| 824 | 91 593 | 91 598 | 91 603 | 91 609 | 91 614 | 91 619 | 91 624 | 91 630 | 91 635 | 91 640 |
| 825 | 91 645 | 91 651 | 91 656 | 91 661 | 91 666 | 91 672 | 91 677 | 91 682 | 91 687 | 91 693 |
| 826 | 91 698 | 91 703 | 91 709 | 91 714 | 91 719 | 91 724 | 91 730 | 91 735 | 91 740 | 91 745 |
| 827 | 91 751 | 91 756 | 91 761 | 91 766 | 91 772 | 91 777 | 91 782 | 91 787 | 91 793 | 91 798 |
| 828 | 91 803 | 91 808 | 91 814 | 91 819 | 91 824 | 91 829 | 91 834 | 91 840 | 91 845 | 91 850 |
| 829 | 91 855 | 91 861 | 91 866 | 91 871 | 91 876 | 91 882 | 91 887 | 91 892 | 91 897 | 91 903 |
| 830 | 91 908 | 91 913 | 91 918 | 91 924 | 91 929 | 91 934 | 91 939 | 91 944 | 91 950 | 91 955 |
| 831 | 91 960 | 91 965 | 91 971 | 91 976 | 91 981 | 91 986 | 91 991 | 91 997 | 92 002 | 92 007 |
| 832 | 92 012 | 92 018 | 92 023 | 92 028 | 92 033 | 92 038 | 92 044 | 92 049 | 92 054 | 92 059 |
| 833 | 92 065 | 92 070 | 92 075 | 92 080 | 92 085 | 92 091 | 92 096 | 92 101 | 92 106 | 92 111 |
| 834 | 92 117 | 92 122 | 92 127 | 92 132 | 92 137 | 92 143 | 92 148 | 92 153 | 92 158 | 92 163 |
| 835 | 92 169 | 92 174 | 92 179 | 92 184 | 92 189 | 92 195 | 92 200 | 92 205 | 92 210 | 92 215 |
| 836 | 92 221 | 92 226 | 92 231 | 92 236 | 92 241 | 92 247 | 92 252 | 92 257 | 92 262 | 92 267 |
| 837 | 92 273 | 92 278 | 92 283 | 92 288 | 92 293 | 92 298 | 92 304 | 92 309 | 92 314 | 92 319 |
| 838 | 92 324 | 92 330 | 92 335 | 92 340 | 92 345 | 92 350 | 92 355 | 92 361 | 92 366 | 92 371 |
| 839 | 92 376 | 92 381 | 92 387 | 92 392 | 92 397 | 92 402 | 92 407 | 92 412 | 92 418 | 92 423 |
| 840 | 92 428 | 92 433 | 92 438 | 92 443 | 92 449 | 92 454 | 92 459 | 92 464 | 92 469 | 92 474 |
| 841 | 92 480 | 92 485 | 92 490 | 92 495 | 92 500 | 92 505 | 92 511 | 92 516 | 92 521 | 92 526 |
| 842 | 92 531 | 92 536 | 92 542 | 92 547 | 92 552 | 92 557 | 92 562 | 92 567 | 92 572 | 92 578 |
| 843 | 92 583 | 92 588 | 92 593 | 92 598 | 92 603 | 92 609 | 92 614 | 92 619 | 92 624 | 92 629 |
| 844 | 92 634 | 92 639 | 92 645 | 92 650 | 92 655 | 92 660 | 92 665 | 92 670 | 92 675 | 92 681 |
| 845 | 92 686 | 92 691 | 92 696 | 92 701 | 92 706 | 92 711 | 92 716 | 92 722 | 92 727 | 92 732 |
| 846 | 92 737 | 92 742 | 92 747 | 92 752 | 92 758 | 92 763 | 92 768 | 92 773 | 92 778 | 92 783 |
| 847 | 92 788 | 92 793 | 92 799 | 92 804 | 92 809 | 92 814 | 92 819 | 92 824 | 92 829 | 92 834 |
| 848 | 92 840 | 92 845 | 92 850 | 92 855 | 92 860 | 92 865 | 92 870 | 92 875 | 92 881 | 92 886 |
| 849 | 92 891 | 92 896 | 92 901 | 92 906 | 92 911 | 92 916 | 92 921 | 92 927 | 92 932 | 92 937 |
| 850 | 92 942 | 92 947 | 92 952 | 92 957 | 92 962 | 92 967 | 92 973 | 92 978 | 92 983 | 92 988 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 850 | 92 942 | 92 947 | 92 952 | 92 957 | 92 962 | 92 967 | 92 973 | 92 978 | 92 983 | 92 988 |
| 851 | 92 993 | 92 998 | 93 003 | 93 008 | 93 013 | 93 018 | 93 024 | 93 029 | 93 034 | 93 039 |
| 852 | 93 044 | 93 049 | 93 054 | 93 059 | 93 064 | 93 069 | 93 075 | 93 080 | 93 085 | 93 090 |
| 853 | 93 095 | 93 100 | 93 105 | 93 110 | 93 115 | 93 120 | 93 125 | 93 131 | 93 136 | 93 141 |
| 854 | 93 146 | 93 151 | 93 156 | 93 161 | 93 166 | 93 171 | 93 176 | 93 181 | 93 186 | 93 192 |
| 855 | 93 197 | 93 202 | 93 207 | 93 212 | 93 217 | 93 222 | 93 227 | 93 232 | 93 237 | 93 242 |
| 856 | 93 247 | 93 252 | 93 258 | 93 263 | 93 268 | 93 273 | 93 278 | 93 283 | 93 288 | 93 293 |
| 857 | 93 298 | 93 303 | 93 308 | 93 313 | 93 318 | 93 323 | 93 328 | 93 334 | 93 339 | 93 344 |
| 858 | 93 349 | 93 354 | 93 359 | 93 364 | 93 369 | 93 374 | 93 379 | 93 384 | 93 389 | 93 394 |
| 859 | 93 399 | 93 404 | 93 409 | 93 414 | 93 420 | 93 425 | 93 430 | 93 435 | 93 440 | 93 445 |
| 860 | 93 450 | 93 455 | 93 460 | 93 465 | 93 470 | 93 475 | 93 480 | 93 485 | 93 490 | 93 495 |
| 861 | 93 500 | 93 505 | 93 510 | 93 515 | 93 520 | 93 526 | 93 531 | 93 536 | 93 541 | 93 546 |
| 862 | 93 551 | 93 556 | 93 561 | 93 566 | 93 571 | 93 576 | 93 581 | 93 586 | 93 591 | 93 596 |
| 863 | 93 601 | 93 606 | 93 611 | 93 616 | 93 621 | 93 626 | 93 631 | 93 636 | 93 641 | 93 646 |
| 864 | 93 651 | 93 656 | 93 661 | 93 666 | 93 671 | 93 676 | 93 682 | 93 687 | 93 692 | 93 697 |
| 865 | 93 702 | 93 707 | 93 712 | 93 717 | 93 722 | 93 727 | 93 732 | 93 737 | 93 742 | 93 747 |
| 866 | 93 752 | 93 757 | 93 762 | 93 767 | 93 772 | 93 777 | 93 782 | 93 787 | 93 792 | 93 797 |
| 867 | 93 802 | 93 807 | 93 812 | 93 817 | 93 822 | 93 827 | 93 832 | 93 837 | 93 842 | 93 847 |
| 868 | 93 852 | 93 857 | 93 862 | 93 867 | 93 872 | 93 877 | 93 882 | 93 887 | 93 892 | 93 897 |
| 869 | 93 902 | 93 907 | 93 912 | 93 917 | 93 922 | 93 927 | 93 932 | 93 937 | 93 942 | 93 947 |
| 870 | 93 952 | 93 957 | 93 962 | 93 967 | 93 972 | 93 977 | 93 982 | 93 987 | 93 992 | 93 997 |
| 871 | 94 002 | 94 007 | 94 012 | 94 017 | 94 022 | 94 027 | 94 032 | 94 037 | 94 042 | 94 047 |
| 872 | 94 052 | 94 057 | 94 062 | 94 067 | 94 072 | 94 077 | 94 082 | 94 086 | 94 091 | 94 096 |
| 873 | 94 101 | 94 106 | 94 111 | 94 116 | 94 121 | 94 126 | 94 131 | 94 136 | 94 141 | 94 146 |
| 874 | 94 151 | 94 156 | 94 161 | 94 166 | 94 171 | 94 176 | 94 181 | 94 186 | 94 191 | 94 196 |
| 875 | 94 201 | 94 206 | 94 211 | 94 216 | 94 221 | 94 226 | 94 231 | 94 236 | 94 240 | 94 245 |
| 876 | 94 250 | 94 255 | 94 260 | 94 265 | 94 270 | 94 275 | 94 280 | 94 285 | 94 290 | 94 295 |
| 877 | 94 300 | 94 305 | 94 310 | 94 315 | 94 320 | 94 325 | 94 330 | 94 335 | 94 340 | 94 345 |
| 878 | 94 349 | 94 354 | 94 359 | 94 364 | 94 369 | 94 374 | 94 379 | 94 384 | 94 389 | 94 394 |
| 879 | 94 399 | 94 404 | 94 409 | 94 414 | 94 419 | 94 424 | 94 429 | 94 433 | 94 438 | 94 443 |
| 880 | 94 448 | 94 453 | 94 458 | 94 463 | 94 468 | 94 473 | 94 478 | 94 483 | 94 488 | 94 493 |
| 881 | 94 498 | 94 503 | 94 507 | 94 512 | 94 517 | 94 522 | 94 527 | 94 532 | 94 537 | 94 542 |
| 882 | 94 547 | 94 552 | 94 557 | 94 562 | 94 567 | 94 571 | 94 576 | 94 581 | 94 586 | 94 591 |
| 883 | 94 596 | 94 601 | 94 606 | 94 611 | 94 616 | 94 621 | 94 626 | 94 630 | 94 635 | 94 640 |
| 884 | 94 645 | 94 650 | 94 655 | 94 660 | 94 665 | 94 670 | 94 675 | 94 680 | 94 685 | 94 689 |
| 885 | 94 694 | 94 699 | 94 704 | 94 709 | 94 714 | 94 719 | 94 724 | 94 729 | 94 734 | 94 738 |
| 886 | 94 743 | 94 748 | 94 753 | 94 758 | 94 763 | 94 768 | 94 773 | 94 778 | 94 783 | 94 787 |
| 887 | 94 792 | 94 797 | 94 802 | 94 807 | 94 812 | 94 817 | 94 822 | 94 827 | 94 832 | 94 836 |
| 888 | 94 841 | 94 846 | 94 851 | 94 856 | 94 861 | 94 866 | 94 871 | 94 876 | 94 880 | 94 885 |
| 889 | 94 890 | 94 895 | 94 900 | 94 905 | 94 910 | 94 915 | 94 919 | 94 924 | 94 929 | 94 934 |
| 890 | 94 939 | 94 944 | 94 949 | 94 954 | 94 959 | 94 963 | 94 968 | 94 973 | 94 978 | 94 983 |
| 891 | 94 988 | 94 993 | 94 998 | 95 002 | 95 007 | 95 012 | 95 017 | 95 022 | 95 027 | 95 032 |
| 892 | 95 036 | 95 041 | 95 046 | 95 051 | 95 056 | 95 061 | 95 066 | 95 071 | 95 075 | 95 080 |
| 893 | 95 085 | 95 090 | 95 095 | 95 100 | 95 105 | 95 109 | 95 114 | 95 119 | 95 124 | 95 129 |
| 894 | 95 134 | 95 139 | 95 143 | 95 148 | 95 153 | 95 158 | 95 163 | 95 168 | 95 173 | 95 177 |
| 895 | 95 182 | 95 187 | 95 192 | 95 197 | 95 202 | 95 207 | 95 211 | 95 216 | 95 221 | 95 226 |
| 896 | 95 231 | 95 236 | 95 240 | 95 245 | 95 250 | 95 255 | 95 260 | 95 265 | 95 270 | 95 274 |
| 897 | 95 279 | 95 284 | 95 289 | 95 294 | 95 299 | 95 303 | 95 308 | 95 313 | 95 318 | 95 323 |
| 898 | 95 328 | 95 332 | 95 337 | 95 342 | 95 347 | 95 352 | 95 357 | 95 361 | 95 366 | 95 371 |
| 899 | 95 376 | 95 381 | 95 386 | 95 390 | 95 395 | 95 400 | 95 405 | 95 410 | 95 415 | 95 419 |
| 900 | 95 424 | 95 429 | 95 434 | 95 439 | 95 444 | 95 448 | 95 453 | 95 458 | 95 463 | 95 468 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 900 | 95 424 | 95 429 | 95 434 | 95 439 | 95 444 | 95 448 | 95 453 | 95 458 | 95 463 | 95 468 |
| 901 | 95 472 | 95 477 | 95 482 | 95 487 | 95 492 | 95 497 | 95 501 | 95 506 | 95 511 | 95 516 |
| 902 | 95 521 | 95 525 | 95 530 | 95 535 | 95 540 | 95 545 | 95 550 | 95 554 | 95 559 | 95 564 |
| 903 | 95 569 | 95 574 | 95 578 | 95 583 | 95 588 | 95 592 | 95 598 | 95 602 | 95 607 | 95 612 |
| 904 | 95 617 | 95 622 | 95 626 | 95 631 | 95 636 | 95 641 | 95 646 | 95 650 | 95 655 | 95 660 |
| 905 | 95 665 | 95 670 | 95 674 | 95 679 | 95 684 | 95 689 | 95 694 | 95 698 | 95 703 | 95 708 |
| 906 | 95 713 | 95 718 | 95 722 | 95 727 | 95 732 | 95 737 | 95 742 | 95 746 | 95 751 | 95 756 |
| 907 | 95 761 | 95 766 | 95 770 | 95 775 | 95 780 | 95 785 | 95 789 | 95 794 | 95 799 | 95 804 |
| 908 | 95 809 | 95 813 | 95 818 | 95 823 | 95 828 | 95 832 | 95 837 | 95 842 | 95 847 | 95 852 |
| 909 | 95 856 | 95 861 | 95 866 | 95 871 | 95 875 | 95 880 | 95 885 | 95 890 | 95 895 | 95 899 |
| 910 | 95 904 | 95 909 | 95 914 | 95 918 | 95 923 | 95 928 | 95 933 | 95 938 | 95 942 | 95 947 |
| 911 | 95 952 | 95 957 | 95 961 | 95 966 | 95 971 | 95 976 | 95 980 | 95 985 | 95 990 | 95 995 |
| 912 | 95 999 | 96 004 | 96 009 | 96 014 | 96 019 | 96 023 | 96 028 | 96 033 | 96 038 | 96 042 |
| 913 | 96 047 | 96 052 | 96 057 | 96 061 | 96 066 | 96 071 | 96 076 | 96 081 | 96 085 | 96 090 |
| 914 | 96 095 | 96 099 | 96 104 | 96 109 | 96 114 | 96 118 | 96 123 | 96 128 | 96 133 | 96 137 |
| 915 | 96 142 | 96 147 | 96 152 | 96 156 | 96 161 | 96 166 | 96 171 | 96 175 | 96 180 | 96 185 |
| 916 | 96 190 | 96 194 | 96 199 | 96 204 | 96 209 | 96 213 | 96 218 | 96 223 | 96 227 | 96 232 |
| 917 | 96 237 | 96 242 | 96 246 | 96 251 | 96 256 | 96 261 | 96 265 | 96 270 | 96 275 | 96 280 |
| 918 | 96 284 | 96 289 | 96 294 | 96 298 | 96 303 | 96 308 | 96 313 | 96 317 | 96 322 | 96 327 |
| 919 | 96 332 | 96 336 | 96 341 | 96 346 | 96 350 | 96 355 | 96 360 | 96 365 | 96 369 | 96 374 |
| 920 | 96 379 | 96 384 | 96 388 | 96 393 | 96 398 | 96 402 | 96 407 | 96 412 | 96 417 | 96 421 |
| 921 | 96 426 | 96 431 | 96 435 | 96 440 | 96 445 | 96 450 | 96 454 | 96 459 | 96 464 | 96 468 |
| 922 | 96 473 | 96 478 | 96 483 | 96 487 | 96 492 | 96 497 | 96 501 | 96 506 | 96 511 | 96 515 |
| 923 | 96 520 | 96 525 | 96 530 | 96 534 | 96 539 | 96 544 | 96 548 | 96 553 | 96 558 | 96 562 |
| 924 | 96 567 | 96 572 | 96 577 | 96 581 | 96 586 | 96 591 | 96 595 | 96 600 | 96 605 | 96 609 |
| 925 | 96 614 | 96 619 | 96 624 | 96 628 | 96 633 | 96 638 | 96 642 | 96 647 | 96 652 | 96 656 |
| 926 | 96 661 | 96 666 | 96 670 | 96 675 | 96 680 | 96 685 | 96 689 | 96 694 | 96 699 | 96 703 |
| 927 | 96 708 | 96 713 | 96 717 | 96 722 | 96 727 | 96 731 | 96 736 | 96 741 | 96 745 | 96 750 |
| 928 | 96 755 | 96 759 | 96 764 | 96 769 | 96 774 | 96 778 | 96 783 | 96 788 | 96 792 | 96 797 |
| 929 | 96 802 | 96 806 | 96 811 | 96 816 | 96 820 | 96 825 | 96 830 | 96 834 | 96 839 | 96 844 |
| 930 | 96 848 | 96 853 | 96 858 | 96 862 | 96 867 | 96 872 | 96 876 | 96 881 | 96 886 | 96 890 |
| 931 | 96 895 | 96 900 | 96 904 | 96 909 | 96 914 | 96 918 | 96 923 | 96 928 | 96 932 | 96 937 |
| 932 | 96 942 | 96 946 | 96 951 | 96 956 | 96 960 | 96 965 | 96 970 | 96 974 | 96 979 | 96 984 |
| 933 | 96 988 | 96 993 | 96 997 | 97 002 | 97 007 | 97 011 | 97 016 | 97 021 | 97 025 | 97 030 |
| 934 | 97 035 | 97 039 | 97 044 | 97 049 | 97 053 | 97 058 | 97 063 | 97 067 | 97 072 | 97 077 |
| 935 | 97 081 | 97 086 | 97 090 | 97 095 | 97 100 | 97 104 | 97 109 | 97 114 | 97 118 | 97 123 |
| 936 | 97 128 | 97 132 | 97 137 | 97 142 | 97 146 | 97 151 | 97 155 | 97 160 | 97 165 | 97 169 |
| 937 | 97 174 | 97 179 | 97 183 | 97 188 | 97 192 | 97 197 | 97 202 | 97 206 | 97 211 | 97 216 |
| 938 | 97 220 | 97 225 | 97 230 | 97 234 | 97 239 | 97 243 | 97 248 | 97 253 | 97 257 | 97 262 |
| 939 | 97 267 | 97 271 | 97 276 | 97 280 | 97 285 | 97 290 | 97 294 | 97 299 | 97 304 | 97 308 |
| 940 | 97 313 | 97 317 | 97 322 | 97 327 | 97 331 | 97 336 | 97 340 | 97 345 | 97 350 | 97 354 |
| 941 | 97 359 | 97 364 | 97 368 | 97 373 | 97 377 | 97 382 | 97 387 | 97 391 | 97 396 | 97 400 |
| 942 | 97 405 | 97 410 | 97 414 | 97 419 | 97 424 | 97 428 | 97 433 | 97 437 | 97 442 | 97 447 |
| 943 | 97 451 | 97 456 | 97 460 | 97 465 | 97 470 | 97 474 | 97 479 | 97 483 | 97 488 | 97 493 |
| 944 | 97 497 | 97 502 | 97 506 | 97 511 | 97 516 | 97 520 | 97 525 | 97 529 | 97 534 | 97 539 |
| 945 | 97 543 | 97 548 | 97 552 | 97 557 | 97 562 | 97 566 | 97 571 | 97 575 | 97 580 | 97 585 |
| 946 | 97 589 | 97 594 | 97 598 | 97 603 | 97 607 | 97 612 | 97 617 | 97 621 | 97 626 | 97 630 |
| 947 | 97 635 | 97 640 | 97 644 | 97 649 | 97 653 | 97 658 | 97 663 | 97 667 | 97 672 | 97 676 |
| 948 | 97 681 | 97 685 | 97 690 | 97 695 | 97 699 | 97 704 | 97 708 | 97 713 | 97 717 | 97 722 |
| 949 | 97 727 | 97 731 | 97 736 | 97 740 | 97 745 | 97 749 | 97 754 | 97 759 | 97 763 | 97 768 |
| 950 | 97 772 | 97 777 | 97 782 | 97 786 | 97 791 | 97 795 | 97 800 | 97 804 | 97 809 | 97 813 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 950 | 97 772 | 97 777 | 97 782 | 97 786 | 97 791 | 97 795 | 97 800 | 97 804 | 97 809 | 97 813 |
| 951 | 97 818 | 97 823 | 97 827 | 97 832 | 97 836 | 97 841 | 97 845 | 97 850 | 97 855 | 97 859 |
| 952 | 97 864 | 97 868 | 97 873 | 97 877 | 97 882 | 97 886 | 97 891 | 97 896 | 97 900 | 97 905 |
| 953 | 97 909 | 97 914 | 97 918 | 97 923 | 97 928 | 97 932 | 97 937 | 97 941 | 97 946 | 97 950 |
| 954 | 97 955 | 97 959 | 97 964 | 97 968 | 97 973 | 97 978 | 97 982 | 97 987 | 97 991 | 97 996 |
| 955 | 98 000 | 98 005 | 98 009 | 98 014 | 98 019 | 98 023 | 98 028 | 98 032 | 98 037 | 98 041 |
| 956 | 98 046 | 98 050 | 98 055 | 98 059 | 98 064 | 98 068 | 98 073 | 98 078 | 98 082 | 98 087 |
| 957 | 98 091 | 98 096 | 98 100 | 98 105 | 98 109 | 98 114 | 98 118 | 98 123 | 98 127 | 98 132 |
| 958 | 98 137 | 98 141 | 98 146 | 98 150 | 98 155 | 98 159 | 98 164 | 98 168 | 98 173 | 98 177 |
| 959 | 98 182 | 98 186 | 98 191 | 98 195 | 98 200 | 98 204 | 98 209 | 98 214 | 98 218 | 98 223 |
| 960 | 98 227 | 98 232 | 98 236 | 98 241 | 98 245 | 98 250 | 98 254 | 98 259 | 98 263 | 98 268 |
| 961 | 98 272 | 98 277 | 98 281 | 98 286 | 98 290 | 98 295 | 98 299 | 98 304 | 98 308 | 98 313 |
| 962 | 98 318 | 98 322 | 98 327 | 98 331 | 98 336 | 98 340 | 98 345 | 98 349 | 98 354 | 98 358 |
| 963 | 98 363 | 98 367 | 98 372 | 98 376 | 98 381 | 98 385 | 98 390 | 98 394 | 98 399 | 98 403 |
| 964 | 98 408 | 98 412 | 98 417 | 98 421 | 98 426 | 98 430 | 98 435 | 98 439 | 98 444 | 98 448 |
| 965 | 98 453 | 98 457 | 98 462 | 98 466 | 98 471 | 98 475 | 98 480 | 98 484 | 98 489 | 98 493 |
| 966 | 98 498 | 98 502 | 98 507 | 98 511 | 98 516 | 98 520 | 98 525 | 98 529 | 98 534 | 98 538 |
| 967 | 98 543 | 98 547 | 98 552 | 98 556 | 98 561 | 98 565 | 98 570 | 98 574 | 98 579 | 98 583 |
| 968 | 98 588 | 98 592 | 98 597 | 98 601 | 98 605 | 98 610 | 98 614 | 98 619 | 98 623 | 98 628 |
| 969 | 98 632 | 98 637 | 98 641 | 98 646 | 98 650 | 98 655 | 98 659 | 98 664 | 98 668 | 98 673 |
| 970 | 98 677 | 98 682 | 98 686 | 98 691 | 98 695 | 98 700 | 98 704 | 98 709 | 98 713 | 98 717 |
| 971 | 98 722 | 98 726 | 98 731 | 98 735 | 98 740 | 98 744 | 98 749 | 98 753 | 98 758 | 98 762 |
| 972 | 98 767 | 98 771 | 98 776 | 98 780 | 98 784 | 98 789 | 98 793 | 98 798 | 98 802 | 98 807 |
| 973 | 98 811 | 98 816 | 98 820 | 98 825 | 98 829 | 98 834 | 98 838 | 98 843 | 98 847 | 98 851 |
| 974 | 98 856 | 98 860 | 98 865 | 98 869 | 98 874 | 98 878 | 98 883 | 98 887 | 98 892 | 98 896 |
| 975 | 98 900 | 98 905 | 98 909 | 98 914 | 98 918 | 98 923 | 98 927 | 98 932 | 98 936 | 98 941 |
| 976 | 98 945 | 98 949 | 98 954 | 98 958 | 98 963 | 98 967 | 98 972 | 98 976 | 98 981 | 98 985 |
| 977 | 98 989 | 98 994 | 98 998 | 99 003 | 99 007 | 99 012 | 99 016 | 99 021 | 99 025 | 99 029 |
| 978 | 99 034 | 99 038 | 99 043 | 99 047 | 99 052 | 99 056 | 99 061 | 99 065 | 99 069 | 99 074 |
| 979 | 99 078 | 99 083 | 99 087 | 99 092 | 99 096 | 99 100 | 99 105 | 99 109 | 99 114 | 99 118 |
| 980 | 99 123 | 99 127 | 99 131 | 99 136 | 99 140 | 99 145 | 99 149 | 99 154 | 99 158 | 99 162 |
| 981 | 99 167 | 99 171 | 99 176 | 99 180 | 99 185 | 99 189 | 99 193 | 99 198 | 99 202 | 99 207 |
| 982 | 99 211 | 99 216 | 99 220 | 99 224 | 99 229 | 99 233 | 99 238 | 99 242 | 99 247 | 99 251 |
| 983 | 99 255 | 99 260 | 99 264 | 99 269 | 99 273 | 99 277 | 99 282 | 99 286 | 99 291 | 99 295 |
| 984 | 99 300 | 99 304 | 99 308 | 99 313 | 99 317 | 99 322 | 99 326 | 99 330 | 99 335 | 99 339 |
| 985 | 99 344 | 99 348 | 99 352 | 99 357 | 99 361 | 99 366 | 99 370 | 99 374 | 99 379 | 99 383 |
| 986 | 99 388 | 99 392 | 99 396 | 99 401 | 99 405 | 99 410 | 99 414 | 99 419 | 99 423 | 99 427 |
| 987 | 99 432 | 99 436 | 99 441 | 99 445 | 99 449 | 99 454 | 99 458 | 99 463 | 99 467 | 99 471 |
| 988 | 99 476 | 99 480 | 99 484 | 99 489 | 99 493 | 99 498 | 99 502 | 99 506 | 99 511 | 99 515 |
| 989 | 99 520 | 99 524 | 99 528 | 99 533 | 99 537 | 99 542 | 99 546 | 99 550 | 99 555 | 99 559 |
| 990 | 99 564 | 99 568 | 99 572 | 99 577 | 99 581 | 99 585 | 99 590 | 99 594 | 99 599 | 99 603 |
| 991 | 99 607 | 99 612 | 99 616 | 99 621 | 99 625 | 99 629 | 99 634 | 99 638 | 99 642 | 99 647 |
| 992 | 99 651 | 99 656 | 99 660 | 99 664 | 99 669 | 99 673 | 99 677 | 99 682 | 99 686 | 99 691 |
| 993 | 99 695 | 99 699 | 99 704 | 99 708 | 99 712 | 99 717 | 99 721 | 99 726 | 99 730 | 99 734 |
| 994 | 99 739 | 99 743 | 99 747 | 99 752 | 99 756 | 99 760 | 99 765 | 99 769 | 99 774 | 99 778 |
| 995 | 99 782 | 99 787 | 99 791 | 99 795 | 99 800 | 99 804 | 99 808 | 99 813 | 99 817 | 99 822 |
| 996 | 99 826 | 99 830 | 99 835 | 99 839 | 99 843 | 99 848 | 99 852 | 99 856 | 99 861 | 99 865 |
| 997 | 99 870 | 99 874 | 99 878 | 99 883 | 99 887 | 99 891 | 99 896 | 99 900 | 99 904 | 99 909 |
| 998 | 99 913 | 99 917 | 99 922 | 99 926 | 99 930 | 99 935 | 99 939 | 99 944 | 99 948 | 99 952 |
| 999 | 99 957 | 99 961 | 99 965 | 99 970 | 99 974 | 99 978 | 99 983 | 99 987 | 99 991 | 99 996 |
| 1000 | 00 000 | 00 004 | 00 009 | 00 013 | 00 017 | 00 022 | 00 026 | 00 030 | 00 035 | 00 039 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

TABLE II

APPROXIMATE EQUATION OF TIME.

| DATE. | MINUTES. | DATE. | MINUTES. | DATE. | MINUTES. | DATE. | MINUTES. |
|---------|----------|--------|----------|---------|----------|---------|----------|
| Jan. 1 | 4 . | Apr. 1 | 4 | Aug. 9 | 5 . | Oct. 27 | 16 . |
| " 3 | 5 . | " 4 | 3 | " 15 | 4 | Nov. 15 | 15 . |
| " 5 | 6 . | " 7 | 2 | " 20 | 3 | " 20 | 14 . |
| " 7 | 7 . | " 11 | 1 | " 24 | 2 | " 24 | 13 . |
| " 9 | 8 . | " 15 | 0 | " 28 | 1 | " 27 | 12 . |
| " 12 | 9 . | " 19 | 1 | " 31 | 0 | " 30 | 11 . |
| " 15 | 10 . | " 24 | 2 | Sept. 3 | 1 . | Dec. 2 | 10 . |
| " 18 | 11 . | " 30 | 3 | " 6 | 2 . | " 5 | 9 . |
| " 21 | 12 . | May 13 | 4 | " 9 | 3 . | " 7 | 8 . |
| " 25 | 13 . | " 29 | 3 | " 12 | 4 . | " 9 | 7 . |
| " 31 | 14 . | June 5 | 2 | " 15 | 5 . | " 11 | 6 . |
| Feb. 10 | 15 . | " 10 | 1 | " 18 | 6 . | " 13 | 5 . |
| " 21 | 14 . | " 15 | 0 | " 21 | 7 . | " 16 | 4 . |
| " 27 | 13 . | " 20 | 1 | " 24 | 8 . | " 18 | 3 . |
| Mar. 4 | 12 . | " 25 | 2 | " 27 | 9 . | " 20 | 2 . |
| " 8 | 11 . | " 29 | 3 | " 30 | 10 . | " 22 | 1 . |
| " 12 | 10 . | July 5 | 4 | Oct. 3 | 11 . | " 24 | 0 . |
| " 15 | 9 . | " 11 | 5 | " 6 | 12 . | " 26 | 1 . |
| " 19 | 8 . | " 28 | 6 | " 10 | 13 . | " 28 | 2 . |
| " 22 | 7 . | | | " 14 | 14 . | " 30 | 3 . |
| " 25 | 6 . | | | " 19 | 15 . | | |
| " 28 | 5 . | | | | | | |

TABLE III.

THE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS :

From 0° to 0° 3', or 89° 57' to 90°, for every second ;
From 0° to 2°, or 88° to 90°, for every ten seconds ;
From 1° to 89°, for every minute.

NOTE. To all the logarithms -10 is to be appended.

| log sin | | | | 0° | | log tan = log sin log cos = 10.00 000 | | | |
|---------|----------|----------|----------|----|----|--|----------|----------|----|
| " | 0' | 1' | 2' | " | " | 0' | 1' | 2' | " |
| 0 | — | 6.46 373 | 6.76 476 | 80 | 80 | 6.16 270 | 6.63 982 | 6.86 167 | 80 |
| 1 | 4.68 557 | 6.47 090 | 6.76 836 | 59 | 81 | 6.17 694 | 6.64 462 | 6.86 455 | 29 |
| 2 | 4.98 660 | 6.47 797 | 6.77 193 | 58 | 82 | 6.19 072 | 6.64 936 | 6.86 742 | 28 |
| 3 | 5.16 270 | 6.48 492 | 6.77 548 | 57 | 83 | 6.20 409 | 6.65 406 | 6.87 027 | 27 |
| 4 | 5.28 763 | 6.49 175 | 6.77 900 | 56 | 84 | 6.21 705 | 6.65 870 | 6.87 310 | 26 |
| 5 | 5.38 454 | 6.49 849 | 6.78 248 | 55 | 85 | 6.22 964 | 6.66 330 | 6.87 591 | 25 |
| 6 | 5.46 373 | 6.50 512 | 6.78 595 | 54 | 86 | 6.24 188 | 6.66 785 | 6.87 870 | 24 |
| 7 | 5.53 067 | 6.51 165 | 6.78 938 | 53 | 87 | 6.25 378 | 6.67 235 | 6.88 147 | 23 |
| 8 | 5.58 866 | 6.51 808 | 6.79 278 | 52 | 88 | 6.26 536 | 6.67 680 | 6.88 423 | 22 |
| 9 | 5.63 982 | 6.52 442 | 6.79 616 | 51 | 89 | 6.27 664 | 6.68 121 | 6.88 697 | 21 |
| 10 | 5.68 557 | 6.53 067 | 6.79 952 | 50 | 40 | 6.28 763 | 6.68 557 | 6.88 969 | 20 |
| 11 | 5.72 697 | 6.53 683 | 6.80 285 | 49 | 41 | 6.29 836 | 6.68 990 | 6.89 240 | 19 |
| 12 | 5.76 476 | 6.54 291 | 6.80 615 | 48 | 42 | 6.30 882 | 6.69 418 | 6.89 509 | 18 |
| 13 | 5.79 952 | 6.54 890 | 6.80 943 | 47 | 43 | 6.31 904 | 6.69 841 | 6.89 776 | 17 |
| 14 | 5.83 170 | 6.55 481 | 6.81 268 | 46 | 44 | 6.32 903 | 6.70 261 | 6.90 042 | 16 |
| 15 | 5.86 167 | 6.56 064 | 6.81 591 | 45 | 45 | 6.33 879 | 6.70 676 | 6.90 306 | 15 |
| 16 | 5.88 969 | 6.56 639 | 6.81 911 | 44 | 46 | 6.34 833 | 6.71 088 | 6.90 568 | 14 |
| 17 | 5.91 602 | 6.57 207 | 6.82 230 | 43 | 47 | 6.35 767 | 6.71 496 | 6.90 829 | 13 |
| 18 | 5.94 085 | 6.57 767 | 6.82 545 | 42 | 48 | 6.36 682 | 6.71 900 | 6.91 088 | 12 |
| 19 | 5.96 433 | 6.58 320 | 6.82 859 | 41 | 49 | 6.37 577 | 6.72 300 | 6.91 346 | 11 |
| 20 | 5.98 660 | 6.58 866 | 6.83 170 | 40 | 50 | 6.38 454 | 6.72 697 | 6.91 602 | 10 |
| 21 | 6.00 779 | 6.59 406 | 6.83 479 | 39 | 51 | 6.39 315 | 6.73 090 | 6.91 857 | 9 |
| 22 | 6.02 800 | 6.59 939 | 6.83 786 | 38 | 52 | 6.40 158 | 6.73 479 | 6.92 110 | 8 |
| 23 | 6.04 730 | 6.60 465 | 6.84 091 | 37 | 53 | 6.40 985 | 6.73 865 | 6.92 362 | 7 |
| 24 | 6.06 579 | 6.60 985 | 6.84 394 | 36 | 54 | 6.41 797 | 6.74 248 | 6.92 612 | 6 |
| 25 | 6.08 351 | 6.61 499 | 6.84 694 | 35 | 55 | 6.42 594 | 6.74 627 | 6.92 861 | 5 |
| 26 | 6.10 055 | 6.62 007 | 6.84 993 | 34 | 56 | 6.43 376 | 6.75 003 | 6.93 109 | 4 |
| 27 | 6.11 694 | 6.62 509 | 6.85 289 | 33 | 57 | 6.44 145 | 6.75 376 | 6.93 355 | 3 |
| 28 | 6.13 273 | 6.63 006 | 6.85 584 | 32 | 58 | 6.44 900 | 6.75 746 | 6.93 599 | 2 |
| 29 | 6.14 797 | 6.63 496 | 6.85 876 | 31 | 59 | 6.45 643 | 6.76 112 | 6.93 843 | 1 |
| 30 | 6.16 270 | 6.63 982 | 6.86 167 | 30 | 60 | 6.46 373 | 6.76 476 | 6.94 085 | 0 |
| " | 59' | 58' | 57' | " | " | 59' | 58' | 57' | " |

log cot = log cos
log sin = 10.00 000

89°

log cos

| ° ' " | log sin | log tan | log sec | ° ' " | ° ' " | log sin | log tan | log sec | ° ' " |
|-------|---------|---------|----------|-------|-------|---------|---------|----------|-------|
| 0 0 | — | — | 10.00000 | 0 00 | 10 0 | 7.46373 | 7.46373 | 10.00000 | 0 50 |
| 10 | 5.68557 | 5.68557 | 10.00000 | 10 | 10 | 7.47090 | 7.47091 | 10.00000 | 50 |
| 20 | 5.98660 | 5.98660 | 10.00000 | 20 | 20 | 7.47797 | 7.47797 | 10.00000 | 40 |
| 30 | 6.16270 | 6.16270 | 10.00000 | 30 | 30 | 7.48491 | 7.48492 | 10.00000 | 30 |
| 40 | 6.28763 | 6.28763 | 10.00000 | 40 | 40 | 7.49175 | 7.49176 | 10.00000 | 20 |
| 50 | 6.38454 | 6.38454 | 10.00000 | 50 | 50 | 7.49849 | 7.49849 | 10.00000 | 10 |
| 1 0 | 6.46373 | 6.46373 | 10.00000 | 0 50 | 11 0 | 7.50512 | 7.50512 | 10.00000 | 0 40 |
| 10 | 6.53067 | 6.53067 | 10.00000 | 50 | 10 | 7.51165 | 7.51165 | 10.00000 | 50 |
| 20 | 6.58866 | 6.58866 | 10.00000 | 40 | 20 | 7.51808 | 7.51809 | 10.00000 | 40 |
| 30 | 6.63982 | 6.63982 | 10.00000 | 30 | 30 | 7.52442 | 7.52443 | 10.00000 | 30 |
| 40 | 6.68557 | 6.68557 | 10.00000 | 20 | 40 | 7.53067 | 7.53067 | 10.00000 | 20 |
| 50 | 6.72697 | 6.72697 | 10.00000 | 10 | 50 | 7.53683 | 7.53683 | 10.00000 | 10 |
| 2 0 | 6.76476 | 6.76476 | 10.00000 | 0 50 | 12 0 | 7.54291 | 7.54291 | 10.00000 | 0 40 |
| 10 | 6.79952 | 6.79952 | 10.00000 | 50 | 10 | 7.54890 | 7.54890 | 10.00000 | 50 |
| 20 | 6.83170 | 6.83170 | 10.00000 | 40 | 20 | 7.55481 | 7.55481 | 10.00000 | 40 |
| 30 | 6.86167 | 6.86167 | 10.00000 | 30 | 30 | 7.56064 | 7.56064 | 10.00000 | 30 |
| 40 | 6.88969 | 6.88969 | 10.00000 | 20 | 40 | 7.56639 | 7.56639 | 10.00000 | 20 |
| 50 | 6.91602 | 6.91602 | 10.00000 | 10 | 50 | 7.57206 | 7.57207 | 10.00000 | 10 |
| 3 0 | 6.94085 | 6.94085 | 10.00000 | 0 57 | 13 0 | 7.57767 | 7.57767 | 10.00000 | 0 47 |
| 10 | 6.96433 | 6.96433 | 10.00000 | 50 | 10 | 7.58320 | 7.58320 | 10.00000 | 50 |
| 20 | 6.98660 | 6.98661 | 10.00000 | 40 | 20 | 7.58866 | 7.58867 | 10.00000 | 40 |
| 30 | 7.00779 | 7.00779 | 10.00000 | 30 | 30 | 7.59406 | 7.59406 | 10.00000 | 30 |
| 40 | 7.02800 | 7.02800 | 10.00000 | 20 | 40 | 7.59939 | 7.59939 | 10.00000 | 20 |
| 50 | 7.04730 | 7.04730 | 10.00000 | 10 | 50 | 7.60465 | 7.60466 | 10.00000 | 10 |
| 4 0 | 7.06579 | 7.06579 | 10.00000 | 0 50 | 14 0 | 7.60985 | 7.60986 | 10.00000 | 0 46 |
| 10 | 7.08351 | 7.08352 | 10.00000 | 50 | 10 | 7.61499 | 7.61500 | 10.00000 | 50 |
| 20 | 7.10055 | 7.10055 | 10.00000 | 40 | 20 | 7.62007 | 7.62008 | 10.00000 | 40 |
| 30 | 7.11694 | 7.11694 | 10.00000 | 30 | 30 | 7.62509 | 7.62510 | 10.00000 | 30 |
| 40 | 7.13273 | 7.13273 | 10.00000 | 20 | 40 | 7.63006 | 7.63006 | 10.00000 | 20 |
| 50 | 7.14797 | 7.14797 | 10.00000 | 10 | 50 | 7.63496 | 7.63497 | 10.00000 | 10 |
| 5 0 | 7.16270 | 7.16270 | 10.00000 | 0 55 | 15 0 | 7.63982 | 7.63982 | 10.00000 | 0 45 |
| 10 | 7.17694 | 7.17694 | 10.00000 | 50 | 10 | 7.64461 | 7.64462 | 10.00000 | 50 |
| 20 | 7.19072 | 7.19073 | 10.00000 | 40 | 20 | 7.64936 | 7.64937 | 10.00000 | 40 |
| 30 | 7.20409 | 7.20409 | 10.00000 | 30 | 30 | 7.65406 | 7.65406 | 10.00000 | 30 |
| 40 | 7.21705 | 7.21705 | 10.00000 | 20 | 40 | 7.65870 | 7.65871 | 10.00000 | 20 |
| 50 | 7.22964 | 7.22964 | 10.00000 | 10 | 50 | 7.66330 | 7.66330 | 10.00000 | 10 |
| 6 0 | 7.24188 | 7.24188 | 10.00000 | 0 54 | 16 0 | 7.66784 | 7.66785 | 10.00000 | 0 44 |
| 10 | 7.25378 | 7.25378 | 10.00000 | 50 | 10 | 7.67235 | 7.67235 | 10.00000 | 50 |
| 20 | 7.26536 | 7.26536 | 10.00000 | 40 | 20 | 7.67680 | 7.67680 | 10.00000 | 40 |
| 30 | 7.27664 | 7.27664 | 10.00000 | 30 | 30 | 7.68121 | 7.68121 | 10.00000 | 30 |
| 40 | 7.28763 | 7.28764 | 10.00000 | 20 | 40 | 7.68557 | 7.68558 | 9.99999 | 20 |
| 50 | 7.29836 | 7.29836 | 10.00000 | 10 | 50 | 7.68989 | 7.68990 | 9.99999 | 10 |
| 7 0 | 7.30882 | 7.30882 | 10.00000 | 0 53 | 17 0 | 7.69417 | 7.69418 | 9.99999 | 0 43 |
| 10 | 7.31904 | 7.31904 | 10.00000 | 50 | 10 | 7.69841 | 7.69842 | 9.99999 | 50 |
| 20 | 7.32903 | 7.32903 | 10.00000 | 40 | 20 | 7.70261 | 7.70261 | 9.99999 | 40 |
| 30 | 7.33879 | 7.33879 | 10.00000 | 30 | 30 | 7.70676 | 7.70677 | 9.99999 | 30 |
| 40 | 7.34833 | 7.34833 | 10.00000 | 20 | 40 | 7.71088 | 7.71088 | 9.99999 | 20 |
| 50 | 7.35767 | 7.35767 | 10.00000 | 10 | 50 | 7.71496 | 7.71496 | 9.99999 | 10 |
| 8 0 | 7.36682 | 7.36682 | 10.00000 | 0 52 | 18 0 | 7.71900 | 7.71900 | 9.99999 | 0 42 |
| 10 | 7.37577 | 7.37577 | 10.00000 | 50 | 10 | 7.72300 | 7.72301 | 9.99999 | 50 |
| 20 | 7.38454 | 7.38455 | 10.00000 | 40 | 20 | 7.72697 | 7.72697 | 9.99999 | 40 |
| 30 | 7.39314 | 7.39315 | 10.00000 | 30 | 30 | 7.73090 | 7.73090 | 9.99999 | 30 |
| 40 | 7.40158 | 7.40158 | 10.00000 | 20 | 40 | 7.73479 | 7.73480 | 9.99999 | 20 |
| 50 | 7.40985 | 7.40985 | 10.00000 | 10 | 50 | 7.73865 | 7.73866 | 9.99999 | 10 |
| 9 0 | 7.41797 | 7.41797 | 10.00000 | 0 51 | 19 0 | 7.74248 | 7.74248 | 9.99999 | 0 41 |
| 10 | 7.42594 | 7.42594 | 10.00000 | 50 | 10 | 7.74627 | 7.74628 | 9.99999 | 50 |
| 20 | 7.43376 | 7.43376 | 10.00000 | 40 | 20 | 7.75003 | 7.75004 | 9.99999 | 40 |
| 30 | 7.44145 | 7.44145 | 10.00000 | 30 | 30 | 7.75376 | 7.75377 | 9.99999 | 30 |
| 40 | 7.44900 | 7.44900 | 10.00000 | 20 | 40 | 7.75745 | 7.75746 | 9.99999 | 20 |
| 50 | 7.45643 | 7.45643 | 10.00000 | 10 | 50 | 7.76112 | 7.76113 | 9.99999 | 10 |
| 10 0 | 7.46373 | 7.46373 | 10.00000 | 0 50 | 20 0 | 7.76475 | 7.76476 | 9.99999 | 0 40 |
| ° ' " | log sin | log tan | log sec | ° ' " | ° ' " | log sin | log tan | log sec | ° ' " |

| ° ' " | log sin | log tan | log cos | ° ' " | ° ' " | log sin | log tan | log cos | ° ' " |
|-------|----------|----------|----------|-------|-------|----------|----------|----------|-------|
| 20 0 | 7.76 475 | 7.76 476 | 9.99 999 | 0 40 | 80 0 | 7.94 084 | 7.94 086 | 9.99 998 | 0 80 |
| 10 | 7.76 836 | 7.76 837 | 9.99 999 | 50 | 10 | 7.94 325 | 7.94 326 | 9.99 998 | 50 |
| 20 | 7.77 193 | 7.77 194 | 9.99 999 | 40 | 20 | 7.94 564 | 7.94 566 | 9.99 998 | 40 |
| 30 | 7.77 548 | 7.77 549 | 9.99 999 | 30 | 30 | 7.94 802 | 7.94 804 | 9.99 998 | 30 |
| 40 | 7.77 899 | 7.77 900 | 9.99 999 | 20 | 40 | 7.95 039 | 7.95 040 | 9.99 998 | 20 |
| 50 | 7.78 248 | 7.78 249 | 9.99 999 | 10 | 50 | 7.95 274 | 7.95 276 | 9.99 998 | 10 |
| 21 0 | 7.78 594 | 7.78 595 | 9.99 999 | 0 39 | 81 0 | 7.95 508 | 7.95 510 | 9.99 998 | 0 29 |
| 10 | 7.78 938 | 7.78 938 | 9.99 999 | 50 | 10 | 7.95 741 | 7.95 743 | 9.99 998 | 50 |
| 20 | 7.79 278 | 7.79 279 | 9.99 999 | 40 | 20 | 7.95 973 | 7.95 974 | 9.99 998 | 40 |
| 30 | 7.79 616 | 7.79 617 | 9.99 999 | 30 | 30 | 7.96 203 | 7.96 205 | 9.99 998 | 30 |
| 40 | 7.79 952 | 7.79 952 | 9.99 999 | 20 | 40 | 7.96 432 | 7.96 434 | 9.99 998 | 20 |
| 50 | 7.80 284 | 7.80 285 | 9.99 999 | 10 | 50 | 7.96 660 | 7.96 662 | 9.99 998 | 10 |
| 22 0 | 7.80 615 | 7.80 615 | 9.99 999 | 0 38 | 82 0 | 7.96 887 | 7.96 889 | 9.99 998 | 0 28 |
| 10 | 7.80 942 | 7.80 943 | 9.99 999 | 50 | 10 | 7.97 113 | 7.97 114 | 9.99 998 | 50 |
| 20 | 7.81 268 | 7.81 269 | 9.99 999 | 40 | 20 | 7.97 337 | 7.97 339 | 9.99 998 | 40 |
| 30 | 7.81 591 | 7.81 591 | 9.99 999 | 30 | 30 | 7.97 560 | 7.97 562 | 9.99 998 | 30 |
| 40 | 7.81 911 | 7.81 912 | 9.99 999 | 20 | 40 | 7.97 782 | 7.97 784 | 9.99 998 | 20 |
| 50 | 7.82 229 | 7.82 230 | 9.99 999 | 10 | 50 | 7.98 003 | 7.98 005 | 9.99 998 | 10 |
| 23 0 | 7.82 545 | 7.82 546 | 9.99 999 | 0 37 | 83 0 | 7.98 223 | 7.98 225 | 9.99 998 | 0 27 |
| 10 | 7.82 859 | 7.82 860 | 9.99 999 | 50 | 10 | 7.98 442 | 7.98 444 | 9.99 998 | 50 |
| 20 | 7.83 170 | 7.83 171 | 9.99 999 | 40 | 20 | 7.98 660 | 7.98 662 | 9.99 998 | 40 |
| 30 | 7.83 479 | 7.83 480 | 9.99 999 | 30 | 30 | 7.98 876 | 7.98 878 | 9.99 998 | 30 |
| 40 | 7.83 786 | 7.83 787 | 9.99 999 | 20 | 40 | 7.99 092 | 7.99 094 | 9.99 998 | 20 |
| 50 | 7.84 091 | 7.84 092 | 9.99 999 | 10 | 50 | 7.99 306 | 7.99 308 | 9.99 998 | 10 |
| 24 0 | 7.84 393 | 7.84 394 | 9.99 999 | 0 36 | 84 0 | 7.99 520 | 7.99 522 | 9.99 998 | 0 26 |
| 10 | 7.84 694 | 7.84 695 | 9.99 999 | 50 | 10 | 7.99 732 | 7.99 734 | 9.99 998 | 50 |
| 20 | 7.84 992 | 7.84 994 | 9.99 999 | 40 | 20 | 7.99 943 | 7.99 946 | 9.99 998 | 40 |
| 30 | 7.85 289 | 7.85 290 | 9.99 999 | 30 | 30 | 8.00 154 | 8.00 156 | 9.99 998 | 30 |
| 40 | 7.85 583 | 7.85 584 | 9.99 999 | 20 | 40 | 8.00 363 | 8.00 365 | 9.99 998 | 20 |
| 50 | 7.85 876 | 7.85 877 | 9.99 999 | 10 | 50 | 8.00 571 | 8.00 574 | 9.99 998 | 10 |
| 25 0 | 7.86 166 | 7.86 167 | 9.99 999 | 0 35 | 85 0 | 8.00 779 | 8.00 781 | 9.99 998 | 0 25 |
| 10 | 7.86 455 | 7.86 456 | 9.99 999 | 50 | 10 | 8.00 985 | 8.00 987 | 9.99 998 | 50 |
| 20 | 7.86 741 | 7.86 743 | 9.99 999 | 40 | 20 | 8.01 190 | 8.01 193 | 9.99 998 | 40 |
| 30 | 7.87 026 | 7.87 027 | 9.99 999 | 30 | 30 | 8.01 395 | 8.01 397 | 9.99 998 | 30 |
| 40 | 7.87 309 | 7.87 310 | 9.99 999 | 20 | 40 | 8.01 598 | 8.01 600 | 9.99 998 | 20 |
| 50 | 7.87 590 | 7.87 591 | 9.99 999 | 10 | 50 | 8.01 801 | 8.01 803 | 9.99 998 | 10 |
| 26 0 | 7.87 870 | 7.87 871 | 9.99 999 | 0 34 | 86 0 | 8.02 002 | 8.02 004 | 9.99 998 | 0 24 |
| 10 | 7.88 147 | 7.88 148 | 9.99 999 | 50 | 10 | 8.02 203 | 8.02 205 | 9.99 998 | 50 |
| 20 | 7.88 423 | 7.88 424 | 9.99 999 | 40 | 20 | 8.02 402 | 8.02 405 | 9.99 998 | 40 |
| 30 | 7.88 697 | 7.88 698 | 9.99 999 | 30 | 30 | 8.02 601 | 8.02 604 | 9.99 998 | 30 |
| 40 | 7.88 969 | 7.88 970 | 9.99 999 | 20 | 40 | 8.02 799 | 8.02 801 | 9.99 998 | 20 |
| 50 | 7.89 240 | 7.89 241 | 9.99 999 | 10 | 50 | 8.02 996 | 8.02 998 | 9.99 998 | 10 |
| 27 0 | 7.89 509 | 7.89 510 | 9.99 999 | 0 33 | 87 0 | 8.03 192 | 8.03 194 | 9.99 997 | 0 23 |
| 10 | 7.89 776 | 7.89 777 | 9.99 999 | 50 | 10 | 8.03 387 | 8.03 390 | 9.99 997 | 50 |
| 20 | 7.90 041 | 7.90 043 | 9.99 999 | 40 | 20 | 8.03 581 | 8.03 584 | 9.99 997 | 40 |
| 30 | 7.90 305 | 7.90 307 | 9.99 999 | 30 | 30 | 8.03 775 | 8.03 777 | 9.99 997 | 30 |
| 40 | 7.90 568 | 7.90 569 | 9.99 999 | 20 | 40 | 8.03 967 | 8.03 970 | 9.99 997 | 20 |
| 50 | 7.90 829 | 7.90 830 | 9.99 999 | 10 | 50 | 8.04 159 | 8.04 162 | 9.99 997 | 10 |
| 28 0 | 7.91 088 | 7.91 089 | 9.99 999 | 0 32 | 88 0 | 8.04 350 | 8.04 353 | 9.99 997 | 0 22 |
| 10 | 7.91 346 | 7.91 347 | 9.99 999 | 50 | 10 | 8.04 540 | 8.04 543 | 9.99 997 | 50 |
| 20 | 7.91 602 | 7.91 603 | 9.99 999 | 40 | 20 | 8.04 729 | 8.04 732 | 9.99 997 | 40 |
| 30 | 7.91 857 | 7.91 858 | 9.99 999 | 30 | 30 | 8.04 918 | 8.04 921 | 9.99 997 | 30 |
| 40 | 7.92 110 | 7.92 111 | 9.99 998 | 20 | 40 | 8.05 105 | 8.05 108 | 9.99 997 | 20 |
| 50 | 7.92 362 | 7.92 363 | 9.99 998 | 10 | 50 | 8.05 292 | 8.05 295 | 9.99 997 | 10 |
| 29 0 | 7.92 612 | 7.92 613 | 9.99 998 | 0 31 | 89 0 | 8.05 478 | 8.05 481 | 9.99 997 | 0 21 |
| 10 | 7.92 861 | 7.92 862 | 9.99 998 | 50 | 10 | 8.05 663 | 8.05 666 | 9.99 997 | 50 |
| 20 | 7.93 108 | 7.93 110 | 9.99 998 | 40 | 20 | 8.05 848 | 8.05 851 | 9.99 997 | 40 |
| 30 | 7.93 354 | 7.93 356 | 9.99 998 | 30 | 30 | 8.06 031 | 8.06 034 | 9.99 997 | 30 |
| 40 | 7.93 599 | 7.93 601 | 9.99 998 | 20 | 40 | 8.06 214 | 8.06 217 | 9.99 997 | 20 |
| 50 | 7.93 842 | 7.93 844 | 9.99 998 | 10 | 50 | 8.06 396 | 8.06 399 | 9.99 997 | 10 |
| 30 0 | 7.94 084 | 7.94 086 | 9.99 998 | 0 30 | 40 0 | 8.06 578 | 8.06 581 | 9.99 997 | 0 20 |
| ° ' " | log cos | log cot | log sin | ° ' " | ° ' " | log cos | log cot | log sin | ° ' " |

| ° ' " | log sin | log tan | log cos | ° ' " | ° ' " | log sin | log tan | log cos | ° ' " |
|-------|---------|---------|---------|-------|-------|---------|---------|---------|-------|
| 40 0 | 8.96578 | 8.96582 | 9.99997 | 0 20 | 50 0 | 8.16268 | 8.16273 | 9.99995 | 1 10 |
| 10 | 8.96758 | 8.96762 | 9.99997 | 50 | 10 | 8.16423 | 8.16427 | 9.99995 | 51 |
| 20 | 8.96938 | 8.96942 | 9.99997 | 40 | 20 | 8.16557 | 8.16562 | 9.99995 | 40 |
| 30 | 8.97117 | 8.97122 | 9.99997 | 30 | 30 | 8.16700 | 8.16705 | 9.99995 | 30 |
| 40 | 8.97295 | 8.97299 | 9.99997 | 20 | 40 | 8.16843 | 8.16848 | 9.99995 | 20 |
| 50 | 8.97473 | 8.97476 | 9.99997 | 10 | 50 | 8.16986 | 8.16992 | 9.99995 | 10 |
| 41 0 | 8.97650 | 8.97653 | 9.99997 | 0 19 | 51 0 | 8.17128 | 8.17133 | 9.99995 | 0 9 |
| 10 | 8.97826 | 8.97829 | 9.99997 | 50 | 10 | 8.17270 | 8.17275 | 9.99995 | 50 |
| 20 | 8.98002 | 8.98005 | 9.99997 | 40 | 20 | 8.17411 | 8.17416 | 9.99995 | 40 |
| 30 | 8.98176 | 8.98180 | 9.99997 | 30 | 30 | 8.17552 | 8.17557 | 9.99995 | 30 |
| 40 | 8.98350 | 8.98354 | 9.99997 | 20 | 40 | 8.17692 | 8.17697 | 9.99995 | 20 |
| 50 | 8.98524 | 8.98527 | 9.99997 | 10 | 50 | 8.17832 | 8.17837 | 9.99995 | 10 |
| 42 0 | 8.98696 | 8.98700 | 9.99997 | 0 18 | 52 0 | 8.17971 | 8.17976 | 9.99995 | 0 8 |
| 10 | 8.98868 | 8.98872 | 9.99997 | 50 | 10 | 8.18110 | 8.18115 | 9.99995 | 50 |
| 20 | 8.99040 | 8.99043 | 9.99997 | 40 | 20 | 8.18249 | 8.18254 | 9.99995 | 40 |
| 30 | 8.99210 | 8.99214 | 9.99997 | 30 | 30 | 8.18387 | 8.18392 | 9.99995 | 30 |
| 40 | 8.99380 | 8.99384 | 9.99997 | 20 | 40 | 8.18524 | 8.18530 | 9.99995 | 20 |
| 50 | 8.99550 | 8.99553 | 9.99997 | 10 | 50 | 8.18662 | 8.18667 | 9.99995 | 10 |
| 43 0 | 8.99718 | 8.99722 | 9.99997 | 0 17 | 53 0 | 8.18798 | 8.18804 | 9.99995 | 0 7 |
| 10 | 8.99886 | 8.99890 | 9.99997 | 50 | 10 | 8.18935 | 8.18940 | 9.99995 | 50 |
| 20 | 8.10054 | 8.10057 | 9.99997 | 40 | 20 | 8.19071 | 8.19076 | 9.99995 | 40 |
| 30 | 8.10220 | 8.10224 | 9.99997 | 30 | 30 | 8.19206 | 8.19212 | 9.99995 | 30 |
| 40 | 8.10386 | 8.10390 | 9.99997 | 20 | 40 | 8.19341 | 8.19347 | 9.99995 | 20 |
| 50 | 8.10552 | 8.10555 | 9.99996 | 10 | 50 | 8.19476 | 8.19481 | 9.99995 | 10 |
| 44 0 | 8.10717 | 8.10720 | 9.99996 | 0 16 | 54 0 | 8.19610 | 8.19616 | 9.99995 | 0 6 |
| 10 | 8.10881 | 8.10884 | 9.99996 | 50 | 10 | 8.19744 | 8.19749 | 9.99995 | 50 |
| 20 | 8.11044 | 8.11048 | 9.99996 | 40 | 20 | 8.19877 | 8.19883 | 9.99995 | 40 |
| 30 | 8.11207 | 8.11211 | 9.99996 | 30 | 30 | 8.20010 | 8.20016 | 9.99995 | 30 |
| 40 | 8.11370 | 8.11373 | 9.99996 | 20 | 40 | 8.20143 | 8.20149 | 9.99995 | 20 |
| 50 | 8.11531 | 8.11535 | 9.99996 | 10 | 50 | 8.20275 | 8.20281 | 9.99994 | 10 |
| 45 0 | 8.11693 | 8.11696 | 9.99996 | 0 15 | 55 0 | 8.20407 | 8.20413 | 9.99994 | 0 5 |
| 10 | 8.11853 | 8.11857 | 9.99996 | 50 | 10 | 8.20538 | 8.20544 | 9.99994 | 50 |
| 20 | 8.12013 | 8.12017 | 9.99996 | 40 | 20 | 8.20669 | 8.20675 | 9.99994 | 40 |
| 30 | 8.12172 | 8.12176 | 9.99996 | 30 | 30 | 8.20800 | 8.20806 | 9.99994 | 30 |
| 40 | 8.12331 | 8.12335 | 9.99996 | 20 | 40 | 8.20930 | 8.20936 | 9.99994 | 20 |
| 50 | 8.12489 | 8.12493 | 9.99996 | 10 | 50 | 8.21060 | 8.21066 | 9.99994 | 10 |
| 46 0 | 8.12647 | 8.12651 | 9.99996 | 0 14 | 56 0 | 8.21189 | 8.21195 | 9.99994 | 0 4 |
| 10 | 8.12804 | 8.12808 | 9.99996 | 50 | 10 | 8.21319 | 8.21324 | 9.99994 | 50 |
| 20 | 8.12961 | 8.12965 | 9.99996 | 40 | 20 | 8.21447 | 8.21453 | 9.99994 | 40 |
| 30 | 8.13117 | 8.13121 | 9.99996 | 30 | 30 | 8.21576 | 8.21581 | 9.99994 | 30 |
| 40 | 8.13272 | 8.13276 | 9.99996 | 20 | 40 | 8.21703 | 8.21709 | 9.99994 | 20 |
| 50 | 8.13427 | 8.13431 | 9.99996 | 10 | 50 | 8.21831 | 8.21837 | 9.99994 | 10 |
| 47 0 | 8.13581 | 8.13585 | 9.99996 | 0 13 | 57 0 | 8.21958 | 8.21964 | 9.99994 | 0 3 |
| 10 | 8.13735 | 8.13739 | 9.99996 | 50 | 10 | 8.22085 | 8.22091 | 9.99994 | 50 |
| 20 | 8.13888 | 8.13892 | 9.99996 | 40 | 20 | 8.22211 | 8.22217 | 9.99994 | 40 |
| 30 | 8.14041 | 8.14045 | 9.99996 | 30 | 30 | 8.22337 | 8.22343 | 9.99994 | 30 |
| 40 | 8.14193 | 8.14197 | 9.99996 | 20 | 40 | 8.22463 | 8.22469 | 9.99994 | 20 |
| 50 | 8.14344 | 8.14348 | 9.99996 | 10 | 50 | 8.22588 | 8.22595 | 9.99994 | 10 |
| 48 0 | 8.14495 | 8.14500 | 9.99996 | 0 12 | 58 0 | 8.22713 | 8.22720 | 9.99994 | 0 2 |
| 10 | 8.14646 | 8.14650 | 9.99996 | 50 | 10 | 8.22838 | 8.22844 | 9.99994 | 50 |
| 20 | 8.14796 | 8.14800 | 9.99996 | 40 | 20 | 8.22962 | 8.22968 | 9.99994 | 40 |
| 30 | 8.14945 | 8.14950 | 9.99996 | 30 | 30 | 8.23086 | 8.23092 | 9.99994 | 30 |
| 40 | 8.15094 | 8.15099 | 9.99996 | 20 | 40 | 8.23210 | 8.23216 | 9.99994 | 20 |
| 50 | 8.15243 | 8.15247 | 9.99996 | 10 | 50 | 8.23333 | 8.23339 | 9.99994 | 10 |
| 49 0 | 8.15391 | 8.15395 | 9.99996 | 0 11 | 59 0 | 8.23456 | 8.23462 | 9.99994 | 0 1 |
| 10 | 8.15538 | 8.15543 | 9.99996 | 50 | 10 | 8.23578 | 8.23585 | 9.99994 | 50 |
| 20 | 8.15685 | 8.15690 | 9.99996 | 40 | 20 | 8.23700 | 8.23707 | 9.99994 | 40 |
| 30 | 8.15832 | 8.15836 | 9.99996 | 30 | 30 | 8.23822 | 8.23829 | 9.99993 | 30 |
| 40 | 8.15978 | 8.15982 | 9.99995 | 20 | 40 | 8.23944 | 8.23950 | 9.99993 | 20 |
| 50 | 8.16123 | 8.16128 | 9.99995 | 10 | 50 | 8.24065 | 8.24071 | 9.99993 | 10 |
| 50 0 | 8.16268 | 8.16273 | 9.99995 | 0 10 | 60 0 | 8.24186 | 8.24192 | 9.99993 | 0 0 |
| ° ' " | log cos | log cot | log sin | ° ' " | ° ' " | log cos | log cot | log sin | ° ' " |

| ° ' " | log sin | log tan | log cos | ° ' " | ° ' " | log sin | log tan | log cos | ° ' " |
|-------|----------|----------|----------|-------|-------|----------|----------|----------|-------|
| 0 0 | 8.24 186 | 8.24 192 | 9.99 993 | 0 60 | 10 0 | 8.30 879 | 8.30 888 | 9.99 991 | 0 50 |
| 10 | 8.24 306 | 8.24 313 | 9.99 993 | 50 | 10 | 8.30 983 | 8.30 992 | 9.99 991 | 50 |
| 20 | 8.24 426 | 8.24 433 | 9.99 993 | 40 | 20 | 8.31 086 | 8.31 095 | 9.99 991 | 40 |
| 30 | 8.24 546 | 8.24 553 | 9.99 993 | 30 | 30 | 8.31 188 | 8.31 198 | 9.99 991 | 30 |
| 40 | 8.24 665 | 8.24 672 | 9.99 993 | 20 | 40 | 8.31 291 | 8.31 300 | 9.99 991 | 20 |
| 50 | 8.24 785 | 8.24 791 | 9.99 993 | 10 | 50 | 8.31 393 | 8.31 403 | 9.99 991 | 10 |
| 1 0 | 8.24 903 | 8.24 910 | 9.99 993 | 0 59 | 11 0 | 8.31 495 | 8.31 505 | 9.99 991 | 0 49 |
| 10 | 8.25 022 | 8.25 029 | 9.99 993 | 50 | 10 | 8.31 597 | 8.31 606 | 9.99 991 | 50 |
| 20 | 8.25 140 | 8.25 147 | 9.99 993 | 40 | 20 | 8.31 699 | 8.31 708 | 9.99 991 | 40 |
| 30 | 8.25 258 | 8.25 265 | 9.99 993 | 30 | 30 | 8.31 800 | 8.31 809 | 9.99 991 | 30 |
| 40 | 8.25 375 | 8.25 382 | 9.99 993 | 20 | 40 | 8.31 901 | 8.31 911 | 9.99 991 | 20 |
| 50 | 8.25 493 | 8.25 500 | 9.99 993 | 10 | 50 | 8.32 002 | 8.32 012 | 9.99 991 | 10 |
| 2 0 | 8.25 609 | 8.25 616 | 9.99 993 | 0 58 | 12 0 | 8.32 103 | 8.32 112 | 9.99 990 | 0 48 |
| 10 | 8.25 726 | 8.25 733 | 9.99 993 | 50 | 10 | 8.32 203 | 8.32 213 | 9.99 990 | 50 |
| 20 | 8.25 842 | 8.25 849 | 9.99 993 | 40 | 20 | 8.32 303 | 8.32 313 | 9.99 990 | 40 |
| 30 | 8.25 958 | 8.25 965 | 9.99 993 | 30 | 30 | 8.32 403 | 8.32 413 | 9.99 990 | 30 |
| 40 | 8.26 074 | 8.26 081 | 9.99 993 | 20 | 40 | 8.32 503 | 8.32 513 | 9.99 990 | 20 |
| 50 | 8.26 189 | 8.26 196 | 9.99 993 | 10 | 50 | 8.32 602 | 8.32 612 | 9.99 990 | 10 |
| 3 0 | 8.26 304 | 8.26 312 | 9.99 993 | 0 57 | 13 0 | 8.32 702 | 8.32 711 | 9.99 990 | 0 47 |
| 10 | 8.26 419 | 8.26 426 | 9.99 993 | 50 | 10 | 8.32 801 | 8.32 811 | 9.99 990 | 50 |
| 20 | 8.26 533 | 8.26 541 | 9.99 993 | 40 | 20 | 8.32 899 | 8.32 909 | 9.99 990 | 40 |
| 30 | 8.26 648 | 8.26 655 | 9.99 993 | 30 | 30 | 8.32 998 | 8.33 008 | 9.99 990 | 30 |
| 40 | 8.26 761 | 8.26 769 | 9.99 993 | 20 | 40 | 8.33 096 | 8.33 106 | 9.99 990 | 20 |
| 50 | 8.26 875 | 8.26 882 | 9.99 993 | 10 | 50 | 8.33 195 | 8.33 205 | 9.99 990 | 10 |
| 4 0 | 8.26 988 | 8.26 996 | 9.99 992 | 0 56 | 14 0 | 8.33 292 | 8.33 302 | 9.99 990 | 0 46 |
| 10 | 8.27 101 | 8.27 109 | 9.99 992 | 50 | 10 | 8.33 390 | 8.33 400 | 9.99 990 | 50 |
| 20 | 8.27 214 | 8.27 221 | 9.99 992 | 40 | 20 | 8.33 488 | 8.33 498 | 9.99 990 | 40 |
| 30 | 8.27 326 | 8.27 334 | 9.99 992 | 30 | 30 | 8.33 585 | 8.33 595 | 9.99 990 | 30 |
| 40 | 8.27 438 | 8.27 446 | 9.99 992 | 20 | 40 | 8.33 682 | 8.33 692 | 9.99 990 | 20 |
| 50 | 8.27 550 | 8.27 558 | 9.99 992 | 10 | 50 | 8.33 779 | 8.33 789 | 9.99 990 | 10 |
| 5 0 | 8.27 661 | 8.27 669 | 9.99 992 | 0 55 | 15 0 | 8.33 875 | 8.33 886 | 9.99 990 | 0 45 |
| 10 | 8.27 773 | 8.27 780 | 9.99 992 | 50 | 10 | 8.33 972 | 8.33 982 | 9.99 990 | 50 |
| 20 | 8.27 883 | 8.27 891 | 9.99 992 | 40 | 20 | 8.34 068 | 8.34 078 | 9.99 990 | 40 |
| 30 | 8.27 994 | 8.28 002 | 9.99 992 | 30 | 30 | 8.34 164 | 8.34 174 | 9.99 990 | 30 |
| 40 | 8.28 104 | 8.28 112 | 9.99 992 | 20 | 40 | 8.34 260 | 8.34 270 | 9.99 989 | 20 |
| 50 | 8.28 215 | 8.28 223 | 9.99 992 | 10 | 50 | 8.34 355 | 8.34 366 | 9.99 989 | 10 |
| 6 0 | 8.28 324 | 8.28 332 | 9.99 992 | 0 54 | 16 0 | 8.34 450 | 8.34 461 | 9.99 989 | 0 44 |
| 10 | 8.28 434 | 8.28 442 | 9.99 992 | 50 | 10 | 8.34 546 | 8.34 556 | 9.99 989 | 50 |
| 20 | 8.28 543 | 8.28 551 | 9.99 992 | 40 | 20 | 8.34 640 | 8.34 651 | 9.99 989 | 40 |
| 30 | 8.28 652 | 8.28 660 | 9.99 992 | 30 | 30 | 8.34 735 | 8.34 746 | 9.99 989 | 30 |
| 40 | 8.28 761 | 8.28 769 | 9.99 992 | 20 | 40 | 8.34 830 | 8.34 840 | 9.99 989 | 20 |
| 50 | 8.28 869 | 8.28 877 | 9.99 992 | 10 | 50 | 8.34 924 | 8.34 935 | 9.99 989 | 10 |
| 7 0 | 8.28 977 | 8.28 986 | 9.99 992 | 0 53 | 17 0 | 8.35 018 | 8.35 029 | 9.99 989 | 0 43 |
| 10 | 8.29 085 | 8.29 094 | 9.99 992 | 50 | 10 | 8.35 112 | 8.35 123 | 9.99 989 | 50 |
| 20 | 8.29 193 | 8.29 201 | 9.99 992 | 40 | 20 | 8.35 206 | 8.35 217 | 9.99 989 | 40 |
| 30 | 8.29 300 | 8.29 309 | 9.99 992 | 30 | 30 | 8.35 299 | 8.35 310 | 9.99 989 | 30 |
| 40 | 8.29 407 | 8.29 416 | 9.99 992 | 20 | 40 | 8.35 392 | 8.35 403 | 9.99 989 | 20 |
| 50 | 8.29 514 | 8.29 523 | 9.99 992 | 10 | 50 | 8.35 485 | 8.35 497 | 9.99 989 | 10 |
| 8 0 | 8.29 621 | 8.29 629 | 9.99 992 | 0 52 | 18 0 | 8.35 578 | 8.35 590 | 9.99 989 | 0 42 |
| 10 | 8.29 727 | 8.29 736 | 9.99 991 | 50 | 10 | 8.35 671 | 8.35 682 | 9.99 989 | 50 |
| 20 | 8.29 833 | 8.29 842 | 9.99 991 | 40 | 20 | 8.35 764 | 8.35 775 | 9.99 989 | 40 |
| 30 | 8.29 939 | 8.29 947 | 9.99 991 | 30 | 30 | 8.35 856 | 8.35 867 | 9.99 989 | 30 |
| 40 | 8.30 044 | 8.30 053 | 9.99 991 | 20 | 40 | 8.35 948 | 8.35 959 | 9.99 989 | 20 |
| 50 | 8.30 150 | 8.30 158 | 9.99 991 | 10 | 50 | 8.36 040 | 8.36 051 | 9.99 989 | 10 |
| 9 0 | 8.30 255 | 8.30 263 | 9.99 991 | 0 51 | 19 0 | 8.36 131 | 8.36 143 | 9.99 989 | 0 41 |
| 10 | 8.30 359 | 8.30 368 | 9.99 991 | 50 | 10 | 8.36 223 | 8.36 235 | 9.99 988 | 50 |
| 20 | 8.30 464 | 8.30 473 | 9.99 991 | 40 | 20 | 8.36 314 | 8.36 326 | 9.99 988 | 40 |
| 30 | 8.30 568 | 8.30 577 | 9.99 991 | 30 | 30 | 8.36 405 | 8.36 417 | 9.99 988 | 30 |
| 40 | 8.30 672 | 8.30 681 | 9.99 991 | 20 | 40 | 8.36 496 | 8.36 508 | 9.99 988 | 20 |
| 50 | 8.30 776 | 8.30 785 | 9.99 991 | 10 | 50 | 8.36 587 | 8.36 599 | 9.99 988 | 10 |
| 10 0 | 8.30 879 | 8.30 888 | 9.99 991 | 0 50 | 20 0 | 8.36 678 | 8.36 689 | 9.99 988 | 0 40 |
| ° ' " | log cos | log cot | log sin | ° ' " | ° ' " | log cos | log cot | log sin | ° ' " |

| ° ' " | log sin | log tan | log cos | ° ' " | ° ' " | log sin | log tan | log cos | ° ' " |
|-------|----------|----------|----------|-------|-------|----------|----------|----------|-------|
| 20 0 | 8.36 678 | 8.36 689 | 9.99 988 | 0 40 | 30 0 | 8.41 792 | 8.41 807 | 9.99 985 | 0 30 |
| 10 | 8.36 768 | 8.36 780 | 9.99 988 | 50 | 10 | 8.41 872 | 8.41 887 | 9.99 985 | 50 |
| 20 | 8.36 858 | 8.36 870 | 9.99 988 | 40 | 20 | 8.41 952 | 8.41 967 | 9.99 985 | 40 |
| 30 | 8.36 948 | 8.36 960 | 9.99 988 | 30 | 30 | 8.42 032 | 8.42 048 | 9.99 985 | 30 |
| 40 | 8.37 038 | 8.37 050 | 9.99 988 | 20 | 40 | 8.42 112 | 8.42 127 | 9.99 985 | 20 |
| 50 | 8.37 128 | 8.37 140 | 9.99 988 | 10 | 50 | 8.42 192 | 8.42 207 | 9.99 985 | 10 |
| 21 0 | 8.37 217 | 8.37 229 | 9.99 988 | 0 39 | 31 0 | 8.42 272 | 8.42 287 | 9.99 985 | 0 29 |
| 10 | 8.37 306 | 8.37 318 | 9.99 988 | 50 | 10 | 8.42 351 | 8.42 366 | 9.99 985 | 50 |
| 20 | 8.37 395 | 8.37 408 | 9.99 988 | 40 | 20 | 8.42 430 | 8.42 446 | 9.99 985 | 40 |
| 30 | 8.37 484 | 8.37 497 | 9.99 988 | 30 | 30 | 8.42 510 | 8.42 525 | 9.99 985 | 30 |
| 40 | 8.37 573 | 8.37 585 | 9.99 988 | 20 | 40 | 8.42 589 | 8.42 604 | 9.99 985 | 20 |
| 50 | 8.37 662 | 8.37 674 | 9.99 988 | 10 | 50 | 8.42 667 | 8.42 683 | 9.99 985 | 10 |
| 22 0 | 8.37 750 | 8.37 762 | 9.99 988 | 0 38 | 32 0 | 8.42 746 | 8.42 762 | 9.99 984 | 0 28 |
| 10 | 8.37 838 | 8.37 850 | 9.99 988 | 50 | 10 | 8.42 825 | 8.42 840 | 9.99 984 | 50 |
| 20 | 8.37 926 | 8.37 938 | 9.99 988 | 40 | 20 | 8.42 903 | 8.42 919 | 9.99 984 | 40 |
| 30 | 8.38 014 | 8.38 026 | 9.99 987 | 30 | 30 | 8.42 982 | 8.42 997 | 9.99 984 | 30 |
| 40 | 8.38 101 | 8.38 114 | 9.99 987 | 20 | 40 | 8.43 060 | 8.43 075 | 9.99 984 | 20 |
| 50 | 8.38 189 | 8.38 202 | 9.99 987 | 10 | 50 | 8.43 138 | 8.43 154 | 9.99 984 | 10 |
| 23 0 | 8.38 276 | 8.38 289 | 9.99 987 | 0 37 | 33 0 | 8.43 216 | 8.43 232 | 9.99 984 | 0 27 |
| 10 | 8.38 363 | 8.38 376 | 9.99 987 | 50 | 10 | 8.43 293 | 8.43 309 | 9.99 984 | 50 |
| 20 | 8.38 450 | 8.38 463 | 9.99 987 | 40 | 20 | 8.43 371 | 8.43 387 | 9.99 984 | 40 |
| 30 | 8.38 537 | 8.38 550 | 9.99 987 | 30 | 30 | 8.43 448 | 8.43 464 | 9.99 984 | 30 |
| 40 | 8.38 624 | 8.38 636 | 9.99 987 | 20 | 40 | 8.43 526 | 8.43 542 | 9.99 984 | 20 |
| 50 | 8.38 710 | 8.38 723 | 9.99 987 | 10 | 50 | 8.43 603 | 8.43 619 | 9.99 984 | 10 |
| 24 0 | 8.38 796 | 8.38 809 | 9.99 987 | 0 36 | 34 0 | 8.43 680 | 8.43 696 | 9.99 984 | 0 26 |
| 10 | 8.38 882 | 8.38 895 | 9.99 987 | 50 | 10 | 8.43 757 | 8.43 773 | 9.99 984 | 50 |
| 20 | 8.38 968 | 8.38 981 | 9.99 987 | 40 | 20 | 8.43 834 | 8.43 850 | 9.99 984 | 40 |
| 30 | 8.39 054 | 8.39 067 | 9.99 987 | 30 | 30 | 8.43 910 | 8.43 927 | 9.99 984 | 30 |
| 40 | 8.39 139 | 8.39 153 | 9.99 987 | 20 | 40 | 8.43 987 | 8.44 003 | 9.99 984 | 20 |
| 50 | 8.39 225 | 8.39 238 | 9.99 987 | 10 | 50 | 8.44 063 | 8.44 080 | 9.99 983 | 10 |
| 25 0 | 8.39 310 | 8.39 323 | 9.99 987 | 0 35 | 35 0 | 8.44 139 | 8.44 156 | 9.99 983 | 0 25 |
| 10 | 8.39 395 | 8.39 408 | 9.99 987 | 50 | 10 | 8.44 216 | 8.44 232 | 9.99 983 | 50 |
| 20 | 8.39 480 | 8.39 493 | 9.99 987 | 40 | 20 | 8.44 292 | 8.44 308 | 9.99 983 | 40 |
| 30 | 8.39 565 | 8.39 578 | 9.99 987 | 30 | 30 | 8.44 367 | 8.44 384 | 9.99 983 | 30 |
| 40 | 8.39 649 | 8.39 663 | 9.99 987 | 20 | 40 | 8.44 443 | 8.44 460 | 9.99 983 | 20 |
| 50 | 8.39 734 | 8.39 747 | 9.99 986 | 10 | 50 | 8.44 519 | 8.44 536 | 9.99 983 | 10 |
| 26 0 | 8.39 818 | 8.39 832 | 9.99 986 | 0 34 | 36 0 | 8.44 594 | 8.44 611 | 9.99 983 | 0 24 |
| 10 | 8.39 902 | 8.39 916 | 9.99 986 | 50 | 10 | 8.44 669 | 8.44 686 | 9.99 983 | 50 |
| 20 | 8.39 986 | 8.40 000 | 9.99 986 | 40 | 20 | 8.44 745 | 8.44 762 | 9.99 983 | 40 |
| 30 | 8.40 070 | 8.40 083 | 9.99 986 | 30 | 30 | 8.44 820 | 8.44 837 | 9.99 983 | 30 |
| 40 | 8.40 153 | 8.40 167 | 9.99 986 | 20 | 40 | 8.44 895 | 8.44 912 | 9.99 983 | 20 |
| 50 | 8.40 237 | 8.40 251 | 9.99 986 | 10 | 50 | 8.44 969 | 8.44 987 | 9.99 983 | 10 |
| 27 0 | 8.40 320 | 8.40 334 | 9.99 986 | 0 33 | 37 0 | 8.45 044 | 8.45 061 | 9.99 983 | 0 23 |
| 10 | 8.40 403 | 8.40 417 | 9.99 986 | 50 | 10 | 8.45 119 | 8.45 136 | 9.99 983 | 50 |
| 20 | 8.40 486 | 8.40 500 | 9.99 986 | 40 | 20 | 8.45 193 | 8.45 210 | 9.99 983 | 40 |
| 30 | 8.40 569 | 8.40 583 | 9.99 986 | 30 | 30 | 8.45 267 | 8.45 285 | 9.99 983 | 30 |
| 40 | 8.40 651 | 8.40 665 | 9.99 986 | 20 | 40 | 8.45 341 | 8.45 359 | 9.99 982 | 20 |
| 50 | 8.40 734 | 8.40 748 | 9.99 986 | 10 | 50 | 8.45 415 | 8.45 433 | 9.99 982 | 10 |
| 28 0 | 8.40 816 | 8.40 830 | 9.99 986 | 0 32 | 38 0 | 8.45 489 | 8.45 507 | 9.99 982 | 0 22 |
| 10 | 8.40 898 | 8.40 913 | 9.99 986 | 50 | 10 | 8.45 563 | 8.45 581 | 9.99 982 | 50 |
| 20 | 8.40 980 | 8.40 995 | 9.99 986 | 40 | 20 | 8.45 637 | 8.45 655 | 9.99 982 | 40 |
| 30 | 8.41 062 | 8.41 077 | 9.99 986 | 30 | 30 | 8.45 710 | 8.45 728 | 9.99 982 | 30 |
| 40 | 8.41 144 | 8.41 158 | 9.99 986 | 20 | 40 | 8.45 784 | 8.45 802 | 9.99 982 | 20 |
| 50 | 8.41 225 | 8.41 240 | 9.99 986 | 10 | 50 | 8.45 857 | 8.45 875 | 9.99 982 | 10 |
| 29 0 | 8.41 307 | 8.41 321 | 9.99 985 | 0 31 | 39 0 | 8.45 930 | 8.45 948 | 9.99 982 | 0 21 |
| 10 | 8.41 388 | 8.41 403 | 9.99 985 | 50 | 10 | 8.46 003 | 8.46 021 | 9.99 982 | 50 |
| 20 | 8.41 469 | 8.41 484 | 9.99 985 | 40 | 20 | 8.46 076 | 8.46 094 | 9.99 982 | 40 |
| 30 | 8.41 550 | 8.41 565 | 9.99 985 | 30 | 30 | 8.46 149 | 8.46 167 | 9.99 982 | 30 |
| 40 | 8.41 631 | 8.41 646 | 9.99 985 | 20 | 40 | 8.46 222 | 8.46 240 | 9.99 982 | 20 |
| 50 | 8.41 711 | 8.41 726 | 9.99 985 | 10 | 50 | 8.46 294 | 8.46 312 | 9.99 982 | 10 |
| 30 0 | 8.41 792 | 8.41 807 | 9.99 985 | 0 30 | 40 0 | 8.46 366 | 8.46 385 | 9.99 982 | 0 20 |
| ° ' " | log cos | log cot | log sin | ° ' " | ° ' " | log cos | log cot | log sin | ° ' " |

| ° ' " | log sin | log tan | log cos | ° ' " | ° ' " | log sin | log tan | log cos | ° ' " |
|-------|---------|---------|---------|-------|-------|---------|---------|---------|-------|
| 40 0 | 8.46366 | 8.46385 | 9.99982 | 0 20 | 50 0 | 8.50504 | 8.50527 | 9.99978 | 0 10 |
| 10 | 8.46439 | 8.46457 | 9.99982 | 50 | 10 | 8.50570 | 8.50593 | 9.99978 | 50 |
| 20 | 8.46511 | 8.46529 | 9.99982 | 40 | 20 | 8.50636 | 8.50658 | 9.99978 | 40 |
| 30 | 8.46583 | 8.46602 | 9.99981 | 30 | 30 | 8.50701 | 8.50724 | 9.99978 | 30 |
| 40 | 8.46655 | 8.46674 | 9.99981 | 20 | 40 | 8.50767 | 8.50789 | 9.99977 | 20 |
| 50 | 8.46727 | 8.46745 | 9.99981 | 10 | 50 | 8.50832 | 8.50855 | 9.99977 | 10 |
| 41 0 | 8.46799 | 8.46817 | 9.99981 | 0 19 | 51 0 | 8.50897 | 8.50920 | 9.99977 | 0 9 |
| 10 | 8.46870 | 8.46889 | 9.99981 | 50 | 10 | 8.50963 | 8.50985 | 9.99977 | 50 |
| 20 | 8.46942 | 8.46960 | 9.99981 | 40 | 20 | 8.51028 | 8.51050 | 9.99977 | 40 |
| 30 | 8.47013 | 8.47032 | 9.99981 | 30 | 30 | 8.51092 | 8.51115 | 9.99977 | 30 |
| 40 | 8.47084 | 8.47103 | 9.99981 | 20 | 40 | 8.51157 | 8.51180 | 9.99977 | 20 |
| 50 | 8.47155 | 8.47174 | 9.99981 | 10 | 50 | 8.51222 | 8.51245 | 9.99977 | 10 |
| 42 0 | 8.47226 | 8.47245 | 9.99981 | 0 18 | 52 0 | 8.51287 | 8.51310 | 9.99977 | 0 8 |
| 10 | 8.47297 | 8.47316 | 9.99981 | 50 | 10 | 8.51351 | 8.51374 | 9.99977 | 50 |
| 20 | 8.47368 | 8.47387 | 9.99981 | 40 | 20 | 8.51416 | 8.51439 | 9.99977 | 40 |
| 30 | 8.47439 | 8.47458 | 9.99981 | 30 | 30 | 8.51480 | 8.51503 | 9.99977 | 30 |
| 40 | 8.47509 | 8.47528 | 9.99981 | 20 | 40 | 8.51544 | 8.51568 | 9.99977 | 20 |
| 50 | 8.47580 | 8.47599 | 9.99981 | 10 | 50 | 8.51609 | 8.51632 | 9.99977 | 10 |
| 43 0 | 8.47650 | 8.47669 | 9.99981 | 0 17 | 53 0 | 8.51673 | 8.51696 | 9.99977 | 0 7 |
| 10 | 8.47720 | 8.47740 | 9.99980 | 50 | 10 | 8.51737 | 8.51760 | 9.99976 | 50 |
| 20 | 8.47790 | 8.47810 | 9.99980 | 40 | 20 | 8.51801 | 8.51824 | 9.99976 | 40 |
| 30 | 8.47860 | 8.47880 | 9.99980 | 30 | 30 | 8.51864 | 8.51888 | 9.99976 | 30 |
| 40 | 8.47930 | 8.47950 | 9.99980 | 20 | 40 | 8.51928 | 8.51952 | 9.99976 | 20 |
| 50 | 8.48000 | 8.48020 | 9.99980 | 10 | 50 | 8.51992 | 8.52015 | 9.99976 | 10 |
| 44 0 | 8.48069 | 8.48090 | 9.99980 | 0 16 | 54 0 | 8.52055 | 8.52079 | 9.99976 | 0 6 |
| 10 | 8.48139 | 8.48159 | 9.99980 | 50 | 10 | 8.52119 | 8.52143 | 9.99976 | 50 |
| 20 | 8.48208 | 8.48228 | 9.99980 | 40 | 20 | 8.52182 | 8.52206 | 9.99976 | 40 |
| 30 | 8.48278 | 8.48298 | 9.99980 | 30 | 30 | 8.52245 | 8.52269 | 9.99976 | 30 |
| 40 | 8.48347 | 8.48367 | 9.99980 | 20 | 40 | 8.52308 | 8.52332 | 9.99976 | 20 |
| 50 | 8.48416 | 8.48436 | 9.99980 | 10 | 50 | 8.52371 | 8.52396 | 9.99976 | 10 |
| 45 0 | 8.48485 | 8.48505 | 9.99980 | 0 15 | 55 0 | 8.52434 | 8.52459 | 9.99976 | 0 5 |
| 10 | 8.48554 | 8.48574 | 9.99980 | 50 | 10 | 8.52497 | 8.52522 | 9.99976 | 50 |
| 20 | 8.48622 | 8.48643 | 9.99980 | 40 | 20 | 8.52560 | 8.52584 | 9.99976 | 40 |
| 30 | 8.48691 | 8.48711 | 9.99980 | 30 | 30 | 8.52623 | 8.52647 | 9.99975 | 30 |
| 40 | 8.48760 | 8.48780 | 9.99979 | 20 | 40 | 8.52685 | 8.52710 | 9.99975 | 20 |
| 50 | 8.48828 | 8.48849 | 9.99979 | 10 | 50 | 8.52748 | 8.52772 | 9.99975 | 10 |
| 46 0 | 8.48896 | 8.48917 | 9.99979 | 0 14 | 56 0 | 8.52810 | 8.52835 | 9.99975 | 0 4 |
| 10 | 8.48965 | 8.48985 | 9.99979 | 50 | 10 | 8.52872 | 8.52897 | 9.99975 | 50 |
| 20 | 8.49033 | 8.49053 | 9.99979 | 40 | 20 | 8.52935 | 8.52960 | 9.99975 | 40 |
| 30 | 8.49101 | 8.49121 | 9.99979 | 30 | 30 | 8.52997 | 8.53022 | 9.99975 | 30 |
| 40 | 8.49169 | 8.49189 | 9.99979 | 20 | 40 | 8.53059 | 8.53084 | 9.99975 | 20 |
| 50 | 8.49236 | 8.49257 | 9.99979 | 10 | 50 | 8.53121 | 8.53146 | 9.99975 | 10 |
| 47 0 | 8.49304 | 8.49325 | 9.99979 | 0 13 | 57 0 | 8.53183 | 8.53208 | 9.99975 | 0 3 |
| 10 | 8.49372 | 8.49393 | 9.99979 | 50 | 10 | 8.53245 | 8.53270 | 9.99975 | 50 |
| 20 | 8.49439 | 8.49460 | 9.99979 | 40 | 20 | 8.53306 | 8.53332 | 9.99975 | 40 |
| 30 | 8.49506 | 8.49528 | 9.99979 | 30 | 30 | 8.53368 | 8.53393 | 9.99975 | 30 |
| 40 | 8.49574 | 8.49595 | 9.99979 | 20 | 40 | 8.53429 | 8.53455 | 9.99975 | 20 |
| 50 | 8.49641 | 8.49662 | 9.99979 | 10 | 50 | 8.53491 | 8.53516 | 9.99974 | 10 |
| 48 0 | 8.49708 | 8.49729 | 9.99979 | 0 12 | 58 0 | 8.53552 | 8.53578 | 9.99974 | 0 2 |
| 10 | 8.49775 | 8.49796 | 9.99979 | 50 | 10 | 8.53614 | 8.53639 | 9.99974 | 50 |
| 20 | 8.49842 | 8.49863 | 9.99978 | 40 | 20 | 8.53675 | 8.53700 | 9.99974 | 40 |
| 30 | 8.49908 | 8.49930 | 9.99978 | 30 | 30 | 8.53736 | 8.53762 | 9.99974 | 30 |
| 40 | 8.49975 | 8.49997 | 9.99978 | 20 | 40 | 8.53797 | 8.53823 | 9.99974 | 20 |
| 50 | 8.50042 | 8.50063 | 9.99978 | 10 | 50 | 8.53858 | 8.53884 | 9.99974 | 10 |
| 49 0 | 8.50108 | 8.50130 | 9.99978 | 0 11 | 59 0 | 8.53919 | 8.53945 | 9.99974 | 0 1 |
| 10 | 8.50174 | 8.50196 | 9.99978 | 50 | 10 | 8.53979 | 8.54005 | 9.99974 | 50 |
| 20 | 8.50241 | 8.50263 | 9.99978 | 40 | 20 | 8.54040 | 8.54066 | 9.99974 | 40 |
| 30 | 8.50307 | 8.50329 | 9.99978 | 30 | 30 | 8.54101 | 8.54127 | 9.99974 | 30 |
| 40 | 8.50373 | 8.50395 | 9.99978 | 20 | 40 | 8.54161 | 8.54187 | 9.99974 | 20 |
| 50 | 8.50439 | 8.50461 | 9.99978 | 10 | 50 | 8.54222 | 8.54248 | 9.99974 | 10 |
| 50 0 | 8.50504 | 8.50527 | 9.99978 | 0 10 | 60 0 | 8.54282 | 8.54308 | 9.99974 | 0 0 |
| ° ' " | log cos | log cot | log sin | ° ' " | ° ' " | log cos | log cot | log sin | ° ' " |

| ' | log sin 8 | log tan 8 | log cot 11 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 24 186 | 24 192 | 75 808 | 99 993 | 60 |
| 1 | 24 903 | 24 910 | 75 090 | 99 993 | 59 |
| 2 | 25 609 | 25 616 | 74 384 | 99 993 | 58 |
| 3 | 26 304 | 26 312 | 73 688 | 99 993 | 57 |
| 4 | 26 988 | 26 996 | 73 004 | 99 992 | 56 |
| 5 | 27 661 | 27 669 | 72 331 | 99 992 | 55 |
| 6 | 28 324 | 28 332 | 71 668 | 99 992 | 54 |
| 7 | 28 977 | 28 986 | 71 014 | 99 992 | 53 |
| 8 | 29 621 | 29 629 | 70 371 | 99 992 | 52 |
| 9 | 30 255 | 30 263 | 69 737 | 99 991 | 51 |
| 10 | 30 879 | 30 888 | 69 112 | 99 991 | 50 |
| 11 | 31 495 | 31 505 | 68 495 | 99 991 | 49 |
| 12 | 32 103 | 32 112 | 67 888 | 99 990 | 48 |
| 13 | 32 702 | 32 711 | 67 289 | 99 990 | 47 |
| 14 | 33 292 | 33 302 | 66 698 | 99 990 | 46 |
| 15 | 33 875 | 33 886 | 66 114 | 99 990 | 45 |
| 16 | 34 450 | 34 461 | 65 539 | 99 989 | 44 |
| 17 | 35 018 | 35 029 | 64 971 | 99 989 | 43 |
| 18 | 35 578 | 35 590 | 64 410 | 99 989 | 42 |
| 19 | 36 131 | 36 143 | 63 857 | 99 989 | 41 |
| 20 | 36 678 | 36 689 | 63 311 | 99 988 | 40 |
| 21 | 37 217 | 37 229 | 62 771 | 99 988 | 39 |
| 22 | 37 750 | 37 762 | 62 238 | 99 988 | 38 |
| 23 | 38 276 | 38 289 | 61 711 | 99 987 | 37 |
| 24 | 38 796 | 38 809 | 61 191 | 99 987 | 36 |
| 25 | 39 310 | 39 323 | 60 677 | 99 987 | 35 |
| 26 | 39 818 | 39 832 | 60 168 | 99 986 | 34 |
| 27 | 40 320 | 40 334 | 59 666 | 99 986 | 33 |
| 28 | 40 816 | 40 830 | 59 170 | 99 986 | 32 |
| 29 | 41 307 | 41 321 | 58 679 | 99 985 | 31 |
| 30 | 41 792 | 41 807 | 58 193 | 99 985 | 30 |
| 31 | 42 272 | 42 287 | 57 713 | 99 985 | 29 |
| 32 | 42 746 | 42 762 | 57 238 | 99 984 | 28 |
| 33 | 43 216 | 43 232 | 56 768 | 99 984 | 27 |
| 34 | 43 680 | 43 696 | 56 304 | 99 984 | 26 |
| 35 | 44 139 | 44 156 | 55 844 | 99 983 | 25 |
| 36 | 44 594 | 44 611 | 55 389 | 99 983 | 24 |
| 37 | 45 044 | 45 061 | 54 939 | 99 983 | 23 |
| 38 | 45 489 | 45 507 | 54 493 | 99 982 | 22 |
| 39 | 45 930 | 45 948 | 54 052 | 99 982 | 21 |
| 40 | 46 366 | 46 385 | 53 615 | 99 982 | 20 |
| 41 | 46 799 | 46 817 | 53 183 | 99 981 | 19 |
| 42 | 47 226 | 47 245 | 52 755 | 99 981 | 18 |
| 43 | 47 650 | 47 669 | 52 331 | 99 981 | 17 |
| 44 | 48 069 | 48 089 | 51 911 | 99 980 | 16 |
| 45 | 48 485 | 48 505 | 51 495 | 99 980 | 15 |
| 46 | 48 896 | 48 917 | 51 083 | 99 979 | 14 |
| 47 | 49 304 | 49 325 | 50 675 | 99 979 | 13 |
| 48 | 49 708 | 49 729 | 50 271 | 99 979 | 12 |
| 49 | 50 108 | 50 130 | 49 870 | 99 978 | 11 |
| 50 | 50 504 | 50 527 | 49 473 | 99 978 | 10 |
| 51 | 50 897 | 50 920 | 49 080 | 99 977 | 9 |
| 52 | 51 287 | 51 310 | 48 690 | 99 977 | 8 |
| 53 | 51 673 | 51 696 | 48 304 | 99 977 | 7 |
| 54 | 52 055 | 52 079 | 47 921 | 99 976 | 6 |
| 55 | 52 434 | 52 459 | 47 541 | 99 976 | 5 |
| 56 | 52 810 | 52 835 | 47 165 | 99 975 | 4 |
| 57 | 53 183 | 53 208 | 46 792 | 99 975 | 3 |
| 58 | 53 552 | 53 578 | 46 422 | 99 974 | 2 |
| 59 | 53 919 | 53 945 | 46 055 | 99 974 | 1 |
| 60 | 54 282 | 54 308 | 45 692 | 99 974 | 0 |
| ' | log cos 8 | log cot 8 | log tan 11 | log sin 9 | ' |

| ' | log sin 8 | log tan 8 | log cot 11 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 54 282 | 54 308 | 45 692 | 99 974 | 60 |
| 1 | 54 642 | 54 669 | 45 331 | 99 973 | 59 |
| 2 | 54 999 | 55 027 | 44 973 | 99 973 | 58 |
| 3 | 55 354 | 55 382 | 44 618 | 99 972 | 57 |
| 4 | 55 705 | 55 734 | 44 266 | 99 972 | 56 |
| 5 | 56 054 | 56 083 | 43 917 | 99 971 | 55 |
| 6 | 56 400 | 56 429 | 43 571 | 99 971 | 54 |
| 7 | 56 743 | 56 773 | 43 227 | 99 970 | 53 |
| 8 | 57 084 | 57 114 | 42 886 | 99 970 | 52 |
| 9 | 57 421 | 57 452 | 42 548 | 99 969 | 51 |
| 10 | 57 757 | 57 788 | 42 212 | 99 969 | 50 |
| 11 | 58 089 | 58 121 | 41 879 | 99 968 | 49 |
| 12 | 58 419 | 58 451 | 41 549 | 99 968 | 48 |
| 13 | 58 747 | 58 779 | 41 221 | 99 967 | 47 |
| 14 | 59 072 | 59 105 | 40 895 | 99 967 | 46 |
| 15 | 59 395 | 59 428 | 40 572 | 99 967 | 45 |
| 16 | 59 715 | 59 749 | 40 251 | 99 966 | 44 |
| 17 | 60 033 | 60 068 | 39 932 | 99 966 | 43 |
| 18 | 60 349 | 60 384 | 39 616 | 99 965 | 42 |
| 19 | 60 662 | 60 698 | 39 302 | 99 964 | 41 |
| 20 | 60 973 | 61 009 | 38 991 | 99 964 | 40 |
| 21 | 61 282 | 61 319 | 38 681 | 99 963 | 39 |
| 22 | 61 589 | 61 626 | 38 374 | 99 963 | 38 |
| 23 | 61 894 | 61 931 | 38 069 | 99 962 | 37 |
| 24 | 62 196 | 62 234 | 37 766 | 99 962 | 36 |
| 25 | 62 497 | 62 535 | 37 465 | 99 961 | 35 |
| 26 | 62 795 | 62 834 | 37 166 | 99 961 | 34 |
| 27 | 63 091 | 63 131 | 36 869 | 99 960 | 33 |
| 28 | 63 385 | 63 426 | 36 574 | 99 960 | 32 |
| 29 | 63 678 | 63 718 | 36 282 | 99 959 | 31 |
| 30 | 63 968 | 64 009 | 35 991 | 99 959 | 30 |
| 31 | 64 256 | 64 298 | 35 702 | 99 958 | 29 |
| 32 | 64 543 | 64 585 | 35 415 | 99 958 | 28 |
| 33 | 64 827 | 64 870 | 35 130 | 99 957 | 27 |
| 34 | 65 110 | 65 154 | 34 846 | 99 956 | 26 |
| 35 | 65 391 | 65 435 | 34 565 | 99 956 | 25 |
| 36 | 65 670 | 65 715 | 34 285 | 99 955 | 24 |
| 37 | 65 947 | 65 993 | 34 007 | 99 955 | 23 |
| 38 | 66 223 | 66 269 | 33 731 | 99 954 | 22 |
| 39 | 66 497 | 66 543 | 33 457 | 99 954 | 21 |
| 40 | 66 769 | 66 816 | 33 184 | 99 953 | 20 |
| 41 | 67 039 | 67 087 | 32 913 | 99 952 | 19 |
| 42 | 67 308 | 67 356 | 32 644 | 99 952 | 18 |
| 43 | 67 575 | 67 624 | 32 376 | 99 951 | 17 |
| 44 | 67 841 | 67 890 | 32 110 | 99 951 | 16 |
| 45 | 68 104 | 68 154 | 31 846 | 99 950 | 15 |
| 46 | 68 367 | 68 417 | 31 583 | 99 949 | 14 |
| 47 | 68 627 | 68 678 | 31 322 | 99 949 | 13 |
| 48 | 68 886 | 68 938 | 31 062 | 99 948 | 12 |
| 49 | 69 144 | 69 196 | 30 804 | 99 948 | 11 |
| 50 | 69 400 | 69 453 | 30 547 | 99 947 | 10 |
| 51 | 69 654 | 69 708 | 30 292 | 99 946 | 9 |
| 52 | 69 907 | 69 962 | 30 038 | 99 946 | 8 |
| 53 | 70 159 | 70 214 | 29 786 | 99 945 | 7 |
| 54 | 70 409 | 70 465 | 29 535 | 99 944 | 6 |
| 55 | 70 658 | 70 714 | 29 286 | 99 944 | 5 |
| 56 | 70 905 | 70 962 | 29 038 | 99 943 | 4 |
| 57 | 71 151 | 71 208 | 28 792 | 99 942 | 3 |
| 58 | 71 395 | 71 453 | 28 547 | 99 942 | 2 |
| 59 | 71 638 | 71 697 | 28 303 | 99 941 | 1 |
| 60 | 71 880 | 71 940 | 28 060 | 99 940 | 0 |
| ' | log cos 8 | log cot 8 | log tan 11 | log sin 9 | ' |

| ' | log sin 8 | log tan 8 | log cot 11 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 71 880 | 71 940 | 28 060 | 99 940 | 60 |
| 1 | 72 120 | 72 181 | 27 819 | 99 940 | 59 |
| 2 | 72 359 | 72 420 | 27 580 | 99 939 | 58 |
| 3 | 72 597 | 72 659 | 27 341 | 99 938 | 57 |
| 4 | 72 834 | 72 896 | 27 104 | 99 938 | 56 |
| 5 | 73 069 | 73 132 | 26 868 | 99 937 | 55 |
| 6 | 73 303 | 73 366 | 26 634 | 99 936 | 54 |
| 7 | 73 535 | 73 600 | 26 400 | 99 936 | 53 |
| 8 | 73 767 | 73 832 | 26 168 | 99 935 | 52 |
| 9 | 73 997 | 74 063 | 25 937 | 99 934 | 51 |
| 10 | 74 226 | 74 292 | 25 708 | 99 934 | 50 |
| 11 | 74 454 | 74 521 | 25 479 | 99 933 | 49 |
| 12 | 74 680 | 74 748 | 25 252 | 99 932 | 48 |
| 13 | 74 906 | 74 974 | 25 026 | 99 932 | 47 |
| 14 | 75 130 | 75 199 | 24 801 | 99 931 | 46 |
| 15 | 75 353 | 75 423 | 24 577 | 99 930 | 45 |
| 16 | 75 575 | 75 645 | 24 355 | 99 929 | 44 |
| 17 | 75 795 | 75 867 | 24 133 | 99 929 | 43 |
| 18 | 76 015 | 76 087 | 23 913 | 99 928 | 42 |
| 19 | 76 234 | 76 306 | 23 694 | 99 927 | 41 |
| 20 | 76 451 | 76 525 | 23 475 | 99 926 | 40 |
| 21 | 76 667 | 76 742 | 23 258 | 99 926 | 39 |
| 22 | 76 883 | 76 958 | 23 042 | 99 925 | 38 |
| 23 | 77 097 | 77 173 | 22 827 | 99 924 | 37 |
| 24 | 77 310 | 77 387 | 22 613 | 99 923 | 36 |
| 25 | 77 522 | 77 600 | 22 400 | 99 923 | 35 |
| 26 | 77 733 | 77 811 | 22 189 | 99 922 | 34 |
| 27 | 77 943 | 78 022 | 21 978 | 99 921 | 33 |
| 28 | 78 152 | 78 232 | 21 768 | 99 920 | 32 |
| 29 | 78 360 | 78 441 | 21 559 | 99 920 | 31 |
| 30 | 78 568 | 78 649 | 21 351 | 99 919 | 30 |
| 31 | 78 774 | 78 855 | 21 145 | 99 918 | 29 |
| 32 | 78 979 | 79 061 | 20 939 | 99 917 | 28 |
| 33 | 79 183 | 79 266 | 20 734 | 99 917 | 27 |
| 34 | 79 386 | 79 470 | 20 530 | 99 916 | 26 |
| 35 | 79 588 | 79 673 | 20 327 | 99 915 | 25 |
| 36 | 79 789 | 79 875 | 20 125 | 99 914 | 24 |
| 37 | 79 990 | 80 076 | 19 924 | 99 913 | 23 |
| 38 | 80 189 | 80 277 | 19 723 | 99 913 | 22 |
| 39 | 80 388 | 80 476 | 19 524 | 99 912 | 21 |
| 40 | 80 585 | 80 674 | 19 326 | 99 911 | 20 |
| 41 | 80 782 | 80 872 | 19 128 | 99 910 | 19 |
| 42 | 80 978 | 81 068 | 18 932 | 99 909 | 18 |
| 43 | 81 173 | 81 264 | 18 736 | 99 909 | 17 |
| 44 | 81 367 | 81 459 | 18 541 | 99 908 | 16 |
| 45 | 81 560 | 81 653 | 18 347 | 99 907 | 15 |
| 46 | 81 752 | 81 846 | 18 154 | 99 906 | 14 |
| 47 | 81 944 | 82 038 | 17 962 | 99 905 | 13 |
| 48 | 82 134 | 82 230 | 17 770 | 99 904 | 12 |
| 49 | 82 324 | 82 420 | 17 580 | 99 904 | 11 |
| 50 | 82 513 | 82 610 | 17 390 | 99 903 | 10 |
| 51 | 82 701 | 82 799 | 17 201 | 99 902 | 9 |
| 52 | 82 888 | 82 987 | 17 013 | 99 901 | 8 |
| 53 | 83 075 | 83 175 | 16 825 | 99 900 | 7 |
| 54 | 83 261 | 83 361 | 16 639 | 99 899 | 6 |
| 55 | 83 446 | 83 547 | 16 453 | 99 898 | 5 |
| 56 | 83 630 | 83 732 | 16 268 | 99 898 | 4 |
| 57 | 83 813 | 83 916 | 16 084 | 99 897 | 3 |
| 58 | 83 996 | 84 100 | 15 900 | 99 896 | 2 |
| 59 | 84 177 | 84 282 | 15 718 | 99 895 | 1 |
| 60 | 84 358 | 84 464 | 15 536 | 99 894 | 0 |
| ' | log cos 8 | log cot 8 | log tan 11 | log sin 9 | ' |

| ' | log sin 8 | log tan 8 | log cot 11 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 84 358 | 84 464 | 15 536 | 99 894 | 60 |
| 1 | 84 539 | 84 646 | 15 354 | 99 893 | 59 |
| 2 | 84 718 | 84 826 | 15 174 | 99 892 | 58 |
| 3 | 84 897 | 85 006 | 14 994 | 99 891 | 57 |
| 4 | 85 075 | 85 185 | 14 815 | 99 891 | 56 |
| 5 | 85 252 | 85 363 | 14 637 | 99 890 | 55 |
| 6 | 85 429 | 85 540 | 14 460 | 99 889 | 54 |
| 7 | 85 605 | 85 717 | 14 283 | 99 888 | 53 |
| 8 | 85 780 | 85 893 | 14 107 | 99 887 | 52 |
| 9 | 85 955 | 86 069 | 13 931 | 99 886 | 51 |
| 10 | 86 128 | 86 243 | 13 757 | 99 885 | 50 |
| 11 | 86 301 | 86 417 | 13 583 | 99 884 | 49 |
| 12 | 86 474 | 86 591 | 13 409 | 99 883 | 48 |
| 13 | 86 645 | 86 763 | 13 237 | 99 882 | 47 |
| 14 | 86 816 | 86 935 | 13 065 | 99 881 | 46 |
| 15 | 86 987 | 87 106 | 12 894 | 99 880 | 45 |
| 16 | 87 156 | 87 277 | 12 723 | 99 879 | 44 |
| 17 | 87 325 | 87 447 | 12 553 | 99 879 | 43 |
| 18 | 87 494 | 87 616 | 12 384 | 99 878 | 42 |
| 19 | 87 661 | 87 785 | 12 215 | 99 877 | 41 |
| 20 | 87 829 | 87 953 | 12 047 | 99 876 | 40 |
| 21 | 87 995 | 88 120 | 11 880 | 99 875 | 39 |
| 22 | 88 161 | 88 287 | 11 713 | 99 874 | 38 |
| 23 | 88 326 | 88 453 | 11 547 | 99 873 | 37 |
| 24 | 88 490 | 88 618 | 11 382 | 99 872 | 36 |
| 25 | 88 654 | 88 783 | 11 217 | 99 871 | 35 |
| 26 | 88 817 | 88 948 | 11 052 | 99 870 | 34 |
| 27 | 88 980 | 89 111 | 10 889 | 99 869 | 33 |
| 28 | 89 142 | 89 274 | 10 726 | 99 868 | 32 |
| 29 | 89 304 | 89 437 | 10 563 | 99 867 | 31 |
| 30 | 89 464 | 89 598 | 10 402 | 99 866 | 30 |
| 31 | 89 625 | 89 760 | 10 240 | 99 865 | 29 |
| 32 | 89 784 | 89 920 | 10 080 | 99 864 | 28 |
| 33 | 89 943 | 90 080 | 09 920 | 99 863 | 27 |
| 34 | 90 102 | 90 240 | 09 760 | 99 862 | 26 |
| 35 | 90 260 | 90 399 | 09 601 | 99 861 | 25 |
| 36 | 90 417 | 90 557 | 09 443 | 99 860 | 24 |
| 37 | 90 574 | 90 715 | 09 285 | 99 859 | 23 |
| 38 | 90 730 | 90 872 | 09 128 | 99 858 | 22 |
| 39 | 90 885 | 91 029 | 08 971 | 99 857 | 21 |
| 40 | 91 040 | 91 185 | 08 815 | 99 856 | 20 |
| 41 | 91 195 | 91 340 | 08 660 | 99 855 | 19 |
| 42 | 91 349 | 91 495 | 08 505 | 99 854 | 18 |
| 43 | 91 502 | 91 650 | 08 350 | 99 853 | 17 |
| 44 | 91 655 | 91 803 | 08 197 | 99 852 | 16 |
| 45 | 91 807 | 91 957 | 08 043 | 99 851 | 15 |
| 46 | 91 959 | 92 110 | 07 890 | 99 850 | 14 |
| 47 | 92 110 | 92 262 | 07 738 | 99 848 | 13 |
| 48 | 92 261 | 92 414 | 07 586 | 99 847 | 12 |
| 49 | 92 411 | 92 565 | 07 435 | 99 846 | 11 |
| 50 | 92 561 | 92 716 | 07 284 | 99 845 | 10 |
| 51 | 92 710 | 92 866 | 07 134 | 99 844 | 9 |
| 52 | 92 859 | 93 016 | 06 984 | 99 843 | 8 |
| 53 | 93 007 | 93 165 | 06 835 | 99 842 | 7 |
| 54 | 93 154 | 93 313 | 06 687 | 99 841 | 6 |
| 55 | 93 301 | 93 462 | 06 538 | 99 840 | 5 |
| 56 | 93 448 | 93 609 | 06 391 | 99 839 | 4 |
| 57 | 93 594 | 93 756 | 06 244 | 99 838 | 3 |
| 58 | 93 740 | 93 903 | 06 097 | 99 837 | 2 |
| 59 | 93 885 | 94 049 | 05 951 | 99 836 | 1 |
| 60 | 94 030 | 94 195 | 05 805 | 99 834 | 0 |
| ' | log cos 8 | log cot 8 | log tan 11 | log sin 9 | ' |

| ' | log sin 8 | log tan 8 | log cot 11 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 94 030 | 94 195 | 05 805 | 99 834 | 60 |
| 1 | 94 174 | 94 340 | 05 660 | 99 833 | 59 |
| 2 | 94 317 | 94 485 | 05 515 | 99 832 | 58 |
| 3 | 94 461 | 94 630 | 05 370 | 99 831 | 57 |
| 4 | 94 603 | 94 773 | 05 227 | 99 830 | 56 |
| 5 | 94 746 | 94 917 | 05 083 | 99 829 | 55 |
| 6 | 94 887 | 95 060 | 04 940 | 99 828 | 54 |
| 7 | 95 029 | 95 202 | 04 798 | 99 827 | 53 |
| 8 | 95 170 | 95 344 | 04 656 | 99 825 | 52 |
| 9 | 95 310 | 95 486 | 04 514 | 99 824 | 51 |
| 10 | 95 450 | 95 627 | 04 373 | 99 823 | 50 |
| 11 | 95 589 | 95 767 | 04 233 | 99 822 | 49 |
| 12 | 95 728 | 95 908 | 04 092 | 99 821 | 48 |
| 13 | 95 867 | 96 047 | 03 953 | 99 820 | 47 |
| 14 | 96 005 | 96 187 | 03 813 | 99 819 | 46 |
| 15 | 96 143 | 96 325 | 03 675 | 99 817 | 45 |
| 16 | 96 280 | 96 464 | 03 536 | 99 816 | 44 |
| 17 | 96 417 | 96 602 | 03 398 | 99 815 | 43 |
| 18 | 96 553 | 96 739 | 03 261 | 99 814 | 42 |
| 19 | 96 689 | 96 877 | 03 123 | 99 813 | 41 |
| 20 | 96 825 | 97 013 | 02 987 | 99 812 | 40 |
| 21 | 96 960 | 97 150 | 02 850 | 99 810 | 39 |
| 22 | 97 095 | 97 285 | 02 715 | 99 809 | 38 |
| 23 | 97 229 | 97 421 | 02 579 | 99 808 | 37 |
| 24 | 97 363 | 97 556 | 02 444 | 99 807 | 36 |
| 25 | 97 496 | 97 691 | 02 309 | 99 806 | 35 |
| 26 | 97 629 | 97 825 | 02 175 | 99 804 | 34 |
| 27 | 97 762 | 97 959 | 02 041 | 99 803 | 33 |
| 28 | 97 894 | 98 092 | 01 908 | 99 802 | 32 |
| 29 | 98 026 | 98 225 | 01 775 | 99 801 | 31 |
| 30 | 98 157 | 98 358 | 01 642 | 99 800 | 30 |
| 31 | 98 288 | 98 490 | 01 510 | 99 798 | 29 |
| 32 | 98 419 | 98 622 | 01 378 | 99 797 | 28 |
| 33 | 98 549 | 98 753 | 01 247 | 99 796 | 27 |
| 34 | 98 679 | 98 884 | 01 116 | 99 795 | 26 |
| 35 | 98 808 | 99 015 | 00 985 | 99 793 | 25 |
| 36 | 98 937 | 99 145 | 00 855 | 99 792 | 24 |
| 37 | 99 066 | 99 275 | 00 725 | 99 791 | 23 |
| 38 | 99 194 | 99 405 | 00 595 | 99 790 | 22 |
| 39 | 99 322 | 99 534 | 00 466 | 99 788 | 21 |
| 40 | 99 450 | 99 662 | 00 338 | 99 787 | 20 |
| 41 | 99 577 | 99 791 | 00 209 | 99 786 | 19 |
| 42 | 99 704 | 99 919 | 00 081 | 99 785 | 18 |
| 43 | 99 830 | 00 046 | 99 954 | 99 783 | 17 |
| 44 | 99 956 | 00 174 | 99 826 | 99 782 | 16 |
| 45 | 00 082 | 00 301 | 99 699 | 99 781 | 15 |
| 46 | 00 207 | 00 427 | 99 573 | 99 780 | 14 |
| 47 | 00 332 | 00 553 | 99 447 | 99 778 | 13 |
| 48 | 00 456 | 00 679 | 99 321 | 99 777 | 12 |
| 49 | 00 581 | 00 805 | 99 195 | 99 776 | 11 |
| 50 | 00 704 | 00 930 | 99 070 | 99 775 | 10 |
| 51 | 00 828 | 01 055 | 98 945 | 99 773 | 9 |
| 52 | 00 951 | 01 179 | 98 821 | 99 772 | 8 |
| 53 | 01 074 | 01 303 | 98 697 | 99 771 | 7 |
| 54 | 01 196 | 01 427 | 98 573 | 99 769 | 6 |
| 55 | 01 318 | 01 550 | 98 450 | 99 768 | 5 |
| 56 | 01 440 | 01 673 | 98 327 | 99 767 | 4 |
| 57 | 01 561 | 01 796 | 98 204 | 99 765 | 3 |
| 58 | 01 682 | 01 918 | 98 082 | 99 764 | 2 |
| 59 | 01 803 | 02 040 | 97 960 | 99 763 | 1 |
| 60 | 01 923 | 02 162 | 97 838 | 99 761 | 0 |
| ' | log cos 9 | log cot 8 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 01 923 | 02 162 | 97 838 | 99 761 | 60 |
| 1 | 02 043 | 02 283 | 97 717 | 99 760 | 59 |
| 2 | 02 163 | 02 404 | 97 596 | 99 759 | 58 |
| 3 | 02 283 | 02 525 | 97 475 | 99 757 | 57 |
| 4 | 02 402 | 02 645 | 97 355 | 99 756 | 56 |
| 5 | 02 520 | 02 766 | 97 234 | 99 755 | 55 |
| 6 | 02 639 | 02 885 | 97 115 | 99 753 | 54 |
| 7 | 02 757 | 03 005 | 96 995 | 99 752 | 53 |
| 8 | 02 874 | 03 124 | 96 876 | 99 751 | 52 |
| 9 | 02 992 | 03 242 | 96 758 | 99 749 | 51 |
| 10 | 03 109 | 03 361 | 96 639 | 99 748 | 50 |
| 11 | 03 226 | 03 479 | 96 521 | 99 747 | 49 |
| 12 | 03 342 | 03 597 | 96 403 | 99 745 | 48 |
| 13 | 03 458 | 03 714 | 96 286 | 99 744 | 47 |
| 14 | 03 574 | 03 832 | 96 168 | 99 742 | 46 |
| 15 | 03 690 | 03 948 | 96 052 | 99 741 | 45 |
| 16 | 03 805 | 04 065 | 95 935 | 99 740 | 44 |
| 17 | 03 920 | 04 181 | 95 819 | 99 738 | 43 |
| 18 | 04 034 | 04 297 | 95 703 | 99 737 | 42 |
| 19 | 04 149 | 04 413 | 95 587 | 99 736 | 41 |
| 20 | 04 262 | 04 528 | 95 472 | 99 734 | 40 |
| 21 | 04 376 | 04 643 | 95 357 | 99 733 | 39 |
| 22 | 04 490 | 04 758 | 95 242 | 99 731 | 38 |
| 23 | 04 603 | 04 873 | 95 127 | 99 730 | 37 |
| 24 | 04 715 | 04 987 | 95 013 | 99 728 | 36 |
| 25 | 04 828 | 05 101 | 94 899 | 99 727 | 35 |
| 26 | 04 940 | 05 214 | 94 786 | 99 726 | 34 |
| 27 | 05 052 | 05 328 | 94 672 | 99 724 | 33 |
| 28 | 05 164 | 05 441 | 94 559 | 99 723 | 32 |
| 29 | 05 275 | 05 553 | 94 447 | 99 721 | 31 |
| 30 | 05 386 | 05 666 | 94 334 | 99 720 | 30 |
| 31 | 05 497 | 05 778 | 94 222 | 99 718 | 29 |
| 32 | 05 607 | 05 890 | 94 110 | 99 717 | 28 |
| 33 | 05 717 | 06 002 | 93 998 | 99 716 | 27 |
| 34 | 05 827 | 06 113 | 93 887 | 99 714 | 26 |
| 35 | 05 937 | 06 224 | 93 776 | 99 713 | 25 |
| 36 | 06 046 | 06 335 | 93 665 | 99 711 | 24 |
| 37 | 06 155 | 06 445 | 93 555 | 99 710 | 23 |
| 38 | 06 264 | 06 556 | 93 444 | 99 708 | 22 |
| 39 | 06 372 | 06 666 | 93 334 | 99 707 | 21 |
| 40 | 06 481 | 06 775 | 93 225 | 99 705 | 20 |
| 41 | 06 589 | 06 885 | 93 115 | 99 704 | 19 |
| 42 | 06 696 | 06 994 | 93 006 | 99 702 | 18 |
| 43 | 06 804 | 07 103 | 92 897 | 99 701 | 17 |
| 44 | 06 911 | 07 211 | 92 789 | 99 699 | 16 |
| 45 | 07 018 | 07 320 | 92 680 | 99 698 | 15 |
| 46 | 07 124 | 07 428 | 92 572 | 99 696 | 14 |
| 47 | 07 231 | 07 536 | 92 464 | 99 695 | 13 |
| 48 | 07 337 | 07 643 | 92 357 | 99 693 | 12 |
| 49 | 07 442 | 07 751 | 92 249 | 99 692 | 11 |
| 50 | 07 548 | 07 858 | 92 142 | 99 690 | 10 |
| 51 | 07 653 | 07 964 | 92 036 | 99 689 | 9 |
| 52 | 07 758 | 08 071 | 91 929 | 99 687 | 8 |
| 53 | 07 863 | 08 177 | 91 823 | 99 686 | 7 |
| 54 | 07 968 | 08 283 | 91 717 | 99 684 | 6 |
| 55 | 08 072 | 08 389 | 91 611 | 99 683 | 5 |
| 56 | 08 176 | 08 495 | 91 505 | 99 681 | 4 |
| 57 | 08 280 | 08 600 | 91 400 | 99 680 | 3 |
| 58 | 08 383 | 08 705 | 91 295 | 99 678 | 2 |
| 59 | 08 486 | 08 810 | 91 190 | 99 677 | 1 |
| 60 | 08 589 | 08 914 | 91 086 | 99 675 | 0 |
| ' | log cos 9 | log cot 8 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 08 589 | 08 914 | 91 086 | 99 675 | 60 |
| 1 | 08 692 | 09 019 | 90 981 | 99 674 | 59 |
| 2 | 08 795 | 09 123 | 90 877 | 99 672 | 58 |
| 3 | 08 897 | 09 227 | 90 773 | 99 670 | 57 |
| 4 | 08 999 | 09 330 | 90 670 | 99 669 | 56 |
| 5 | 09 101 | 09 434 | 90 566 | 99 667 | 55 |
| 6 | 09 202 | 09 537 | 90 463 | 99 666 | 54 |
| 7 | 09 304 | 09 640 | 90 360 | 99 664 | 53 |
| 8 | 09 405 | 09 742 | 90 258 | 99 663 | 52 |
| 9 | 09 506 | 09 845 | 90 155 | 99 661 | 51 |
| 10 | 09 606 | 09 947 | 90 053 | 99 659 | 50 |
| 11 | 09 707 | 10 049 | 89 951 | 99 658 | 49 |
| 12 | 09 807 | 10 150 | 89 850 | 99 656 | 48 |
| 13 | 09 907 | 10 252 | 89 748 | 99 655 | 47 |
| 14 | 10 006 | 10 353 | 89 647 | 99 653 | 46 |
| 15 | 10 106 | 10 454 | 89 546 | 99 651 | 45 |
| 16 | 10 205 | 10 555 | 89 445 | 99 650 | 44 |
| 17 | 10 304 | 10 656 | 89 344 | 99 648 | 43 |
| 18 | 10 402 | 10 756 | 89 244 | 99 647 | 42 |
| 19 | 10 501 | 10 856 | 89 144 | 99 645 | 41 |
| 20 | 10 599 | 10 956 | 89 044 | 99 643 | 40 |
| 21 | 10 697 | 11 056 | 88 944 | 99 642 | 39 |
| 22 | 10 795 | 11 155 | 88 845 | 99 640 | 38 |
| 23 | 10 893 | 11 254 | 88 746 | 99 638 | 37 |
| 24 | 10 990 | 11 353 | 88 647 | 99 637 | 36 |
| 25 | 11 087 | 11 452 | 88 548 | 99 635 | 35 |
| 26 | 11 184 | 11 551 | 88 449 | 99 633 | 34 |
| 27 | 11 281 | 11 649 | 88 351 | 99 632 | 33 |
| 28 | 11 377 | 11 747 | 88 253 | 99 630 | 32 |
| 29 | 11 474 | 11 845 | 88 155 | 99 629 | 31 |
| 30 | 11 570 | 11 943 | 88 057 | 99 627 | 30 |
| 31 | 11 666 | 12 040 | 87 960 | 99 625 | 29 |
| 32 | 11 761 | 12 138 | 87 862 | 99 624 | 28 |
| 33 | 11 857 | 12 235 | 87 765 | 99 622 | 27 |
| 34 | 11 952 | 12 332 | 87 668 | 99 620 | 26 |
| 35 | 12 047 | 12 428 | 87 572 | 99 618 | 25 |
| 36 | 12 142 | 12 525 | 87 475 | 99 617 | 24 |
| 37 | 12 236 | 12 621 | 87 379 | 99 615 | 23 |
| 38 | 12 331 | 12 717 | 87 283 | 99 613 | 22 |
| 39 | 12 425 | 12 813 | 87 187 | 99 612 | 21 |
| 40 | 12 519 | 12 909 | 87 091 | 99 610 | 20 |
| 41 | 12 612 | 13 004 | 86 996 | 99 608 | 19 |
| 42 | 12 706 | 13 099 | 86 901 | 99 607 | 18 |
| 43 | 12 799 | 13 194 | 86 806 | 99 605 | 17 |
| 44 | 12 892 | 13 289 | 86 711 | 99 603 | 16 |
| 45 | 12 985 | 13 384 | 86 616 | 99 601 | 15 |
| 46 | 13 078 | 13 478 | 86 522 | 99 600 | 14 |
| 47 | 13 171 | 13 573 | 86 427 | 99 598 | 13 |
| 48 | 13 263 | 13 667 | 86 333 | 99 596 | 12 |
| 49 | 13 355 | 13 761 | 86 239 | 99 595 | 11 |
| 50 | 13 447 | 13 854 | 86 146 | 99 593 | 10 |
| 51 | 13 539 | 13 948 | 86 052 | 99 591 | 9 |
| 52 | 13 630 | 14 041 | 85 959 | 99 589 | 8 |
| 53 | 13 722 | 14 134 | 85 866 | 99 588 | 7 |
| 54 | 13 813 | 14 227 | 85 773 | 99 586 | 6 |
| 55 | 13 904 | 14 320 | 85 680 | 99 584 | 5 |
| 56 | 13 994 | 14 412 | 85 588 | 99 582 | 4 |
| 57 | 14 085 | 14 504 | 85 496 | 99 581 | 3 |
| 58 | 14 175 | 14 597 | 85 403 | 99 579 | 2 |
| 59 | 14 266 | 14 688 | 85 312 | 99 577 | 1 |
| 60 | 14 356 | 14 780 | 85 220 | 99 575 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 14 356 | 14 780 | 85 220 | 99 575 | 60 |
| 1 | 14 445 | 14 872 | 85 128 | 99 574 | 59 |
| 2 | 14 535 | 14 963 | 85 037 | 99 572 | 58 |
| 3 | 14 624 | 15 054 | 84 946 | 99 570 | 57 |
| 4 | 14 714 | 15 145 | 84 855 | 99 568 | 56 |
| 5 | 14 803 | 15 236 | 84 764 | 99 566 | 55 |
| 6 | 14 891 | 15 327 | 84 673 | 99 565 | 54 |
| 7 | 14 980 | 15 417 | 84 583 | 99 563 | 53 |
| 8 | 15 069 | 15 508 | 84 492 | 99 561 | 52 |
| 9 | 15 157 | 15 598 | 84 402 | 99 559 | 51 |
| 10 | 15 245 | 15 688 | 84 312 | 99 557 | 50 |
| 11 | 15 333 | 15 777 | 84 223 | 99 556 | 49 |
| 12 | 15 421 | 15 867 | 84 133 | 99 554 | 48 |
| 13 | 15 508 | 15 956 | 84 044 | 99 552 | 47 |
| 14 | 15 596 | 16 046 | 83 954 | 99 550 | 46 |
| 15 | 15 683 | 16 135 | 83 865 | 99 548 | 45 |
| 16 | 15 770 | 16 224 | 83 776 | 99 546 | 44 |
| 17 | 15 857 | 16 312 | 83 688 | 99 545 | 43 |
| 18 | 15 944 | 16 401 | 83 599 | 99 543 | 42 |
| 19 | 16 030 | 16 489 | 83 511 | 99 541 | 41 |
| 20 | 16 116 | 16 577 | 83 423 | 99 539 | 40 |
| 21 | 16 203 | 16 665 | 83 335 | 99 537 | 39 |
| 22 | 16 289 | 16 753 | 83 247 | 99 535 | 38 |
| 23 | 16 374 | 16 841 | 83 159 | 99 533 | 37 |
| 24 | 16 460 | 16 928 | 83 072 | 99 532 | 36 |
| 25 | 16 545 | 17 016 | 82 984 | 99 530 | 35 |
| 26 | 16 631 | 17 103 | 82 897 | 99 528 | 34 |
| 27 | 16 716 | 17 190 | 82 810 | 99 526 | 33 |
| 28 | 16 801 | 17 277 | 82 723 | 99 524 | 32 |
| 29 | 16 886 | 17 363 | 82 637 | 99 522 | 31 |
| 30 | 16 970 | 17 450 | 82 550 | 99 520 | 30 |
| 31 | 17 055 | 17 536 | 82 464 | 99 518 | 29 |
| 32 | 17 139 | 17 622 | 82 378 | 99 517 | 28 |
| 33 | 17 223 | 17 708 | 82 292 | 99 515 | 27 |
| 34 | 17 307 | 17 794 | 82 206 | 99 513 | 26 |
| 35 | 17 391 | 17 880 | 82 120 | 99 511 | 25 |
| 36 | 17 474 | 17 965 | 82 035 | 99 509 | 24 |
| 37 | 17 558 | 18 051 | 81 949 | 99 507 | 23 |
| 38 | 17 641 | 18 136 | 81 864 | 99 505 | 22 |
| 39 | 17 724 | 18 221 | 81 779 | 99 503 | 21 |
| 40 | 17 807 | 18 306 | 81 694 | 99 501 | 20 |
| 41 | 17 890 | 18 391 | 81 609 | 99 499 | 19 |
| 42 | 17 973 | 18 475 | 81 525 | 99 497 | 18 |
| 43 | 18 055 | 18 560 | 81 440 | 99 495 | 17 |
| 44 | 18 137 | 18 644 | 81 356 | 99 494 | 16 |
| 45 | 18 220 | 18 728 | 81 272 | 99 492 | 15 |
| 46 | 18 302 | 18 812 | 81 188 | 99 490 | 14 |
| 47 | 18 383 | 18 896 | 81 104 | 99 488 | 13 |
| 48 | 18 465 | 18 979 | 81 021 | 99 486 | 12 |
| 49 | 18 547 | 19 063 | 80 937 | 99 484 | 11 |
| 50 | 18 628 | 19 146 | 80 854 | 99 482 | 10 |
| 51 | 18 709 | 19 229 | 80 771 | 99 480 | 9 |
| 52 | 18 790 | 19 312 | 80 688 | 99 478 | 8 |
| 53 | 18 871 | 19 395 | 80 605 | 99 476 | 7 |
| 54 | 18 952 | 19 478 | 80 522 | 99 474 | 6 |
| 55 | 19 033 | 19 561 | 80 439 | 99 472 | 5 |
| 56 | 19 113 | 19 643 | 80 357 | 99 470 | 4 |
| 57 | 19 193 | 19 725 | 80 275 | 99 468 | 3 |
| 58 | 19 273 | 19 807 | 80 193 | 99 466 | 2 |
| 59 | 19 353 | 19 889 | 80 111 | 99 464 | 1 |
| 60 | 19 433 | 19 971 | 80 029 | 99 462 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 19 433 | 19 971 | 80 029 | 99 462 | 60 |
| 1 | 19 513 | 20 053 | 79 947 | 99 460 | 59 |
| 2 | 19 592 | 20 134 | 79 866 | 99 458 | 58 |
| 3 | 19 672 | 20 216 | 79 784 | 99 456 | 57 |
| 4 | 19 751 | 20 297 | 79 703 | 99 454 | 56 |
| 5 | 19 830 | 20 378 | 79 622 | 99 452 | 55 |
| 6 | 19 909 | 20 459 | 79 541 | 99 450 | 54 |
| 7 | 19 988 | 20 540 | 79 460 | 99 448 | 53 |
| 8 | 20 067 | 20 621 | 79 379 | 99 446 | 52 |
| 9 | 20 145 | 20 701 | 79 299 | 99 444 | 51 |
| 10 | 20 223 | 20 782 | 79 218 | 99 442 | 50 |
| 11 | 20 302 | 20 862 | 79 138 | 99 440 | 49 |
| 12 | 20 380 | 20 942 | 79 058 | 99 438 | 48 |
| 13 | 20 458 | 21 022 | 78 978 | 99 436 | 47 |
| 14 | 20 535 | 21 102 | 78 898 | 99 434 | 46 |
| 15 | 20 613 | 21 182 | 78 818 | 99 432 | 45 |
| 16 | 20 691 | 21 261 | 78 739 | 99 429 | 44 |
| 17 | 20 768 | 21 341 | 78 659 | 99 427 | 43 |
| 18 | 20 845 | 21 420 | 78 580 | 99 425 | 42 |
| 19 | 20 922 | 21 499 | 78 501 | 99 423 | 41 |
| 20 | 20 999 | 21 578 | 78 422 | 99 421 | 40 |
| 21 | 21 076 | 21 657 | 78 343 | 99 419 | 39 |
| 22 | 21 153 | 21 736 | 78 264 | 99 417 | 38 |
| 23 | 21 229 | 21 814 | 78 186 | 99 415 | 37 |
| 24 | 21 306 | 21 893 | 78 107 | 99 413 | 36 |
| 25 | 21 382 | 21 971 | 78 029 | 99 411 | 35 |
| 26 | 21 458 | 22 049 | 77 951 | 99 409 | 34 |
| 27 | 21 534 | 22 127 | 77 873 | 99 407 | 33 |
| 28 | 21 610 | 22 205 | 77 795 | 99 404 | 32 |
| 29 | 21 685 | 22 283 | 77 717 | 99 402 | 31 |
| 30 | 21 761 | 22 361 | 77 639 | 99 400 | 30 |
| 31 | 21 836 | 22 438 | 77 562 | 99 398 | 29 |
| 32 | 21 912 | 22 516 | 77 484 | 99 396 | 28 |
| 33 | 21 987 | 22 593 | 77 407 | 99 394 | 27 |
| 34 | 22 062 | 22 670 | 77 330 | 99 392 | 26 |
| 35 | 22 137 | 22 747 | 77 253 | 99 390 | 25 |
| 36 | 22 211 | 22 824 | 77 176 | 99 388 | 24 |
| 37 | 22 286 | 22 901 | 77 099 | 99 385 | 23 |
| 38 | 22 361 | 22 977 | 77 023 | 99 383 | 22 |
| 39 | 22 435 | 23 054 | 76 946 | 99 381 | 21 |
| 40 | 22 509 | 23 130 | 76 870 | 99 379 | 20 |
| 41 | 22 583 | 23 206 | 76 794 | 99 377 | 19 |
| 42 | 22 657 | 23 283 | 76 717 | 99 375 | 18 |
| 43 | 22 731 | 23 359 | 76 641 | 99 372 | 17 |
| 44 | 22 805 | 23 435 | 76 565 | 99 370 | 16 |
| 45 | 22 878 | 23 510 | 76 490 | 99 368 | 15 |
| 46 | 22 952 | 23 586 | 76 414 | 99 366 | 14 |
| 47 | 23 025 | 23 661 | 76 339 | 99 364 | 13 |
| 48 | 23 098 | 23 737 | 76 263 | 99 362 | 12 |
| 49 | 23 171 | 23 812 | 76 188 | 99 359 | 11 |
| 50 | 23 244 | 23 887 | 76 113 | 99 357 | 10 |
| 51 | 23 317 | 23 962 | 76 038 | 99 355 | 9 |
| 52 | 23 390 | 24 037 | 75 963 | 99 353 | 8 |
| 53 | 23 462 | 24 112 | 75 888 | 99 351 | 7 |
| 54 | 23 535 | 24 186 | 75 814 | 99 348 | 6 |
| 55 | 23 607 | 24 261 | 75 739 | 99 346 | 5 |
| 56 | 23 679 | 24 335 | 75 665 | 99 344 | 4 |
| 57 | 23 752 | 24 410 | 75 590 | 99 342 | 3 |
| 58 | 23 823 | 24 484 | 75 516 | 99 340 | 2 |
| 59 | 23 895 | 24 558 | 75 442 | 99 337 | 1 |
| 60 | 23 967 | 24 632 | 75 368 | 99 335 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 23 967 | 24 632 | 75 368 | 99 335 | 60 |
| 1 | 24 039 | 24 706 | 75 294 | 99 333 | 59 |
| 2 | 24 110 | 24 779 | 75 221 | 99 331 | 58 |
| 3 | 24 181 | 24 853 | 75 147 | 99 328 | 57 |
| 4 | 24 253 | 24 926 | 75 074 | 99 326 | 56 |
| 5 | 24 324 | 25 000 | 75 000 | 99 324 | 55 |
| 6 | 24 395 | 25 073 | 74 927 | 99 322 | 54 |
| 7 | 24 466 | 25 146 | 74 854 | 99 319 | 53 |
| 8 | 24 536 | 25 219 | 74 781 | 99 317 | 52 |
| 9 | 24 607 | 25 292 | 74 708 | 99 315 | 51 |
| 10 | 24 677 | 25 365 | 74 635 | 99 313 | 50 |
| 11 | 24 748 | 25 437 | 74 563 | 99 310 | 49 |
| 12 | 24 818 | 25 510 | 74 490 | 99 308 | 48 |
| 13 | 24 888 | 25 582 | 74 418 | 99 306 | 47 |
| 14 | 24 958 | 25 655 | 74 345 | 99 304 | 46 |
| 15 | 25 028 | 25 727 | 74 273 | 99 301 | 45 |
| 16 | 25 098 | 25 799 | 74 201 | 99 299 | 44 |
| 17 | 25 168 | 25 871 | 74 129 | 99 297 | 43 |
| 18 | 25 237 | 25 943 | 74 057 | 99 294 | 42 |
| 19 | 25 307 | 26 015 | 73 985 | 99 292 | 41 |
| 20 | 25 376 | 26 086 | 73 914 | 99 290 | 40 |
| 21 | 25 445 | 26 158 | 73 842 | 99 288 | 39 |
| 22 | 25 514 | 26 229 | 73 771 | 99 285 | 38 |
| 23 | 25 583 | 26 301 | 73 699 | 99 283 | 37 |
| 24 | 25 652 | 26 372 | 73 628 | 99 281 | 36 |
| 25 | 25 721 | 26 443 | 73 557 | 99 278 | 35 |
| 26 | 25 790 | 26 514 | 73 486 | 99 276 | 34 |
| 27 | 25 858 | 26 585 | 73 415 | 99 274 | 33 |
| 28 | 25 927 | 26 655 | 73 345 | 99 271 | 32 |
| 29 | 25 995 | 26 726 | 73 274 | 99 269 | 31 |
| 30 | 26 063 | 26 797 | 73 203 | 99 267 | 30 |
| 31 | 26 131 | 26 867 | 73 133 | 99 264 | 29 |
| 32 | 26 199 | 26 937 | 73 063 | 99 262 | 28 |
| 33 | 26 267 | 27 008 | 72 992 | 99 260 | 27 |
| 34 | 26 335 | 27 078 | 72 922 | 99 257 | 26 |
| 35 | 26 403 | 27 148 | 72 852 | 99 255 | 25 |
| 36 | 26 470 | 27 218 | 72 782 | 99 252 | 24 |
| 37 | 26 538 | 27 288 | 72 712 | 99 250 | 23 |
| 38 | 26 605 | 27 357 | 72 643 | 99 248 | 22 |
| 39 | 26 672 | 27 427 | 72 573 | 99 245 | 21 |
| 40 | 26 739 | 27 496 | 72 504 | 99 243 | 20 |
| 41 | 26 806 | 27 566 | 72 434 | 99 241 | 19 |
| 42 | 26 873 | 27 635 | 72 365 | 99 238 | 18 |
| 43 | 26 940 | 27 704 | 72 296 | 99 236 | 17 |
| 44 | 27 007 | 27 773 | 72 227 | 99 233 | 16 |
| 45 | 27 073 | 27 842 | 72 158 | 99 231 | 15 |
| 46 | 27 140 | 27 911 | 72 089 | 99 229 | 14 |
| 47 | 27 206 | 27 980 | 72 020 | 99 226 | 13 |
| 48 | 27 273 | 28 049 | 71 951 | 99 224 | 12 |
| 49 | 27 339 | 28 117 | 71 883 | 99 221 | 11 |
| 50 | 27 405 | 28 186 | 71 814 | 99 219 | 10 |
| 51 | 27 471 | 28 254 | 71 746 | 99 217 | 9 |
| 52 | 27 537 | 28 323 | 71 677 | 99 214 | 8 |
| 53 | 27 602 | 28 391 | 71 609 | 99 212 | 7 |
| 54 | 27 668 | 28 459 | 71 541 | 99 209 | 6 |
| 55 | 27 734 | 28 527 | 71 473 | 99 207 | 5 |
| 56 | 27 799 | 28 595 | 71 405 | 99 204 | 4 |
| 57 | 27 864 | 28 662 | 71 338 | 99 202 | 3 |
| 58 | 27 930 | 28 730 | 71 270 | 99 200 | 2 |
| 59 | 27 995 | 28 798 | 71 202 | 99 197 | 1 |
| 60 | 28 060 | 28 865 | 71 135 | 99 195 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 28 060 | 28 865 | 71 135 | 99 195 | 60 |
| 1 | 28 125 | 28 933 | 71 067 | 99 192 | 59 |
| 2 | 28 190 | 29 000 | 71 000 | 99 190 | 58 |
| 3 | 28 254 | 29 067 | 70 933 | 99 187 | 57 |
| 4 | 28 319 | 29 134 | 70 866 | 99 185 | 56 |
| 5 | 28 384 | 29 201 | 70 799 | 99 182 | 55 |
| 6 | 28 448 | 29 268 | 70 732 | 99 180 | 54 |
| 7 | 28 512 | 29 335 | 70 665 | 99 177 | 53 |
| 8 | 28 577 | 29 402 | 70 598 | 99 175 | 52 |
| 9 | 28 641 | 29 468 | 70 532 | 99 172 | 51 |
| 10 | 28 705 | 29 535 | 70 465 | 99 170 | 50 |
| 11 | 28 769 | 29 601 | 70 399 | 99 167 | 49 |
| 12 | 28 833 | 29 668 | 70 332 | 99 165 | 48 |
| 13 | 28 896 | 29 734 | 70 266 | 99 162 | 47 |
| 14 | 28 960 | 29 800 | 70 200 | 99 160 | 46 |
| 15 | 29 024 | 29 866 | 70 134 | 99 157 | 45 |
| 16 | 29 087 | 29 932 | 70 068 | 99 155 | 44 |
| 17 | 29 150 | 29 998 | 70 002 | 99 152 | 43 |
| 18 | 29 214 | 30 064 | 69 936 | 99 150 | 42 |
| 19 | 29 277 | 30 130 | 69 870 | 99 147 | 41 |
| 20 | 29 340 | 30 195 | 69 805 | 99 145 | 40 |
| 21 | 29 403 | 30 261 | 69 739 | 99 142 | 39 |
| 22 | 29 466 | 30 326 | 69 674 | 99 140 | 38 |
| 23 | 29 529 | 30 391 | 69 609 | 99 137 | 37 |
| 24 | 29 591 | 30 457 | 69 543 | 99 135 | 36 |
| 25 | 29 654 | 30 522 | 69 478 | 99 132 | 35 |
| 26 | 29 716 | 30 587 | 69 413 | 99 130 | 34 |
| 27 | 29 779 | 30 652 | 69 348 | 99 127 | 33 |
| 28 | 29 841 | 30 717 | 69 283 | 99 124 | 32 |
| 29 | 29 903 | 30 782 | 69 218 | 99 122 | 31 |
| 30 | 29 966 | 30 846 | 69 154 | 99 119 | 30 |
| 31 | 30 028 | 30 911 | 69 089 | 99 117 | 29 |
| 32 | 30 090 | 30 975 | 69 025 | 99 114 | 28 |
| 33 | 30 151 | 31 040 | 68 960 | 99 112 | 27 |
| 34 | 30 213 | 31 104 | 68 896 | 99 109 | 26 |
| 35 | 30 275 | 31 168 | 68 832 | 99 106 | 25 |
| 36 | 30 336 | 31 233 | 68 767 | 99 104 | 24 |
| 37 | 30 398 | 31 297 | 68 703 | 99 101 | 23 |
| 38 | 30 459 | 31 361 | 68 639 | 99 099 | 22 |
| 39 | 30 521 | 31 425 | 68 575 | 99 096 | 21 |
| 40 | 30 582 | 31 489 | 68 511 | 99 093 | 20 |
| 41 | 30 643 | 31 552 | 68 448 | 99 091 | 19 |
| 42 | 30 704 | 31 616 | 68 384 | 99 088 | 18 |
| 43 | 30 765 | 31 679 | 68 321 | 99 086 | 17 |
| 44 | 30 826 | 31 743 | 68 257 | 99 083 | 16 |
| 45 | 30 887 | 31 806 | 68 194 | 99 080 | 15 |
| 46 | 30 947 | 31 870 | 68 130 | 99 078 | 14 |
| 47 | 31 008 | 31 933 | 68 067 | 99 075 | 13 |
| 48 | 31 068 | 31 996 | 68 004 | 99 072 | 12 |
| 49 | 31 129 | 32 059 | 67 941 | 99 070 | 11 |
| 50 | 31 189 | 32 122 | 67 878 | 99 067 | 10 |
| 51 | 31 250 | 32 185 | 67 815 | 99 064 | 9 |
| 52 | 31 310 | 32 248 | 67 752 | 99 062 | 8 |
| 53 | 31 370 | 32 311 | 67 689 | 99 059 | 7 |
| 54 | 31 430 | 32 373 | 67 627 | 99 056 | 6 |
| 55 | 31 490 | 32 436 | 67 564 | 99 054 | 5 |
| 56 | 31 549 | 32 498 | 67 502 | 99 051 | 4 |
| 57 | 31 609 | 32 561 | 67 439 | 99 048 | 3 |
| 58 | 31 669 | 32 623 | 67 377 | 99 046 | 2 |
| 59 | 31 728 | 32 685 | 67 315 | 99 043 | 1 |
| 60 | 31 788 | 32 747 | 67 253 | 99 040 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 0 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 31 788 | 32 747 | 67 253 | 99 040 | 60 |
| 1 | 31 847 | 32 810 | 67 190 | 99 038 | 59 |
| 2 | 31 907 | 32 872 | 67 128 | 99 035 | 58 |
| 3 | 31 966 | 32 933 | 67 067 | 99 032 | 57 |
| 4 | 32 025 | 32 995 | 67 005 | 99 030 | 56 |
| 5 | 32 084 | 33 057 | 66 943 | 99 027 | 55 |
| 6 | 32 143 | 33 119 | 66 881 | 99 024 | 54 |
| 7 | 32 202 | 33 180 | 66 820 | 99 022 | 53 |
| 8 | 32 261 | 33 242 | 66 758 | 99 019 | 52 |
| 9 | 32 319 | 33 303 | 66 697 | 99 016 | 51 |
| 10 | 32 378 | 33 365 | 66 635 | 99 013 | 50 |
| 11 | 32 437 | 33 426 | 66 574 | 99 011 | 49 |
| 12 | 32 495 | 33 487 | 66 513 | 99 008 | 48 |
| 13 | 32 553 | 33 548 | 66 452 | 99 005 | 47 |
| 14 | 32 612 | 33 609 | 66 391 | 99 002 | 46 |
| 15 | 32 670 | 33 670 | 66 330 | 99 000 | 45 |
| 16 | 32 728 | 33 731 | 66 269 | 98 997 | 44 |
| 17 | 32 786 | 33 792 | 66 208 | 98 994 | 43 |
| 18 | 32 844 | 33 853 | 66 147 | 98 991 | 42 |
| 19 | 32 902 | 33 913 | 66 087 | 98 989 | 41 |
| 20 | 32 960 | 33 974 | 66 026 | 98 986 | 40 |
| 21 | 33 018 | 34 034 | 65 966 | 98 983 | 39 |
| 22 | 33 075 | 34 095 | 65 905 | 98 980 | 38 |
| 23 | 33 133 | 34 155 | 65 845 | 98 978 | 37 |
| 24 | 33 190 | 34 215 | 65 785 | 98 975 | 36 |
| 25 | 33 248 | 34 276 | 65 724 | 98 972 | 35 |
| 26 | 33 305 | 34 336 | 65 664 | 98 969 | 34 |
| 27 | 33 362 | 34 396 | 65 604 | 98 967 | 33 |
| 28 | 33 420 | 34 456 | 65 544 | 98 964 | 32 |
| 29 | 33 477 | 34 516 | 65 484 | 98 961 | 31 |
| 30 | 33 534 | 34 576 | 65 424 | 98 958 | 30 |
| 31 | 33 591 | 34 635 | 65 365 | 98 955 | 29 |
| 32 | 33 647 | 34 695 | 65 305 | 98 953 | 28 |
| 33 | 33 704 | 34 755 | 65 245 | 98 950 | 27 |
| 34 | 33 761 | 34 814 | 65 186 | 98 947 | 26 |
| 35 | 33 818 | 34 874 | 65 126 | 98 944 | 25 |
| 36 | 33 874 | 34 933 | 65 067 | 98 941 | 24 |
| 37 | 33 931 | 34 992 | 65 008 | 98 938 | 23 |
| 38 | 33 987 | 35 051 | 64 949 | 98 936 | 22 |
| 39 | 34 043 | 35 111 | 64 889 | 98 933 | 21 |
| 40 | 34 100 | 35 170 | 64 830 | 98 930 | 20 |
| 41 | 34 156 | 35 229 | 64 771 | 98 927 | 19 |
| 42 | 34 212 | 35 288 | 64 712 | 98 924 | 18 |
| 43 | 34 268 | 35 347 | 64 653 | 98 921 | 17 |
| 44 | 34 324 | 35 405 | 64 595 | 98 919 | 16 |
| 45 | 34 380 | 35 464 | 64 536 | 98 916 | 15 |
| 46 | 34 436 | 35 523 | 64 477 | 98 913 | 14 |
| 47 | 34 491 | 35 581 | 64 419 | 98 910 | 13 |
| 48 | 34 547 | 35 640 | 64 360 | 98 907 | 12 |
| 49 | 34 602 | 35 698 | 64 302 | 98 904 | 11 |
| 50 | 34 658 | 35 757 | 64 243 | 98 901 | 10 |
| 51 | 34 713 | 35 815 | 64 185 | 98 898 | 9 |
| 52 | 34 769 | 35 873 | 64 127 | 98 896 | 8 |
| 53 | 34 824 | 35 931 | 64 069 | 98 893 | 7 |
| 54 | 34 879 | 35 989 | 64 011 | 98 890 | 6 |
| 55 | 34 934 | 36 047 | 63 953 | 98 887 | 5 |
| 56 | 34 989 | 36 105 | 63 895 | 98 884 | 4 |
| 57 | 35 044 | 36 163 | 63 837 | 98 881 | 3 |
| 58 | 35 099 | 36 221 | 63 779 | 98 878 | 2 |
| 59 | 35 154 | 36 279 | 63 721 | 98 875 | 1 |
| 60 | 35 209 | 36 336 | 63 664 | 98 872 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 0 | ' |

| ' | log sin | log tan | log cot | log cos | ' |
|----|---------|---------|---------|---------|----|
| 0 | 35 209 | 36 336 | 63 664 | 95 572 | 90 |
| 1 | 35 263 | 36 394 | 63 606 | 95 569 | 89 |
| 2 | 35 315 | 36 452 | 63 548 | 95 567 | 88 |
| 3 | 35 373 | 36 509 | 63 491 | 95 564 | 87 |
| 4 | 35 427 | 36 566 | 63 434 | 95 561 | 86 |
| 5 | 35 481 | 36 624 | 63 376 | 95 558 | 85 |
| 6 | 35 536 | 36 681 | 63 319 | 95 555 | 84 |
| 7 | 35 590 | 36 738 | 63 262 | 95 552 | 83 |
| 8 | 35 644 | 36 795 | 63 205 | 95 549 | 82 |
| 9 | 35 698 | 36 852 | 63 148 | 95 546 | 81 |
| 10 | 35 752 | 36 909 | 63 091 | 95 543 | 80 |
| 11 | 35 806 | 36 966 | 63 034 | 95 540 | 79 |
| 12 | 35 860 | 37 023 | 62 977 | 95 537 | 78 |
| 13 | 35 914 | 37 080 | 62 920 | 95 534 | 77 |
| 14 | 35 968 | 37 137 | 62 863 | 95 531 | 76 |
| 15 | 36 022 | 37 193 | 62 807 | 95 528 | 75 |
| 16 | 36 075 | 37 250 | 62 750 | 95 525 | 74 |
| 17 | 36 129 | 37 306 | 62 694 | 95 522 | 73 |
| 18 | 36 182 | 37 363 | 62 637 | 95 519 | 72 |
| 19 | 36 236 | 37 419 | 62 581 | 95 516 | 71 |
| 20 | 36 289 | 37 476 | 62 524 | 95 513 | 70 |
| 21 | 36 342 | 37 532 | 62 468 | 95 510 | 69 |
| 22 | 36 395 | 37 588 | 62 412 | 95 507 | 68 |
| 23 | 36 449 | 37 644 | 62 356 | 95 504 | 67 |
| 24 | 36 502 | 37 700 | 62 300 | 95 501 | 66 |
| 25 | 36 555 | 37 756 | 62 244 | 95 498 | 65 |
| 26 | 36 608 | 37 812 | 62 188 | 95 495 | 64 |
| 27 | 36 660 | 37 868 | 62 132 | 95 492 | 63 |
| 28 | 36 713 | 37 924 | 62 076 | 95 489 | 62 |
| 29 | 36 766 | 37 980 | 62 020 | 95 486 | 61 |
| 30 | 36 819 | 38 035 | 61 965 | 95 483 | 60 |
| 31 | 36 871 | 38 091 | 61 909 | 95 480 | 59 |
| 32 | 36 924 | 38 147 | 61 853 | 95 477 | 58 |
| 33 | 36 976 | 38 202 | 61 798 | 95 474 | 57 |
| 34 | 37 028 | 38 257 | 61 743 | 95 471 | 56 |
| 35 | 37 081 | 38 313 | 61 687 | 95 468 | 55 |
| 36 | 37 133 | 38 368 | 61 632 | 95 465 | 54 |
| 37 | 37 185 | 38 423 | 61 577 | 95 462 | 53 |
| 38 | 37 237 | 38 479 | 61 521 | 95 459 | 52 |
| 39 | 37 289 | 38 534 | 61 466 | 95 456 | 51 |
| 40 | 37 341 | 38 589 | 61 411 | 95 453 | 50 |
| 41 | 37 393 | 38 644 | 61 356 | 95 450 | 49 |
| 42 | 37 445 | 38 699 | 61 301 | 95 446 | 48 |
| 43 | 37 497 | 38 754 | 61 246 | 95 443 | 47 |
| 44 | 37 549 | 38 808 | 61 192 | 95 440 | 46 |
| 45 | 37 600 | 38 863 | 61 137 | 95 437 | 45 |
| 46 | 37 652 | 38 918 | 61 082 | 95 434 | 44 |
| 47 | 37 703 | 38 972 | 61 028 | 95 431 | 43 |
| 48 | 37 755 | 39 027 | 60 973 | 95 428 | 42 |
| 49 | 37 806 | 39 082 | 60 918 | 95 425 | 41 |
| 50 | 37 858 | 39 136 | 60 864 | 95 422 | 40 |
| 51 | 37 909 | 39 190 | 60 810 | 95 419 | 39 |
| 52 | 37 960 | 39 245 | 60 755 | 95 415 | 38 |
| 53 | 38 011 | 39 299 | 60 701 | 95 412 | 37 |
| 54 | 38 062 | 39 353 | 60 647 | 95 409 | 36 |
| 55 | 38 113 | 39 407 | 60 593 | 95 406 | 35 |
| 56 | 38 164 | 39 461 | 60 539 | 95 403 | 34 |
| 57 | 38 215 | 39 515 | 60 485 | 95 400 | 33 |
| 58 | 38 266 | 39 569 | 60 431 | 95 397 | 32 |
| 59 | 38 317 | 39 623 | 60 377 | 95 394 | 31 |
| 60 | 38 368 | 39 677 | 60 323 | 95 390 | 30 |
| | — 9 — | — 9 — | — 10 — | — 9 — | |
| ' | log cos | log cot | log tan | log sin | ' |

| ' | log sin | log tan | log cot | log cos | ' |
|----|---------|---------|---------|---------|----|
| 0 | 35 366 | 39 677 | 60 323 | 95 690 | 90 |
| 1 | 35 418 | 39 731 | 60 269 | 95 687 | 89 |
| 2 | 35 469 | 39 785 | 60 215 | 95 684 | 88 |
| 3 | 35 519 | 39 838 | 60 162 | 95 681 | 87 |
| 4 | 35 570 | 39 892 | 60 108 | 95 678 | 86 |
| 5 | 35 620 | 39 945 | 60 055 | 95 675 | 85 |
| 6 | 35 670 | 39 999 | 60 001 | 95 671 | 84 |
| 7 | 35 721 | 40 052 | 59 948 | 95 668 | 83 |
| 8 | 35 771 | 40 106 | 59 894 | 95 665 | 82 |
| 9 | 35 821 | 40 159 | 59 841 | 95 662 | 81 |
| 10 | 35 871 | 40 212 | 59 788 | 95 659 | 80 |
| 11 | 35 921 | 40 266 | 59 734 | 95 656 | 79 |
| 12 | 35 971 | 40 319 | 59 681 | 95 652 | 78 |
| 13 | 39 021 | 40 372 | 59 628 | 95 649 | 77 |
| 14 | 39 071 | 40 425 | 59 575 | 95 646 | 76 |
| 15 | 39 121 | 40 478 | 59 522 | 95 643 | 75 |
| 16 | 39 170 | 40 531 | 59 469 | 95 640 | 74 |
| 17 | 39 220 | 40 584 | 59 416 | 95 636 | 73 |
| 18 | 39 270 | 40 636 | 59 364 | 95 633 | 72 |
| 19 | 39 319 | 40 689 | 59 311 | 95 630 | 71 |
| 20 | 39 369 | 40 742 | 59 258 | 95 627 | 70 |
| 21 | 39 418 | 40 795 | 59 205 | 95 623 | 69 |
| 22 | 39 467 | 40 847 | 59 153 | 95 620 | 68 |
| 23 | 39 517 | 40 900 | 59 100 | 95 617 | 67 |
| 24 | 39 566 | 40 952 | 59 048 | 95 614 | 66 |
| 25 | 39 615 | 41 005 | 58 995 | 95 610 | 65 |
| 26 | 39 664 | 41 057 | 58 943 | 95 607 | 64 |
| 27 | 39 713 | 41 109 | 58 891 | 95 604 | 63 |
| 28 | 39 762 | 41 161 | 58 839 | 95 601 | 62 |
| 29 | 39 811 | 41 214 | 58 786 | 95 597 | 61 |
| 30 | 39 860 | 41 266 | 58 734 | 95 594 | 60 |
| 31 | 39 909 | 41 318 | 58 682 | 95 591 | 59 |
| 32 | 39 958 | 41 370 | 58 630 | 95 588 | 58 |
| 33 | 40 006 | 41 422 | 58 578 | 95 584 | 57 |
| 34 | 40 055 | 41 474 | 58 526 | 95 581 | 56 |
| 35 | 40 103 | 41 526 | 58 474 | 95 578 | 55 |
| 36 | 40 152 | 41 578 | 58 422 | 95 574 | 54 |
| 37 | 40 200 | 41 629 | 58 371 | 95 571 | 53 |
| 38 | 40 249 | 41 681 | 58 319 | 95 568 | 52 |
| 39 | 40 297 | 41 733 | 58 267 | 95 565 | 51 |
| 40 | 40 346 | 41 784 | 58 216 | 95 561 | 50 |
| 41 | 40 394 | 41 836 | 58 164 | 95 558 | 49 |
| 42 | 40 442 | 41 887 | 58 113 | 95 555 | 48 |
| 43 | 40 490 | 41 939 | 58 061 | 95 551 | 47 |
| 44 | 40 538 | 41 990 | 58 010 | 95 548 | 46 |
| 45 | 40 586 | 42 041 | 57 959 | 95 545 | 45 |
| 46 | 40 634 | 42 093 | 57 907 | 95 541 | 44 |
| 47 | 40 682 | 42 144 | 57 856 | 95 538 | 43 |
| 48 | 40 730 | 42 195 | 57 805 | 95 535 | 42 |
| 49 | 40 778 | 42 246 | 57 754 | 95 531 | 41 |
| 50 | 40 825 | 42 297 | 57 703 | 95 528 | 40 |
| 51 | 40 873 | 42 348 | 57 652 | 95 525 | 39 |
| 52 | 40 921 | 42 399 | 57 601 | 95 521 | 38 |
| 53 | 40 968 | 42 450 | 57 550 | 95 518 | 37 |
| 54 | 41 016 | 42 501 | 57 499 | 95 515 | 36 |
| 55 | 41 063 | 42 552 | 57 448 | 95 511 | 35 |
| 56 | 41 111 | 42 603 | 57 397 | 95 508 | 34 |
| 57 | 41 158 | 42 653 | 57 347 | 95 505 | 33 |
| 58 | 41 205 | 42 704 | 57 296 | 95 501 | 32 |
| 59 | 41 252 | 42 755 | 57 245 | 95 498 | 31 |
| 60 | 41 300 | 42 805 | 57 195 | 95 494 | 30 |
| | — 9 — | — 9 — | — 10 — | — 9 — | |
| ' | log cos | log cot | log tan | log sin | ' |

15°

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 41 300 | 42 805 | 57 195 | 98 494 | 60 |
| 1 | 41 347 | 42 856 | 57 144 | 98 491 | 59 |
| 2 | 41 394 | 42 906 | 57 094 | 98 488 | 58 |
| 3 | 41 441 | 42 957 | 57 043 | 98 484 | 57 |
| 4 | 41 488 | 43 007 | 56 993 | 98 481 | 56 |
| 5 | 41 535 | 43 057 | 56 943 | 98 477 | 55 |
| 6 | 41 582 | 43 108 | 56 892 | 98 474 | 54 |
| 7 | 41 628 | 43 158 | 56 842 | 98 471 | 53 |
| 8 | 41 675 | 43 208 | 56 792 | 98 467 | 52 |
| 9 | 41 722 | 43 258 | 56 742 | 98 464 | 51 |
| 10 | 41 768 | 43 308 | 56 692 | 98 460 | 50 |
| 11 | 41 815 | 43 358 | 56 642 | 98 457 | 49 |
| 12 | 41 861 | 43 408 | 56 592 | 98 453 | 48 |
| 13 | 41 908 | 43 458 | 56 542 | 98 450 | 47 |
| 14 | 41 954 | 43 508 | 56 492 | 98 447 | 46 |
| 15 | 42 001 | 43 558 | 56 442 | 98 443 | 45 |
| 16 | 42 047 | 43 607 | 56 393 | 98 440 | 44 |
| 17 | 42 093 | 43 657 | 56 343 | 98 436 | 43 |
| 18 | 42 140 | 43 707 | 56 293 | 98 433 | 42 |
| 19 | 42 186 | 43 756 | 56 244 | 98 429 | 41 |
| 20 | 42 232 | 43 806 | 56 194 | 98 426 | 40 |
| 21 | 42 278 | 43 855 | 56 145 | 98 422 | 39 |
| 22 | 42 324 | 43 905 | 56 095 | 98 419 | 38 |
| 23 | 42 370 | 43 954 | 56 046 | 98 415 | 37 |
| 24 | 42 416 | 44 004 | 55 996 | 98 412 | 36 |
| 25 | 42 461 | 44 053 | 55 947 | 98 409 | 35 |
| 26 | 42 507 | 44 102 | 55 898 | 98 405 | 34 |
| 27 | 42 553 | 44 151 | 55 849 | 98 402 | 33 |
| 28 | 42 599 | 44 201 | 55 799 | 98 398 | 32 |
| 29 | 42 644 | 44 250 | 55 750 | 98 395 | 31 |
| 30 | 42 690 | 44 299 | 55 701 | 98 391 | 30 |
| 31 | 42 735 | 44 348 | 55 652 | 98 388 | 29 |
| 32 | 42 781 | 44 397 | 55 603 | 98 384 | 28 |
| 33 | 42 826 | 44 446 | 55 554 | 98 381 | 27 |
| 34 | 42 872 | 44 495 | 55 505 | 98 377 | 26 |
| 35 | 42 917 | 44 544 | 55 456 | 98 373 | 25 |
| 36 | 42 962 | 44 592 | 55 408 | 98 370 | 24 |
| 37 | 43 008 | 44 641 | 55 359 | 98 366 | 23 |
| 38 | 43 053 | 44 690 | 55 310 | 98 363 | 22 |
| 39 | 43 098 | 44 738 | 55 262 | 98 359 | 21 |
| 40 | 43 143 | 44 787 | 55 213 | 98 356 | 20 |
| 41 | 43 188 | 44 836 | 55 164 | 98 352 | 19 |
| 42 | 43 233 | 44 884 | 55 116 | 98 349 | 18 |
| 43 | 43 278 | 44 933 | 55 067 | 98 345 | 17 |
| 44 | 43 323 | 44 981 | 55 019 | 98 342 | 16 |
| 45 | 43 367 | 45 029 | 54 971 | 98 338 | 15 |
| 46 | 43 412 | 45 078 | 54 922 | 98 334 | 14 |
| 47 | 43 457 | 45 126 | 54 874 | 98 331 | 13 |
| 48 | 43 502 | 45 174 | 54 826 | 98 327 | 12 |
| 49 | 43 546 | 45 222 | 54 778 | 98 324 | 11 |
| 50 | 43 591 | 45 271 | 54 729 | 98 320 | 10 |
| 51 | 43 635 | 45 319 | 54 681 | 98 317 | 9 |
| 52 | 43 680 | 45 367 | 54 633 | 98 313 | 8 |
| 53 | 43 724 | 45 415 | 54 585 | 98 309 | 7 |
| 54 | 43 769 | 45 463 | 54 537 | 98 306 | 6 |
| 55 | 43 813 | 45 511 | 54 489 | 98 302 | 5 |
| 56 | 43 857 | 45 559 | 54 441 | 98 299 | 4 |
| 57 | 43 901 | 45 606 | 54 394 | 98 295 | 3 |
| 58 | 43 946 | 45 654 | 54 346 | 98 291 | 2 |
| 59 | 43 990 | 45 702 | 54 298 | 98 288 | 1 |
| 60 | 44 034 | 45 750 | 54 250 | 98 284 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

74°

16°

85

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 44 034 | 45 750 | 54 250 | 98 284 | 60 |
| 1 | 44 078 | 45 797 | 54 203 | 98 281 | 59 |
| 2 | 44 122 | 45 845 | 54 155 | 98 277 | 58 |
| 3 | 44 166 | 45 892 | 54 108 | 98 273 | 57 |
| 4 | 44 210 | 45 940 | 54 060 | 98 270 | 56 |
| 5 | 44 253 | 45 987 | 54 013 | 98 266 | 55 |
| 6 | 44 297 | 46 035 | 53 965 | 98 262 | 54 |
| 7 | 44 341 | 46 082 | 53 918 | 98 259 | 53 |
| 8 | 44 385 | 46 130 | 53 870 | 98 255 | 52 |
| 9 | 44 428 | 46 177 | 53 823 | 98 251 | 51 |
| 10 | 44 472 | 46 224 | 53 776 | 98 248 | 50 |
| 11 | 44 516 | 46 271 | 53 729 | 98 244 | 49 |
| 12 | 44 559 | 46 319 | 53 681 | 98 240 | 48 |
| 13 | 44 602 | 46 366 | 53 634 | 98 237 | 47 |
| 14 | 44 646 | 46 413 | 53 587 | 98 233 | 46 |
| 15 | 44 689 | 46 460 | 53 540 | 98 229 | 45 |
| 16 | 44 733 | 46 507 | 53 493 | 98 226 | 44 |
| 17 | 44 776 | 46 554 | 53 446 | 98 222 | 43 |
| 18 | 44 819 | 46 601 | 53 399 | 98 218 | 42 |
| 19 | 44 862 | 46 648 | 53 352 | 98 215 | 41 |
| 20 | 44 905 | 46 694 | 53 306 | 98 211 | 40 |
| 21 | 44 948 | 46 741 | 53 259 | 98 207 | 39 |
| 22 | 44 992 | 46 788 | 53 212 | 98 204 | 38 |
| 23 | 45 035 | 46 835 | 53 165 | 98 200 | 37 |
| 24 | 45 077 | 46 881 | 53 119 | 98 196 | 36 |
| 25 | 45 120 | 46 928 | 53 072 | 98 192 | 35 |
| 26 | 45 163 | 46 975 | 53 025 | 98 189 | 34 |
| 27 | 45 206 | 47 021 | 52 979 | 98 185 | 33 |
| 28 | 45 249 | 47 068 | 52 932 | 98 181 | 32 |
| 29 | 45 292 | 47 114 | 52 886 | 98 177 | 31 |
| 30 | 45 334 | 47 160 | 52 840 | 98 174 | 30 |
| 31 | 45 377 | 47 207 | 52 793 | 98 170 | 29 |
| 32 | 45 419 | 47 253 | 52 747 | 98 166 | 28 |
| 33 | 45 462 | 47 299 | 52 701 | 98 162 | 27 |
| 34 | 45 504 | 47 346 | 52 654 | 98 159 | 26 |
| 35 | 45 547 | 47 392 | 52 608 | 98 155 | 25 |
| 36 | 45 589 | 47 438 | 52 562 | 98 151 | 24 |
| 37 | 45 632 | 47 484 | 52 516 | 98 147 | 23 |
| 38 | 45 674 | 47 530 | 52 470 | 98 144 | 22 |
| 39 | 45 716 | 47 576 | 52 424 | 98 140 | 21 |
| 40 | 45 758 | 47 622 | 52 378 | 98 136 | 20 |
| 41 | 45 801 | 47 668 | 52 332 | 98 132 | 19 |
| 42 | 45 843 | 47 714 | 52 286 | 98 129 | 18 |
| 43 | 45 885 | 47 760 | 52 240 | 98 125 | 17 |
| 44 | 45 927 | 47 806 | 52 194 | 98 121 | 16 |
| 45 | 45 969 | 47 852 | 52 148 | 98 117 | 15 |
| 46 | 46 011 | 47 897 | 52 103 | 98 113 | 14 |
| 47 | 46 053 | 47 943 | 52 057 | 98 110 | 13 |
| 48 | 46 095 | 47 989 | 52 011 | 98 106 | 12 |
| 49 | 46 136 | 48 035 | 51 965 | 98 102 | 11 |
| 50 | 46 178 | 48 080 | 51 920 | 98 098 | 10 |
| 51 | 46 220 | 48 126 | 51 874 | 98 094 | 9 |
| 52 | 46 262 | 48 171 | 51 829 | 98 090 | 8 |
| 53 | 46 303 | 48 217 | 51 783 | 98 087 | 7 |
| 54 | 46 345 | 48 262 | 51 738 | 98 083 | 6 |
| 55 | 46 386 | 48 307 | 51 693 | 98 079 | 5 |
| 56 | 46 428 | 48 353 | 51 647 | 98 075 | 4 |
| 57 | 46 469 | 48 398 | 51 602 | 98 071 | 3 |
| 58 | 46 511 | 48 443 | 51 557 | 98 067 | 2 |
| 59 | 46 552 | 48 489 | 51 511 | 98 063 | 1 |
| 60 | 46 594 | 48 534 | 51 466 | 98 060 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

73°

| ' | log sin 0 | log tan 0 | log cot -10 | log cos 0 | ' |
|----|--------------|----------------|----------------|--------------|----|
| 0 | 46 594 | 48 534 | 51 466 | 98 060 | 60 |
| 1 | 46 635 | 48 579 | 51 421 | 98 056 | 59 |
| 2 | 46 676 | 48 624 | 51 376 | 98 052 | 58 |
| 3 | 46 717 | 48 669 | 51 331 | 98 048 | 57 |
| 4 | 46 758 | 48 714 | 51 286 | 98 044 | 56 |
| 5 | 46 800 | 48 759 | 51 241 | 98 040 | 55 |
| 6 | 46 841 | 48 804 | 51 196 | 98 036 | 54 |
| 7 | 46 882 | 48 849 | 51 151 | 98 032 | 53 |
| 8 | 46 923 | 48 894 | 51 106 | 98 029 | 52 |
| 9 | 46 964 | 48 939 | 51 061 | 98 025 | 51 |
| 10 | 47 005 | 48 984 | 51 016 | 98 021 | 50 |
| 11 | 47 045 | 49 029 | 50 971 | 98 017 | 49 |
| 12 | 47 086 | 49 073 | 50 927 | 98 013 | 48 |
| 13 | 47 127 | 49 118 | 50 882 | 98 009 | 47 |
| 14 | 47 168 | 49 163 | 50 837 | 98 005 | 46 |
| 15 | 47 209 | 49 207 | 50 793 | 98 001 | 45 |
| 16 | 47 249 | 49 252 | 50 748 | 97 997 | 44 |
| 17 | 47 290 | 49 296 | 50 704 | 97 993 | 43 |
| 18 | 47 330 | 49 341 | 50 659 | 97 989 | 42 |
| 19 | 47 371 | 49 385 | 50 615 | 97 986 | 41 |
| 20 | 47 411 | 49 430 | 50 570 | 97 982 | 40 |
| 21 | 47 452 | 49 474 | 50 526 | 97 978 | 39 |
| 22 | 47 492 | 49 519 | 50 481 | 97 974 | 38 |
| 23 | 47 533 | 49 563 | 50 437 | 97 970 | 37 |
| 24 | 47 573 | 49 607 | 50 393 | 97 966 | 36 |
| 25 | 47 613 | 49 652 | 50 348 | 97 962 | 35 |
| 26 | 47 654 | 49 696 | 50 304 | 97 958 | 34 |
| 27 | 47 694 | 49 740 | 50 260 | 97 954 | 33 |
| 28 | 47 734 | 49 784 | 50 216 | 97 950 | 32 |
| 29 | 47 774 | 49 828 | 50 172 | 97 946 | 31 |
| 30 | 47 814 | 49 872 | 50 128 | 97 942 | 30 |
| 31 | 47 854 | 49 916 | 50 084 | 97 938 | 29 |
| 32 | 47 894 | 49 960 | 50 040 | 97 934 | 28 |
| 33 | 47 934 | 50 004 | 49 996 | 97 930 | 27 |
| 34 | 47 974 | 50 048 | 49 952 | 97 926 | 26 |
| 35 | 48 014 | 50 092 | 49 908 | 97 922 | 25 |
| 36 | 48 054 | 50 136 | 49 864 | 97 918 | 24 |
| 37 | 48 094 | 50 180 | 49 820 | 97 914 | 23 |
| 38 | 48 133 | 50 223 | 49 777 | 97 910 | 22 |
| 39 | 48 173 | 50 267 | 49 733 | 97 906 | 21 |
| 40 | 48 213 | 50 311 | 49 689 | 97 902 | 20 |
| 41 | 48 252 | 50 355 | 49 645 | 97 898 | 19 |
| 42 | 48 292 | 50 398 | 49 602 | 97 894 | 18 |
| 43 | 48 332 | 50 442 | 49 558 | 97 890 | 17 |
| 44 | 48 371 | 50 485 | 49 515 | 97 886 | 16 |
| 45 | 48 411 | 50 529 | 49 471 | 97 882 | 15 |
| 46 | 48 450 | 50 572 | 49 428 | 97 878 | 14 |
| 47 | 48 490 | 50 616 | 49 384 | 97 874 | 13 |
| 48 | 48 529 | 50 659 | 49 341 | 97 870 | 12 |
| 49 | 48 568 | 50 703 | 49 297 | 97 866 | 11 |
| 50 | 48 607 | 50 746 | 49 254 | 97 861 | 10 |
| 51 | 48 647 | 50 789 | 49 211 | 97 857 | 9 |
| 52 | 48 686 | 50 833 | 49 167 | 97 853 | 8 |
| 53 | 48 725 | 50 876 | 49 124 | 97 849 | 7 |
| 54 | 48 764 | 50 919 | 49 081 | 97 845 | 6 |
| 55 | 48 803 | 50 962 | 49 038 | 97 841 | 5 |
| 56 | 48 842 | 51 005 | 48 995 | 97 837 | 4 |
| 57 | 48 881 | 51 048 | 48 952 | 97 833 | 3 |
| 58 | 48 920 | 51 092 | 48 908 | 97 829 | 2 |
| 59 | 48 959 | 51 135 | 48 865 | 97 825 | 1 |
| 60 | 48 998 | 51 178 | 48 822 | 97 821 | 0 |
| ' | log cos 0 | log cot -10 | log tan 0 | log sin 0 | ' |

| ' | log sin 0 | log tan 0 | log cot -10 | log cos 0 | ' |
|----|--------------|----------------|----------------|--------------|----|
| 0 | 48 998 | 51 178 | 48 822 | 97 821 | 60 |
| 1 | 49 037 | 51 221 | 48 779 | 97 817 | 59 |
| 2 | 49 076 | 51 264 | 48 736 | 97 812 | 58 |
| 3 | 49 115 | 51 306 | 48 694 | 97 808 | 57 |
| 4 | 49 153 | 51 349 | 48 651 | 97 804 | 56 |
| 5 | 49 192 | 51 392 | 48 608 | 97 800 | 55 |
| 6 | 49 231 | 51 435 | 48 565 | 97 796 | 54 |
| 7 | 49 269 | 51 478 | 48 522 | 97 792 | 53 |
| 8 | 49 308 | 51 520 | 48 480 | 97 788 | 52 |
| 9 | 49 347 | 51 563 | 48 437 | 97 784 | 51 |
| 10 | 49 385 | 51 606 | 48 394 | 97 779 | 50 |
| 11 | 49 424 | 51 648 | 48 352 | 97 775 | 49 |
| 12 | 49 462 | 51 691 | 48 309 | 97 771 | 48 |
| 13 | 49 500 | 51 734 | 48 266 | 97 767 | 47 |
| 14 | 49 539 | 51 776 | 48 224 | 97 763 | 46 |
| 15 | 49 577 | 51 819 | 48 181 | 97 759 | 45 |
| 16 | 49 615 | 51 861 | 48 139 | 97 754 | 44 |
| 17 | 49 654 | 51 903 | 48 097 | 97 750 | 43 |
| 18 | 49 692 | 51 946 | 48 054 | 97 746 | 42 |
| 19 | 49 730 | 51 988 | 48 012 | 97 742 | 41 |
| 20 | 49 768 | 52 031 | 47 969 | 97 738 | 40 |
| 21 | 49 806 | 52 073 | 47 927 | 97 734 | 39 |
| 22 | 49 844 | 52 115 | 47 885 | 97 729 | 38 |
| 23 | 49 882 | 52 157 | 47 843 | 97 725 | 37 |
| 24 | 49 920 | 52 200 | 47 800 | 97 721 | 36 |
| 25 | 49 958 | 52 242 | 47 758 | 97 717 | 35 |
| 26 | 49 996 | 52 284 | 47 716 | 97 713 | 34 |
| 27 | 50 034 | 52 326 | 47 674 | 97 708 | 33 |
| 28 | 50 072 | 52 368 | 47 632 | 97 704 | 32 |
| 29 | 50 110 | 52 410 | 47 590 | 97 700 | 31 |
| 30 | 50 148 | 52 452 | 47 548 | 97 696 | 30 |
| 31 | 50 185 | 52 494 | 47 506 | 97 691 | 29 |
| 32 | 50 223 | 52 536 | 47 464 | 97 687 | 28 |
| 33 | 50 261 | 52 578 | 47 422 | 97 683 | 27 |
| 34 | 50 298 | 52 620 | 47 380 | 97 679 | 26 |
| 35 | 50 336 | 52 661 | 47 339 | 97 674 | 25 |
| 36 | 50 374 | 52 703 | 47 297 | 97 670 | 24 |
| 37 | 50 411 | 52 745 | 47 255 | 97 666 | 23 |
| 38 | 50 449 | 52 787 | 47 213 | 97 662 | 22 |
| 39 | 50 486 | 52 829 | 47 171 | 97 657 | 21 |
| 40 | 50 523 | 52 870 | 47 130 | 97 653 | 20 |
| 41 | 50 561 | 52 912 | 47 088 | 97 649 | 19 |
| 42 | 50 598 | 52 953 | 47 047 | 97 645 | 18 |
| 43 | 50 635 | 52 995 | 47 005 | 97 640 | 17 |
| 44 | 50 673 | 53 037 | 46 963 | 97 636 | 16 |
| 45 | 50 710 | 53 078 | 46 922 | 97 632 | 15 |
| 46 | 50 747 | 53 120 | 46 880 | 97 628 | 14 |
| 47 | 50 784 | 53 161 | 46 839 | 97 623 | 13 |
| 48 | 50 821 | 53 202 | 46 798 | 97 619 | 12 |
| 49 | 50 858 | 53 244 | 46 756 | 97 615 | 11 |
| 50 | 50 896 | 53 285 | 46 715 | 97 610 | 10 |
| 51 | 50 933 | 53 327 | 46 673 | 97 606 | 9 |
| 52 | 50 970 | 53 368 | 46 632 | 97 602 | 8 |
| 53 | 51 007 | 53 409 | 46 591 | 97 597 | 7 |
| 54 | 51 043 | 53 450 | 46 550 | 97 593 | 6 |
| 55 | 51 080 | 53 492 | 46 508 | 97 589 | 5 |
| 56 | 51 117 | 53 533 | 46 467 | 97 584 | 4 |
| 57 | 51 154 | 53 574 | 46 426 | 97 580 | 3 |
| 58 | 51 191 | 53 615 | 46 385 | 97 576 | 2 |
| 59 | 51 227 | 53 656 | 46 344 | 97 571 | 1 |
| 60 | 51 264 | 53 697 | 46 303 | 97 567 | 0 |
| ' | log cos 0 | log cot -10 | log tan 0 | log sin 0 | ' |

| / | log sin | log tan | log cot | log cos | / |
|----|---------|---------|---------|---------|----|
| | 9 | 9 | 10 | 9 | |
| 0 | 51 264 | 53 697 | 46 303 | 97 567 | 60 |
| 1 | 51 301 | 53 738 | 46 262 | 97 563 | 59 |
| 2 | 51 338 | 53 779 | 46 221 | 97 558 | 58 |
| 3 | 51 374 | 53 820 | 46 180 | 97 554 | 57 |
| 4 | 51 411 | 53 861 | 46 139 | 97 555 | 56 |
| 5 | 51 447 | 53 902 | 46 098 | 97 545 | 55 |
| 6 | 51 484 | 53 943 | 46 057 | 97 541 | 54 |
| 7 | 51 520 | 53 984 | 46 016 | 97 536 | 53 |
| 8 | 51 557 | 54 025 | 45 975 | 97 532 | 52 |
| 9 | 51 593 | 54 065 | 45 935 | 97 528 | 51 |
| 10 | 51 629 | 54 106 | 45 894 | 97 523 | 50 |
| 11 | 51 666 | 54 147 | 45 853 | 97 519 | 49 |
| 12 | 51 702 | 54 187 | 45 813 | 97 515 | 48 |
| 13 | 51 738 | 54 228 | 45 772 | 97 510 | 47 |
| 14 | 51 774 | 54 269 | 45 731 | 97 506 | 46 |
| 15 | 51 811 | 54 309 | 45 691 | 97 501 | 45 |
| 16 | 51 847 | 54 350 | 45 650 | 97 497 | 44 |
| 17 | 51 883 | 54 390 | 45 610 | 97 492 | 43 |
| 18 | 51 919 | 54 431 | 45 569 | 97 488 | 42 |
| 19 | 51 955 | 54 471 | 45 529 | 97 484 | 41 |
| 20 | 51 991 | 54 512 | 45 488 | 97 479 | 40 |
| 21 | 52 027 | 54 552 | 45 448 | 97 475 | 39 |
| 22 | 52 063 | 54 593 | 45 407 | 97 470 | 38 |
| 23 | 52 099 | 54 633 | 45 367 | 97 466 | 37 |
| 24 | 52 135 | 54 673 | 45 327 | 97 461 | 36 |
| 25 | 52 171 | 54 714 | 45 286 | 97 457 | 35 |
| 26 | 52 207 | 54 754 | 45 246 | 97 453 | 34 |
| 27 | 52 242 | 54 794 | 45 206 | 97 448 | 33 |
| 28 | 52 278 | 54 835 | 45 165 | 97 444 | 32 |
| 29 | 52 314 | 54 875 | 45 125 | 97 439 | 31 |
| 30 | 52 350 | 54 915 | 45 085 | 97 435 | 30 |
| 31 | 52 385 | 54 955 | 45 045 | 97 430 | 29 |
| 32 | 52 421 | 54 995 | 45 005 | 97 426 | 28 |
| 33 | 52 456 | 55 035 | 44 965 | 97 421 | 27 |
| 34 | 52 492 | 55 075 | 44 925 | 97 417 | 26 |
| 35 | 52 527 | 55 115 | 44 885 | 97 412 | 25 |
| 36 | 52 563 | 55 155 | 44 845 | 97 408 | 24 |
| 37 | 52 598 | 55 195 | 44 805 | 97 403 | 23 |
| 38 | 52 634 | 55 235 | 44 765 | 97 399 | 22 |
| 39 | 52 669 | 55 275 | 44 725 | 97 394 | 21 |
| 40 | 52 705 | 55 315 | 44 685 | 97 390 | 20 |
| 41 | 52 740 | 55 355 | 44 645 | 97 385 | 19 |
| 42 | 52 775 | 55 395 | 44 605 | 97 381 | 18 |
| 43 | 52 811 | 55 434 | 44 566 | 97 376 | 17 |
| 44 | 52 846 | 55 474 | 44 526 | 97 372 | 16 |
| 45 | 52 881 | 55 514 | 44 486 | 97 367 | 15 |
| 46 | 52 916 | 55 554 | 44 446 | 97 363 | 14 |
| 47 | 52 951 | 55 593 | 44 407 | 97 358 | 13 |
| 48 | 52 986 | 55 633 | 44 367 | 97 353 | 12 |
| 49 | 53 021 | 55 673 | 44 327 | 97 349 | 11 |
| 50 | 53 056 | 55 712 | 44 288 | 97 344 | 10 |
| 51 | 53 092 | 55 752 | 44 248 | 97 340 | 9 |
| 52 | 53 126 | 55 791 | 44 209 | 97 335 | 8 |
| 53 | 53 161 | 55 831 | 44 169 | 97 331 | 7 |
| 54 | 53 196 | 55 870 | 44 130 | 97 326 | 6 |
| 55 | 53 231 | 55 910 | 44 090 | 97 322 | 5 |
| 56 | 53 266 | 55 949 | 44 051 | 97 317 | 4 |
| 57 | 53 301 | 55 989 | 44 011 | 97 312 | 3 |
| 58 | 53 336 | 56 028 | 43 972 | 97 308 | 2 |
| 59 | 53 370 | 56 067 | 43 933 | 97 303 | 1 |
| 60 | 53 405 | 56 107 | 43 893 | 97 299 | 0 |
| / | log cos | log cot | log tan | log sin | / |
| | 9 | 9 | 10 | 9 | |

| / | log sin | log tan | log cot | log cos | / |
|----|---------|---------|---------|---------|----|
| | 9 | 9 | 10 | 9 | |
| 0 | 53 405 | 56 107 | 43 893 | 97 299 | 60 |
| 1 | 53 440 | 56 146 | 43 854 | 97 294 | 59 |
| 2 | 53 475 | 56 185 | 43 815 | 97 289 | 58 |
| 3 | 53 509 | 56 224 | 43 776 | 97 285 | 57 |
| 4 | 53 544 | 56 264 | 43 736 | 97 280 | 56 |
| 5 | 53 578 | 56 303 | 43 697 | 97 276 | 55 |
| 6 | 53 613 | 56 342 | 43 658 | 97 271 | 54 |
| 7 | 53 647 | 56 381 | 43 619 | 97 266 | 53 |
| 8 | 53 682 | 56 420 | 43 580 | 97 262 | 52 |
| 9 | 53 716 | 56 459 | 43 541 | 97 257 | 51 |
| 10 | 53 751 | 56 498 | 43 502 | 97 252 | 50 |
| 11 | 53 785 | 56 537 | 43 463 | 97 248 | 49 |
| 12 | 53 819 | 56 576 | 43 424 | 97 243 | 48 |
| 13 | 53 854 | 56 615 | 43 385 | 97 238 | 47 |
| 14 | 53 888 | 56 654 | 43 346 | 97 234 | 46 |
| 15 | 53 922 | 56 693 | 43 307 | 97 229 | 45 |
| 16 | 53 957 | 56 732 | 43 268 | 97 224 | 44 |
| 17 | 53 991 | 56 771 | 43 229 | 97 220 | 43 |
| 18 | 54 025 | 56 810 | 43 190 | 97 215 | 42 |
| 19 | 54 059 | 56 849 | 43 151 | 97 210 | 41 |
| 20 | 54 093 | 56 887 | 43 113 | 97 206 | 40 |
| 21 | 54 127 | 56 926 | 43 074 | 97 201 | 39 |
| 22 | 54 161 | 56 965 | 43 035 | 97 196 | 38 |
| 23 | 54 195 | 57 004 | 42 996 | 97 192 | 37 |
| 24 | 54 229 | 57 042 | 42 958 | 97 187 | 36 |
| 25 | 54 263 | 57 081 | 42 919 | 97 182 | 35 |
| 26 | 54 297 | 57 120 | 42 880 | 97 178 | 34 |
| 27 | 54 331 | 57 158 | 42 842 | 97 173 | 33 |
| 28 | 54 365 | 57 197 | 42 803 | 97 168 | 32 |
| 29 | 54 399 | 57 235 | 42 765 | 97 163 | 31 |
| 30 | 54 433 | 57 274 | 42 726 | 97 159 | 30 |
| 31 | 54 466 | 57 312 | 42 688 | 97 154 | 29 |
| 32 | 54 500 | 57 351 | 42 649 | 97 149 | 28 |
| 33 | 54 534 | 57 389 | 42 611 | 97 145 | 27 |
| 34 | 54 567 | 57 428 | 42 572 | 97 140 | 26 |
| 35 | 54 601 | 57 466 | 42 534 | 97 135 | 25 |
| 36 | 54 635 | 57 504 | 42 496 | 97 130 | 24 |
| 37 | 54 668 | 57 543 | 42 457 | 97 126 | 23 |
| 38 | 54 702 | 57 581 | 42 419 | 97 121 | 22 |
| 39 | 54 735 | 57 619 | 42 381 | 97 116 | 21 |
| 40 | 54 769 | 57 658 | 42 342 | 97 111 | 20 |
| 41 | 54 802 | 57 696 | 42 304 | 97 107 | 19 |
| 42 | 54 836 | 57 734 | 42 266 | 97 102 | 18 |
| 43 | 54 869 | 57 772 | 42 228 | 97 097 | 17 |
| 44 | 54 903 | 57 810 | 42 190 | 97 092 | 16 |
| 45 | 54 936 | 57 849 | 42 151 | 97 087 | 15 |
| 46 | 54 969 | 57 887 | 42 113 | 97 083 | 14 |
| 47 | 55 003 | 57 925 | 42 075 | 97 078 | 13 |
| 48 | 55 036 | 57 963 | 42 037 | 97 073 | 12 |
| 49 | 55 069 | 58 001 | 41 999 | 97 068 | 11 |
| 50 | 55 102 | 58 039 | 41 961 | 97 063 | 10 |
| 51 | 55 136 | 58 077 | 41 923 | 97 059 | 9 |
| 52 | 55 169 | 58 115 | 41 885 | 97 054 | 8 |
| 53 | 55 202 | 58 153 | 41 847 | 97 049 | 7 |
| 54 | 55 235 | 58 191 | 41 809 | 97 044 | 6 |
| 55 | 55 268 | 58 229 | 41 771 | 97 039 | 5 |
| 56 | 55 301 | 58 267 | 41 733 | 97 035 | 4 |
| 57 | 55 334 | 58 304 | 41 696 | 97 030 | 3 |
| 58 | 55 367 | 58 342 | 41 658 | 97 025 | 2 |
| 59 | 55 400 | 58 380 | 41 620 | 97 020 | 1 |
| 60 | 55 433 | 58 418 | 41 582 | 97 015 | 0 |
| / | log cos | log cot | log tan | log sin | / |
| | 9 | 9 | 10 | 9 | |

| / | log sin 0 | log tan 0 | log cot 10 | log cos 0 | / |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 55 433 | 58 418 | 41 582 | 97 015 | 60 |
| 1 | 55 466 | 58 455 | 41 545 | 97 010 | 59 |
| 2 | 55 499 | 58 493 | 41 507 | 97 005 | 58 |
| 3 | 55 532 | 58 531 | 41 469 | 97 001 | 57 |
| 4 | 55 564 | 58 569 | 41 431 | 96 996 | 56 |
| 5 | 55 597 | 58 606 | 41 394 | 96 991 | 55 |
| 6 | 55 630 | 58 644 | 41 356 | 96 986 | 54 |
| 7 | 55 663 | 58 681 | 41 319 | 96 981 | 53 |
| 8 | 55 695 | 58 719 | 41 281 | 96 976 | 52 |
| 9 | 55 728 | 58 757 | 41 243 | 96 971 | 51 |
| 10 | 55 761 | 58 794 | 41 206 | 96 966 | 50 |
| 11 | 55 793 | 58 832 | 41 168 | 96 962 | 49 |
| 12 | 55 826 | 58 869 | 41 131 | 96 957 | 48 |
| 13 | 55 858 | 58 907 | 41 093 | 96 952 | 47 |
| 14 | 55 891 | 58 944 | 41 056 | 96 947 | 46 |
| 15 | 55 923 | 58 981 | 41 019 | 96 942 | 45 |
| 16 | 55 956 | 59 019 | 40 981 | 96 937 | 44 |
| 17 | 55 988 | 59 056 | 40 944 | 96 932 | 43 |
| 18 | 56 021 | 59 094 | 40 906 | 96 927 | 42 |
| 19 | 56 053 | 59 131 | 40 869 | 96 922 | 41 |
| 20 | 56 085 | 59 168 | 40 832 | 96 917 | 40 |
| 21 | 56 118 | 59 205 | 40 795 | 96 912 | 39 |
| 22 | 56 150 | 59 243 | 40 757 | 96 907 | 38 |
| 23 | 56 182 | 59 280 | 40 720 | 96 903 | 37 |
| 24 | 56 215 | 59 317 | 40 683 | 96 898 | 36 |
| 25 | 56 247 | 59 354 | 40 646 | 96 893 | 35 |
| 26 | 56 279 | 59 391 | 40 609 | 96 888 | 34 |
| 27 | 56 311 | 59 429 | 40 571 | 96 883 | 33 |
| 28 | 56 343 | 59 466 | 40 534 | 96 878 | 32 |
| 29 | 56 375 | 59 503 | 40 497 | 96 873 | 31 |
| 30 | 56 408 | 59 540 | 40 460 | 96 868 | 30 |
| 31 | 56 440 | 59 577 | 40 423 | 96 863 | 29 |
| 32 | 56 472 | 59 614 | 40 386 | 96 858 | 28 |
| 33 | 56 504 | 59 651 | 40 349 | 96 853 | 27 |
| 34 | 56 536 | 59 688 | 40 312 | 96 848 | 26 |
| 35 | 56 568 | 59 725 | 40 275 | 96 843 | 25 |
| 36 | 56 599 | 59 762 | 40 238 | 96 838 | 24 |
| 37 | 56 631 | 59 799 | 40 201 | 96 833 | 23 |
| 38 | 56 663 | 59 835 | 40 165 | 96 828 | 22 |
| 39 | 56 695 | 59 872 | 40 128 | 96 823 | 21 |
| 40 | 56 727 | 59 909 | 40 091 | 96 818 | 20 |
| 41 | 56 759 | 59 946 | 40 054 | 96 813 | 19 |
| 42 | 56 790 | 59 983 | 40 017 | 96 808 | 18 |
| 43 | 56 822 | 60 019 | 39 981 | 96 803 | 17 |
| 44 | 56 854 | 60 056 | 39 944 | 96 798 | 16 |
| 45 | 56 886 | 60 093 | 39 907 | 96 793 | 15 |
| 46 | 56 917 | 60 130 | 39 870 | 96 788 | 14 |
| 47 | 56 949 | 60 166 | 39 834 | 96 783 | 13 |
| 48 | 56 980 | 60 203 | 39 797 | 96 778 | 12 |
| 49 | 57 012 | 60 240 | 39 760 | 96 772 | 11 |
| 50 | 57 044 | 60 276 | 39 724 | 96 767 | 10 |
| 51 | 57 075 | 60 313 | 39 687 | 96 762 | 9 |
| 52 | 57 107 | 60 349 | 39 651 | 96 757 | 8 |
| 53 | 57 138 | 60 386 | 39 614 | 96 752 | 7 |
| 54 | 57 169 | 60 422 | 39 578 | 96 747 | 6 |
| 55 | 57 201 | 60 459 | 39 541 | 96 742 | 5 |
| 56 | 57 232 | 60 495 | 39 505 | 96 737 | 4 |
| 57 | 57 264 | 60 532 | 39 468 | 96 732 | 3 |
| 58 | 57 295 | 60 568 | 39 432 | 96 727 | 2 |
| 59 | 57 326 | 60 605 | 39 395 | 96 722 | 1 |
| 60 | 57 358 | 60 641 | 39 359 | 96 717 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| / | log sin 0 | log tan 0 | log cot 10 | log cos 0 | / |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 57 358 | 60 641 | 39 359 | 96 717 | 60 |
| 1 | 57 389 | 60 677 | 39 323 | 96 711 | 59 |
| 2 | 57 420 | 60 714 | 39 286 | 96 706 | 58 |
| 3 | 57 451 | 60 750 | 39 250 | 96 701 | 57 |
| 4 | 57 482 | 60 786 | 39 214 | 96 696 | 56 |
| 5 | 57 514 | 60 823 | 39 177 | 96 691 | 55 |
| 6 | 57 545 | 60 859 | 39 141 | 96 686 | 54 |
| 7 | 57 576 | 60 895 | 39 105 | 96 681 | 53 |
| 8 | 57 607 | 60 931 | 39 069 | 96 676 | 52 |
| 9 | 57 638 | 60 967 | 39 033 | 96 670 | 51 |
| 10 | 57 669 | 61 004 | 38 996 | 96 665 | 50 |
| 11 | 57 700 | 61 040 | 38 960 | 96 660 | 49 |
| 12 | 57 731 | 61 076 | 38 924 | 96 655 | 48 |
| 13 | 57 762 | 61 112 | 38 888 | 96 650 | 47 |
| 14 | 57 793 | 61 148 | 38 852 | 96 645 | 46 |
| 15 | 57 824 | 61 184 | 38 816 | 96 640 | 45 |
| 16 | 57 855 | 61 220 | 38 780 | 96 634 | 44 |
| 17 | 57 885 | 61 256 | 38 744 | 96 629 | 43 |
| 18 | 57 916 | 61 292 | 38 708 | 96 624 | 42 |
| 19 | 57 947 | 61 328 | 38 672 | 96 619 | 41 |
| 20 | 57 978 | 61 364 | 38 636 | 96 614 | 40 |
| 21 | 58 008 | 61 400 | 38 600 | 96 608 | 39 |
| 22 | 58 039 | 61 436 | 38 564 | 96 603 | 38 |
| 23 | 58 070 | 61 472 | 38 528 | 96 598 | 37 |
| 24 | 58 101 | 61 508 | 38 492 | 96 593 | 36 |
| 25 | 58 131 | 61 544 | 38 456 | 96 588 | 35 |
| 26 | 58 162 | 61 579 | 38 421 | 96 582 | 34 |
| 27 | 58 192 | 61 615 | 38 385 | 96 577 | 33 |
| 28 | 58 223 | 61 651 | 38 349 | 96 572 | 32 |
| 29 | 58 253 | 61 687 | 38 313 | 96 567 | 31 |
| 30 | 58 284 | 61 722 | 38 278 | 96 562 | 30 |
| 31 | 58 314 | 61 758 | 38 242 | 96 556 | 29 |
| 32 | 58 345 | 61 794 | 38 206 | 96 551 | 28 |
| 33 | 58 375 | 61 830 | 38 170 | 96 546 | 27 |
| 34 | 58 406 | 61 865 | 38 135 | 96 541 | 26 |
| 35 | 58 436 | 61 901 | 38 099 | 96 535 | 25 |
| 36 | 58 467 | 61 936 | 38 064 | 96 530 | 24 |
| 37 | 58 497 | 61 972 | 38 028 | 96 525 | 23 |
| 38 | 58 527 | 62 008 | 37 992 | 96 520 | 22 |
| 39 | 58 557 | 62 043 | 37 957 | 96 514 | 21 |
| 40 | 58 588 | 62 079 | 37 921 | 96 509 | 20 |
| 41 | 58 618 | 62 114 | 37 886 | 96 504 | 19 |
| 42 | 58 648 | 62 150 | 37 850 | 96 498 | 18 |
| 43 | 58 678 | 62 185 | 37 815 | 96 493 | 17 |
| 44 | 58 709 | 62 221 | 37 779 | 96 488 | 16 |
| 45 | 58 739 | 62 256 | 37 744 | 96 483 | 15 |
| 46 | 58 769 | 62 292 | 37 708 | 96 477 | 14 |
| 47 | 58 799 | 62 327 | 37 673 | 96 472 | 13 |
| 48 | 58 829 | 62 362 | 37 638 | 96 467 | 12 |
| 49 | 58 859 | 62 398 | 37 602 | 96 461 | 11 |
| 50 | 58 889 | 62 433 | 37 567 | 96 456 | 10 |
| 51 | 58 919 | 62 468 | 37 532 | 96 451 | 9 |
| 52 | 58 949 | 62 504 | 37 496 | 96 445 | 8 |
| 53 | 58 979 | 62 539 | 37 461 | 96 440 | 7 |
| 54 | 59 009 | 62 574 | 37 426 | 96 435 | 6 |
| 55 | 59 039 | 62 609 | 37 391 | 96 429 | 5 |
| 56 | 59 069 | 62 645 | 37 355 | 96 424 | 4 |
| 57 | 59 098 | 62 680 | 37 320 | 96 419 | 3 |
| 58 | 59 128 | 62 715 | 37 285 | 96 413 | 2 |
| 59 | 59 158 | 62 750 | 37 250 | 96 408 | 1 |
| 60 | 59 188 | 62 785 | 37 215 | 96 403 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| / | log sin 9 | log tan 9 | log cot 10 | log cos 9 | / |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 59 188 | 62 785 | 37 215 | 96 403 | 60 |
| 1 | 59 218 | 62 820 | 37 180 | 96 397 | 59 |
| 2 | 59 247 | 62 855 | 37 145 | 96 392 | 58 |
| 3 | 59 277 | 62 890 | 37 110 | 96 387 | 57 |
| 4 | 59 307 | 62 926 | 37 074 | 96 381 | 56 |
| 5 | 59 336 | 62 961 | 37 039 | 96 376 | 55 |
| 6 | 59 366 | 62 996 | 37 004 | 96 370 | 54 |
| 7 | 59 396 | 63 031 | 36 969 | 96 365 | 53 |
| 8 | 59 425 | 63 066 | 36 934 | 96 360 | 52 |
| 9 | 59 455 | 63 101 | 36 899 | 96 354 | 51 |
| 10 | 59 484 | 63 135 | 36 865 | 96 349 | 50 |
| 11 | 59 514 | 63 170 | 36 830 | 96 343 | 49 |
| 12 | 59 543 | 63 205 | 36 795 | 96 338 | 48 |
| 13 | 59 573 | 63 240 | 36 760 | 96 333 | 47 |
| 14 | 59 602 | 63 275 | 36 725 | 96 327 | 46 |
| 15 | 59 632 | 63 310 | 36 690 | 96 322 | 45 |
| 16 | 59 661 | 63 345 | 36 655 | 96 316 | 44 |
| 17 | 59 690 | 63 379 | 36 621 | 96 311 | 43 |
| 18 | 59 720 | 63 414 | 36 586 | 96 305 | 42 |
| 19 | 59 749 | 63 449 | 36 551 | 96 300 | 41 |
| 20 | 59 778 | 63 484 | 36 516 | 96 294 | 40 |
| 21 | 59 808 | 63 519 | 36 481 | 96 289 | 39 |
| 22 | 59 837 | 63 553 | 36 447 | 96 284 | 38 |
| 23 | 59 866 | 63 588 | 36 412 | 96 278 | 37 |
| 24 | 59 895 | 63 623 | 36 377 | 96 273 | 36 |
| 25 | 59 924 | 63 657 | 36 343 | 96 267 | 35 |
| 26 | 59 954 | 63 692 | 36 308 | 96 262 | 34 |
| 27 | 59 983 | 63 726 | 36 274 | 96 256 | 33 |
| 28 | 60 012 | 63 761 | 36 239 | 96 251 | 32 |
| 29 | 60 041 | 63 796 | 36 204 | 96 245 | 31 |
| 30 | 60 070 | 63 830 | 36 170 | 96 240 | 30 |
| 31 | 60 099 | 63 865 | 36 135 | 96 234 | 29 |
| 32 | 60 128 | 63 899 | 36 101 | 96 229 | 28 |
| 33 | 60 157 | 63 934 | 36 066 | 96 223 | 27 |
| 34 | 60 186 | 63 968 | 36 032 | 96 218 | 26 |
| 35 | 60 215 | 64 003 | 35 997 | 96 212 | 25 |
| 36 | 60 244 | 64 037 | 35 963 | 96 207 | 24 |
| 37 | 60 273 | 64 072 | 35 928 | 96 201 | 23 |
| 38 | 60 302 | 64 106 | 35 894 | 96 196 | 22 |
| 39 | 60 331 | 64 140 | 35 860 | 96 190 | 21 |
| 40 | 60 359 | 64 175 | 35 825 | 96 185 | 20 |
| 41 | 60 388 | 64 209 | 35 791 | 96 179 | 19 |
| 42 | 60 417 | 64 243 | 35 757 | 96 174 | 18 |
| 43 | 60 446 | 64 278 | 35 722 | 96 168 | 17 |
| 44 | 60 474 | 64 312 | 35 688 | 96 162 | 16 |
| 45 | 60 503 | 64 346 | 35 654 | 96 157 | 15 |
| 46 | 60 532 | 64 381 | 35 619 | 96 151 | 14 |
| 47 | 60 561 | 64 415 | 35 585 | 96 146 | 13 |
| 48 | 60 589 | 64 449 | 35 551 | 96 140 | 12 |
| 49 | 60 618 | 64 483 | 35 517 | 96 135 | 11 |
| 50 | 60 646 | 64 517 | 35 483 | 96 129 | 10 |
| 51 | 60 675 | 64 552 | 35 448 | 96 123 | 9 |
| 52 | 60 704 | 64 586 | 35 414 | 96 118 | 8 |
| 53 | 60 732 | 64 620 | 35 380 | 96 112 | 7 |
| 54 | 60 761 | 64 654 | 35 346 | 96 107 | 6 |
| 55 | 60 789 | 64 688 | 35 312 | 96 101 | 5 |
| 56 | 60 818 | 64 722 | 35 278 | 96 095 | 4 |
| 57 | 60 846 | 64 756 | 35 244 | 96 090 | 3 |
| 58 | 60 875 | 64 790 | 35 210 | 96 084 | 2 |
| 59 | 60 903 | 64 824 | 35 176 | 96 079 | 1 |
| 60 | 60 931 | 64 858 | 35 142 | 96 073 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| / | log sin 9 | log tan 9 | log cot 10 | log cos 9 | / |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 60 831 | 64 858 | 35 142 | 96 073 | 60 |
| 1 | 60 960 | 64 892 | 35 108 | 96 067 | 59 |
| 2 | 60 988 | 64 926 | 35 074 | 96 062 | 58 |
| 3 | 61 016 | 64 960 | 35 040 | 96 056 | 57 |
| 4 | 61 045 | 64 994 | 35 006 | 96 050 | 56 |
| 5 | 61 073 | 65 028 | 34 972 | 96 045 | 55 |
| 6 | 61 101 | 65 062 | 34 938 | 96 039 | 54 |
| 7 | 61 129 | 65 096 | 34 904 | 96 034 | 53 |
| 8 | 61 158 | 65 130 | 34 870 | 96 028 | 52 |
| 9 | 61 186 | 65 164 | 34 836 | 96 022 | 51 |
| 10 | 61 214 | 65 197 | 34 803 | 96 017 | 50 |
| 11 | 61 242 | 65 231 | 34 769 | 96 011 | 49 |
| 12 | 61 270 | 65 265 | 34 735 | 96 005 | 48 |
| 13 | 61 298 | 65 299 | 34 701 | 96 000 | 47 |
| 14 | 61 326 | 65 333 | 34 667 | 95 994 | 46 |
| 15 | 61 354 | 65 366 | 34 634 | 95 988 | 45 |
| 16 | 61 382 | 65 400 | 34 600 | 95 982 | 44 |
| 17 | 61 411 | 65 434 | 34 566 | 95 977 | 43 |
| 18 | 61 438 | 65 467 | 34 533 | 95 971 | 42 |
| 19 | 61 466 | 65 501 | 34 499 | 95 965 | 41 |
| 20 | 61 494 | 65 535 | 34 465 | 95 960 | 40 |
| 21 | 61 522 | 65 568 | 34 432 | 95 954 | 39 |
| 22 | 61 550 | 65 602 | 34 398 | 95 948 | 38 |
| 23 | 61 578 | 65 636 | 34 364 | 95 942 | 37 |
| 24 | 61 606 | 65 669 | 34 331 | 95 937 | 36 |
| 25 | 61 634 | 65 703 | 34 297 | 95 931 | 35 |
| 26 | 61 662 | 65 736 | 34 264 | 95 925 | 34 |
| 27 | 61 689 | 65 770 | 34 230 | 95 920 | 33 |
| 28 | 61 717 | 65 803 | 34 197 | 95 914 | 32 |
| 29 | 61 745 | 65 837 | 34 163 | 95 908 | 31 |
| 30 | 61 773 | 65 870 | 34 130 | 95 902 | 30 |
| 31 | 61 800 | 65 904 | 34 096 | 95 897 | 29 |
| 32 | 61 828 | 65 937 | 34 063 | 95 891 | 28 |
| 33 | 61 856 | 65 971 | 34 029 | 95 885 | 27 |
| 34 | 61 883 | 66 004 | 33 996 | 95 879 | 26 |
| 35 | 61 911 | 66 038 | 33 962 | 95 873 | 25 |
| 36 | 61 939 | 66 071 | 33 929 | 95 868 | 24 |
| 37 | 61 966 | 66 104 | 33 896 | 95 862 | 23 |
| 38 | 61 994 | 66 138 | 33 862 | 95 856 | 22 |
| 39 | 62 021 | 66 171 | 33 829 | 95 850 | 21 |
| 40 | 62 049 | 66 204 | 33 796 | 95 844 | 20 |
| 41 | 62 076 | 66 238 | 33 762 | 95 839 | 19 |
| 42 | 62 104 | 66 271 | 33 729 | 95 833 | 18 |
| 43 | 62 131 | 66 304 | 33 696 | 95 827 | 17 |
| 44 | 62 159 | 66 337 | 33 663 | 95 821 | 16 |
| 45 | 62 186 | 66 371 | 33 629 | 95 815 | 15 |
| 46 | 62 214 | 66 404 | 33 596 | 95 810 | 14 |
| 47 | 62 241 | 66 437 | 33 563 | 95 804 | 13 |
| 48 | 62 268 | 66 470 | 33 530 | 95 798 | 12 |
| 49 | 62 296 | 66 503 | 33 497 | 95 792 | 11 |
| 50 | 62 323 | 66 537 | 33 463 | 95 786 | 10 |
| 51 | 62 350 | 66 570 | 33 430 | 95 780 | 9 |
| 52 | 62 377 | 66 603 | 33 397 | 95 775 | 8 |
| 53 | 62 405 | 66 636 | 33 364 | 95 769 | 7 |
| 54 | 62 432 | 66 669 | 33 331 | 95 763 | 6 |
| 55 | 62 459 | 66 702 | 33 298 | 95 757 | 5 |
| 56 | 62 486 | 66 735 | 33 265 | 95 751 | 4 |
| 57 | 62 513 | 66 768 | 33 232 | 95 745 | 3 |
| 58 | 62 541 | 66 801 | 33 199 | 95 739 | 2 |
| 59 | 62 568 | 66 834 | 33 166 | 95 733 | 1 |
| 60 | 62 595 | 66 867 | 33 133 | 95 728 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 62 595 | 66 867 | 33 133 | 95 728 | 60 |
| 1 | 62 622 | 66 900 | 33 100 | 95 722 | 59 |
| 2 | 62 649 | 66 933 | 33 067 | 95 716 | 58 |
| 3 | 62 676 | 66 966 | 33 034 | 95 710 | 57 |
| 4 | 62 703 | 66 999 | 33 001 | 95 704 | 56 |
| 5 | 62 730 | 67 032 | 32 968 | 95 698 | 55 |
| 6 | 62 757 | 67 065 | 32 935 | 95 692 | 54 |
| 7 | 62 784 | 67 098 | 32 902 | 95 686 | 53 |
| 8 | 62 811 | 67 131 | 32 869 | 95 680 | 52 |
| 9 | 62 838 | 67 163 | 32 837 | 95 674 | 51 |
| 10 | 62 865 | 67 196 | 32 804 | 95 668 | 50 |
| 11 | 62 892 | 67 229 | 32 771 | 95 663 | 49 |
| 12 | 62 918 | 67 262 | 32 738 | 95 657 | 48 |
| 13 | 62 945 | 67 295 | 32 705 | 95 651 | 47 |
| 14 | 62 972 | 67 327 | 32 673 | 95 645 | 46 |
| 15 | 62 999 | 67 360 | 32 640 | 95 639 | 45 |
| 16 | 63 026 | 67 393 | 32 607 | 95 633 | 44 |
| 17 | 63 052 | 67 426 | 32 574 | 95 627 | 43 |
| 18 | 63 079 | 67 458 | 32 542 | 95 621 | 42 |
| 19 | 63 106 | 67 491 | 32 509 | 95 615 | 41 |
| 20 | 63 133 | 67 524 | 32 476 | 95 609 | 40 |
| 21 | 63 159 | 67 556 | 32 444 | 95 603 | 39 |
| 22 | 63 186 | 67 589 | 32 411 | 95 597 | 38 |
| 23 | 63 213 | 67 622 | 32 378 | 95 591 | 37 |
| 24 | 63 239 | 67 654 | 32 346 | 95 585 | 36 |
| 25 | 63 266 | 67 687 | 32 313 | 95 579 | 35 |
| 26 | 63 292 | 67 719 | 32 281 | 95 573 | 34 |
| 27 | 63 319 | 67 752 | 32 248 | 95 567 | 33 |
| 28 | 63 345 | 67 785 | 32 215 | 95 561 | 32 |
| 29 | 63 372 | 67 817 | 32 183 | 95 555 | 31 |
| 30 | 63 398 | 67 850 | 32 150 | 95 549 | 30 |
| 31 | 63 425 | 67 882 | 32 118 | 95 543 | 29 |
| 32 | 63 451 | 67 915 | 32 085 | 95 537 | 28 |
| 33 | 63 478 | 67 947 | 32 053 | 95 531 | 27 |
| 34 | 63 504 | 67 980 | 32 020 | 95 525 | 26 |
| 35 | 63 531 | 68 012 | 31 988 | 95 519 | 25 |
| 36 | 63 557 | 68 044 | 31 956 | 95 513 | 24 |
| 37 | 63 583 | 68 077 | 31 923 | 95 507 | 23 |
| 38 | 63 610 | 68 109 | 31 891 | 95 500 | 22 |
| 39 | 63 636 | 68 142 | 31 858 | 95 494 | 21 |
| 40 | 63 662 | 68 174 | 31 826 | 95 488 | 20 |
| 41 | 63 689 | 68 206 | 31 794 | 95 482 | 19 |
| 42 | 63 715 | 68 239 | 31 761 | 95 476 | 18 |
| 43 | 63 741 | 68 271 | 31 729 | 95 470 | 17 |
| 44 | 63 767 | 68 303 | 31 697 | 95 464 | 16 |
| 45 | 63 794 | 68 336 | 31 664 | 95 458 | 15 |
| 46 | 63 820 | 68 368 | 31 632 | 95 452 | 14 |
| 47 | 63 846 | 68 400 | 31 600 | 95 446 | 13 |
| 48 | 63 872 | 68 432 | 31 568 | 95 440 | 12 |
| 49 | 63 898 | 68 465 | 31 535 | 95 434 | 11 |
| 50 | 63 924 | 68 497 | 31 503 | 95 427 | 10 |
| 51 | 63 950 | 68 529 | 31 471 | 95 421 | 9 |
| 52 | 63 976 | 68 561 | 31 439 | 95 415 | 8 |
| 53 | 64 002 | 68 593 | 31 407 | 95 409 | 7 |
| 54 | 64 028 | 68 626 | 31 374 | 95 403 | 6 |
| 55 | 64 054 | 68 658 | 31 342 | 95 397 | 5 |
| 56 | 64 080 | 68 690 | 31 310 | 95 391 | 4 |
| 57 | 64 106 | 68 722 | 31 278 | 95 384 | 3 |
| 58 | 64 132 | 68 754 | 31 246 | 95 378 | 2 |
| 59 | 64 158 | 68 786 | 31 214 | 95 372 | 1 |
| 60 | 64 184 | 68 818 | 31 182 | 95 366 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 64 184 | 68 818 | 31 182 | 95 366 | 60 |
| 1 | 64 210 | 68 850 | 31 150 | 95 360 | 59 |
| 2 | 64 236 | 68 882 | 31 118 | 95 354 | 58 |
| 3 | 64 262 | 68 914 | 31 086 | 95 348 | 57 |
| 4 | 64 288 | 68 946 | 31 054 | 95 341 | 56 |
| 5 | 64 313 | 68 978 | 31 022 | 95 335 | 55 |
| 6 | 64 339 | 69 010 | 30 990 | 95 329 | 54 |
| 7 | 64 365 | 69 042 | 30 958 | 95 323 | 53 |
| 8 | 64 391 | 69 074 | 30 926 | 95 317 | 52 |
| 9 | 64 417 | 69 106 | 30 894 | 95 310 | 51 |
| 10 | 64 442 | 69 138 | 30 862 | 95 304 | 50 |
| 11 | 64 468 | 69 170 | 30 830 | 95 298 | 49 |
| 12 | 64 494 | 69 202 | 30 798 | 95 292 | 48 |
| 13 | 64 519 | 69 234 | 30 766 | 95 286 | 47 |
| 14 | 64 545 | 69 266 | 30 734 | 95 279 | 46 |
| 15 | 64 571 | 69 298 | 30 702 | 95 273 | 45 |
| 16 | 64 596 | 69 329 | 30 671 | 95 267 | 44 |
| 17 | 64 622 | 69 361 | 30 639 | 95 261 | 43 |
| 18 | 64 647 | 69 393 | 30 607 | 95 254 | 42 |
| 19 | 64 673 | 69 425 | 30 575 | 95 248 | 41 |
| 20 | 64 698 | 69 457 | 30 543 | 95 242 | 40 |
| 21 | 64 724 | 69 488 | 30 512 | 95 236 | 39 |
| 22 | 64 749 | 69 520 | 30 480 | 95 229 | 38 |
| 23 | 64 775 | 69 552 | 30 448 | 95 223 | 37 |
| 24 | 64 800 | 69 584 | 30 416 | 95 217 | 36 |
| 25 | 64 826 | 69 615 | 30 385 | 95 211 | 35 |
| 26 | 64 851 | 69 647 | 30 353 | 95 204 | 34 |
| 27 | 64 877 | 69 679 | 30 321 | 95 198 | 33 |
| 28 | 64 902 | 69 710 | 30 290 | 95 192 | 32 |
| 29 | 64 927 | 69 742 | 30 258 | 95 185 | 31 |
| 30 | 64 953 | 69 774 | 30 226 | 95 179 | 30 |
| 31 | 64 978 | 69 805 | 30 195 | 95 173 | 29 |
| 32 | 65 003 | 69 837 | 30 163 | 95 167 | 28 |
| 33 | 65 029 | 69 868 | 30 132 | 95 160 | 27 |
| 34 | 65 054 | 69 900 | 30 100 | 95 154 | 26 |
| 35 | 65 079 | 69 932 | 30 068 | 95 148 | 25 |
| 36 | 65 104 | 69 963 | 30 037 | 95 141 | 24 |
| 37 | 65 130 | 69 995 | 30 005 | 95 135 | 23 |
| 38 | 65 155 | 70 026 | 29 974 | 95 129 | 22 |
| 39 | 65 180 | 70 058 | 29 942 | 95 122 | 21 |
| 40 | 65 205 | 70 089 | 29 911 | 95 116 | 20 |
| 41 | 65 230 | 70 121 | 29 879 | 95 110 | 19 |
| 42 | 65 255 | 70 152 | 29 848 | 95 103 | 18 |
| 43 | 65 281 | 70 184 | 29 816 | 95 097 | 17 |
| 44 | 65 306 | 70 215 | 29 785 | 95 090 | 16 |
| 45 | 65 331 | 70 247 | 29 753 | 95 084 | 15 |
| 46 | 65 356 | 70 278 | 29 722 | 95 078 | 14 |
| 47 | 65 381 | 70 309 | 29 691 | 95 071 | 13 |
| 48 | 65 406 | 70 341 | 29 659 | 95 065 | 12 |
| 49 | 65 431 | 70 372 | 29 628 | 95 059 | 11 |
| 50 | 65 456 | 70 404 | 29 596 | 95 052 | 10 |
| 51 | 65 481 | 70 435 | 29 565 | 95 046 | 9 |
| 52 | 65 506 | 70 466 | 29 534 | 95 039 | 8 |
| 53 | 65 531 | 70 498 | 29 502 | 95 033 | 7 |
| 54 | 65 556 | 70 529 | 29 471 | 95 027 | 6 |
| 55 | 65 580 | 70 560 | 29 440 | 95 020 | 5 |
| 56 | 65 605 | 70 592 | 29 408 | 95 014 | 4 |
| 57 | 65 630 | 70 623 | 29 377 | 95 007 | 3 |
| 58 | 65 655 | 70 654 | 29 346 | 95 001 | 2 |
| 59 | 65 680 | 70 685 | 29 315 | 94 995 | 1 |
| 60 | 65 705 | 70 717 | 29 283 | 94 988 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

27°

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 65 705 | 70 717 | 29 283 | 94 988 | 60 |
| 1 | 65 729 | 70 748 | 29 252 | 94 982 | 59 |
| 2 | 65 754 | 70 779 | 29 221 | 94 975 | 58 |
| 3 | 65 779 | 70 810 | 29 190 | 94 969 | 57 |
| 4 | 65 804 | 70 841 | 29 159 | 94 962 | 56 |
| 5 | 65 828 | 70 873 | 29 127 | 94 956 | 55 |
| 6 | 65 853 | 70 904 | 29 096 | 94 949 | 54 |
| 7 | 65 878 | 70 935 | 29 065 | 94 943 | 53 |
| 8 | 65 902 | 70 966 | 29 034 | 94 936 | 52 |
| 9 | 65 927 | 70 997 | 29 003 | 94 930 | 51 |
| 10 | 65 952 | 71 028 | 28 972 | 94 923 | 50 |
| 11 | 65 976 | 71 059 | 28 941 | 94 917 | 49 |
| 12 | 66 001 | 71 090 | 28 910 | 94 911 | 48 |
| 13 | 66 025 | 71 121 | 28 879 | 94 904 | 47 |
| 14 | 66 050 | 71 153 | 28 847 | 94 898 | 46 |
| 15 | 66 075 | 71 184 | 28 816 | 94 891 | 45 |
| 16 | 66 099 | 71 215 | 28 785 | 94 885 | 44 |
| 17 | 66 124 | 71 246 | 28 754 | 94 878 | 43 |
| 18 | 66 148 | 71 277 | 28 723 | 94 871 | 42 |
| 19 | 66 173 | 71 308 | 28 692 | 94 865 | 41 |
| 20 | 66 197 | 71 339 | 28 661 | 94 858 | 40 |
| 21 | 66 221 | 71 370 | 28 630 | 94 852 | 39 |
| 22 | 66 246 | 71 401 | 28 599 | 94 845 | 38 |
| 23 | 66 270 | 71 431 | 28 569 | 94 839 | 37 |
| 24 | 66 295 | 71 462 | 28 538 | 94 832 | 36 |
| 25 | 66 319 | 71 493 | 28 507 | 94 826 | 35 |
| 26 | 66 343 | 71 524 | 28 476 | 94 819 | 34 |
| 27 | 66 368 | 71 555 | 28 445 | 94 813 | 33 |
| 28 | 66 392 | 71 586 | 28 414 | 94 806 | 32 |
| 29 | 66 416 | 71 617 | 28 383 | 94 799 | 31 |
| 30 | 66 441 | 71 648 | 28 352 | 94 793 | 30 |
| 31 | 66 465 | 71 679 | 28 321 | 94 786 | 29 |
| 32 | 66 489 | 71 709 | 28 291 | 94 780 | 28 |
| 33 | 66 513 | 71 740 | 28 260 | 94 773 | 27 |
| 34 | 66 537 | 71 771 | 28 229 | 94 767 | 26 |
| 35 | 66 562 | 71 802 | 28 198 | 94 760 | 25 |
| 36 | 66 586 | 71 833 | 28 167 | 94 753 | 24 |
| 37 | 66 610 | 71 863 | 28 137 | 94 747 | 23 |
| 38 | 66 634 | 71 894 | 28 106 | 94 740 | 22 |
| 39 | 66 658 | 71 925 | 28 075 | 94 734 | 21 |
| 40 | 66 682 | 71 955 | 28 045 | 94 727 | 20 |
| 41 | 66 706 | 71 986 | 28 014 | 94 720 | 19 |
| 42 | 66 731 | 72 017 | 27 983 | 94 714 | 18 |
| 43 | 66 755 | 72 048 | 27 952 | 94 707 | 17 |
| 44 | 66 779 | 72 078 | 27 922 | 94 700 | 16 |
| 45 | 66 803 | 72 109 | 27 891 | 94 694 | 15 |
| 46 | 66 827 | 72 140 | 27 860 | 94 687 | 14 |
| 47 | 66 851 | 72 170 | 27 830 | 94 680 | 13 |
| 48 | 66 875 | 72 201 | 27 799 | 94 674 | 12 |
| 49 | 66 899 | 72 231 | 27 769 | 94 667 | 11 |
| 50 | 66 922 | 72 262 | 27 738 | 94 660 | 10 |
| 51 | 66 946 | 72 293 | 27 707 | 94 654 | 9 |
| 52 | 66 970 | 72 323 | 27 677 | 94 647 | 8 |
| 53 | 66 994 | 72 354 | 27 646 | 94 640 | 7 |
| 54 | 67 018 | 72 384 | 27 616 | 94 634 | 6 |
| 55 | 67 042 | 72 415 | 27 585 | 94 627 | 5 |
| 56 | 67 066 | 72 445 | 27 555 | 94 620 | 4 |
| 57 | 67 090 | 72 476 | 27 524 | 94 614 | 3 |
| 58 | 67 113 | 72 506 | 27 494 | 94 607 | 2 |
| 59 | 67 137 | 72 537 | 27 463 | 94 600 | 1 |
| 60 | 67 161 | 72 567 | 27 433 | 94 593 | 0 |
| ' | log cos | log cot | log tan | log sin | ' |

62°

28°

41

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 67 161 | 72 567 | 27 433 | 94 593 | 60 |
| 1 | 67 185 | 72 598 | 27 402 | 94 587 | 59 |
| 2 | 67 208 | 72 628 | 27 372 | 94 580 | 58 |
| 3 | 67 232 | 72 659 | 27 341 | 94 573 | 57 |
| 4 | 67 256 | 72 689 | 27 311 | 94 567 | 56 |
| 5 | 67 280 | 72 720 | 27 280 | 94 560 | 55 |
| 6 | 67 303 | 72 750 | 27 250 | 94 553 | 54 |
| 7 | 67 327 | 72 780 | 27 220 | 94 546 | 53 |
| 8 | 67 350 | 72 811 | 27 189 | 94 540 | 52 |
| 9 | 67 374 | 72 841 | 27 159 | 94 533 | 51 |
| 10 | 67 398 | 72 872 | 27 128 | 94 526 | 50 |
| 11 | 67 421 | 72 902 | 27 098 | 94 519 | 49 |
| 12 | 67 445 | 72 932 | 27 068 | 94 513 | 48 |
| 13 | 67 468 | 72 963 | 27 037 | 94 506 | 47 |
| 14 | 67 492 | 72 993 | 27 007 | 94 499 | 46 |
| 15 | 67 515 | 73 023 | 26 977 | 94 492 | 45 |
| 16 | 67 539 | 73 054 | 26 946 | 94 485 | 44 |
| 17 | 67 562 | 73 084 | 26 916 | 94 479 | 43 |
| 18 | 67 586 | 73 114 | 26 886 | 94 472 | 42 |
| 19 | 67 609 | 73 144 | 26 856 | 94 465 | 41 |
| 20 | 67 633 | 73 175 | 26 825 | 94 458 | 40 |
| 21 | 67 656 | 73 205 | 26 795 | 94 451 | 39 |
| 22 | 67 680 | 73 235 | 26 765 | 94 445 | 38 |
| 23 | 67 703 | 73 265 | 26 735 | 94 438 | 37 |
| 24 | 67 726 | 73 295 | 26 705 | 94 431 | 36 |
| 25 | 67 750 | 73 326 | 26 674 | 94 424 | 35 |
| 26 | 67 773 | 73 356 | 26 644 | 94 417 | 34 |
| 27 | 67 796 | 73 386 | 26 614 | 94 410 | 33 |
| 28 | 67 820 | 73 416 | 26 584 | 94 404 | 32 |
| 29 | 67 843 | 73 446 | 26 554 | 94 397 | 31 |
| 30 | 67 866 | 73 476 | 26 524 | 94 390 | 30 |
| 31 | 67 890 | 73 507 | 26 493 | 94 383 | 29 |
| 32 | 67 913 | 73 537 | 26 463 | 94 376 | 28 |
| 33 | 67 936 | 73 567 | 26 433 | 94 369 | 27 |
| 34 | 67 959 | 73 597 | 26 403 | 94 362 | 26 |
| 35 | 67 982 | 73 627 | 26 373 | 94 355 | 25 |
| 36 | 68 006 | 73 657 | 26 343 | 94 349 | 24 |
| 37 | 68 029 | 73 687 | 26 313 | 94 342 | 23 |
| 38 | 68 052 | 73 717 | 26 283 | 94 335 | 22 |
| 39 | 68 075 | 73 747 | 26 253 | 94 328 | 21 |
| 40 | 68 098 | 73 777 | 26 223 | 94 321 | 20 |
| 41 | 68 121 | 73 807 | 26 193 | 94 314 | 19 |
| 42 | 68 144 | 73 837 | 26 163 | 94 307 | 18 |
| 43 | 68 167 | 73 867 | 26 133 | 94 300 | 17 |
| 44 | 68 190 | 73 897 | 26 103 | 94 293 | 16 |
| 45 | 68 213 | 73 927 | 26 073 | 94 286 | 15 |
| 46 | 68 237 | 73 957 | 26 043 | 94 279 | 14 |
| 47 | 68 260 | 73 987 | 26 013 | 94 273 | 13 |
| 48 | 68 283 | 74 017 | 25 983 | 94 266 | 12 |
| 49 | 68 305 | 74 047 | 25 953 | 94 259 | 11 |
| 50 | 68 328 | 74 077 | 25 923 | 94 252 | 10 |
| 51 | 68 351 | 74 107 | 25 893 | 94 245 | 9 |
| 52 | 68 374 | 74 137 | 25 863 | 94 238 | 8 |
| 53 | 68 397 | 74 166 | 25 834 | 94 231 | 7 |
| 54 | 68 420 | 74 196 | 25 804 | 94 224 | 6 |
| 55 | 68 443 | 74 226 | 25 774 | 94 217 | 5 |
| 56 | 68 466 | 74 256 | 25 744 | 94 210 | 4 |
| 57 | 68 489 | 74 286 | 25 714 | 94 203 | 3 |
| 58 | 68 512 | 74 316 | 25 684 | 94 196 | 2 |
| 59 | 68 534 | 74 345 | 25 655 | 94 189 | 1 |
| 60 | 68 557 | 74 375 | 25 625 | 94 182 | 0 |
| ' | log cos | log cot | log tan | log sin | ' |

61°

| ' | log sin 0 | log tan 0 | log cot 1 | log cos | ' |
|----|--------------|--------------|--------------|---------|----|
| 0 | 19 433 | 19 971 | 80 029 | 99 462 | 60 |
| 1 | 19 513 | 20 053 | 79 947 | 99 460 | 59 |
| 2 | 19 592 | 20 134 | 79 866 | 99 458 | 58 |
| 3 | 19 672 | 20 216 | 79 784 | 99 456 | 57 |
| 4 | 19 751 | 20 297 | 79 703 | 99 454 | 56 |
| 5 | 19 830 | 20 378 | 79 622 | 99 452 | 55 |
| 6 | 19 909 | 20 459 | 79 541 | 99 450 | 54 |
| 7 | 19 988 | 20 540 | 79 460 | 99 448 | 53 |
| 8 | 20 067 | 20 621 | 79 379 | 99 446 | 52 |
| 9 | 20 145 | 20 701 | 79 299 | 99 444 | 51 |
| 10 | 20 223 | 20 782 | 79 218 | 99 442 | 50 |
| 11 | 20 302 | 20 862 | 79 138 | 99 440 | 49 |
| 12 | 20 380 | 20 942 | 79 058 | 99 438 | 48 |
| 13 | 20 458 | 21 022 | 78 978 | 99 436 | 47 |
| 14 | 20 535 | 21 102 | 78 898 | 99 434 | 46 |
| 15 | 20 613 | 21 182 | 78 818 | 99 432 | 45 |
| 16 | 20 691 | 21 261 | 78 739 | 99 429 | 44 |
| 17 | 20 768 | 21 341 | 78 659 | 99 427 | 43 |
| 18 | 20 845 | 21 420 | 78 580 | 99 425 | 42 |
| 19 | 20 922 | 21 499 | 78 501 | 99 423 | 41 |
| 20 | 20 999 | 21 578 | 78 422 | 99 421 | 40 |
| 21 | 21 076 | 21 657 | 78 343 | 99 419 | 39 |
| 22 | 21 153 | 21 736 | 78 264 | 99 417 | 38 |
| 23 | 21 229 | 21 814 | 78 186 | 99 415 | 37 |
| 24 | 21 306 | 21 893 | 78 107 | 99 413 | 36 |
| 25 | 21 382 | 21 971 | 78 029 | 99 411 | 35 |
| 26 | 21 458 | 22 049 | 77 951 | 99 409 | 34 |
| 27 | 21 534 | 22 127 | 77 873 | 99 407 | 33 |
| 28 | 21 610 | 22 205 | 77 795 | 99 404 | 32 |
| 29 | 21 685 | 22 283 | 77 717 | 99 402 | 31 |
| 30 | 21 761 | 22 361 | 77 639 | 99 400 | 30 |
| 31 | 21 836 | 22 438 | 77 562 | 99 398 | 29 |
| 32 | 21 912 | 22 516 | 77 484 | 99 396 | 28 |
| 33 | 21 987 | 22 593 | 77 407 | 99 394 | 27 |
| 34 | 22 062 | 22 670 | 77 330 | 99 392 | 26 |
| 35 | 22 137 | 22 747 | 77 253 | 99 390 | 25 |
| 36 | 22 211 | 22 824 | 77 176 | 99 388 | 24 |
| 37 | 22 286 | 22 901 | 77 099 | 99 385 | 23 |
| 38 | 22 361 | 22 977 | 77 023 | 99 383 | 22 |
| 39 | 22 435 | 23 054 | 76 946 | 99 381 | 21 |
| 40 | 22 509 | 23 130 | 76 870 | 99 379 | 20 |
| 41 | 22 583 | 23 206 | 76 794 | 99 377 | 19 |
| 42 | 22 657 | 23 283 | 76 717 | 99 375 | 18 |
| 43 | 22 731 | 23 359 | 76 641 | 99 372 | 17 |
| 44 | 22 805 | 23 435 | 76 565 | 99 370 | 16 |
| 45 | 22 878 | 23 510 | 76 490 | 99 368 | 15 |
| 46 | 22 952 | 23 586 | 76 414 | 99 366 | 14 |
| 47 | 23 025 | 23 661 | 76 339 | 99 364 | 13 |
| 48 | 23 098 | 23 737 | 76 263 | 99 362 | 12 |
| 49 | 23 171 | 23 812 | 76 188 | 99 359 | 11 |
| 50 | 23 244 | 23 887 | 76 113 | 99 357 | 10 |
| 51 | 23 317 | 23 962 | 76 038 | 99 355 | 9 |
| 52 | 23 390 | 24 037 | 75 963 | 99 353 | 8 |
| 53 | 23 462 | 24 112 | 75 888 | 99 351 | 7 |
| 54 | 23 535 | 24 186 | 75 814 | 99 348 | 6 |
| 55 | 23 607 | 24 261 | 75 739 | 99 346 | 5 |
| 56 | 23 679 | 24 335 | 75 665 | 99 344 | 4 |
| 57 | 23 752 | 24 410 | 75 590 | 99 342 | 3 |
| 58 | 23 823 | 24 484 | 75 516 | 99 340 | 2 |
| 59 | 23 895 | 24 558 | 75 442 | 99 337 | 1 |
| 60 | 23 967 | 24 632 | 75 368 | 99 335 | 0 |
| ' | log cos | log cot | log tan | log sin | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 23 967 | 24 632 | 75 368 | 99 335 | 60 |
| 1 | 24 039 | 24 706 | 75 294 | 99 333 | 59 |
| 2 | 24 110 | 24 779 | 75 221 | 99 331 | 58 |
| 3 | 24 181 | 24 853 | 75 147 | 99 328 | 57 |
| 4 | 24 253 | 24 926 | 75 074 | 99 326 | 56 |
| 5 | 24 324 | 25 000 | 75 000 | 99 324 | 55 |
| 6 | 24 395 | 25 073 | 74 927 | 99 322 | 54 |
| 7 | 24 466 | 25 146 | 74 854 | 99 319 | 53 |
| 8 | 24 536 | 25 219 | 74 781 | 99 317 | 52 |
| 9 | 24 607 | 25 292 | 74 708 | 99 315 | 51 |
| 10 | 24 677 | 25 365 | 74 635 | 99 313 | 50 |
| 11 | 24 748 | 25 437 | 74 563 | 99 310 | 49 |
| 12 | 24 818 | 25 510 | 74 490 | 99 308 | 48 |
| 13 | 24 888 | 25 582 | 74 418 | 99 306 | 47 |
| 14 | 24 958 | 25 655 | 74 345 | 99 304 | 46 |
| 15 | 25 028 | 25 727 | 74 273 | 99 301 | 45 |
| 16 | 25 098 | 25 799 | 74 201 | 99 299 | 44 |
| 17 | 25 168 | 25 871 | 74 129 | 99 297 | 43 |
| 18 | 25 237 | 25 943 | 74 057 | 99 294 | 42 |
| 19 | 25 307 | 26 015 | 73 985 | 99 292 | 41 |
| 20 | 25 376 | 26 086 | 73 914 | 99 290 | 40 |
| 21 | 25 445 | 26 158 | 73 842 | 99 288 | 39 |
| 22 | 25 514 | 26 229 | 73 771 | 99 285 | 38 |
| 23 | 25 583 | 26 301 | 73 699 | 99 283 | 37 |
| 24 | 25 652 | 26 372 | 73 628 | 99 281 | 36 |
| 25 | 25 721 | 26 443 | 73 557 | 99 278 | 35 |
| 26 | 25 790 | 26 514 | 73 486 | 99 276 | 34 |
| 27 | 25 858 | 26 585 | 73 415 | 99 274 | 33 |
| 28 | 25 927 | 26 655 | 73 345 | 99 271 | 32 |
| 29 | 25 995 | 26 726 | 73 274 | 99 269 | 31 |
| 30 | 26 063 | 26 797 | 73 203 | 99 267 | 30 |
| 31 | 26 131 | 26 867 | 73 133 | 99 264 | 29 |
| 32 | 26 199 | 26 937 | 73 063 | 99 262 | 28 |
| 33 | 26 267 | 27 008 | 72 992 | 99 260 | 27 |
| 34 | 26 335 | 27 078 | 72 922 | 99 257 | 26 |
| 35 | 26 403 | 27 148 | 72 852 | 99 255 | 25 |
| 36 | 26 470 | 27 218 | 72 782 | 99 252 | 24 |
| 37 | 26 538 | 27 288 | 72 712 | 99 250 | 23 |
| 38 | 26 605 | 27 357 | 72 643 | 99 248 | 22 |
| 39 | 26 672 | 27 427 | 72 573 | 99 245 | 21 |
| 40 | 26 739 | 27 496 | 72 504 | 99 243 | 20 |
| 41 | 26 806 | 27 566 | 72 434 | 99 241 | 19 |
| 42 | 26 873 | 27 635 | 72 365 | 99 238 | 18 |
| 43 | 26 940 | 27 704 | 72 296 | 99 236 | 17 |
| 44 | 27 007 | 27 773 | 72 227 | 99 233 | 16 |
| 45 | 27 073 | 27 842 | 72 158 | 99 231 | 15 |
| 46 | 27 140 | 27 911 | 72 089 | 99 229 | 14 |
| 47 | 27 206 | 27 980 | 72 020 | 99 226 | 13 |
| 48 | 27 273 | 28 049 | 71 951 | 99 224 | 12 |
| 49 | 27 339 | 28 117 | 71 883 | 99 221 | 11 |
| 50 | 27 405 | 28 186 | 71 814 | 99 219 | 10 |
| 51 | 27 471 | 28 254 | 71 746 | 99 217 | 9 |
| 52 | 27 537 | 28 323 | 71 677 | 99 214 | 8 |
| 53 | 27 602 | 28 391 | 71 609 | 99 212 | 7 |
| 54 | 27 668 | 28 459 | 71 541 | 99 209 | 6 |
| 55 | 27 734 | 28 527 | 71 473 | 99 207 | 5 |
| 56 | 27 799 | 28 595 | 71 405 | 99 204 | 4 |
| 57 | 27 864 | 28 662 | 71 338 | 99 202 | 3 |
| 58 | 27 930 | 28 730 | 71 270 | 99 200 | 2 |
| 59 | 27 995 | 28 798 | 71 202 | 99 197 | 1 |
| 60 | 28 060 | 28 865 | 71 135 | 99 195 | 0 |
| ' | log cos | log cot | log tan | log sin | ' |

11°

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 28 060 | 28 865 | 71 135 | 99 195 | 60 |
| 1 | 28 125 | 28 933 | 71 067 | 99 192 | 59 |
| 2 | 28 190 | 29 000 | 71 000 | 99 190 | 58 |
| 3 | 28 254 | 29 067 | 70 933 | 99 187 | 57 |
| 4 | 28 319 | 29 134 | 70 866 | 99 185 | 56 |
| 5 | 28 384 | 29 201 | 70 799 | 99 182 | 55 |
| 6 | 28 448 | 29 268 | 70 732 | 99 180 | 54 |
| 7 | 28 512 | 29 335 | 70 665 | 99 177 | 53 |
| 8 | 28 577 | 29 402 | 70 598 | 99 175 | 52 |
| 9 | 28 641 | 29 468 | 70 532 | 99 172 | 51 |
| 10 | 28 705 | 29 535 | 70 465 | 99 170 | 50 |
| 11 | 28 769 | 29 601 | 70 399 | 99 167 | 49 |
| 12 | 28 833 | 29 668 | 70 332 | 99 165 | 48 |
| 13 | 28 896 | 29 734 | 70 266 | 99 162 | 47 |
| 14 | 28 960 | 29 800 | 70 200 | 99 160 | 46 |
| 15 | 29 024 | 29 866 | 70 134 | 99 157 | 45 |
| 16 | 29 087 | 29 932 | 70 068 | 99 155 | 44 |
| 17 | 29 150 | 29 998 | 70 002 | 99 152 | 43 |
| 18 | 29 214 | 30 064 | 69 936 | 99 150 | 42 |
| 19 | 29 277 | 30 130 | 69 870 | 99 147 | 41 |
| 20 | 29 340 | 30 195 | 69 805 | 99 145 | 40 |
| 21 | 29 403 | 30 261 | 69 739 | 99 142 | 39 |
| 22 | 29 466 | 30 326 | 69 674 | 99 140 | 38 |
| 23 | 29 529 | 30 391 | 69 609 | 99 137 | 37 |
| 24 | 29 591 | 30 457 | 69 543 | 99 135 | 36 |
| 25 | 29 654 | 30 522 | 69 478 | 99 132 | 35 |
| 26 | 29 716 | 30 587 | 69 413 | 99 130 | 34 |
| 27 | 29 779 | 30 652 | 69 348 | 99 127 | 33 |
| 28 | 29 841 | 30 717 | 69 283 | 99 124 | 32 |
| 29 | 29 903 | 30 782 | 69 218 | 99 122 | 31 |
| 30 | 29 966 | 30 846 | 69 154 | 99 119 | 30 |
| 31 | 30 028 | 30 911 | 69 089 | 99 117 | 29 |
| 32 | 30 090 | 30 975 | 69 025 | 99 114 | 28 |
| 33 | 30 151 | 31 040 | 68 960 | 99 112 | 27 |
| 34 | 30 213 | 31 104 | 68 896 | 99 109 | 26 |
| 35 | 30 275 | 31 168 | 68 832 | 99 106 | 25 |
| 36 | 30 336 | 31 233 | 68 767 | 99 104 | 24 |
| 37 | 30 398 | 31 297 | 68 703 | 99 101 | 23 |
| 38 | 30 459 | 31 361 | 68 639 | 99 099 | 22 |
| 39 | 30 521 | 31 425 | 68 575 | 99 096 | 21 |
| 40 | 30 582 | 31 489 | 68 511 | 99 093 | 20 |
| 41 | 30 643 | 31 552 | 68 448 | 99 091 | 19 |
| 42 | 30 704 | 31 616 | 68 384 | 99 088 | 18 |
| 43 | 30 765 | 31 679 | 68 321 | 99 086 | 17 |
| 44 | 30 826 | 31 743 | 68 257 | 99 083 | 16 |
| 45 | 30 887 | 31 806 | 68 194 | 99 080 | 15 |
| 46 | 30 947 | 31 870 | 68 130 | 99 078 | 14 |
| 47 | 31 008 | 31 933 | 68 067 | 99 075 | 13 |
| 48 | 31 068 | 31 996 | 68 004 | 99 072 | 12 |
| 49 | 31 129 | 32 059 | 67 941 | 99 070 | 11 |
| 50 | 31 189 | 32 122 | 67 878 | 99 067 | 10 |
| 51 | 31 250 | 32 185 | 67 815 | 99 064 | 9 |
| 52 | 31 310 | 32 248 | 67 752 | 99 062 | 8 |
| 53 | 31 370 | 32 311 | 67 689 | 99 059 | 7 |
| 54 | 31 430 | 32 373 | 67 627 | 99 056 | 6 |
| 55 | 31 490 | 32 436 | 67 564 | 99 054 | 5 |
| 56 | 31 549 | 32 498 | 67 502 | 99 051 | 4 |
| 57 | 31 609 | 32 561 | 67 439 | 99 048 | 3 |
| 58 | 31 669 | 32 623 | 67 377 | 99 046 | 2 |
| 59 | 31 728 | 32 685 | 67 315 | 99 043 | 1 |
| 60 | 31 788 | 32 747 | 67 253 | 99 040 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

78°

12°

33

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 31 788 | 32 747 | 67 253 | 99 040 | 60 |
| 1 | 31 847 | 32 810 | 67 190 | 99 038 | 59 |
| 2 | 31 907 | 32 872 | 67 128 | 99 035 | 58 |
| 3 | 31 966 | 32 933 | 67 067 | 99 032 | 57 |
| 4 | 32 025 | 32 995 | 67 005 | 99 030 | 56 |
| 5 | 32 084 | 33 057 | 66 943 | 99 027 | 55 |
| 6 | 32 143 | 33 119 | 66 881 | 99 024 | 54 |
| 7 | 32 202 | 33 180 | 66 820 | 99 022 | 53 |
| 8 | 32 261 | 33 242 | 66 758 | 99 019 | 52 |
| 9 | 32 319 | 33 303 | 66 697 | 99 016 | 51 |
| 10 | 32 378 | 33 365 | 66 635 | 99 013 | 50 |
| 11 | 32 437 | 33 426 | 66 574 | 99 011 | 49 |
| 12 | 32 495 | 33 487 | 66 513 | 99 008 | 48 |
| 13 | 32 553 | 33 548 | 66 452 | 99 005 | 47 |
| 14 | 32 612 | 33 609 | 66 391 | 99 002 | 46 |
| 15 | 32 670 | 33 670 | 66 330 | 99 000 | 45 |
| 16 | 32 728 | 33 731 | 66 269 | 98 997 | 44 |
| 17 | 32 786 | 33 792 | 66 208 | 98 994 | 43 |
| 18 | 32 844 | 33 853 | 66 147 | 98 991 | 42 |
| 19 | 32 902 | 33 913 | 66 087 | 98 989 | 41 |
| 20 | 32 960 | 33 974 | 66 026 | 98 986 | 40 |
| 21 | 33 018 | 34 034 | 65 966 | 98 983 | 39 |
| 22 | 33 075 | 34 095 | 65 905 | 98 980 | 38 |
| 23 | 33 133 | 34 155 | 65 845 | 98 978 | 37 |
| 24 | 33 190 | 34 215 | 65 785 | 98 975 | 36 |
| 25 | 33 248 | 34 276 | 65 724 | 98 972 | 35 |
| 26 | 33 305 | 34 336 | 65 664 | 98 969 | 34 |
| 27 | 33 362 | 34 396 | 65 604 | 98 967 | 33 |
| 28 | 33 420 | 34 456 | 65 544 | 98 964 | 32 |
| 29 | 33 477 | 34 516 | 65 484 | 98 961 | 31 |
| 30 | 33 534 | 34 576 | 65 424 | 98 958 | 30 |
| 31 | 33 591 | 34 635 | 65 365 | 98 955 | 29 |
| 32 | 33 647 | 34 695 | 65 305 | 98 953 | 28 |
| 33 | 33 704 | 34 755 | 65 245 | 98 950 | 27 |
| 34 | 33 761 | 34 814 | 65 186 | 98 947 | 26 |
| 35 | 33 818 | 34 874 | 65 126 | 98 944 | 25 |
| 36 | 33 874 | 34 933 | 65 067 | 98 941 | 24 |
| 37 | 33 931 | 34 992 | 65 008 | 98 938 | 23 |
| 38 | 33 987 | 35 051 | 64 949 | 98 936 | 22 |
| 39 | 34 043 | 35 111 | 64 889 | 98 933 | 21 |
| 40 | 34 100 | 35 170 | 64 830 | 98 930 | 20 |
| 41 | 34 156 | 35 229 | 64 771 | 98 927 | 19 |
| 42 | 34 212 | 35 288 | 64 712 | 98 924 | 18 |
| 43 | 34 268 | 35 347 | 64 653 | 98 921 | 17 |
| 44 | 34 324 | 35 405 | 64 595 | 98 919 | 16 |
| 45 | 34 380 | 35 464 | 64 536 | 98 916 | 15 |
| 46 | 34 436 | 35 523 | 64 477 | 98 913 | 14 |
| 47 | 34 491 | 35 581 | 64 419 | 98 910 | 13 |
| 48 | 34 547 | 35 640 | 64 360 | 98 907 | 12 |
| 49 | 34 602 | 35 698 | 64 302 | 98 904 | 11 |
| 50 | 34 658 | 35 757 | 64 243 | 98 901 | 10 |
| 51 | 34 713 | 35 815 | 64 185 | 98 898 | 9 |
| 52 | 34 769 | 35 873 | 64 127 | 98 896 | 8 |
| 53 | 34 824 | 35 931 | 64 069 | 98 893 | 7 |
| 54 | 34 879 | 35 989 | 64 011 | 98 890 | 6 |
| 55 | 34 934 | 36 047 | 63 953 | 98 887 | 5 |
| 56 | 34 989 | 36 105 | 63 895 | 98 884 | 4 |
| 57 | 35 044 | 36 163 | 63 837 | 98 881 | 3 |
| 58 | 35 099 | 36 221 | 63 779 | 98 878 | 2 |
| 59 | 35 154 | 36 279 | 63 721 | 98 875 | 1 |
| 60 | 35 209 | 36 336 | 63 664 | 98 872 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

77°

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 35 209 | 36 336 | 63 664 | 98 872 | 60 |
| 1 | 35 263 | 36 394 | 63 606 | 98 869 | 59 |
| 2 | 35 318 | 36 452 | 63 548 | 98 867 | 58 |
| 3 | 35 373 | 36 509 | 63 491 | 98 864 | 57 |
| 4 | 35 427 | 36 566 | 63 434 | 98 861 | 56 |
| 5 | 35 481 | 36 624 | 63 376 | 98 858 | 55 |
| 6 | 35 536 | 36 681 | 63 319 | 98 855 | 54 |
| 7 | 35 590 | 36 738 | 63 262 | 98 852 | 53 |
| 8 | 35 644 | 36 795 | 63 205 | 98 849 | 52 |
| 9 | 35 698 | 36 852 | 63 148 | 98 846 | 51 |
| 10 | 35 752 | 36 909 | 63 091 | 98 843 | 50 |
| 11 | 35 806 | 36 966 | 63 034 | 98 840 | 49 |
| 12 | 35 860 | 37 023 | 62 977 | 98 837 | 48 |
| 13 | 35 914 | 37 080 | 62 920 | 98 834 | 47 |
| 14 | 35 968 | 37 137 | 62 863 | 98 831 | 46 |
| 15 | 36 022 | 37 193 | 62 807 | 98 828 | 45 |
| 16 | 36 075 | 37 250 | 62 750 | 98 825 | 44 |
| 17 | 36 129 | 37 306 | 62 694 | 98 822 | 43 |
| 18 | 36 182 | 37 363 | 62 637 | 98 819 | 42 |
| 19 | 36 236 | 37 419 | 62 581 | 98 816 | 41 |
| 20 | 36 289 | 37 476 | 62 524 | 98 813 | 40 |
| 21 | 36 342 | 37 532 | 62 468 | 98 810 | 39 |
| 22 | 36 395 | 37 588 | 62 412 | 98 807 | 38 |
| 23 | 36 449 | 37 644 | 62 356 | 98 804 | 37 |
| 24 | 36 502 | 37 700 | 62 300 | 98 801 | 36 |
| 25 | 36 555 | 37 756 | 62 244 | 98 798 | 35 |
| 26 | 36 608 | 37 812 | 62 188 | 98 795 | 34 |
| 27 | 36 660 | 37 868 | 62 132 | 98 792 | 33 |
| 28 | 36 713 | 37 924 | 62 076 | 98 789 | 32 |
| 29 | 36 766 | 37 980 | 62 020 | 98 786 | 31 |
| 30 | 36 819 | 38 035 | 61 965 | 98 783 | 30 |
| 31 | 36 871 | 38 091 | 61 909 | 98 780 | 29 |
| 32 | 36 924 | 38 147 | 61 853 | 98 777 | 28 |
| 33 | 36 976 | 38 202 | 61 798 | 98 774 | 27 |
| 34 | 37 028 | 38 257 | 61 743 | 98 771 | 26 |
| 35 | 37 081 | 38 313 | 61 687 | 98 768 | 25 |
| 36 | 37 133 | 38 368 | 61 632 | 98 765 | 24 |
| 37 | 37 185 | 38 423 | 61 577 | 98 762 | 23 |
| 38 | 37 237 | 38 479 | 61 521 | 98 759 | 22 |
| 39 | 37 289 | 38 534 | 61 466 | 98 756 | 21 |
| 40 | 37 341 | 38 589 | 61 411 | 98 753 | 20 |
| 41 | 37 393 | 38 644 | 61 356 | 98 750 | 19 |
| 42 | 37 445 | 38 699 | 61 301 | 98 746 | 18 |
| 43 | 37 497 | 38 754 | 61 246 | 98 743 | 17 |
| 44 | 37 549 | 38 808 | 61 192 | 98 740 | 16 |
| 45 | 37 600 | 38 863 | 61 137 | 98 737 | 15 |
| 46 | 37 652 | 38 918 | 61 082 | 98 734 | 14 |
| 47 | 37 703 | 38 972 | 61 028 | 98 731 | 13 |
| 48 | 37 755 | 39 027 | 60 973 | 98 728 | 12 |
| 49 | 37 806 | 39 082 | 60 918 | 98 725 | 11 |
| 50 | 37 858 | 39 136 | 60 864 | 98 722 | 10 |
| 51 | 37 909 | 39 190 | 60 810 | 98 719 | 9 |
| 52 | 37 960 | 39 245 | 60 755 | 98 715 | 8 |
| 53 | 38 011 | 39 299 | 60 701 | 98 712 | 7 |
| 54 | 38 062 | 39 353 | 60 647 | 98 709 | 6 |
| 55 | 38 113 | 39 407 | 60 593 | 98 706 | 5 |
| 56 | 38 164 | 39 461 | 60 539 | 98 703 | 4 |
| 57 | 38 215 | 39 515 | 60 485 | 98 700 | 3 |
| 58 | 38 266 | 39 569 | 60 431 | 98 697 | 2 |
| 59 | 38 317 | 39 623 | 60 377 | 98 694 | 1 |
| 60 | 38 368 | 39 677 | 60 323 | 98 690 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 0 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 38 368 | 39 677 | 60 323 | 98 690 | 60 |
| 1 | 38 418 | 39 731 | 60 269 | 98 687 | 59 |
| 2 | 38 469 | 39 785 | 60 215 | 98 684 | 58 |
| 3 | 38 519 | 39 838 | 60 162 | 98 681 | 57 |
| 4 | 38 570 | 39 892 | 60 108 | 98 678 | 56 |
| 5 | 38 620 | 39 945 | 60 055 | 98 675 | 55 |
| 6 | 38 670 | 39 999 | 60 001 | 98 671 | 54 |
| 7 | 38 721 | 40 052 | 59 948 | 98 668 | 53 |
| 8 | 38 771 | 40 106 | 59 894 | 98 665 | 52 |
| 9 | 38 821 | 40 159 | 59 841 | 98 662 | 51 |
| 10 | 38 871 | 40 212 | 59 788 | 98 659 | 50 |
| 11 | 38 921 | 40 266 | 59 734 | 98 656 | 49 |
| 12 | 38 971 | 40 319 | 59 681 | 98 652 | 48 |
| 13 | 39 021 | 40 372 | 59 628 | 98 649 | 47 |
| 14 | 39 071 | 40 425 | 59 575 | 98 646 | 46 |
| 15 | 39 121 | 40 478 | 59 522 | 98 643 | 45 |
| 16 | 39 170 | 40 531 | 59 469 | 98 640 | 44 |
| 17 | 39 220 | 40 584 | 59 416 | 98 636 | 43 |
| 18 | 39 270 | 40 636 | 59 364 | 98 633 | 42 |
| 19 | 39 319 | 40 689 | 59 311 | 98 630 | 41 |
| 20 | 39 369 | 40 742 | 59 258 | 98 627 | 40 |
| 21 | 39 418 | 40 795 | 59 205 | 98 623 | 39 |
| 22 | 39 467 | 40 847 | 59 153 | 98 620 | 38 |
| 23 | 39 517 | 40 900 | 59 100 | 98 617 | 37 |
| 24 | 39 566 | 40 952 | 59 048 | 98 614 | 36 |
| 25 | 39 615 | 41 005 | 58 995 | 98 610 | 35 |
| 26 | 39 664 | 41 057 | 58 943 | 98 607 | 34 |
| 27 | 39 713 | 41 109 | 58 891 | 98 604 | 33 |
| 28 | 39 762 | 41 161 | 58 839 | 98 601 | 32 |
| 29 | 39 811 | 41 214 | 58 786 | 98 597 | 31 |
| 30 | 39 860 | 41 266 | 58 734 | 98 594 | 30 |
| 31 | 39 909 | 41 318 | 58 682 | 98 591 | 29 |
| 32 | 39 958 | 41 370 | 58 630 | 98 588 | 28 |
| 33 | 40 006 | 41 422 | 58 578 | 98 584 | 27 |
| 34 | 40 055 | 41 474 | 58 526 | 98 581 | 26 |
| 35 | 40 103 | 41 526 | 58 474 | 98 578 | 25 |
| 36 | 40 152 | 41 578 | 58 422 | 98 574 | 24 |
| 37 | 40 200 | 41 629 | 58 371 | 98 571 | 23 |
| 38 | 40 249 | 41 681 | 58 319 | 98 568 | 22 |
| 39 | 40 297 | 41 733 | 58 267 | 98 565 | 21 |
| 40 | 40 346 | 41 784 | 58 216 | 98 561 | 20 |
| 41 | 40 394 | 41 836 | 58 164 | 98 558 | 19 |
| 42 | 40 442 | 41 887 | 58 113 | 98 555 | 18 |
| 43 | 40 490 | 41 939 | 58 061 | 98 551 | 17 |
| 44 | 40 538 | 41 990 | 58 010 | 98 548 | 16 |
| 45 | 40 586 | 42 041 | 57 959 | 98 545 | 15 |
| 46 | 40 634 | 42 093 | 57 907 | 98 541 | 14 |
| 47 | 40 682 | 42 144 | 57 856 | 98 538 | 13 |
| 48 | 40 730 | 42 195 | 57 805 | 98 535 | 12 |
| 49 | 40 778 | 42 246 | 57 754 | 98 531 | 11 |
| 50 | 40 825 | 42 297 | 57 703 | 98 528 | 10 |
| 51 | 40 873 | 42 348 | 57 652 | 98 525 | 9 |
| 52 | 40 921 | 42 399 | 57 601 | 98 521 | 8 |
| 53 | 40 968 | 42 450 | 57 550 | 98 518 | 7 |
| 54 | 41 016 | 42 501 | 57 499 | 98 515 | 6 |
| 55 | 41 063 | 42 552 | 57 448 | 98 511 | 5 |
| 56 | 41 111 | 42 603 | 57 397 | 98 508 | 4 |
| 57 | 41 158 | 42 653 | 57 347 | 98 505 | 3 |
| 58 | 41 205 | 42 704 | 57 296 | 98 501 | 2 |
| 59 | 41 252 | 42 755 | 57 245 | 98 498 | 1 |
| 60 | 41 300 | 42 805 | 57 195 | 98 494 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 0 | ' |

15°

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 41 300 | 42 805 | 57 195 | 98 494 | 60 |
| 1 | 41 347 | 42 856 | 57 144 | 98 491 | 59 |
| 2 | 41 394 | 42 906 | 57 094 | 98 488 | 58 |
| 3 | 41 441 | 42 957 | 57 043 | 98 484 | 57 |
| 4 | 41 488 | 43 007 | 56 993 | 98 481 | 56 |
| 5 | 41 535 | 43 057 | 56 943 | 98 477 | 55 |
| 6 | 41 582 | 43 108 | 56 892 | 98 474 | 54 |
| 7 | 41 628 | 43 158 | 56 842 | 98 471 | 53 |
| 8 | 41 675 | 43 208 | 56 792 | 98 467 | 52 |
| 9 | 41 722 | 43 258 | 56 742 | 98 464 | 51 |
| 10 | 41 768 | 43 308 | 56 692 | 98 460 | 50 |
| 11 | 41 815 | 43 358 | 56 642 | 98 457 | 49 |
| 12 | 41 861 | 43 408 | 56 592 | 98 453 | 48 |
| 13 | 41 908 | 43 458 | 56 542 | 98 450 | 47 |
| 14 | 41 954 | 43 508 | 56 492 | 98 447 | 46 |
| 15 | 42 001 | 43 558 | 56 442 | 98 443 | 45 |
| 16 | 42 047 | 43 607 | 56 393 | 98 440 | 44 |
| 17 | 42 093 | 43 657 | 56 343 | 98 436 | 43 |
| 18 | 42 140 | 43 707 | 56 293 | 98 433 | 42 |
| 19 | 42 186 | 43 756 | 56 244 | 98 429 | 41 |
| 20 | 42 232 | 43 806 | 56 194 | 98 426 | 40 |
| 21 | 42 278 | 43 855 | 56 145 | 98 422 | 39 |
| 22 | 42 324 | 43 905 | 56 095 | 98 419 | 38 |
| 23 | 42 370 | 43 954 | 56 046 | 98 415 | 37 |
| 24 | 42 416 | 44 004 | 55 996 | 98 412 | 36 |
| 25 | 42 461 | 44 053 | 55 947 | 98 409 | 35 |
| 26 | 42 507 | 44 102 | 55 898 | 98 405 | 34 |
| 27 | 42 553 | 44 151 | 55 849 | 98 402 | 33 |
| 28 | 42 599 | 44 201 | 55 799 | 98 398 | 32 |
| 29 | 42 644 | 44 250 | 55 750 | 98 395 | 31 |
| 30 | 42 690 | 44 299 | 55 701 | 98 391 | 30 |
| 31 | 42 735 | 44 348 | 55 652 | 98 388 | 29 |
| 32 | 42 781 | 44 397 | 55 603 | 98 384 | 28 |
| 33 | 42 826 | 44 446 | 55 554 | 98 381 | 27 |
| 34 | 42 872 | 44 495 | 55 505 | 98 377 | 26 |
| 35 | 42 917 | 44 544 | 55 456 | 98 373 | 25 |
| 36 | 42 962 | 44 592 | 55 408 | 98 370 | 24 |
| 37 | 43 008 | 44 641 | 55 359 | 98 366 | 23 |
| 38 | 43 053 | 44 690 | 55 310 | 98 363 | 22 |
| 39 | 43 098 | 44 738 | 55 262 | 98 359 | 21 |
| 40 | 43 143 | 44 787 | 55 213 | 98 356 | 20 |
| 41 | 43 188 | 44 836 | 55 164 | 98 352 | 19 |
| 42 | 43 233 | 44 884 | 55 116 | 98 349 | 18 |
| 43 | 43 278 | 44 933 | 55 067 | 98 345 | 17 |
| 44 | 43 323 | 44 981 | 55 019 | 98 342 | 16 |
| 45 | 43 367 | 45 029 | 54 971 | 98 338 | 15 |
| 46 | 43 412 | 45 078 | 54 922 | 98 334 | 14 |
| 47 | 43 457 | 45 126 | 54 874 | 98 331 | 13 |
| 48 | 43 502 | 45 174 | 54 826 | 98 327 | 12 |
| 49 | 43 546 | 45 222 | 54 778 | 98 324 | 11 |
| 50 | 43 591 | 45 271 | 54 729 | 98 320 | 10 |
| 51 | 43 635 | 45 319 | 54 681 | 98 317 | 9 |
| 52 | 43 680 | 45 367 | 54 633 | 98 313 | 8 |
| 53 | 43 724 | 45 415 | 54 585 | 98 309 | 7 |
| 54 | 43 769 | 45 463 | 54 537 | 98 306 | 6 |
| 55 | 43 813 | 45 511 | 54 489 | 98 302 | 5 |
| 56 | 43 857 | 45 559 | 54 441 | 98 299 | 4 |
| 57 | 43 901 | 45 606 | 54 394 | 98 295 | 3 |
| 58 | 43 946 | 45 654 | 54 346 | 98 291 | 2 |
| 59 | 43 990 | 45 702 | 54 298 | 98 288 | 1 |
| 60 | 44 034 | 45 750 | 54 250 | 98 284 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

74°

16°

85

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 44 034 | 45 750 | 54 250 | 98 284 | 60 |
| 1 | 44 078 | 45 797 | 54 203 | 98 281 | 59 |
| 2 | 44 122 | 45 845 | 54 155 | 98 277 | 58 |
| 3 | 44 166 | 45 892 | 54 108 | 98 273 | 57 |
| 4 | 44 210 | 45 940 | 54 060 | 98 270 | 56 |
| 5 | 44 253 | 45 987 | 54 013 | 98 266 | 55 |
| 6 | 44 297 | 46 035 | 53 965 | 98 262 | 54 |
| 7 | 44 341 | 46 082 | 53 918 | 98 259 | 53 |
| 8 | 44 385 | 46 130 | 53 870 | 98 255 | 52 |
| 9 | 44 428 | 46 177 | 53 823 | 98 251 | 51 |
| 10 | 44 472 | 46 224 | 53 776 | 98 248 | 50 |
| 11 | 44 516 | 46 271 | 53 729 | 98 244 | 49 |
| 12 | 44 559 | 46 319 | 53 681 | 98 240 | 48 |
| 13 | 44 602 | 46 366 | 53 634 | 98 237 | 47 |
| 14 | 44 646 | 46 413 | 53 587 | 98 233 | 46 |
| 15 | 44 689 | 46 460 | 53 540 | 98 229 | 45 |
| 16 | 44 733 | 46 507 | 53 493 | 98 226 | 44 |
| 17 | 44 776 | 46 554 | 53 446 | 98 222 | 43 |
| 18 | 44 819 | 46 601 | 53 399 | 98 218 | 42 |
| 19 | 44 862 | 46 648 | 53 352 | 98 215 | 41 |
| 20 | 44 905 | 46 694 | 53 306 | 98 211 | 40 |
| 21 | 44 948 | 46 741 | 53 259 | 98 207 | 39 |
| 22 | 44 992 | 46 788 | 53 212 | 98 204 | 38 |
| 23 | 45 035 | 46 835 | 53 165 | 98 200 | 37 |
| 24 | 45 077 | 46 881 | 53 119 | 98 196 | 36 |
| 25 | 45 120 | 46 928 | 53 072 | 98 192 | 35 |
| 26 | 45 163 | 46 975 | 53 025 | 98 189 | 34 |
| 27 | 45 206 | 47 021 | 52 979 | 98 185 | 33 |
| 28 | 45 249 | 47 068 | 52 932 | 98 181 | 32 |
| 29 | 45 292 | 47 114 | 52 886 | 98 177 | 31 |
| 30 | 45 334 | 47 160 | 52 840 | 98 174 | 30 |
| 31 | 45 377 | 47 207 | 52 793 | 98 170 | 29 |
| 32 | 45 419 | 47 253 | 52 747 | 98 166 | 28 |
| 33 | 45 462 | 47 299 | 52 701 | 98 162 | 27 |
| 34 | 45 504 | 47 346 | 52 654 | 98 159 | 26 |
| 35 | 45 547 | 47 392 | 52 608 | 98 155 | 25 |
| 36 | 45 589 | 47 438 | 52 562 | 98 151 | 24 |
| 37 | 45 632 | 47 484 | 52 516 | 98 147 | 23 |
| 38 | 45 674 | 47 530 | 52 470 | 98 144 | 22 |
| 39 | 45 716 | 47 576 | 52 424 | 98 140 | 21 |
| 40 | 45 758 | 47 622 | 52 378 | 98 136 | 20 |
| 41 | 45 801 | 47 668 | 52 332 | 98 132 | 19 |
| 42 | 45 843 | 47 714 | 52 286 | 98 129 | 18 |
| 43 | 45 885 | 47 760 | 52 240 | 98 125 | 17 |
| 44 | 45 927 | 47 806 | 52 194 | 98 121 | 16 |
| 45 | 45 969 | 47 852 | 52 148 | 98 117 | 15 |
| 46 | 46 011 | 47 897 | 52 103 | 98 113 | 14 |
| 47 | 46 053 | 47 943 | 52 057 | 98 110 | 13 |
| 48 | 46 095 | 47 989 | 52 011 | 98 106 | 12 |
| 49 | 46 136 | 48 035 | 51 965 | 98 102 | 11 |
| 50 | 46 178 | 48 080 | 51 920 | 98 098 | 10 |
| 51 | 46 220 | 48 126 | 51 874 | 98 094 | 9 |
| 52 | 46 262 | 48 171 | 51 829 | 98 090 | 8 |
| 53 | 46 303 | 48 217 | 51 783 | 98 087 | 7 |
| 54 | 46 345 | 48 262 | 51 738 | 98 083 | 6 |
| 55 | 46 386 | 48 307 | 51 693 | 98 079 | 5 |
| 56 | 46 428 | 48 353 | 51 647 | 98 075 | 4 |
| 57 | 46 469 | 48 398 | 51 602 | 98 071 | 3 |
| 58 | 46 511 | 48 443 | 51 557 | 98 067 | 2 |
| 59 | 46 552 | 48 489 | 51 511 | 98 063 | 1 |
| 60 | 46 594 | 48 534 | 51 466 | 98 060 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

73°

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 46 594 | 48 534 | 51 466 | 98 060 | 60 |
| 1 | 46 635 | 48 579 | 51 421 | 98 056 | 59 |
| 2 | 46 676 | 48 624 | 51 376 | 98 052 | 58 |
| 3 | 46 717 | 48 669 | 51 331 | 98 048 | 57 |
| 4 | 46 758 | 48 714 | 51 286 | 98 044 | 56 |
| 5 | 46 800 | 48 759 | 51 241 | 98 040 | 55 |
| 6 | 46 841 | 48 804 | 51 196 | 98 036 | 54 |
| 7 | 46 882 | 48 849 | 51 151 | 98 032 | 53 |
| 8 | 46 923 | 48 894 | 51 106 | 98 029 | 52 |
| 9 | 46 964 | 48 939 | 51 061 | 98 025 | 51 |
| 10 | 47 005 | 48 984 | 51 016 | 98 021 | 50 |
| 11 | 47 045 | 49 029 | 50 971 | 98 017 | 49 |
| 12 | 47 086 | 49 073 | 50 927 | 98 013 | 48 |
| 13 | 47 127 | 49 118 | 50 882 | 98 009 | 47 |
| 14 | 47 168 | 49 163 | 50 837 | 98 005 | 46 |
| 15 | 47 209 | 49 207 | 50 793 | 98 001 | 45 |
| 16 | 47 249 | 49 252 | 50 748 | 97 997 | 44 |
| 17 | 47 290 | 49 296 | 50 704 | 97 993 | 43 |
| 18 | 47 330 | 49 341 | 50 659 | 97 989 | 42 |
| 19 | 47 371 | 49 385 | 50 615 | 97 986 | 41 |
| 20 | 47 411 | 49 430 | 50 570 | 97 982 | 40 |
| 21 | 47 452 | 49 474 | 50 526 | 97 978 | 39 |
| 22 | 47 492 | 49 519 | 50 481 | 97 974 | 38 |
| 23 | 47 533 | 49 563 | 50 437 | 97 970 | 37 |
| 24 | 47 573 | 49 607 | 50 393 | 97 966 | 36 |
| 25 | 47 613 | 49 652 | 50 348 | 97 962 | 35 |
| 26 | 47 654 | 49 696 | 50 304 | 97 958 | 34 |
| 27 | 47 694 | 49 740 | 50 260 | 97 954 | 33 |
| 28 | 47 734 | 49 784 | 50 216 | 97 950 | 32 |
| 29 | 47 774 | 49 828 | 50 172 | 97 946 | 31 |
| 30 | 47 814 | 49 872 | 50 128 | 97 942 | 30 |
| 31 | 47 854 | 49 916 | 50 084 | 97 938 | 29 |
| 32 | 47 894 | 49 960 | 50 040 | 97 934 | 28 |
| 33 | 47 934 | 50 004 | 49 996 | 97 930 | 27 |
| 34 | 47 974 | 50 048 | 49 952 | 97 926 | 26 |
| 35 | 48 014 | 50 092 | 49 908 | 97 922 | 25 |
| 36 | 48 054 | 50 136 | 49 864 | 97 918 | 24 |
| 37 | 48 094 | 50 180 | 49 820 | 97 914 | 23 |
| 38 | 48 133 | 50 223 | 49 777 | 97 910 | 22 |
| 39 | 48 173 | 50 267 | 49 733 | 97 906 | 21 |
| 40 | 48 213 | 50 311 | 49 689 | 97 902 | 20 |
| 41 | 48 252 | 50 355 | 49 645 | 97 898 | 19 |
| 42 | 48 292 | 50 398 | 49 602 | 97 894 | 18 |
| 43 | 48 332 | 50 442 | 49 558 | 97 890 | 17 |
| 44 | 48 371 | 50 485 | 49 515 | 97 886 | 16 |
| 45 | 48 411 | 50 529 | 49 471 | 97 882 | 15 |
| 46 | 48 450 | 50 572 | 49 428 | 97 878 | 14 |
| 47 | 48 490 | 50 616 | 49 384 | 97 874 | 13 |
| 48 | 48 529 | 50 659 | 49 341 | 97 870 | 12 |
| 49 | 48 568 | 50 703 | 49 297 | 97 866 | 11 |
| 50 | 48 607 | 50 746 | 49 254 | 97 861 | 10 |
| 51 | 48 647 | 50 789 | 49 211 | 97 857 | 9 |
| 52 | 48 686 | 50 833 | 49 167 | 97 853 | 8 |
| 53 | 48 725 | 50 876 | 49 124 | 97 849 | 7 |
| 54 | 48 764 | 50 919 | 49 081 | 97 845 | 6 |
| 55 | 48 803 | 50 962 | 49 038 | 97 841 | 5 |
| 56 | 48 842 | 51 005 | 48 995 | 97 837 | 4 |
| 57 | 48 881 | 51 048 | 48 952 | 97 833 | 3 |
| 58 | 48 920 | 51 092 | 48 908 | 97 829 | 2 |
| 59 | 48 959 | 51 135 | 48 865 | 97 825 | 1 |
| 60 | 48 998 | 51 178 | 48 822 | 97 821 | 0 |
| ' | log cos 0 | log cot 10 | log tan 0 | log sin 0 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 48 998 | 51 178 | 48 822 | 97 821 | 60 |
| 1 | 49 037 | 51 221 | 48 779 | 97 817 | 59 |
| 2 | 49 076 | 51 264 | 48 736 | 97 812 | 58 |
| 3 | 49 115 | 51 306 | 48 694 | 97 808 | 57 |
| 4 | 49 153 | 51 349 | 48 651 | 97 804 | 56 |
| 5 | 49 192 | 51 392 | 48 608 | 97 800 | 55 |
| 6 | 49 231 | 51 435 | 48 565 | 97 796 | 54 |
| 7 | 49 269 | 51 478 | 48 522 | 97 792 | 53 |
| 8 | 49 308 | 51 520 | 48 480 | 97 788 | 52 |
| 9 | 49 347 | 51 563 | 48 437 | 97 784 | 51 |
| 10 | 49 385 | 51 606 | 48 394 | 97 779 | 50 |
| 11 | 49 424 | 51 648 | 48 352 | 97 775 | 49 |
| 12 | 49 462 | 51 691 | 48 309 | 97 771 | 48 |
| 13 | 49 500 | 51 734 | 48 266 | 97 767 | 47 |
| 14 | 49 539 | 51 776 | 48 224 | 97 763 | 46 |
| 15 | 49 577 | 51 819 | 48 181 | 97 759 | 45 |
| 16 | 49 615 | 51 861 | 48 139 | 97 754 | 44 |
| 17 | 49 654 | 51 903 | 48 097 | 97 750 | 43 |
| 18 | 49 692 | 51 946 | 48 054 | 97 746 | 42 |
| 19 | 49 730 | 51 988 | 48 012 | 97 742 | 41 |
| 20 | 49 768 | 52 031 | 47 969 | 97 738 | 40 |
| 21 | 49 806 | 52 073 | 47 927 | 97 734 | 39 |
| 22 | 49 844 | 52 115 | 47 885 | 97 729 | 38 |
| 23 | 49 882 | 52 157 | 47 843 | 97 725 | 37 |
| 24 | 49 920 | 52 200 | 47 800 | 97 721 | 36 |
| 25 | 49 958 | 52 242 | 47 758 | 97 717 | 35 |
| 26 | 49 996 | 52 284 | 47 716 | 97 713 | 34 |
| 27 | 50 034 | 52 326 | 47 674 | 97 708 | 33 |
| 28 | 50 072 | 52 368 | 47 632 | 97 704 | 32 |
| 29 | 50 110 | 52 410 | 47 590 | 97 700 | 31 |
| 30 | 50 148 | 52 452 | 47 548 | 97 696 | 30 |
| 31 | 50 185 | 52 494 | 47 506 | 97 691 | 29 |
| 32 | 50 223 | 52 536 | 47 464 | 97 687 | 28 |
| 33 | 50 261 | 52 578 | 47 422 | 97 683 | 27 |
| 34 | 50 298 | 52 620 | 47 380 | 97 679 | 26 |
| 35 | 50 336 | 52 661 | 47 339 | 97 674 | 25 |
| 36 | 50 374 | 52 703 | 47 297 | 97 670 | 24 |
| 37 | 50 411 | 52 745 | 47 255 | 97 666 | 23 |
| 38 | 50 449 | 52 787 | 47 213 | 97 662 | 22 |
| 39 | 50 486 | 52 829 | 47 171 | 97 657 | 21 |
| 40 | 50 523 | 52 870 | 47 130 | 97 653 | 20 |
| 41 | 50 561 | 52 912 | 47 088 | 97 649 | 19 |
| 42 | 50 598 | 52 953 | 47 047 | 97 645 | 18 |
| 43 | 50 635 | 52 995 | 47 005 | 97 640 | 17 |
| 44 | 50 673 | 53 037 | 46 963 | 97 636 | 16 |
| 45 | 50 710 | 53 078 | 46 922 | 97 632 | 15 |
| 46 | 50 747 | 53 120 | 46 880 | 97 628 | 14 |
| 47 | 50 784 | 53 161 | 46 839 | 97 623 | 13 |
| 48 | 50 821 | 53 202 | 46 798 | 97 619 | 12 |
| 49 | 50 858 | 53 244 | 46 756 | 97 615 | 11 |
| 50 | 50 896 | 53 285 | 46 715 | 97 610 | 10 |
| 51 | 50 933 | 53 327 | 46 673 | 97 606 | 9 |
| 52 | 50 970 | 53 368 | 46 632 | 97 602 | 8 |
| 53 | 51 007 | 53 409 | 46 591 | 97 597 | 7 |
| 54 | 51 043 | 53 450 | 46 550 | 97 593 | 6 |
| 55 | 51 080 | 53 492 | 46 508 | 97 589 | 5 |
| 56 | 51 117 | 53 533 | 46 467 | 97 584 | 4 |
| 57 | 51 154 | 53 574 | 46 426 | 97 580 | 3 |
| 58 | 51 191 | 53 615 | 46 385 | 97 576 | 2 |
| 59 | 51 227 | 53 656 | 46 344 | 97 571 | 1 |
| 60 | 51 264 | 53 697 | 46 303 | 97 567 | 0 |
| ' | log cos 0 | log cot 10 | log tan 0 | log sin 0 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 51 264 | 53 697 | 46 303 | 97 567 | 60 |
| 1 | 51 301 | 53 738 | 46 262 | 97 563 | 59 |
| 2 | 51 338 | 53 779 | 46 221 | 97 558 | 58 |
| 3 | 51 374 | 53 820 | 46 180 | 97 554 | 57 |
| 4 | 51 411 | 53 861 | 46 139 | 97 555 | 56 |
| 5 | 51 447 | 53 902 | 46 098 | 97 545 | 55 |
| 6 | 51 484 | 53 943 | 46 057 | 97 541 | 54 |
| 7 | 51 520 | 53 984 | 46 016 | 97 536 | 53 |
| 8 | 51 557 | 54 025 | 45 975 | 97 532 | 52 |
| 9 | 51 593 | 54 065 | 45 935 | 97 528 | 51 |
| 10 | 51 629 | 54 106 | 45 894 | 97 523 | 50 |
| 11 | 51 666 | 54 147 | 45 853 | 97 519 | 49 |
| 12 | 51 702 | 54 187 | 45 813 | 97 515 | 48 |
| 13 | 51 738 | 54 228 | 45 772 | 97 510 | 47 |
| 14 | 51 774 | 54 269 | 45 731 | 97 506 | 46 |
| 15 | 51 811 | 54 309 | 45 691 | 97 501 | 45 |
| 16 | 51 847 | 54 350 | 45 650 | 97 497 | 44 |
| 17 | 51 883 | 54 390 | 45 610 | 97 492 | 43 |
| 18 | 51 919 | 54 431 | 45 569 | 97 488 | 42 |
| 19 | 51 955 | 54 471 | 45 529 | 97 484 | 41 |
| 20 | 51 991 | 54 512 | 45 488 | 97 479 | 40 |
| 21 | 52 027 | 54 552 | 45 448 | 97 475 | 39 |
| 22 | 52 063 | 54 593 | 45 407 | 97 470 | 38 |
| 23 | 52 099 | 54 633 | 45 367 | 97 466 | 37 |
| 24 | 52 135 | 54 673 | 45 327 | 97 461 | 36 |
| 25 | 52 171 | 54 714 | 45 286 | 97 457 | 35 |
| 26 | 52 207 | 54 754 | 45 246 | 97 453 | 34 |
| 27 | 52 242 | 54 794 | 45 206 | 97 448 | 33 |
| 28 | 52 278 | 54 835 | 45 165 | 97 444 | 32 |
| 29 | 52 314 | 54 875 | 45 125 | 97 439 | 31 |
| 30 | 52 350 | 54 915 | 45 085 | 97 435 | 30 |
| 31 | 52 385 | 54 955 | 45 045 | 97 430 | 29 |
| 32 | 52 421 | 54 995 | 45 005 | 97 426 | 28 |
| 33 | 52 456 | 55 035 | 44 965 | 97 421 | 27 |
| 34 | 52 492 | 55 075 | 44 925 | 97 417 | 26 |
| 35 | 52 527 | 55 115 | 44 885 | 97 412 | 25 |
| 36 | 52 563 | 55 155 | 44 845 | 97 408 | 24 |
| 37 | 52 598 | 55 195 | 44 805 | 97 403 | 23 |
| 38 | 52 634 | 55 235 | 44 765 | 97 399 | 22 |
| 39 | 52 669 | 55 275 | 44 725 | 97 394 | 21 |
| 40 | 52 705 | 55 315 | 44 685 | 97 390 | 20 |
| 41 | 52 740 | 55 355 | 44 645 | 97 385 | 19 |
| 42 | 52 775 | 55 395 | 44 605 | 97 381 | 18 |
| 43 | 52 811 | 55 434 | 44 566 | 97 376 | 17 |
| 44 | 52 846 | 55 474 | 44 526 | 97 372 | 16 |
| 45 | 52 881 | 55 514 | 44 486 | 97 367 | 15 |
| 46 | 52 916 | 55 554 | 44 446 | 97 363 | 14 |
| 47 | 52 951 | 55 593 | 44 407 | 97 358 | 13 |
| 48 | 52 986 | 55 633 | 44 367 | 97 353 | 12 |
| 49 | 53 021 | 55 673 | 44 327 | 97 349 | 11 |
| 50 | 53 056 | 55 712 | 44 288 | 97 344 | 10 |
| 51 | 53 092 | 55 752 | 44 248 | 97 340 | 9 |
| 52 | 53 126 | 55 791 | 44 209 | 97 335 | 8 |
| 53 | 53 161 | 55 831 | 44 169 | 97 331 | 7 |
| 54 | 53 196 | 55 870 | 44 130 | 97 326 | 6 |
| 55 | 53 231 | 55 910 | 44 090 | 97 322 | 5 |
| 56 | 53 266 | 55 949 | 44 051 | 97 317 | 4 |
| 57 | 53 301 | 55 989 | 44 011 | 97 312 | 3 |
| 58 | 53 336 | 56 028 | 43 972 | 97 308 | 2 |
| 59 | 53 370 | 56 067 | 43 933 | 97 303 | 1 |
| 60 | 53 405 | 56 107 | 43 893 | 97 299 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 53 405 | 56 107 | 43 893 | 97 299 | 60 |
| 1 | 53 440 | 56 146 | 43 854 | 97 294 | 59 |
| 2 | 53 475 | 56 185 | 43 815 | 97 289 | 58 |
| 3 | 53 509 | 56 224 | 43 776 | 97 285 | 57 |
| 4 | 53 544 | 56 264 | 43 736 | 97 280 | 56 |
| 5 | 53 578 | 56 303 | 43 697 | 97 276 | 55 |
| 6 | 53 613 | 56 342 | 43 658 | 97 271 | 54 |
| 7 | 53 647 | 56 381 | 43 619 | 97 266 | 53 |
| 8 | 53 682 | 56 420 | 43 580 | 97 262 | 52 |
| 9 | 53 716 | 56 459 | 43 541 | 97 257 | 51 |
| 10 | 53 751 | 56 498 | 43 502 | 97 252 | 50 |
| 11 | 53 785 | 56 537 | 43 463 | 97 248 | 49 |
| 12 | 53 819 | 56 576 | 43 424 | 97 243 | 48 |
| 13 | 53 854 | 56 615 | 43 385 | 97 238 | 47 |
| 14 | 53 888 | 56 654 | 43 346 | 97 234 | 46 |
| 15 | 53 922 | 56 693 | 43 307 | 97 229 | 45 |
| 16 | 53 957 | 56 732 | 43 268 | 97 224 | 44 |
| 17 | 53 991 | 56 771 | 43 229 | 97 220 | 43 |
| 18 | 54 025 | 56 810 | 43 190 | 97 215 | 42 |
| 19 | 54 059 | 56 849 | 43 151 | 97 210 | 41 |
| 20 | 54 093 | 56 887 | 43 113 | 97 206 | 40 |
| 21 | 54 127 | 56 926 | 43 074 | 97 201 | 39 |
| 22 | 54 161 | 56 965 | 43 035 | 97 196 | 38 |
| 23 | 54 195 | 57 004 | 42 996 | 97 192 | 37 |
| 24 | 54 229 | 57 042 | 42 958 | 97 187 | 36 |
| 25 | 54 263 | 57 081 | 42 919 | 97 182 | 35 |
| 26 | 54 297 | 57 120 | 42 880 | 97 178 | 34 |
| 27 | 54 331 | 57 158 | 42 842 | 97 173 | 33 |
| 28 | 54 365 | 57 197 | 42 803 | 97 168 | 32 |
| 29 | 54 399 | 57 235 | 42 765 | 97 163 | 31 |
| 30 | 54 433 | 57 274 | 42 726 | 97 159 | 30 |
| 31 | 54 466 | 57 312 | 42 688 | 97 154 | 29 |
| 32 | 54 500 | 57 351 | 42 649 | 97 149 | 28 |
| 33 | 54 534 | 57 389 | 42 611 | 97 145 | 27 |
| 34 | 54 567 | 57 428 | 42 572 | 97 140 | 26 |
| 35 | 54 601 | 57 466 | 42 534 | 97 135 | 25 |
| 36 | 54 635 | 57 504 | 42 496 | 97 130 | 24 |
| 37 | 54 668 | 57 543 | 42 457 | 97 126 | 23 |
| 38 | 54 702 | 57 581 | 42 419 | 97 121 | 22 |
| 39 | 54 735 | 57 619 | 42 381 | 97 116 | 21 |
| 40 | 54 769 | 57 658 | 42 342 | 97 111 | 20 |
| 41 | 54 802 | 57 696 | 42 304 | 97 107 | 19 |
| 42 | 54 836 | 57 734 | 42 266 | 97 102 | 18 |
| 43 | 54 869 | 57 772 | 42 228 | 97 097 | 17 |
| 44 | 54 903 | 57 810 | 42 190 | 97 092 | 16 |
| 45 | 54 936 | 57 849 | 42 151 | 97 087 | 15 |
| 46 | 54 969 | 57 887 | 42 113 | 97 083 | 14 |
| 47 | 55 003 | 57 925 | 42 075 | 97 078 | 13 |
| 48 | 55 036 | 57 963 | 42 037 | 97 073 | 12 |
| 49 | 55 069 | 58 001 | 41 999 | 97 068 | 11 |
| 50 | 55 102 | 58 039 | 41 961 | 97 063 | 10 |
| 51 | 55 136 | 58 077 | 41 923 | 97 059 | 9 |
| 52 | 55 169 | 58 115 | 41 885 | 97 054 | 8 |
| 53 | 55 202 | 58 153 | 41 847 | 97 049 | 7 |
| 54 | 55 235 | 58 191 | 41 809 | 97 044 | 6 |
| 55 | 55 268 | 58 229 | 41 771 | 97 039 | 5 |
| 56 | 55 301 | 58 267 | 41 733 | 97 035 | 4 |
| 57 | 55 334 | 58 304 | 41 696 | 97 030 | 3 |
| 58 | 55 367 | 58 342 | 41 658 | 97 025 | 2 |
| 59 | 55 400 | 58 380 | 41 620 | 97 020 | 1 |
| 60 | 55 433 | 58 418 | 41 582 | 97 015 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

| / | log sin 9 | log tan | log cot 10 | log cos 9 | / |
|----|--------------|---------|---------------|--------------|----|
| 0 | 55 433 | 58 418 | 41 582 | 97 015 | 60 |
| 1 | 55 466 | 58 455 | 41 545 | 97 010 | 59 |
| 2 | 55 499 | 58 493 | 41 507 | 97 005 | 58 |
| 3 | 55 532 | 58 531 | 41 469 | 97 001 | 57 |
| 4 | 55 564 | 58 569 | 41 431 | 96 996 | 56 |
| 5 | 55 597 | 58 606 | 41 394 | 96 991 | 55 |
| 6 | 55 630 | 58 644 | 41 356 | 96 986 | 54 |
| 7 | 55 663 | 58 681 | 41 319 | 96 981 | 53 |
| 8 | 55 695 | 58 719 | 41 281 | 96 976 | 52 |
| 9 | 55 728 | 58 757 | 41 243 | 96 971 | 51 |
| 10 | 55 761 | 58 794 | 41 206 | 96 966 | 50 |
| 11 | 55 793 | 58 832 | 41 168 | 96 962 | 49 |
| 12 | 55 826 | 58 869 | 41 131 | 96 957 | 48 |
| 13 | 55 858 | 58 907 | 41 093 | 96 952 | 47 |
| 14 | 55 891 | 58 944 | 41 056 | 96 947 | 46 |
| 15 | 55 923 | 58 981 | 41 019 | 96 942 | 45 |
| 16 | 55 956 | 59 019 | 40 981 | 96 937 | 44 |
| 17 | 55 988 | 59 056 | 40 944 | 96 932 | 43 |
| 18 | 56 021 | 59 094 | 40 906 | 96 927 | 42 |
| 19 | 56 053 | 59 131 | 40 869 | 96 922 | 41 |
| 20 | 56 085 | 59 168 | 40 832 | 96 917 | 40 |
| 21 | 56 118 | 59 205 | 40 795 | 96 912 | 39 |
| 22 | 56 150 | 59 243 | 40 757 | 96 907 | 38 |
| 23 | 56 182 | 59 280 | 40 720 | 96 903 | 37 |
| 24 | 56 215 | 59 317 | 40 683 | 96 898 | 36 |
| 25 | 56 247 | 59 354 | 40 646 | 96 893 | 35 |
| 26 | 56 279 | 59 391 | 40 609 | 96 888 | 34 |
| 27 | 56 311 | 59 429 | 40 571 | 96 883 | 33 |
| 28 | 56 343 | 59 466 | 40 534 | 96 878 | 32 |
| 29 | 56 375 | 59 503 | 40 497 | 96 873 | 31 |
| 30 | 56 408 | 59 540 | 40 460 | 96 868 | 30 |
| 31 | 56 440 | 59 577 | 40 423 | 96 863 | 29 |
| 32 | 56 472 | 59 614 | 40 386 | 96 858 | 28 |
| 33 | 56 504 | 59 651 | 40 349 | 96 853 | 27 |
| 34 | 56 536 | 59 688 | 40 312 | 96 848 | 26 |
| 35 | 56 568 | 59 725 | 40 275 | 96 843 | 25 |
| 36 | 56 599 | 59 762 | 40 238 | 96 838 | 24 |
| 37 | 56 631 | 59 799 | 40 201 | 96 833 | 23 |
| 38 | 56 663 | 59 835 | 40 165 | 96 828 | 22 |
| 39 | 56 695 | 59 872 | 40 128 | 96 823 | 21 |
| 40 | 56 727 | 59 909 | 40 091 | 96 818 | 20 |
| 41 | 56 759 | 59 946 | 40 054 | 96 813 | 19 |
| 42 | 56 790 | 59 983 | 40 017 | 96 808 | 18 |
| 43 | 56 822 | 60 019 | 39 981 | 96 803 | 17 |
| 44 | 56 854 | 60 056 | 39 944 | 96 798 | 16 |
| 45 | 56 886 | 60 093 | 39 907 | 96 793 | 15 |
| 46 | 56 917 | 60 130 | 39 870 | 96 788 | 14 |
| 47 | 56 949 | 60 166 | 39 834 | 96 783 | 13 |
| 48 | 56 980 | 60 203 | 39 797 | 96 778 | 12 |
| 49 | 57 012 | 60 240 | 39 760 | 96 772 | 11 |
| 50 | 57 044 | 60 276 | 39 724 | 96 767 | 10 |
| 51 | 57 075 | 60 313 | 39 687 | 96 762 | 9 |
| 52 | 57 107 | 60 349 | 39 651 | 96 757 | 8 |
| 53 | 57 138 | 60 386 | 39 614 | 96 752 | 7 |
| 54 | 57 169 | 60 422 | 39 578 | 96 747 | 6 |
| 55 | 57 201 | 60 459 | 39 541 | 96 742 | 5 |
| 56 | 57 232 | 60 495 | 39 505 | 96 737 | 4 |
| 57 | 57 264 | 60 532 | 39 468 | 96 732 | 3 |
| 58 | 57 295 | 60 568 | 39 432 | 96 727 | 2 |
| 59 | 57 326 | 60 605 | 39 395 | 96 722 | 1 |
| 60 | 57 358 | 60 641 | 39 359 | 96 717 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| / | log sin 9 | log tan 9 | log cot 10 | log cos 9 | / |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 57 358 | 60 641 | 39 359 | 96 717 | 60 |
| 1 | 57 389 | 60 677 | 39 323 | 96 711 | 59 |
| 2 | 57 420 | 60 714 | 39 286 | 96 706 | 58 |
| 3 | 57 451 | 60 750 | 39 250 | 96 701 | 57 |
| 4 | 57 482 | 60 786 | 39 214 | 96 696 | 56 |
| 5 | 57 514 | 60 823 | 39 177 | 96 691 | 55 |
| 6 | 57 545 | 60 859 | 39 141 | 96 686 | 54 |
| 7 | 57 576 | 60 895 | 39 105 | 96 681 | 53 |
| 8 | 57 607 | 60 931 | 39 069 | 96 676 | 52 |
| 9 | 57 638 | 60 967 | 39 033 | 96 670 | 51 |
| 10 | 57 669 | 61 004 | 38 996 | 96 665 | 50 |
| 11 | 57 700 | 61 040 | 38 960 | 96 660 | 49 |
| 12 | 57 731 | 61 076 | 38 924 | 96 655 | 48 |
| 13 | 57 762 | 61 112 | 38 888 | 96 650 | 47 |
| 14 | 57 793 | 61 148 | 38 852 | 96 645 | 46 |
| 15 | 57 824 | 61 184 | 38 816 | 96 640 | 45 |
| 16 | 57 855 | 61 220 | 38 780 | 96 634 | 44 |
| 17 | 57 885 | 61 256 | 38 744 | 96 629 | 43 |
| 18 | 57 916 | 61 292 | 38 708 | 96 624 | 42 |
| 19 | 57 947 | 61 328 | 38 672 | 96 619 | 41 |
| 20 | 57 978 | 61 364 | 38 636 | 96 614 | 40 |
| 21 | 58 008 | 61 400 | 38 600 | 96 608 | 39 |
| 22 | 58 039 | 61 436 | 38 564 | 96 603 | 38 |
| 23 | 58 070 | 61 472 | 38 528 | 96 598 | 37 |
| 24 | 58 101 | 61 508 | 38 492 | 96 593 | 36 |
| 25 | 58 131 | 61 544 | 38 456 | 96 588 | 35 |
| 26 | 58 162 | 61 579 | 38 421 | 96 582 | 34 |
| 27 | 58 192 | 61 615 | 38 385 | 96 577 | 33 |
| 28 | 58 223 | 61 651 | 38 349 | 96 572 | 32 |
| 29 | 58 253 | 61 687 | 38 313 | 96 567 | 31 |
| 30 | 58 284 | 61 722 | 38 278 | 96 562 | 30 |
| 31 | 58 314 | 61 758 | 38 242 | 96 556 | 29 |
| 32 | 58 345 | 61 794 | 38 206 | 96 551 | 28 |
| 33 | 58 375 | 61 830 | 38 170 | 96 546 | 27 |
| 34 | 58 406 | 61 865 | 38 135 | 96 541 | 26 |
| 35 | 58 436 | 61 901 | 38 099 | 96 535 | 25 |
| 36 | 58 467 | 61 936 | 38 064 | 96 530 | 24 |
| 37 | 58 497 | 61 972 | 38 028 | 96 525 | 23 |
| 38 | 58 527 | 62 008 | 37 992 | 96 520 | 22 |
| 39 | 58 557 | 62 043 | 37 957 | 96 514 | 21 |
| 40 | 58 588 | 62 079 | 37 921 | 96 509 | 20 |
| 41 | 58 618 | 62 114 | 37 886 | 96 504 | 19 |
| 42 | 58 648 | 62 150 | 37 850 | 96 498 | 18 |
| 43 | 58 678 | 62 185 | 37 815 | 96 493 | 17 |
| 44 | 58 709 | 62 221 | 37 779 | 96 488 | 16 |
| 45 | 58 739 | 62 256 | 37 744 | 96 483 | 15 |
| 46 | 58 769 | 62 292 | 37 708 | 96 477 | 14 |
| 47 | 58 799 | 62 327 | 37 673 | 96 472 | 13 |
| 48 | 58 829 | 62 362 | 37 638 | 96 467 | 12 |
| 49 | 58 859 | 62 398 | 37 602 | 96 461 | 11 |
| 50 | 58 889 | 62 433 | 37 567 | 96 456 | 10 |
| 51 | 58 919 | 62 468 | 37 532 | 96 451 | 9 |
| 52 | 58 949 | 62 504 | 37 496 | 96 445 | 8 |
| 53 | 58 979 | 62 539 | 37 461 | 96 440 | 7 |
| 54 | 59 009 | 62 574 | 37 426 | 96 435 | 6 |
| 55 | 59 039 | 62 609 | 37 391 | 96 429 | 5 |
| 56 | 59 069 | 62 645 | 37 355 | 96 424 | 4 |
| 57 | 59 098 | 62 680 | 37 320 | 96 419 | 3 |
| 58 | 59 128 | 62 715 | 37 285 | 96 413 | 2 |
| 59 | 59 158 | 62 750 | 37 250 | 96 408 | 1 |
| 60 | 59 188 | 62 785 | 37 215 | 96 403 | 0 |
| / | log cos | log cot | log tan | log sin | / |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 59 188 | 62 785 | 37 215 | 96 403 | 60 |
| 1 | 59 218 | 62 820 | 37 180 | 96 397 | 59 |
| 2 | 59 247 | 62 855 | 37 145 | 96 392 | 58 |
| 3 | 59 277 | 62 890 | 37 110 | 96 387 | 57 |
| 4 | 59 307 | 62 926 | 37 074 | 96 381 | 56 |
| 5 | 59 336 | 62 961 | 37 039 | 96 376 | 55 |
| 6 | 59 366 | 62 996 | 37 004 | 96 370 | 54 |
| 7 | 59 396 | 63 031 | 36 969 | 96 365 | 53 |
| 8 | 59 425 | 63 066 | 36 934 | 96 360 | 52 |
| 9 | 59 455 | 63 101 | 36 899 | 96 354 | 51 |
| 10 | 59 484 | 63 135 | 36 865 | 96 349 | 50 |
| 11 | 59 514 | 63 170 | 36 830 | 96 343 | 49 |
| 12 | 59 543 | 63 205 | 36 795 | 96 338 | 48 |
| 13 | 59 573 | 63 240 | 36 760 | 96 333 | 47 |
| 14 | 59 602 | 63 275 | 36 725 | 96 327 | 46 |
| 15 | 59 632 | 63 310 | 36 690 | 96 322 | 45 |
| 16 | 59 661 | 63 345 | 36 655 | 96 316 | 44 |
| 17 | 59 690 | 63 379 | 36 621 | 96 311 | 43 |
| 18 | 59 720 | 63 414 | 36 586 | 96 305 | 42 |
| 19 | 59 749 | 63 449 | 36 551 | 96 300 | 41 |
| 20 | 59 778 | 63 484 | 36 516 | 96 294 | 40 |
| 21 | 59 808 | 63 519 | 36 481 | 96 289 | 39 |
| 22 | 59 837 | 63 553 | 36 447 | 96 284 | 38 |
| 23 | 59 866 | 63 588 | 36 412 | 96 278 | 37 |
| 24 | 59 895 | 63 623 | 36 377 | 96 273 | 36 |
| 25 | 59 924 | 63 657 | 36 343 | 96 267 | 35 |
| 26 | 59 954 | 63 692 | 36 308 | 96 262 | 34 |
| 27 | 59 983 | 63 726 | 36 274 | 96 256 | 33 |
| 28 | 60 012 | 63 761 | 36 239 | 96 251 | 32 |
| 29 | 60 041 | 63 796 | 36 204 | 96 245 | 31 |
| 30 | 60 070 | 63 830 | 36 170 | 96 240 | 30 |
| 31 | 60 099 | 63 865 | 36 135 | 96 234 | 29 |
| 32 | 60 128 | 63 899 | 36 101 | 96 229 | 28 |
| 33 | 60 157 | 63 934 | 36 066 | 96 223 | 27 |
| 34 | 60 186 | 63 968 | 36 032 | 96 218 | 26 |
| 35 | 60 215 | 64 003 | 35 997 | 96 212 | 25 |
| 36 | 60 244 | 64 037 | 35 963 | 96 207 | 24 |
| 37 | 60 273 | 64 072 | 35 928 | 96 201 | 23 |
| 38 | 60 302 | 64 106 | 35 894 | 96 196 | 22 |
| 39 | 60 331 | 64 140 | 35 860 | 96 190 | 21 |
| 40 | 60 359 | 64 175 | 35 825 | 96 185 | 20 |
| 41 | 60 388 | 64 209 | 35 791 | 96 179 | 19 |
| 42 | 60 417 | 64 243 | 35 757 | 96 174 | 18 |
| 43 | 60 446 | 64 278 | 35 722 | 96 168 | 17 |
| 44 | 60 474 | 64 312 | 35 688 | 96 162 | 16 |
| 45 | 60 503 | 64 346 | 35 654 | 96 157 | 15 |
| 46 | 60 532 | 64 381 | 35 619 | 96 151 | 14 |
| 47 | 60 561 | 64 415 | 35 585 | 96 146 | 13 |
| 48 | 60 589 | 64 449 | 35 551 | 96 140 | 12 |
| 49 | 60 618 | 64 483 | 35 517 | 96 135 | 11 |
| 50 | 60 646 | 64 517 | 35 483 | 96 129 | 10 |
| 51 | 60 675 | 64 552 | 35 448 | 96 123 | 9 |
| 52 | 60 704 | 64 586 | 35 414 | 96 118 | 8 |
| 53 | 60 732 | 64 620 | 35 380 | 96 112 | 7 |
| 54 | 60 761 | 64 654 | 35 346 | 96 107 | 6 |
| 55 | 60 789 | 64 688 | 35 312 | 96 101 | 5 |
| 56 | 60 818 | 64 722 | 35 278 | 96 095 | 4 |
| 57 | 60 846 | 64 756 | 35 244 | 96 090 | 3 |
| 58 | 60 875 | 64 790 | 35 210 | 96 084 | 2 |
| 59 | 60 903 | 64 824 | 35 176 | 96 079 | 1 |
| 60 | 60 931 | 64 858 | 35 142 | 96 073 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 60 931 | 64 858 | 35 142 | 96 073 | 60 |
| 1 | 60 960 | 64 892 | 35 108 | 96 067 | 59 |
| 2 | 60 988 | 64 926 | 35 074 | 96 062 | 58 |
| 3 | 61 016 | 64 960 | 35 040 | 96 056 | 57 |
| 4 | 61 045 | 64 994 | 35 006 | 96 050 | 56 |
| 5 | 61 073 | 65 028 | 34 972 | 96 045 | 55 |
| 6 | 61 101 | 65 062 | 34 938 | 96 039 | 54 |
| 7 | 61 129 | 65 096 | 34 904 | 96 034 | 53 |
| 8 | 61 158 | 65 130 | 34 870 | 96 028 | 52 |
| 9 | 61 186 | 65 164 | 34 836 | 96 022 | 51 |
| 10 | 61 214 | 65 197 | 34 803 | 96 017 | 50 |
| 11 | 61 242 | 65 231 | 34 769 | 96 011 | 49 |
| 12 | 61 270 | 65 265 | 34 735 | 96 005 | 48 |
| 13 | 61 298 | 65 299 | 34 701 | 96 000 | 47 |
| 14 | 61 326 | 65 333 | 34 667 | 95 994 | 46 |
| 15 | 61 354 | 65 366 | 34 634 | 95 988 | 45 |
| 16 | 61 382 | 65 400 | 34 600 | 95 982 | 44 |
| 17 | 61 411 | 65 434 | 34 566 | 95 977 | 43 |
| 18 | 61 438 | 65 467 | 34 533 | 95 971 | 42 |
| 19 | 61 466 | 65 501 | 34 499 | 95 965 | 41 |
| 20 | 61 494 | 65 535 | 34 465 | 95 960 | 40 |
| 21 | 61 522 | 65 568 | 34 432 | 95 954 | 39 |
| 22 | 61 550 | 65 602 | 34 398 | 95 948 | 38 |
| 23 | 61 578 | 65 636 | 34 364 | 95 942 | 37 |
| 24 | 61 606 | 65 669 | 34 331 | 95 937 | 36 |
| 25 | 61 634 | 65 703 | 34 297 | 95 931 | 35 |
| 26 | 61 662 | 65 736 | 34 264 | 95 925 | 34 |
| 27 | 61 689 | 65 770 | 34 230 | 95 920 | 33 |
| 28 | 61 717 | 65 803 | 34 197 | 95 914 | 32 |
| 29 | 61 745 | 65 837 | 34 163 | 95 908 | 31 |
| 30 | 61 773 | 65 870 | 34 130 | 95 902 | 30 |
| 31 | 61 800 | 65 904 | 34 096 | 95 897 | 29 |
| 32 | 61 828 | 65 937 | 34 063 | 95 891 | 28 |
| 33 | 61 856 | 65 971 | 34 029 | 95 885 | 27 |
| 34 | 61 883 | 66 004 | 33 996 | 95 879 | 26 |
| 35 | 61 911 | 66 038 | 33 962 | 95 873 | 25 |
| 36 | 61 939 | 66 071 | 33 929 | 95 868 | 24 |
| 37 | 61 966 | 66 104 | 33 896 | 95 862 | 23 |
| 38 | 61 994 | 66 138 | 33 862 | 95 856 | 22 |
| 39 | 62 021 | 66 171 | 33 829 | 95 850 | 21 |
| 40 | 62 049 | 66 204 | 33 796 | 95 844 | 20 |
| 41 | 62 076 | 66 238 | 33 762 | 95 839 | 19 |
| 42 | 62 104 | 66 271 | 33 729 | 95 833 | 18 |
| 43 | 62 131 | 66 304 | 33 696 | 95 827 | 17 |
| 44 | 62 159 | 66 337 | 33 663 | 95 821 | 16 |
| 45 | 62 186 | 66 371 | 33 629 | 95 815 | 15 |
| 46 | 62 214 | 66 404 | 33 596 | 95 810 | 14 |
| 47 | 62 241 | 66 437 | 33 563 | 95 804 | 13 |
| 48 | 62 268 | 66 470 | 33 530 | 95 798 | 12 |
| 49 | 62 296 | 66 503 | 33 497 | 95 792 | 11 |
| 50 | 62 323 | 66 537 | 33 463 | 95 786 | 10 |
| 51 | 62 350 | 66 570 | 33 430 | 95 780 | 9 |
| 52 | 62 377 | 66 603 | 33 397 | 95 775 | 8 |
| 53 | 62 405 | 66 636 | 33 364 | 95 769 | 7 |
| 54 | 62 432 | 66 669 | 33 331 | 95 763 | 6 |
| 55 | 62 459 | 66 702 | 33 298 | 95 757 | 5 |
| 56 | 62 486 | 66 735 | 33 265 | 95 751 | 4 |
| 57 | 62 513 | 66 768 | 33 232 | 95 745 | 3 |
| 58 | 62 541 | 66 801 | 33 199 | 95 739 | 2 |
| 59 | 62 568 | 66 834 | 33 166 | 95 733 | 1 |
| 60 | 62 595 | 66 867 | 33 133 | 95 728 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ° | log sin | log tan | log cot | log cos | ° |
|----|---------|---------|---------|---------|----|
| 0 | 0 | 0 | 10 | 0 | |
| 0 | 62 595 | 66 867 | 33 133 | 95 728 | 60 |
| 1 | 62 622 | 66 900 | 33 100 | 95 722 | 59 |
| 2 | 62 649 | 66 933 | 33 067 | 95 716 | 58 |
| 3 | 62 676 | 66 966 | 33 034 | 95 710 | 57 |
| 4 | 62 703 | 66 999 | 33 001 | 95 704 | 56 |
| 5 | 62 730 | 67 032 | 32 968 | 95 698 | 55 |
| 6 | 62 757 | 67 065 | 32 935 | 95 692 | 54 |
| 7 | 62 784 | 67 098 | 32 902 | 95 686 | 53 |
| 8 | 62 811 | 67 131 | 32 869 | 95 680 | 52 |
| 9 | 62 838 | 67 163 | 32 837 | 95 674 | 51 |
| 10 | 62 865 | 67 196 | 32 804 | 95 668 | 50 |
| 11 | 62 892 | 67 229 | 32 771 | 95 663 | 49 |
| 12 | 62 918 | 67 262 | 32 738 | 95 657 | 48 |
| 13 | 62 945 | 67 295 | 32 705 | 95 651 | 47 |
| 14 | 62 972 | 67 327 | 32 673 | 95 645 | 46 |
| 15 | 62 999 | 67 360 | 32 640 | 95 639 | 45 |
| 16 | 63 026 | 67 393 | 32 607 | 95 633 | 44 |
| 17 | 63 052 | 67 426 | 32 574 | 95 627 | 43 |
| 18 | 63 079 | 67 458 | 32 542 | 95 621 | 42 |
| 19 | 63 106 | 67 491 | 32 509 | 95 615 | 41 |
| 20 | 63 133 | 67 524 | 32 476 | 95 609 | 40 |
| 21 | 63 159 | 67 556 | 32 444 | 95 603 | 39 |
| 22 | 63 186 | 67 589 | 32 411 | 95 597 | 38 |
| 23 | 63 213 | 67 622 | 32 378 | 95 591 | 37 |
| 24 | 63 239 | 67 654 | 32 346 | 95 585 | 36 |
| 25 | 63 266 | 67 687 | 32 313 | 95 579 | 35 |
| 26 | 63 292 | 67 719 | 32 281 | 95 573 | 34 |
| 27 | 63 319 | 67 752 | 32 248 | 95 567 | 33 |
| 28 | 63 345 | 67 785 | 32 215 | 95 561 | 32 |
| 29 | 63 372 | 67 817 | 32 183 | 95 555 | 31 |
| 30 | 63 398 | 67 850 | 32 150 | 95 549 | 30 |
| 31 | 63 425 | 67 882 | 32 118 | 95 543 | 29 |
| 32 | 63 451 | 67 915 | 32 085 | 95 537 | 28 |
| 33 | 63 478 | 67 947 | 32 053 | 95 531 | 27 |
| 34 | 63 504 | 67 980 | 32 020 | 95 525 | 26 |
| 35 | 63 531 | 68 012 | 31 988 | 95 519 | 25 |
| 36 | 63 557 | 68 044 | 31 956 | 95 513 | 24 |
| 37 | 63 583 | 68 077 | 31 923 | 95 507 | 23 |
| 38 | 63 610 | 68 109 | 31 891 | 95 500 | 22 |
| 39 | 63 636 | 68 142 | 31 858 | 95 494 | 21 |
| 40 | 63 662 | 68 174 | 31 826 | 95 488 | 20 |
| 41 | 63 689 | 68 206 | 31 794 | 95 482 | 19 |
| 42 | 63 715 | 68 239 | 31 761 | 95 476 | 18 |
| 43 | 63 741 | 68 271 | 31 729 | 95 470 | 17 |
| 44 | 63 767 | 68 303 | 31 697 | 95 464 | 16 |
| 45 | 63 794 | 68 336 | 31 664 | 95 458 | 15 |
| 46 | 63 820 | 68 368 | 31 632 | 95 452 | 14 |
| 47 | 63 846 | 68 400 | 31 600 | 95 446 | 13 |
| 48 | 63 872 | 68 432 | 31 568 | 95 440 | 12 |
| 49 | 63 898 | 68 465 | 31 535 | 95 434 | 11 |
| 50 | 63 924 | 68 497 | 31 503 | 95 427 | 10 |
| 51 | 63 950 | 68 529 | 31 471 | 95 421 | 9 |
| 52 | 63 976 | 68 561 | 31 439 | 95 415 | 8 |
| 53 | 64 002 | 68 593 | 31 407 | 95 409 | 7 |
| 54 | 64 028 | 68 626 | 31 374 | 95 403 | 6 |
| 55 | 64 054 | 68 658 | 31 342 | 95 397 | 5 |
| 56 | 64 080 | 68 690 | 31 310 | 95 391 | 4 |
| 57 | 64 106 | 68 722 | 31 278 | 95 384 | 3 |
| 58 | 64 132 | 68 754 | 31 246 | 95 378 | 2 |
| 59 | 64 158 | 68 786 | 31 214 | 95 372 | 1 |
| 60 | 64 184 | 68 818 | 31 182 | 95 366 | 0 |
| ° | log cos | log cot | log tan | log sin | ° |

| ° | log sin | log tan | log cot | log cos | ° |
|----|---------|---------|---------|---------|----|
| 0 | 0 | 0 | 10 | 0 | |
| 0 | 64 184 | 68 818 | 31 182 | 95 366 | 60 |
| 1 | 64 210 | 68 850 | 31 150 | 95 360 | 59 |
| 2 | 64 236 | 68 882 | 31 118 | 95 354 | 58 |
| 3 | 64 262 | 68 914 | 31 086 | 95 348 | 57 |
| 4 | 64 288 | 68 946 | 31 054 | 95 341 | 56 |
| 5 | 64 313 | 68 978 | 31 022 | 95 335 | 55 |
| 6 | 64 339 | 69 010 | 30 990 | 95 329 | 54 |
| 7 | 64 365 | 69 042 | 30 958 | 95 323 | 53 |
| 8 | 64 391 | 69 074 | 30 926 | 95 317 | 52 |
| 9 | 64 417 | 69 106 | 30 894 | 95 310 | 51 |
| 10 | 64 442 | 69 138 | 30 862 | 95 304 | 50 |
| 11 | 64 468 | 69 170 | 30 830 | 95 298 | 49 |
| 12 | 64 494 | 69 202 | 30 798 | 95 292 | 48 |
| 13 | 64 519 | 69 234 | 30 766 | 95 286 | 47 |
| 14 | 64 545 | 69 266 | 30 734 | 95 279 | 46 |
| 15 | 64 571 | 69 298 | 30 702 | 95 273 | 45 |
| 16 | 64 596 | 69 329 | 30 671 | 95 267 | 44 |
| 17 | 64 622 | 69 361 | 30 639 | 95 261 | 43 |
| 18 | 64 647 | 69 393 | 30 607 | 95 254 | 42 |
| 19 | 64 673 | 69 425 | 30 575 | 95 248 | 41 |
| 20 | 64 698 | 69 457 | 30 543 | 95 242 | 40 |
| 21 | 64 724 | 69 488 | 30 512 | 95 236 | 39 |
| 22 | 64 749 | 69 520 | 30 480 | 95 229 | 38 |
| 23 | 64 775 | 69 552 | 30 448 | 95 223 | 37 |
| 24 | 64 800 | 69 584 | 30 416 | 95 217 | 36 |
| 25 | 64 826 | 69 615 | 30 385 | 95 211 | 35 |
| 26 | 64 851 | 69 647 | 30 353 | 95 204 | 34 |
| 27 | 64 877 | 69 679 | 30 321 | 95 198 | 33 |
| 28 | 64 902 | 69 710 | 30 290 | 95 192 | 32 |
| 29 | 64 927 | 69 742 | 30 258 | 95 185 | 31 |
| 30 | 64 953 | 69 774 | 30 226 | 95 179 | 30 |
| 31 | 64 978 | 69 805 | 30 195 | 95 173 | 29 |
| 32 | 65 003 | 69 837 | 30 163 | 95 167 | 28 |
| 33 | 65 029 | 69 868 | 30 132 | 95 160 | 27 |
| 34 | 65 054 | 69 900 | 30 100 | 95 154 | 26 |
| 35 | 65 079 | 69 932 | 30 068 | 95 148 | 25 |
| 36 | 65 104 | 69 963 | 30 037 | 95 141 | 24 |
| 37 | 65 130 | 69 995 | 30 005 | 95 135 | 23 |
| 38 | 65 155 | 70 026 | 29 974 | 95 129 | 22 |
| 39 | 65 180 | 70 058 | 29 942 | 95 122 | 21 |
| 40 | 65 205 | 70 089 | 29 911 | 95 116 | 20 |
| 41 | 65 230 | 70 121 | 29 879 | 95 110 | 19 |
| 42 | 65 255 | 70 152 | 29 848 | 95 103 | 18 |
| 43 | 65 281 | 70 184 | 29 816 | 95 097 | 17 |
| 44 | 65 306 | 70 215 | 29 785 | 95 090 | 16 |
| 45 | 65 331 | 70 247 | 29 753 | 95 084 | 15 |
| 46 | 65 356 | 70 278 | 29 722 | 95 078 | 14 |
| 47 | 65 381 | 70 309 | 29 691 | 95 071 | 13 |
| 48 | 65 406 | 70 341 | 29 659 | 95 065 | 12 |
| 49 | 65 431 | 70 372 | 29 628 | 95 059 | 11 |
| 50 | 65 456 | 70 404 | 29 596 | 95 052 | 10 |
| 51 | 65 481 | 70 435 | 29 565 | 95 046 | 9 |
| 52 | 65 506 | 70 466 | 29 534 | 95 039 | 8 |
| 53 | 65 531 | 70 498 | 29 502 | 95 033 | 7 |
| 54 | 65 556 | 70 529 | 29 471 | 95 027 | 6 |
| 55 | 65 580 | 70 560 | 29 440 | 95 020 | 5 |
| 56 | 65 605 | 70 592 | 29 408 | 95 014 | 4 |
| 57 | 65 630 | 70 623 | 29 377 | 95 007 | 3 |
| 58 | 65 655 | 70 654 | 29 346 | 95 001 | 2 |
| 59 | 65 680 | 70 685 | 29 315 | 94 995 | 1 |
| 60 | 65 705 | 70 717 | 29 283 | 94 988 | 0 |
| ° | log cos | log cot | log tan | log sin | ° |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 65 705 | 70 717 | 29 283 | 94 988 | 60 |
| 1 | 65 729 | 70 748 | 29 252 | 94 982 | 59 |
| 2 | 65 754 | 70 779 | 29 221 | 94 975 | 58 |
| 3 | 65 779 | 70 810 | 29 190 | 94 969 | 57 |
| 4 | 65 804 | 70 841 | 29 159 | 94 962 | 56 |
| 5 | 65 828 | 70 873 | 29 127 | 94 956 | 55 |
| 6 | 65 853 | 70 904 | 29 096 | 94 949 | 54 |
| 7 | 65 878 | 70 935 | 29 065 | 94 943 | 53 |
| 8 | 65 902 | 70 966 | 29 034 | 94 936 | 52 |
| 9 | 65 927 | 70 997 | 29 003 | 94 930 | 51 |
| 10 | 65 952 | 71 028 | 28 972 | 94 923 | 50 |
| 11 | 65 976 | 71 059 | 28 941 | 94 917 | 49 |
| 12 | 66 001 | 71 090 | 28 910 | 94 911 | 48 |
| 13 | 66 025 | 71 121 | 28 879 | 94 904 | 47 |
| 14 | 66 050 | 71 153 | 28 847 | 94 898 | 46 |
| 15 | 66 075 | 71 184 | 28 816 | 94 891 | 45 |
| 16 | 66 099 | 71 215 | 28 785 | 94 885 | 44 |
| 17 | 66 124 | 71 246 | 28 754 | 94 878 | 43 |
| 18 | 66 148 | 71 277 | 28 723 | 94 871 | 42 |
| 19 | 66 173 | 71 308 | 28 692 | 94 865 | 41 |
| 20 | 66 197 | 71 339 | 28 661 | 94 858 | 40 |
| 21 | 66 221 | 71 370 | 28 630 | 94 852 | 39 |
| 22 | 66 246 | 71 401 | 28 599 | 94 845 | 38 |
| 23 | 66 270 | 71 431 | 28 569 | 94 839 | 37 |
| 24 | 66 295 | 71 462 | 28 538 | 94 832 | 36 |
| 25 | 66 319 | 71 493 | 28 507 | 94 826 | 35 |
| 26 | 66 343 | 71 524 | 28 476 | 94 819 | 34 |
| 27 | 66 368 | 71 555 | 28 445 | 94 813 | 33 |
| 28 | 66 392 | 71 586 | 28 414 | 94 806 | 32 |
| 29 | 66 416 | 71 617 | 28 383 | 94 799 | 31 |
| 30 | 66 441 | 71 648 | 28 352 | 94 793 | 30 |
| 31 | 66 465 | 71 679 | 28 321 | 94 786 | 29 |
| 32 | 66 489 | 71 709 | 28 291 | 94 780 | 28 |
| 33 | 66 513 | 71 740 | 28 260 | 94 773 | 27 |
| 34 | 66 537 | 71 771 | 28 229 | 94 767 | 26 |
| 35 | 66 562 | 71 802 | 28 198 | 94 760 | 25 |
| 36 | 66 586 | 71 833 | 28 167 | 94 753 | 24 |
| 37 | 66 610 | 71 863 | 28 137 | 94 747 | 23 |
| 38 | 66 634 | 71 894 | 28 106 | 94 740 | 22 |
| 39 | 66 658 | 71 925 | 28 075 | 94 734 | 21 |
| 40 | 66 682 | 71 955 | 28 045 | 94 727 | 20 |
| 41 | 66 706 | 71 986 | 28 014 | 94 720 | 19 |
| 42 | 66 731 | 72 017 | 27 983 | 94 714 | 18 |
| 43 | 66 755 | 72 048 | 27 952 | 94 707 | 17 |
| 44 | 66 779 | 72 078 | 27 922 | 94 700 | 16 |
| 45 | 66 803 | 72 109 | 27 891 | 94 694 | 15 |
| 46 | 66 827 | 72 140 | 27 860 | 94 687 | 14 |
| 47 | 66 851 | 72 170 | 27 830 | 94 680 | 13 |
| 48 | 66 875 | 72 201 | 27 799 | 94 674 | 12 |
| 49 | 66 899 | 72 231 | 27 769 | 94 667 | 11 |
| 50 | 66 922 | 72 262 | 27 738 | 94 660 | 10 |
| 51 | 66 946 | 72 293 | 27 707 | 94 654 | 9 |
| 52 | 66 970 | 72 323 | 27 677 | 94 647 | 8 |
| 53 | 66 994 | 72 354 | 27 646 | 94 640 | 7 |
| 54 | 67 018 | 72 384 | 27 616 | 94 634 | 6 |
| 55 | 67 042 | 72 415 | 27 585 | 94 627 | 5 |
| 56 | 67 066 | 72 445 | 27 555 | 94 620 | 4 |
| 57 | 67 090 | 72 476 | 27 524 | 94 614 | 3 |
| 58 | 67 113 | 72 506 | 27 494 | 94 607 | 2 |
| 59 | 67 137 | 72 537 | 27 463 | 94 600 | 1 |
| 60 | 67 161 | 72 567 | 27 433 | 94 593 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 67 161 | 72 567 | 27 433 | 94 593 | 60 |
| 1 | 67 185 | 72 598 | 27 402 | 94 587 | 59 |
| 2 | 67 208 | 72 628 | 27 372 | 94 580 | 58 |
| 3 | 67 232 | 72 659 | 27 341 | 94 573 | 57 |
| 4 | 67 256 | 72 689 | 27 311 | 94 567 | 56 |
| 5 | 67 280 | 72 720 | 27 280 | 94 560 | 55 |
| 6 | 67 303 | 72 750 | 27 250 | 94 553 | 54 |
| 7 | 67 327 | 72 780 | 27 220 | 94 546 | 53 |
| 8 | 67 350 | 72 811 | 27 189 | 94 540 | 52 |
| 9 | 67 374 | 72 841 | 27 159 | 94 533 | 51 |
| 10 | 67 398 | 72 872 | 27 128 | 94 526 | 50 |
| 11 | 67 421 | 72 902 | 27 098 | 94 519 | 49 |
| 12 | 67 445 | 72 932 | 27 068 | 94 513 | 48 |
| 13 | 67 468 | 72 963 | 27 037 | 94 506 | 47 |
| 14 | 67 492 | 72 993 | 27 007 | 94 499 | 46 |
| 15 | 67 515 | 73 023 | 26 977 | 94 492 | 45 |
| 16 | 67 539 | 73 054 | 26 946 | 94 485 | 44 |
| 17 | 67 562 | 73 084 | 26 916 | 94 479 | 43 |
| 18 | 67 586 | 73 114 | 26 886 | 94 472 | 42 |
| 19 | 67 609 | 73 144 | 26 856 | 94 465 | 41 |
| 20 | 67 633 | 73 175 | 26 825 | 94 458 | 40 |
| 21 | 67 656 | 73 205 | 26 795 | 94 451 | 39 |
| 22 | 67 680 | 73 235 | 26 765 | 94 445 | 38 |
| 23 | 67 703 | 73 265 | 26 735 | 94 438 | 37 |
| 24 | 67 726 | 73 295 | 26 705 | 94 431 | 36 |
| 25 | 67 750 | 73 326 | 26 674 | 94 424 | 35 |
| 26 | 67 773 | 73 356 | 26 644 | 94 417 | 34 |
| 27 | 67 796 | 73 386 | 26 614 | 94 410 | 33 |
| 28 | 67 820 | 73 416 | 26 584 | 94 404 | 32 |
| 29 | 67 843 | 73 446 | 26 554 | 94 397 | 31 |
| 30 | 67 866 | 73 476 | 26 524 | 94 390 | 30 |
| 31 | 67 890 | 73 507 | 26 493 | 94 383 | 29 |
| 32 | 67 913 | 73 537 | 26 463 | 94 376 | 28 |
| 33 | 67 936 | 73 567 | 26 433 | 94 369 | 27 |
| 34 | 67 959 | 73 597 | 26 403 | 94 362 | 26 |
| 35 | 67 982 | 73 627 | 26 373 | 94 355 | 25 |
| 36 | 68 006 | 73 657 | 26 343 | 94 349 | 24 |
| 37 | 68 029 | 73 687 | 26 313 | 94 342 | 23 |
| 38 | 68 052 | 73 717 | 26 283 | 94 335 | 22 |
| 39 | 68 075 | 73 747 | 26 253 | 94 328 | 21 |
| 40 | 68 098 | 73 777 | 26 223 | 94 321 | 20 |
| 41 | 68 121 | 73 807 | 26 193 | 94 314 | 19 |
| 42 | 68 144 | 73 837 | 26 163 | 94 307 | 18 |
| 43 | 68 167 | 73 867 | 26 133 | 94 300 | 17 |
| 44 | 68 190 | 73 897 | 26 103 | 94 293 | 16 |
| 45 | 68 213 | 73 927 | 26 073 | 94 286 | 15 |
| 46 | 68 237 | 73 957 | 26 043 | 94 279 | 14 |
| 47 | 68 260 | 73 987 | 26 013 | 94 273 | 13 |
| 48 | 68 283 | 74 017 | 25 983 | 94 266 | 12 |
| 49 | 68 305 | 74 047 | 25 953 | 94 259 | 11 |
| 50 | 68 328 | 74 077 | 25 923 | 94 252 | 10 |
| 51 | 68 351 | 74 107 | 25 893 | 94 245 | 9 |
| 52 | 68 374 | 74 137 | 25 863 | 94 238 | 8 |
| 53 | 68 397 | 74 166 | 25 834 | 94 231 | 7 |
| 54 | 68 420 | 74 196 | 25 804 | 94 224 | 6 |
| 55 | 68 443 | 74 226 | 25 774 | 94 217 | 5 |
| 56 | 68 466 | 74 256 | 25 744 | 94 210 | 4 |
| 57 | 68 489 | 74 286 | 25 714 | 94 203 | 3 |
| 58 | 68 512 | 74 316 | 25 684 | 94 196 | 2 |
| 59 | 68 534 | 74 345 | 25 655 | 94 189 | 1 |
| 60 | 68 557 | 74 375 | 25 625 | 94 182 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 68 557 | 74 375 | 25 625 | 94 182 | 60 |
| 1 | 68 580 | 74 405 | 25 595 | 94 175 | 59 |
| 2 | 68 603 | 74 435 | 25 565 | 94 168 | 58 |
| 3 | 68 625 | 74 465 | 25 535 | 94 161 | 57 |
| 4 | 68 648 | 74 494 | 25 506 | 94 154 | 56 |
| 5 | 68 671 | 74 524 | 25 476 | 94 147 | 55 |
| 6 | 68 694 | 74 554 | 25 446 | 94 140 | 54 |
| 7 | 68 716 | 74 583 | 25 417 | 94 133 | 53 |
| 8 | 68 739 | 74 613 | 25 387 | 94 126 | 52 |
| 9 | 68 762 | 74 643 | 25 357 | 94 119 | 51 |
| 10 | 68 784 | 74 673 | 25 327 | 94 112 | 50 |
| 11 | 68 807 | 74 702 | 25 298 | 94 105 | 49 |
| 12 | 68 829 | 74 732 | 25 268 | 94 098 | 48 |
| 13 | 68 852 | 74 762 | 25 238 | 94 090 | 47 |
| 14 | 68 875 | 74 791 | 25 209 | 94 083 | 46 |
| 15 | 68 897 | 74 821 | 25 179 | 94 076 | 45 |
| 16 | 68 920 | 74 851 | 25 149 | 94 069 | 44 |
| 17 | 68 942 | 74 880 | 25 120 | 94 062 | 43 |
| 18 | 68 965 | 74 910 | 25 090 | 94 055 | 42 |
| 19 | 68 987 | 74 939 | 25 061 | 94 048 | 41 |
| 20 | 69 010 | 74 969 | 25 031 | 94 041 | 40 |
| 21 | 69 032 | 74 998 | 25 002 | 94 034 | 39 |
| 22 | 69 055 | 75 028 | 24 972 | 94 027 | 38 |
| 23 | 69 077 | 75 058 | 24 942 | 94 020 | 37 |
| 24 | 69 100 | 75 087 | 24 913 | 94 012 | 36 |
| 25 | 69 122 | 75 117 | 24 883 | 94 005 | 35 |
| 26 | 69 144 | 75 146 | 24 854 | 93 998 | 34 |
| 27 | 69 167 | 75 176 | 24 824 | 93 991 | 33 |
| 28 | 69 189 | 75 205 | 24 795 | 93 984 | 32 |
| 29 | 69 212 | 75 235 | 24 765 | 93 977 | 31 |
| 30 | 69 234 | 75 264 | 24 736 | 93 970 | 30 |
| 31 | 69 256 | 75 294 | 24 706 | 93 963 | 29 |
| 32 | 69 279 | 75 323 | 24 677 | 93 955 | 28 |
| 33 | 69 301 | 75 353 | 24 647 | 93 948 | 27 |
| 34 | 69 323 | 75 382 | 24 618 | 93 941 | 26 |
| 35 | 69 345 | 75 411 | 24 589 | 93 934 | 25 |
| 36 | 69 368 | 75 441 | 24 559 | 93 927 | 24 |
| 37 | 69 390 | 75 470 | 24 530 | 93 920 | 23 |
| 38 | 69 412 | 75 500 | 24 500 | 93 912 | 22 |
| 39 | 69 434 | 75 529 | 24 471 | 93 905 | 21 |
| 40 | 69 456 | 75 558 | 24 442 | 93 898 | 20 |
| 41 | 69 479 | 75 588 | 24 412 | 93 891 | 19 |
| 42 | 69 501 | 75 617 | 24 383 | 93 884 | 18 |
| 43 | 69 523 | 75 647 | 24 353 | 93 876 | 17 |
| 44 | 69 545 | 75 676 | 24 324 | 93 869 | 16 |
| 45 | 69 567 | 75 705 | 24 295 | 93 862 | 15 |
| 46 | 69 589 | 75 735 | 24 265 | 93 855 | 14 |
| 47 | 69 611 | 75 764 | 24 236 | 93 847 | 13 |
| 48 | 69 633 | 75 793 | 24 207 | 93 840 | 12 |
| 49 | 69 655 | 75 822 | 24 178 | 93 833 | 11 |
| 50 | 69 677 | 75 852 | 24 148 | 93 826 | 10 |
| 51 | 69 699 | 75 881 | 24 119 | 93 819 | 9 |
| 52 | 69 721 | 75 910 | 24 090 | 93 811 | 8 |
| 53 | 69 743 | 75 939 | 24 061 | 93 804 | 7 |
| 54 | 69 765 | 75 969 | 24 031 | 93 797 | 6 |
| 55 | 69 787 | 75 998 | 24 002 | 93 789 | 5 |
| 56 | 69 809 | 76 027 | 23 973 | 93 782 | 4 |
| 57 | 69 831 | 76 056 | 23 944 | 93 775 | 3 |
| 58 | 69 853 | 76 086 | 23 914 | 93 768 | 2 |
| 59 | 69 875 | 76 115 | 23 885 | 93 760 | 1 |
| 60 | 69 897 | 76 144 | 23 856 | 93 753 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 69 897 | 76 144 | 23 856 | 93 753 | 60 |
| 1 | 69 919 | 76 173 | 23 827 | 93 746 | 59 |
| 2 | 69 941 | 76 202 | 23 798 | 93 738 | 58 |
| 3 | 69 963 | 76 231 | 23 769 | 93 731 | 57 |
| 4 | 69 984 | 76 261 | 23 739 | 93 724 | 56 |
| 5 | 70 006 | 76 290 | 23 710 | 93 717 | 55 |
| 6 | 70 028 | 76 319 | 23 681 | 93 709 | 54 |
| 7 | 70 050 | 76 348 | 23 652 | 93 702 | 53 |
| 8 | 70 072 | 76 377 | 23 623 | 93 695 | 52 |
| 9 | 70 093 | 76 406 | 23 594 | 93 687 | 51 |
| 10 | 70 115 | 76 435 | 23 565 | 93 680 | 50 |
| 11 | 70 137 | 76 464 | 23 536 | 93 673 | 49 |
| 12 | 70 159 | 76 493 | 23 507 | 93 665 | 48 |
| 13 | 70 180 | 76 522 | 23 478 | 93 658 | 47 |
| 14 | 70 202 | 76 551 | 23 449 | 93 650 | 46 |
| 15 | 70 224 | 76 580 | 23 420 | 93 643 | 45 |
| 16 | 70 245 | 76 609 | 23 391 | 93 636 | 44 |
| 17 | 70 267 | 76 639 | 23 361 | 93 628 | 43 |
| 18 | 70 288 | 76 668 | 23 332 | 93 621 | 42 |
| 19 | 70 310 | 76 697 | 23 303 | 93 614 | 41 |
| 20 | 70 332 | 76 725 | 23 275 | 93 606 | 40 |
| 21 | 70 353 | 76 754 | 23 246 | 93 599 | 39 |
| 22 | 70 375 | 76 783 | 23 217 | 93 591 | 38 |
| 23 | 70 396 | 76 812 | 23 188 | 93 584 | 37 |
| 24 | 70 418 | 76 841 | 23 159 | 93 577 | 36 |
| 25 | 70 439 | 76 870 | 23 130 | 93 569 | 35 |
| 26 | 70 461 | 76 899 | 23 101 | 93 562 | 34 |
| 27 | 70 482 | 76 928 | 23 072 | 93 554 | 33 |
| 28 | 70 504 | 76 957 | 23 043 | 93 547 | 32 |
| 29 | 70 525 | 76 986 | 23 014 | 93 539 | 31 |
| 30 | 70 547 | 77 015 | 22 985 | 93 532 | 30 |
| 31 | 70 568 | 77 044 | 22 956 | 93 525 | 29 |
| 32 | 70 590 | 77 073 | 22 927 | 93 517 | 28 |
| 33 | 70 611 | 77 101 | 22 899 | 93 510 | 27 |
| 34 | 70 633 | 77 130 | 22 870 | 93 502 | 26 |
| 35 | 70 654 | 77 159 | 22 841 | 93 495 | 25 |
| 36 | 70 675 | 77 188 | 22 812 | 93 487 | 24 |
| 37 | 70 697 | 77 217 | 22 783 | 93 480 | 23 |
| 38 | 70 718 | 77 246 | 22 754 | 93 472 | 22 |
| 39 | 70 739 | 77 274 | 22 726 | 93 465 | 21 |
| 40 | 70 761 | 77 303 | 22 697 | 93 457 | 20 |
| 41 | 70 782 | 77 332 | 22 668 | 93 450 | 19 |
| 42 | 70 803 | 77 361 | 22 639 | 93 442 | 18 |
| 43 | 70 824 | 77 390 | 22 610 | 93 435 | 17 |
| 44 | 70 846 | 77 418 | 22 582 | 93 427 | 16 |
| 45 | 70 867 | 77 447 | 22 553 | 93 420 | 15 |
| 46 | 70 888 | 77 476 | 22 524 | 93 412 | 14 |
| 47 | 70 909 | 77 505 | 22 495 | 93 405 | 13 |
| 48 | 70 931 | 77 533 | 22 467 | 93 397 | 12 |
| 49 | 70 952 | 77 562 | 22 438 | 93 390 | 11 |
| 50 | 70 973 | 77 591 | 22 409 | 93 382 | 10 |
| 51 | 70 994 | 77 619 | 22 381 | 93 375 | 9 |
| 52 | 71 015 | 77 648 | 22 352 | 93 367 | 8 |
| 53 | 71 036 | 77 677 | 22 323 | 93 360 | 7 |
| 54 | 71 058 | 77 706 | 22 294 | 93 352 | 6 |
| 55 | 71 079 | 77 734 | 22 266 | 93 344 | 5 |
| 56 | 71 100 | 77 763 | 22 237 | 93 337 | 4 |
| 57 | 71 121 | 77 791 | 22 209 | 93 329 | 3 |
| 58 | 71 142 | 77 820 | 22 180 | 93 322 | 2 |
| 59 | 71 163 | 77 849 | 22 151 | 93 314 | 1 |
| 60 | 71 184 | 77 877 | 22 123 | 93 307 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 71 184 | 77 877 | 22 123 | 93 307 | 60 |
| 1 | 71 205 | 77 906 | 22 094 | 93 299 | 59 |
| 2 | 71 226 | 77 935 | 22 065 | 93 291 | 58 |
| 3 | 71 247 | 77 963 | 22 037 | 93 284 | 57 |
| 4 | 71 268 | 77 992 | 22 008 | 93 276 | 56 |
| 5 | 71 289 | 78 020 | 21 980 | 93 269 | 55 |
| 6 | 71 310 | 78 049 | 21 951 | 93 261 | 54 |
| 7 | 71 331 | 78 077 | 21 923 | 93 253 | 53 |
| 8 | 71 352 | 78 106 | 21 894 | 93 246 | 52 |
| 9 | 71 373 | 78 135 | 21 865 | 93 238 | 51 |
| 10 | 71 393 | 78 163 | 21 837 | 93 230 | 50 |
| 11 | 71 414 | 78 192 | 21 808 | 93 223 | 49 |
| 12 | 71 435 | 78 220 | 21 780 | 93 215 | 48 |
| 13 | 71 456 | 78 249 | 21 751 | 93 207 | 47 |
| 14 | 71 477 | 78 277 | 21 723 | 93 200 | 46 |
| 15 | 71 498 | 78 306 | 21 694 | 93 192 | 45 |
| 16 | 71 519 | 78 334 | 21 666 | 93 184 | 44 |
| 17 | 71 539 | 78 363 | 21 637 | 93 177 | 43 |
| 18 | 71 560 | 78 391 | 21 609 | 93 169 | 42 |
| 19 | 71 581 | 78 419 | 21 581 | 93 161 | 41 |
| 20 | 71 602 | 78 448 | 21 552 | 93 154 | 40 |
| 21 | 71 622 | 78 476 | 21 524 | 93 146 | 39 |
| 22 | 71 643 | 78 505 | 21 495 | 93 138 | 38 |
| 23 | 71 664 | 78 533 | 21 467 | 93 131 | 37 |
| 24 | 71 685 | 78 562 | 21 438 | 93 123 | 36 |
| 25 | 71 705 | 78 590 | 21 410 | 93 115 | 35 |
| 26 | 71 726 | 78 618 | 21 382 | 93 108 | 34 |
| 27 | 71 747 | 78 647 | 21 353 | 93 100 | 33 |
| 28 | 71 767 | 78 675 | 21 325 | 93 092 | 32 |
| 29 | 71 788 | 78 704 | 21 296 | 93 084 | 31 |
| 30 | 71 809 | 78 732 | 21 268 | 93 077 | 30 |
| 31 | 71 829 | 78 760 | 21 240 | 93 069 | 29 |
| 32 | 71 850 | 78 789 | 21 211 | 93 061 | 28 |
| 33 | 71 870 | 78 817 | 21 183 | 93 053 | 27 |
| 34 | 71 891 | 78 845 | 21 155 | 93 046 | 26 |
| 35 | 71 911 | 78 874 | 21 126 | 93 038 | 25 |
| 36 | 71 932 | 78 902 | 21 098 | 93 030 | 24 |
| 37 | 71 952 | 78 930 | 21 070 | 93 022 | 23 |
| 38 | 71 973 | 78 959 | 21 041 | 93 014 | 22 |
| 39 | 71 994 | 78 987 | 21 013 | 93 007 | 21 |
| 40 | 72 014 | 79 015 | 20 985 | 92 999 | 20 |
| 41 | 72 034 | 79 043 | 20 957 | 92 991 | 19 |
| 42 | 72 055 | 79 072 | 20 928 | 92 983 | 18 |
| 43 | 72 075 | 79 100 | 20 900 | 92 976 | 17 |
| 44 | 72 096 | 79 128 | 20 872 | 92 968 | 16 |
| 45 | 72 116 | 79 156 | 20 844 | 92 960 | 15 |
| 46 | 72 137 | 79 185 | 20 815 | 92 952 | 14 |
| 47 | 72 157 | 79 213 | 20 787 | 92 944 | 13 |
| 48 | 72 177 | 79 241 | 20 759 | 92 936 | 12 |
| 49 | 72 198 | 79 269 | 20 731 | 92 929 | 11 |
| 50 | 72 218 | 79 297 | 20 703 | 92 921 | 10 |
| 51 | 72 238 | 79 326 | 20 674 | 92 913 | 9 |
| 52 | 72 259 | 79 354 | 20 646 | 92 905 | 8 |
| 53 | 72 279 | 79 382 | 20 618 | 92 897 | 7 |
| 54 | 72 299 | 79 410 | 20 590 | 92 889 | 6 |
| 55 | 72 320 | 79 438 | 20 562 | 92 881 | 5 |
| 56 | 72 340 | 79 466 | 20 534 | 92 874 | 4 |
| 57 | 72 360 | 79 495 | 20 505 | 92 866 | 3 |
| 58 | 72 381 | 79 523 | 20 477 | 92 858 | 2 |
| 59 | 72 401 | 79 551 | 20 449 | 92 850 | 1 |
| 60 | 72 421 | 79 579 | 20 421 | 92 842 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 72 421 | 79 579 | 20 421 | 92 842 | 60 |
| 1 | 72 441 | 79 607 | 20 393 | 92 834 | 59 |
| 2 | 72 461 | 79 635 | 20 365 | 92 826 | 58 |
| 3 | 72 482 | 79 663 | 20 337 | 92 818 | 57 |
| 4 | 72 502 | 79 691 | 20 309 | 92 810 | 56 |
| 5 | 72 522 | 79 719 | 20 281 | 92 803 | 55 |
| 6 | 72 542 | 79 747 | 20 253 | 92 795 | 54 |
| 7 | 72 562 | 79 776 | 20 224 | 92 787 | 53 |
| 8 | 72 582 | 79 804 | 20 196 | 92 779 | 52 |
| 9 | 72 602 | 79 832 | 20 168 | 92 771 | 51 |
| 10 | 72 622 | 79 860 | 20 140 | 92 763 | 50 |
| 11 | 72 643 | 79 888 | 20 112 | 92 755 | 49 |
| 12 | 72 663 | 79 916 | 20 084 | 92 747 | 48 |
| 13 | 72 683 | 79 944 | 20 056 | 92 739 | 47 |
| 14 | 72 703 | 79 972 | 20 028 | 92 731 | 46 |
| 15 | 72 723 | 80 000 | 20 000 | 92 723 | 45 |
| 16 | 72 743 | 80 028 | 19 972 | 92 715 | 44 |
| 17 | 72 763 | 80 056 | 19 944 | 92 707 | 43 |
| 18 | 72 783 | 80 084 | 19 916 | 92 699 | 42 |
| 19 | 72 803 | 80 112 | 19 888 | 92 691 | 41 |
| 20 | 72 823 | 80 140 | 19 860 | 92 683 | 40 |
| 21 | 72 843 | 80 168 | 19 832 | 92 675 | 39 |
| 22 | 72 863 | 80 195 | 19 805 | 92 667 | 38 |
| 23 | 72 883 | 80 223 | 19 777 | 92 659 | 37 |
| 24 | 72 902 | 80 251 | 19 749 | 92 651 | 36 |
| 25 | 72 922 | 80 279 | 19 721 | 92 643 | 35 |
| 26 | 72 942 | 80 307 | 19 693 | 92 635 | 34 |
| 27 | 72 962 | 80 335 | 19 665 | 92 627 | 33 |
| 28 | 72 982 | 80 363 | 19 637 | 92 619 | 32 |
| 29 | 73 002 | 80 391 | 19 609 | 92 611 | 31 |
| 30 | 73 022 | 80 419 | 19 581 | 92 603 | 30 |
| 31 | 73 041 | 80 447 | 19 553 | 92 595 | 29 |
| 32 | 73 061 | 80 474 | 19 526 | 92 587 | 28 |
| 33 | 73 081 | 80 502 | 19 498 | 92 579 | 27 |
| 34 | 73 101 | 80 530 | 19 470 | 92 571 | 26 |
| 35 | 73 121 | 80 558 | 19 442 | 92 563 | 25 |
| 36 | 73 140 | 80 586 | 19 414 | 92 555 | 24 |
| 37 | 73 160 | 80 614 | 19 386 | 92 546 | 23 |
| 38 | 73 180 | 80 642 | 19 358 | 92 538 | 22 |
| 39 | 73 200 | 80 669 | 19 331 | 92 530 | 21 |
| 40 | 73 219 | 80 697 | 19 303 | 92 522 | 20 |
| 41 | 73 239 | 80 725 | 19 275 | 92 514 | 19 |
| 42 | 73 259 | 80 753 | 19 247 | 92 506 | 18 |
| 43 | 73 278 | 80 781 | 19 219 | 92 498 | 17 |
| 44 | 73 298 | 80 808 | 19 192 | 92 490 | 16 |
| 45 | 73 318 | 80 836 | 19 164 | 92 482 | 15 |
| 46 | 73 337 | 80 864 | 19 136 | 92 473 | 14 |
| 47 | 73 357 | 80 892 | 19 108 | 92 465 | 13 |
| 48 | 73 377 | 80 919 | 19 081 | 92 457 | 12 |
| 49 | 73 396 | 80 947 | 19 053 | 92 449 | 11 |
| 50 | 73 416 | 80 975 | 19 025 | 92 441 | 10 |
| 51 | 73 435 | 81 003 | 18 997 | 92 433 | 9 |
| 52 | 73 455 | 81 030 | 18 970 | 92 425 | 8 |
| 53 | 73 474 | 81 058 | 18 942 | 92 416 | 7 |
| 54 | 73 494 | 81 086 | 18 914 | 92 408 | 6 |
| 55 | 73 513 | 81 113 | 18 887 | 92 400 | 5 |
| 56 | 73 533 | 81 141 | 18 859 | 92 392 | 4 |
| 57 | 73 552 | 81 169 | 18 831 | 92 384 | 3 |
| 58 | 73 572 | 81 196 | 18 804 | 92 376 | 2 |
| 59 | 73 591 | 81 224 | 18 776 | 92 367 | 1 |
| 60 | 73 611 | 81 252 | 18 748 | 92 359 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 73 611 | 81 252 | 18 748 | 92 359 | 60 |
| 1 | 73 630 | 81 279 | 18 721 | 92 351 | 59 |
| 2 | 73 650 | 81 307 | 18 693 | 92 343 | 58 |
| 3 | 73 669 | 81 335 | 18 665 | 92 335 | 57 |
| 4 | 73 689 | 81 362 | 18 638 | 92 326 | 56 |
| 5 | 73 708 | 81 390 | 18 610 | 92 318 | 55 |
| 6 | 73 727 | 81 418 | 18 582 | 92 310 | 54 |
| 7 | 73 747 | 81 445 | 18 555 | 92 302 | 53 |
| 8 | 73 766 | 81 473 | 18 527 | 92 293 | 52 |
| 9 | 73 785 | 81 500 | 18 500 | 92 285 | 51 |
| 10 | 73 805 | 81 528 | 18 472 | 92 277 | 50 |
| 11 | 73 824 | 81 556 | 18 444 | 92 269 | 49 |
| 12 | 73 843 | 81 583 | 18 417 | 92 260 | 48 |
| 13 | 73 863 | 81 611 | 18 389 | 92 252 | 47 |
| 14 | 73 882 | 81 638 | 18 362 | 92 244 | 46 |
| 15 | 73 901 | 81 666 | 18 334 | 92 235 | 45 |
| 16 | 73 921 | 81 693 | 18 307 | 92 227 | 44 |
| 17 | 73 940 | 81 721 | 18 279 | 92 219 | 43 |
| 18 | 73 959 | 81 748 | 18 252 | 92 211 | 42 |
| 19 | 73 978 | 81 776 | 18 224 | 92 202 | 41 |
| 20 | 73 997 | 81 803 | 18 197 | 92 194 | 40 |
| 21 | 74 017 | 81 831 | 18 169 | 92 186 | 39 |
| 22 | 74 036 | 81 858 | 18 142 | 92 177 | 38 |
| 23 | 74 055 | 81 886 | 18 114 | 92 169 | 37 |
| 24 | 74 074 | 81 913 | 18 087 | 92 161 | 36 |
| 25 | 74 093 | 81 941 | 18 059 | 92 152 | 35 |
| 26 | 74 113 | 81 968 | 18 032 | 92 144 | 34 |
| 27 | 74 132 | 81 996 | 18 004 | 92 136 | 33 |
| 28 | 74 151 | 82 023 | 17 977 | 92 127 | 32 |
| 29 | 74 170 | 82 051 | 17 949 | 92 119 | 31 |
| 30 | 74 189 | 82 078 | 17 922 | 92 111 | 30 |
| 31 | 74 208 | 82 106 | 17 894 | 92 102 | 29 |
| 32 | 74 227 | 82 133 | 17 867 | 92 094 | 28 |
| 33 | 74 246 | 82 161 | 17 839 | 92 086 | 27 |
| 34 | 74 265 | 82 188 | 17 812 | 92 077 | 26 |
| 35 | 74 284 | 82 215 | 17 785 | 92 069 | 25 |
| 36 | 74 303 | 82 243 | 17 757 | 92 060 | 24 |
| 37 | 74 322 | 82 270 | 17 730 | 92 052 | 23 |
| 38 | 74 341 | 82 298 | 17 702 | 92 044 | 22 |
| 39 | 74 360 | 82 325 | 17 675 | 92 035 | 21 |
| 40 | 74 379 | 82 352 | 17 648 | 92 027 | 20 |
| 41 | 74 398 | 82 380 | 17 620 | 92 018 | 19 |
| 42 | 74 417 | 82 407 | 17 593 | 92 010 | 18 |
| 43 | 74 436 | 82 435 | 17 565 | 92 002 | 17 |
| 44 | 74 455 | 82 462 | 17 538 | 91 993 | 16 |
| 45 | 74 474 | 82 489 | 17 511 | 91 985 | 15 |
| 46 | 74 493 | 82 517 | 17 483 | 91 976 | 14 |
| 47 | 74 512 | 82 544 | 17 456 | 91 968 | 13 |
| 48 | 74 531 | 82 571 | 17 429 | 91 959 | 12 |
| 49 | 74 549 | 82 599 | 17 401 | 91 951 | 11 |
| 50 | 74 568 | 82 626 | 17 374 | 91 942 | 10 |
| 51 | 74 587 | 82 653 | 17 347 | 91 934 | 9 |
| 52 | 74 606 | 82 681 | 17 319 | 91 925 | 8 |
| 53 | 74 625 | 82 708 | 17 292 | 91 917 | 7 |
| 54 | 74 644 | 82 735 | 17 265 | 91 908 | 6 |
| 55 | 74 662 | 82 762 | 17 238 | 91 900 | 5 |
| 56 | 74 681 | 82 790 | 17 210 | 91 891 | 4 |
| 57 | 74 700 | 82 817 | 17 183 | 91 883 | 3 |
| 58 | 74 719 | 82 844 | 17 156 | 91 874 | 2 |
| 59 | 74 737 | 82 871 | 17 129 | 91 866 | 1 |
| 60 | 74 756 | 82 899 | 17 101 | 91 857 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 74 756 | 82 899 | 17 101 | 91 857 | 60 |
| 1 | 74 775 | 82 926 | 17 074 | 91 849 | 59 |
| 2 | 74 794 | 82 953 | 17 047 | 91 840 | 58 |
| 3 | 74 812 | 82 980 | 17 020 | 91 832 | 57 |
| 4 | 74 831 | 83 008 | 16 992 | 91 823 | 56 |
| 5 | 74 850 | 83 035 | 16 965 | 91 815 | 55 |
| 6 | 74 868 | 83 062 | 16 938 | 91 806 | 54 |
| 7 | 74 887 | 83 089 | 16 911 | 91 798 | 53 |
| 8 | 74 906 | 83 117 | 16 883 | 91 789 | 52 |
| 9 | 74 924 | 83 144 | 16 856 | 91 781 | 51 |
| 10 | 74 943 | 83 171 | 16 829 | 91 772 | 50 |
| 11 | 74 961 | 83 198 | 16 802 | 91 763 | 49 |
| 12 | 74 980 | 83 225 | 16 775 | 91 755 | 48 |
| 13 | 74 999 | 83 252 | 16 748 | 91 746 | 47 |
| 14 | 75 017 | 83 280 | 16 720 | 91 738 | 46 |
| 15 | 75 036 | 83 307 | 16 693 | 91 729 | 45 |
| 16 | 75 054 | 83 334 | 16 666 | 91 720 | 44 |
| 17 | 75 073 | 83 361 | 16 639 | 91 712 | 43 |
| 18 | 75 091 | 83 388 | 16 612 | 91 703 | 42 |
| 19 | 75 110 | 83 415 | 16 585 | 91 695 | 41 |
| 20 | 75 128 | 83 442 | 16 558 | 91 686 | 40 |
| 21 | 75 147 | 83 470 | 16 530 | 91 677 | 39 |
| 22 | 75 165 | 83 497 | 16 503 | 91 669 | 38 |
| 23 | 75 184 | 83 524 | 16 476 | 91 660 | 37 |
| 24 | 75 202 | 83 551 | 16 449 | 91 651 | 36 |
| 25 | 75 221 | 83 578 | 16 422 | 91 643 | 35 |
| 26 | 75 239 | 83 605 | 16 395 | 91 634 | 34 |
| 27 | 75 258 | 83 632 | 16 368 | 91 625 | 33 |
| 28 | 75 276 | 83 659 | 16 341 | 91 617 | 32 |
| 29 | 75 294 | 83 686 | 16 314 | 91 608 | 31 |
| 30 | 75 313 | 83 713 | 16 287 | 91 599 | 30 |
| 31 | 75 331 | 83 740 | 16 260 | 91 591 | 29 |
| 32 | 75 350 | 83 768 | 16 232 | 91 582 | 28 |
| 33 | 75 368 | 83 795 | 16 205 | 91 573 | 27 |
| 34 | 75 386 | 83 822 | 16 178 | 91 565 | 26 |
| 35 | 75 405 | 83 849 | 16 151 | 91 556 | 25 |
| 36 | 75 423 | 83 876 | 16 124 | 91 547 | 24 |
| 37 | 75 441 | 83 903 | 16 097 | 91 538 | 23 |
| 38 | 75 459 | 83 930 | 16 070 | 91 530 | 22 |
| 39 | 75 478 | 83 957 | 16 043 | 91 521 | 21 |
| 40 | 75 496 | 83 984 | 16 016 | 91 512 | 20 |
| 41 | 75 514 | 84 011 | 15 989 | 91 504 | 19 |
| 42 | 75 533 | 84 038 | 15 962 | 91 495 | 18 |
| 43 | 75 551 | 84 065 | 15 935 | 91 486 | 17 |
| 44 | 75 569 | 84 092 | 15 908 | 91 477 | 16 |
| 45 | 75 587 | 84 119 | 15 881 | 91 469 | 15 |
| 46 | 75 605 | 84 146 | 15 854 | 91 460 | 14 |
| 47 | 75 624 | 84 173 | 15 827 | 91 451 | 13 |
| 48 | 75 642 | 84 200 | 15 800 | 91 442 | 12 |
| 49 | 75 660 | 84 227 | 15 773 | 91 433 | 11 |
| 50 | 75 678 | 84 254 | 15 746 | 91 425 | 10 |
| 51 | 75 696 | 84 280 | 15 720 | 91 416 | 9 |
| 52 | 75 714 | 84 307 | 15 693 | 91 407 | 8 |
| 53 | 75 733 | 84 334 | 15 666 | 91 398 | 7 |
| 54 | 75 751 | 84 361 | 15 639 | 91 389 | 6 |
| 55 | 75 769 | 84 388 | 15 612 | 91 381 | 5 |
| 56 | 75 787 | 84 415 | 15 585 | 91 372 | 4 |
| 57 | 75 805 | 84 442 | 15 558 | 91 363 | 3 |
| 58 | 75 823 | 84 469 | 15 531 | 91 354 | 2 |
| 59 | 75 841 | 84 496 | 15 504 | 91 345 | 1 |
| 60 | 75 859 | 84 523 | 15 477 | 91 336 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

| ' | log sin | log tan | log cot | log cos | ' |
|----|---------|---------|---------|---------|----|
| | | 9 | 10 | 9 | |
| 0 | 75 859 | 84 523 | 15 477 | 91 336 | 60 |
| 1 | 75 877 | 84 550 | 15 450 | 91 328 | 59 |
| 2 | 75 895 | 84 576 | 15 424 | 91 319 | 58 |
| 3 | 75 913 | 84 603 | 15 397 | 91 310 | 57 |
| 4 | 75 931 | 84 630 | 15 370 | 91 301 | 56 |
| 5 | 75 949 | 84 657 | 15 343 | 91 292 | 55 |
| 6 | 75 967 | 84 684 | 15 316 | 91 283 | 54 |
| 7 | 75 985 | 84 711 | 15 289 | 91 274 | 53 |
| 8 | 76 003 | 84 738 | 15 262 | 91 266 | 52 |
| 9 | 76 021 | 84 764 | 15 236 | 91 257 | 51 |
| 10 | 76 039 | 84 791 | 15 209 | 91 248 | 50 |
| 11 | 76 057 | 84 818 | 15 182 | 91 239 | 49 |
| 12 | 76 075 | 84 845 | 15 155 | 91 230 | 48 |
| 13 | 76 093 | 84 872 | 15 128 | 91 221 | 47 |
| 14 | 76 111 | 84 899 | 15 101 | 91 212 | 46 |
| 15 | 76 129 | 84 925 | 15 075 | 91 203 | 45 |
| 16 | 76 146 | 84 952 | 15 048 | 91 194 | 44 |
| 17 | 76 164 | 84 979 | 15 021 | 91 185 | 43 |
| 18 | 76 182 | 85 006 | 14 994 | 91 176 | 42 |
| 19 | 76 200 | 85 033 | 14 967 | 91 167 | 41 |
| 20 | 76 218 | 85 059 | 14 941 | 91 158 | 40 |
| 21 | 76 236 | 85 086 | 14 914 | 91 149 | 39 |
| 22 | 76 253 | 85 113 | 14 887 | 91 141 | 38 |
| 23 | 76 271 | 85 140 | 14 860 | 91 132 | 37 |
| 24 | 76 289 | 85 166 | 14 834 | 91 123 | 36 |
| 25 | 76 307 | 85 193 | 14 807 | 91 114 | 35 |
| 26 | 76 324 | 85 220 | 14 780 | 91 105 | 34 |
| 27 | 76 342 | 85 247 | 14 753 | 91 096 | 33 |
| 28 | 76 360 | 85 273 | 14 727 | 91 087 | 32 |
| 29 | 76 378 | 85 300 | 14 700 | 91 078 | 31 |
| 30 | 76 395 | 85 327 | 14 673 | 91 069 | 30 |
| 31 | 76 413 | 85 354 | 14 646 | 91 060 | 29 |
| 32 | 76 431 | 85 380 | 14 620 | 91 051 | 28 |
| 33 | 76 448 | 85 407 | 14 593 | 91 042 | 27 |
| 34 | 76 466 | 85 434 | 14 566 | 91 033 | 26 |
| 35 | 76 484 | 85 460 | 14 540 | 91 023 | 25 |
| 36 | 76 501 | 85 487 | 14 513 | 91 014 | 24 |
| 37 | 76 519 | 85 514 | 14 486 | 91 005 | 23 |
| 38 | 76 537 | 85 540 | 14 460 | 90 996 | 22 |
| 39 | 76 554 | 85 567 | 14 433 | 90 987 | 21 |
| 40 | 76 572 | 85 594 | 14 406 | 90 978 | 20 |
| 41 | 76 590 | 85 620 | 14 380 | 90 969 | 19 |
| 42 | 76 607 | 85 647 | 14 353 | 90 960 | 18 |
| 43 | 76 625 | 85 674 | 14 326 | 90 951 | 17 |
| 44 | 76 642 | 85 700 | 14 300 | 90 942 | 16 |
| 45 | 76 660 | 85 727 | 14 273 | 90 933 | 15 |
| 46 | 76 677 | 85 754 | 14 246 | 90 924 | 14 |
| 47 | 76 695 | 85 780 | 14 220 | 90 915 | 13 |
| 48 | 76 712 | 85 807 | 14 193 | 90 906 | 12 |
| 49 | 76 730 | 85 834 | 14 166 | 90 896 | 11 |
| 50 | 76 747 | 85 860 | 14 140 | 90 887 | 10 |
| 51 | 76 765 | 85 887 | 14 113 | 90 878 | 9 |
| 52 | 76 782 | 85 913 | 14 087 | 90 869 | 8 |
| 53 | 76 800 | 85 940 | 14 060 | 90 860 | 7 |
| 54 | 76 817 | 85 967 | 14 033 | 90 851 | 6 |
| 55 | 76 835 | 85 993 | 14 007 | 90 842 | 5 |
| 56 | 76 852 | 86 020 | 13 980 | 90 832 | 4 |
| 57 | 76 870 | 86 046 | 13 954 | 90 823 | 3 |
| 58 | 76 887 | 86 073 | 13 927 | 90 814 | 2 |
| 59 | 76 904 | 86 100 | 13 900 | 90 805 | 1 |
| 60 | 76 922 | 86 126 | 13 874 | 90 796 | 0 |
| | | | 10 | | |
| ' | log cos | log cot | log tan | log sin | ' |

| ' | log sin | log tan | log cot | log cos | ' |
|----|---------|---------|---------|---------|----|
| | 9 | 9 | 10 | 9 | |
| 0 | 76 922 | 86 126 | 13 874 | 90 796 | 60 |
| 1 | 76 939 | 86 153 | 13 847 | 90 787 | 59 |
| 2 | 76 957 | 86 179 | 13 821 | 90 777 | 58 |
| 3 | 76 974 | 86 206 | 13 794 | 90 768 | 57 |
| 4 | 76 991 | 86 232 | 13 768 | 90 759 | 56 |
| 5 | 77 009 | 86 259 | 13 741 | 90 750 | 55 |
| 6 | 77 026 | 86 285 | 13 715 | 90 741 | 54 |
| 7 | 77 043 | 86 312 | 13 688 | 90 731 | 53 |
| 8 | 77 061 | 86 338 | 13 662 | 90 722 | 52 |
| 9 | 77 078 | 86 365 | 13 635 | 90 713 | 51 |
| 10 | 77 095 | 86 392 | 13 608 | 90 704 | 50 |
| 11 | 77 112 | 86 418 | 13 582 | 90 694 | 49 |
| 12 | 77 130 | 86 445 | 13 555 | 90 685 | 48 |
| 13 | 77 147 | 86 471 | 13 529 | 90 676 | 47 |
| 14 | 77 164 | 86 498 | 13 502 | 90 667 | 46 |
| 15 | 77 181 | 86 524 | 13 476 | 90 657 | 45 |
| 16 | 77 199 | 86 551 | 13 449 | 90 648 | 44 |
| 17 | 77 216 | 86 577 | 13 423 | 90 639 | 43 |
| 18 | 77 233 | 86 603 | 13 397 | 90 630 | 42 |
| 19 | 77 250 | 86 630 | 13 370 | 90 620 | 41 |
| 20 | 77 268 | 86 656 | 13 344 | 90 611 | 40 |
| 21 | 77 285 | 86 683 | 13 317 | 90 602 | 39 |
| 22 | 77 302 | 86 709 | 13 291 | 90 592 | 38 |
| 23 | 77 319 | 86 736 | 13 264 | 90 583 | 37 |
| 24 | 77 336 | 86 762 | 13 238 | 90 574 | 36 |
| 25 | 77 353 | 86 789 | 13 211 | 90 565 | 35 |
| 26 | 77 370 | 86 815 | 13 185 | 90 555 | 34 |
| 27 | 77 387 | 86 842 | 13 158 | 90 546 | 33 |
| 28 | 77 405 | 86 868 | 13 132 | 90 537 | 32 |
| 29 | 77 422 | 86 894 | 13 106 | 90 527 | 31 |
| 30 | 77 439 | 86 921 | 13 079 | 90 518 | 30 |
| 31 | 77 456 | 86 947 | 13 053 | 90 509 | 29 |
| 32 | 77 473 | 86 974 | 13 026 | 90 499 | 28 |
| 33 | 77 490 | 87 000 | 13 000 | 90 490 | 27 |
| 34 | 77 507 | 87 027 | 12 973 | 90 480 | 26 |
| 35 | 77 524 | 87 053 | 12 947 | 90 471 | 25 |
| 36 | 77 541 | 87 079 | 12 921 | 90 462 | 24 |
| 37 | 77 558 | 87 106 | 12 894 | 90 452 | 23 |
| 38 | 77 575 | 87 132 | 12 868 | 90 443 | 22 |
| 39 | 77 592 | 87 158 | 12 842 | 90 434 | 21 |
| 40 | 77 609 | 87 185 | 12 815 | 90 424 | 20 |
| 41 | 77 626 | 87 211 | 12 789 | 90 415 | 19 |
| 42 | 77 643 | 87 238 | 12 762 | 90 405 | 18 |
| 43 | 77 660 | 87 264 | 12 736 | 90 396 | 17 |
| 44 | 77 677 | 87 290 | 12 710 | 90 386 | 16 |
| 45 | 77 694 | 87 317 | 12 683 | 90 377 | 15 |
| 46 | 77 711 | 87 343 | 12 657 | 90 368 | 14 |
| 47 | 77 728 | 87 369 | 12 631 | 90 358 | 13 |
| 48 | 77 744 | 87 396 | 12 604 | 90 349 | 12 |
| 49 | 77 761 | 87 422 | 12 578 | 90 339 | 11 |
| 50 | 77 778 | 87 448 | 12 552 | 90 330 | 10 |
| 51 | 77 795 | 87 475 | 12 525 | 90 320 | 9 |
| 52 | 77 812 | 87 501 | 12 499 | 90 311 | 8 |
| 53 | 77 829 | 87 527 | 12 473 | 90 301 | 7 |
| 54 | 77 846 | 87 554 | 12 446 | 90 292 | 6 |
| 55 | 77 862 | 87 580 | 12 420 | 90 282 | 5 |
| 56 | 77 879 | 87 606 | 12 394 | 90 273 | 4 |
| 57 | 77 896 | 87 633 | 12 367 | 90 263 | 3 |
| 58 | 77 913 | 87 659 | 12 341 | 90 254 | 2 |
| 59 | 77 930 | 87 685 | 12 315 | 90 244 | 1 |
| 60 | 77 946 | 87 711 | 12 289 | 90 235 | 0 |
| | 9 | 9 | 10 | 9 | |
| ' | log cos | log cot | log tan | log sin | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 77 946 | 87 711 | 12 289 | 90 235 | 60 |
| 1 | 77 963 | 87 738 | 12 262 | 90 225 | 59 |
| 2 | 77 980 | 87 764 | 12 236 | 90 216 | 58 |
| 3 | 77 997 | 87 790 | 12 210 | 90 206 | 57 |
| 4 | 78 013 | 87 817 | 12 183 | 90 197 | 56 |
| 5 | 78 030 | 87 843 | 12 157 | 90 187 | 55 |
| 6 | 78 047 | 87 869 | 12 131 | 90 178 | 54 |
| 7 | 78 063 | 87 895 | 12 105 | 90 168 | 53 |
| 8 | 78 080 | 87 922 | 12 078 | 90 159 | 52 |
| 9 | 78 097 | 87 948 | 12 052 | 90 149 | 51 |
| 10 | 78 113 | 87 974 | 12 026 | 90 139 | 50 |
| 11 | 78 130 | 88 000 | 12 000 | 90 130 | 49 |
| 12 | 78 147 | 88 027 | 11 973 | 90 120 | 48 |
| 13 | 78 163 | 88 053 | 11 947 | 90 111 | 47 |
| 14 | 78 180 | 88 079 | 11 921 | 90 101 | 46 |
| 15 | 78 197 | 88 105 | 11 895 | 90 091 | 45 |
| 16 | 78 213 | 88 131 | 11 869 | 90 082 | 44 |
| 17 | 78 230 | 88 158 | 11 842 | 90 072 | 43 |
| 18 | 78 246 | 88 184 | 11 816 | 90 063 | 42 |
| 19 | 78 263 | 88 210 | 11 790 | 90 053 | 41 |
| 20 | 78 280 | 88 236 | 11 764 | 90 043 | 40 |
| 21 | 78 296 | 88 262 | 11 738 | 90 034 | 39 |
| 22 | 78 313 | 88 289 | 11 711 | 90 024 | 38 |
| 23 | 78 329 | 88 315 | 11 685 | 90 014 | 37 |
| 24 | 78 346 | 88 341 | 11 659 | 90 005 | 36 |
| 25 | 78 362 | 88 367 | 11 633 | 89 995 | 35 |
| 26 | 78 379 | 88 393 | 11 607 | 89 985 | 34 |
| 27 | 78 395 | 88 420 | 11 580 | 89 976 | 33 |
| 28 | 78 412 | 88 446 | 11 554 | 89 966 | 32 |
| 29 | 78 428 | 88 472 | 11 528 | 89 956 | 31 |
| 30 | 78 445 | 88 498 | 11 502 | 89 947 | 30 |
| 31 | 78 461 | 88 524 | 11 476 | 89 937 | 29 |
| 32 | 78 478 | 88 550 | 11 450 | 89 927 | 28 |
| 33 | 78 494 | 88 577 | 11 423 | 89 918 | 27 |
| 34 | 78 510 | 88 603 | 11 397 | 89 908 | 26 |
| 35 | 78 527 | 88 629 | 11 371 | 89 898 | 25 |
| 36 | 78 543 | 88 655 | 11 345 | 89 888 | 24 |
| 37 | 78 560 | 88 681 | 11 319 | 89 879 | 23 |
| 38 | 78 576 | 88 707 | 11 293 | 89 869 | 22 |
| 39 | 78 592 | 88 733 | 11 267 | 89 859 | 21 |
| 40 | 78 609 | 88 759 | 11 241 | 89 849 | 20 |
| 41 | 78 625 | 88 786 | 11 214 | 89 840 | 19 |
| 42 | 78 642 | 88 812 | 11 188 | 89 830 | 18 |
| 43 | 78 658 | 88 838 | 11 162 | 89 820 | 17 |
| 44 | 78 674 | 88 864 | 11 136 | 89 810 | 16 |
| 45 | 78 691 | 88 890 | 11 110 | 89 801 | 15 |
| 46 | 78 707 | 88 916 | 11 084 | 89 791 | 14 |
| 47 | 78 723 | 88 942 | 11 058 | 89 781 | 13 |
| 48 | 78 739 | 88 968 | 11 032 | 89 771 | 12 |
| 49 | 78 756 | 88 994 | 11 006 | 89 761 | 11 |
| 50 | 78 772 | 89 020 | 10 980 | 89 752 | 10 |
| 51 | 78 788 | 89 046 | 10 954 | 89 742 | 9 |
| 52 | 78 805 | 89 073 | 10 927 | 89 732 | 8 |
| 53 | 78 821 | 89 099 | 10 901 | 89 722 | 7 |
| 54 | 78 837 | 89 125 | 10 875 | 89 712 | 6 |
| 55 | 78 853 | 89 151 | 10 849 | 89 702 | 5 |
| 56 | 78 869 | 89 177 | 10 823 | 89 693 | 4 |
| 57 | 78 886 | 89 203 | 10 797 | 89 683 | 3 |
| 58 | 78 902 | 89 229 | 10 771 | 89 673 | 2 |
| 59 | 78 918 | 89 255 | 10 745 | 89 663 | 1 |
| 60 | 78 934 | 89 281 | 10 719 | 89 653 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 9 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 78 934 | 89 281 | 10 719 | 89 653 | 60 |
| 1 | 78 950 | 89 307 | 10 693 | 89 643 | 59 |
| 2 | 78 967 | 89 333 | 10 667 | 89 633 | 58 |
| 3 | 78 983 | 89 359 | 10 641 | 89 624 | 57 |
| 4 | 78 999 | 89 385 | 10 615 | 89 614 | 56 |
| 5 | 79 015 | 89 411 | 10 589 | 89 604 | 55 |
| 6 | 79 031 | 89 437 | 10 563 | 89 594 | 54 |
| 7 | 79 047 | 89 463 | 10 537 | 89 584 | 53 |
| 8 | 79 063 | 89 489 | 10 511 | 89 574 | 52 |
| 9 | 79 079 | 89 515 | 10 485 | 89 564 | 51 |
| 10 | 79 095 | 89 541 | 10 459 | 89 554 | 50 |
| 11 | 79 111 | 89 567 | 10 433 | 89 544 | 49 |
| 12 | 79 128 | 89 593 | 10 407 | 89 534 | 48 |
| 13 | 79 144 | 89 619 | 10 381 | 89 524 | 47 |
| 14 | 79 160 | 89 645 | 10 355 | 89 514 | 46 |
| 15 | 79 176 | 89 671 | 10 329 | 89 504 | 45 |
| 16 | 79 192 | 89 697 | 10 303 | 89 495 | 44 |
| 17 | 79 208 | 89 723 | 10 277 | 89 485 | 43 |
| 18 | 79 224 | 89 749 | 10 251 | 89 475 | 42 |
| 19 | 79 240 | 89 775 | 10 225 | 89 465 | 41 |
| 20 | 79 256 | 89 801 | 10 199 | 89 455 | 40 |
| 21 | 79 272 | 89 827 | 10 173 | 89 445 | 39 |
| 22 | 79 288 | 89 853 | 10 147 | 89 435 | 38 |
| 23 | 79 304 | 89 879 | 10 121 | 89 425 | 37 |
| 24 | 79 319 | 89 905 | 10 095 | 89 415 | 36 |
| 25 | 79 335 | 89 931 | 10 069 | 89 405 | 35 |
| 26 | 79 351 | 89 957 | 10 043 | 89 395 | 34 |
| 27 | 79 367 | 89 983 | 10 017 | 89 385 | 33 |
| 28 | 79 383 | 90 009 | 09 991 | 89 375 | 32 |
| 29 | 79 399 | 90 035 | 09 965 | 89 364 | 31 |
| 30 | 79 415 | 90 061 | 09 939 | 89 354 | 30 |
| 31 | 79 431 | 90 086 | 09 914 | 89 344 | 29 |
| 32 | 79 447 | 90 112 | 09 888 | 89 334 | 28 |
| 33 | 79 463 | 90 138 | 09 862 | 89 324 | 27 |
| 34 | 79 478 | 90 164 | 09 836 | 89 314 | 26 |
| 35 | 79 494 | 90 190 | 09 810 | 89 304 | 25 |
| 36 | 79 510 | 90 216 | 09 784 | 89 294 | 24 |
| 37 | 79 526 | 90 242 | 09 758 | 89 284 | 23 |
| 38 | 79 542 | 90 268 | 09 732 | 89 274 | 22 |
| 39 | 79 558 | 90 294 | 09 706 | 89 264 | 21 |
| 40 | 79 573 | 90 320 | 09 680 | 89 254 | 20 |
| 41 | 79 589 | 90 346 | 09 654 | 89 244 | 19 |
| 42 | 79 605 | 90 371 | 09 629 | 89 233 | 18 |
| 43 | 79 621 | 90 397 | 09 603 | 89 223 | 17 |
| 44 | 79 636 | 90 423 | 09 577 | 89 213 | 16 |
| 45 | 79 652 | 90 449 | 09 551 | 89 203 | 15 |
| 46 | 79 668 | 90 475 | 09 525 | 89 193 | 14 |
| 47 | 79 684 | 90 501 | 09 499 | 89 183 | 13 |
| 48 | 79 699 | 90 527 | 09 473 | 89 173 | 12 |
| 49 | 79 715 | 90 553 | 09 447 | 89 162 | 11 |
| 50 | 79 731 | 90 578 | 09 422 | 89 152 | 10 |
| 51 | 79 746 | 90 604 | 09 396 | 89 142 | 9 |
| 52 | 79 762 | 90 630 | 09 370 | 89 132 | 8 |
| 53 | 79 778 | 90 656 | 09 344 | 89 122 | 7 |
| 54 | 79 793 | 90 682 | 09 318 | 89 112 | 6 |
| 55 | 79 809 | 90 708 | 09 292 | 89 101 | 5 |
| 56 | 79 825 | 90 734 | 09 266 | 89 091 | 4 |
| 57 | 79 840 | 90 759 | 09 241 | 89 081 | 3 |
| 58 | 79 856 | 90 785 | 09 215 | 89 071 | 2 |
| 59 | 79 872 | 90 811 | 09 189 | 89 060 | 1 |
| 60 | 79 887 | 90 837 | 09 163 | 89 050 | 0 |
| ' | log cos 0 | log cot 0 | log tan 10 | log sin 9 | ' |

39°

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 79 887 | 90 837 | 09 163 | 89 050 | 60 |
| 1 | 79 903 | 90 863 | 09 137 | 89 040 | 59 |
| 2 | 79 918 | 90 889 | 09 111 | 89 030 | 58 |
| 3 | 79 934 | 90 914 | 09 086 | 89 020 | 57 |
| 4 | 79 950 | 90 940 | 09 060 | 89 009 | 56 |
| 5 | 79 965 | 90 966 | 09 034 | 88 999 | 55 |
| 6 | 79 981 | 90 992 | 09 008 | 88 989 | 54 |
| 7 | 79 996 | 91 018 | 08 982 | 88 978 | 53 |
| 8 | 80 012 | 91 043 | 08 957 | 88 968 | 52 |
| 9 | 80 027 | 91 069 | 08 931 | 88 958 | 51 |
| 10 | 80 043 | 91 095 | 08 905 | 88 948 | 50 |
| 11 | 80 058 | 91 121 | 08 879 | 88 937 | 49 |
| 12 | 80 074 | 91 147 | 08 853 | 88 927 | 48 |
| 13 | 80 089 | 91 172 | 08 828 | 88 917 | 47 |
| 14 | 80 105 | 91 198 | 08 802 | 88 906 | 46 |
| 15 | 80 120 | 91 224 | 08 776 | 88 896 | 45 |
| 16 | 80 136 | 91 250 | 08 750 | 88 886 | 44 |
| 17 | 80 151 | 91 276 | 08 724 | 88 875 | 43 |
| 18 | 80 166 | 91 301 | 08 699 | 88 865 | 42 |
| 19 | 80 182 | 91 327 | 08 673 | 88 855 | 41 |
| 20 | 80 197 | 91 353 | 08 647 | 88 844 | 40 |
| 21 | 80 213 | 91 379 | 08 621 | 88 834 | 39 |
| 22 | 80 228 | 91 404 | 08 596 | 88 824 | 38 |
| 23 | 80 244 | 91 430 | 08 570 | 88 813 | 37 |
| 24 | 80 259 | 91 456 | 08 544 | 88 803 | 36 |
| 25 | 80 274 | 91 482 | 08 518 | 88 793 | 35 |
| 26 | 80 290 | 91 507 | 08 493 | 88 782 | 34 |
| 27 | 80 305 | 91 533 | 08 467 | 88 772 | 33 |
| 28 | 80 320 | 91 559 | 08 441 | 88 761 | 32 |
| 29 | 80 336 | 91 585 | 08 415 | 88 751 | 31 |
| 30 | 80 351 | 91 610 | 08 390 | 88 741 | 30 |
| 31 | 80 366 | 91 636 | 08 364 | 88 730 | 29 |
| 32 | 80 382 | 91 662 | 08 338 | 88 720 | 28 |
| 33 | 80 397 | 91 688 | 08 312 | 88 709 | 27 |
| 34 | 80 412 | 91 713 | 08 287 | 88 699 | 26 |
| 35 | 80 428 | 91 739 | 08 261 | 88 688 | 25 |
| 36 | 80 443 | 91 765 | 08 235 | 88 678 | 24 |
| 37 | 80 458 | 91 791 | 08 209 | 88 668 | 23 |
| 38 | 80 473 | 91 816 | 08 184 | 88 657 | 22 |
| 39 | 80 489 | 91 842 | 08 158 | 88 647 | 21 |
| 40 | 80 504 | 91 868 | 08 132 | 88 636 | 20 |
| 41 | 80 519 | 91 893 | 08 107 | 88 626 | 19 |
| 42 | 80 534 | 91 919 | 08 081 | 88 615 | 18 |
| 43 | 80 550 | 91 945 | 08 055 | 88 605 | 17 |
| 44 | 80 565 | 91 971 | 08 029 | 88 594 | 16 |
| 45 | 80 580 | 91 996 | 08 004 | 88 584 | 15 |
| 46 | 80 595 | 92 022 | 07 978 | 88 573 | 14 |
| 47 | 80 610 | 92 048 | 07 952 | 88 563 | 13 |
| 48 | 80 625 | 92 073 | 07 927 | 88 552 | 12 |
| 49 | 80 641 | 92 099 | 07 901 | 88 542 | 11 |
| 50 | 80 656 | 92 125 | 07 875 | 88 531 | 10 |
| 51 | 80 671 | 92 150 | 07 850 | 88 521 | 9 |
| 52 | 80 686 | 92 176 | 07 824 | 88 510 | 8 |
| 53 | 80 701 | 92 202 | 07 798 | 88 499 | 7 |
| 54 | 80 716 | 92 227 | 07 773 | 88 489 | 6 |
| 55 | 80 731 | 92 253 | 07 747 | 88 478 | 5 |
| 56 | 80 746 | 92 279 | 07 721 | 88 468 | 4 |
| 57 | 80 762 | 92 304 | 07 696 | 88 457 | 3 |
| 58 | 80 777 | 92 330 | 07 670 | 88 447 | 2 |
| 59 | 80 792 | 92 356 | 07 644 | 88 436 | 1 |
| 60 | 80 807 | 92 381 | 07 619 | 88 425 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

50°

40°

47

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|--------------|---------------|--------------|----|
| 0 | 80 807 | 92 381 | 07 619 | 88 425 | 60 |
| 1 | 80 822 | 92 407 | 07 593 | 88 415 | 59 |
| 2 | 80 837 | 92 433 | 07 567 | 88 404 | 58 |
| 3 | 80 852 | 92 458 | 07 542 | 88 394 | 57 |
| 4 | 80 867 | 92 484 | 07 516 | 88 383 | 56 |
| 5 | 80 882 | 92 510 | 07 490 | 88 372 | 55 |
| 6 | 80 897 | 92 535 | 07 465 | 88 362 | 54 |
| 7 | 80 912 | 92 561 | 07 439 | 88 351 | 53 |
| 8 | 80 927 | 92 587 | 07 413 | 88 340 | 52 |
| 9 | 80 942 | 92 612 | 07 388 | 88 330 | 51 |
| 10 | 80 957 | 92 638 | 07 362 | 88 319 | 50 |
| 11 | 80 972 | 92 663 | 07 337 | 88 308 | 49 |
| 12 | 80 987 | 92 689 | 07 311 | 88 298 | 48 |
| 13 | 81 002 | 92 715 | 07 285 | 88 287 | 47 |
| 14 | 81 017 | 92 740 | 07 260 | 88 276 | 46 |
| 15 | 81 032 | 92 766 | 07 234 | 88 266 | 45 |
| 16 | 81 047 | 92 792 | 07 208 | 88 255 | 44 |
| 17 | 81 061 | 92 817 | 07 183 | 88 244 | 43 |
| 18 | 81 076 | 92 843 | 07 157 | 88 234 | 42 |
| 19 | 81 091 | 92 868 | 07 132 | 88 223 | 41 |
| 20 | 81 106 | 92 894 | 07 106 | 88 212 | 40 |
| 21 | 81 121 | 92 920 | 07 080 | 88 201 | 39 |
| 22 | 81 136 | 92 945 | 07 055 | 88 191 | 38 |
| 23 | 81 151 | 92 971 | 07 029 | 88 180 | 37 |
| 24 | 81 166 | 92 996 | 07 004 | 88 169 | 36 |
| 25 | 81 180 | 93 022 | 06 978 | 88 158 | 35 |
| 26 | 81 195 | 93 048 | 06 952 | 88 148 | 34 |
| 27 | 81 210 | 93 073 | 06 927 | 88 137 | 33 |
| 28 | 81 225 | 93 099 | 06 901 | 88 126 | 32 |
| 29 | 81 240 | 93 124 | 06 876 | 88 115 | 31 |
| 30 | 81 254 | 93 150 | 06 850 | 88 105 | 30 |
| 31 | 81 269 | 93 175 | 06 825 | 88 094 | 29 |
| 32 | 81 284 | 93 201 | 06 799 | 88 083 | 28 |
| 33 | 81 299 | 93 227 | 06 773 | 88 072 | 27 |
| 34 | 81 314 | 93 252 | 06 748 | 88 061 | 26 |
| 35 | 81 328 | 93 278 | 06 722 | 88 051 | 25 |
| 36 | 81 343 | 93 303 | 06 697 | 88 040 | 24 |
| 37 | 81 358 | 93 329 | 06 671 | 88 029 | 23 |
| 38 | 81 372 | 93 354 | 06 646 | 88 018 | 22 |
| 39 | 81 387 | 93 380 | 06 620 | 88 007 | 21 |
| 40 | 81 402 | 93 406 | 06 594 | 87 996 | 20 |
| 41 | 81 417 | 93 431 | 06 569 | 87 985 | 19 |
| 42 | 81 431 | 93 457 | 06 543 | 87 975 | 18 |
| 43 | 81 446 | 93 482 | 06 518 | 87 964 | 17 |
| 44 | 81 461 | 93 508 | 06 492 | 87 953 | 16 |
| 45 | 81 475 | 93 533 | 06 467 | 87 942 | 15 |
| 46 | 81 490 | 93 559 | 06 441 | 87 931 | 14 |
| 47 | 81 505 | 93 584 | 06 416 | 87 920 | 13 |
| 48 | 81 519 | 93 610 | 06 390 | 87 909 | 12 |
| 49 | 81 534 | 93 636 | 06 364 | 87 898 | 11 |
| 50 | 81 549 | 93 661 | 06 339 | 87 887 | 10 |
| 51 | 81 563 | 93 687 | 06 313 | 87 877 | 9 |
| 52 | 81 578 | 93 712 | 06 288 | 87 866 | 8 |
| 53 | 81 592 | 93 738 | 06 262 | 87 855 | 7 |
| 54 | 81 607 | 93 763 | 06 237 | 87 844 | 6 |
| 55 | 81 622 | 93 789 | 06 211 | 87 833 | 5 |
| 56 | 81 636 | 93 814 | 06 186 | 87 822 | 4 |
| 57 | 81 651 | 93 840 | 06 160 | 87 811 | 3 |
| 58 | 81 665 | 93 865 | 06 135 | 87 800 | 2 |
| 59 | 81 680 | 93 891 | 06 109 | 87 789 | 1 |
| 60 | 81 694 | 93 916 | 06 084 | 87 778 | 0 |
| ' | log cos 9 | log cot 9 | log tan 10 | log sin 9 | ' |

49°

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 81 694 | 93 916 | 06 084 | 87 778 | 60 |
| 1 | 81 709 | 93 942 | 06 058 | 87 767 | 59 |
| 2 | 81 723 | 93 967 | 06 033 | 87 756 | 58 |
| 3 | 81 738 | 93 993 | 06 007 | 87 745 | 57 |
| 4 | 81 752 | 94 018 | 05 982 | 87 734 | 56 |
| 5 | 81 767 | 94 044 | 05 956 | 87 723 | 55 |
| 6 | 81 781 | 94 069 | 05 931 | 87 712 | 54 |
| 7 | 81 796 | 94 095 | 05 905 | 87 701 | 53 |
| 8 | 81 810 | 94 120 | 05 880 | 87 690 | 52 |
| 9 | 81 825 | 94 146 | 05 854 | 87 679 | 51 |
| 10 | 81 839 | 94 171 | 05 829 | 87 668 | 50 |
| 11 | 81 854 | 94 197 | 05 803 | 87 657 | 49 |
| 12 | 81 868 | 94 222 | 05 778 | 87 646 | 48 |
| 13 | 81 882 | 94 248 | 05 752 | 87 635 | 47 |
| 14 | 81 897 | 94 273 | 05 727 | 87 624 | 46 |
| 15 | 81 911 | 94 299 | 05 701 | 87 613 | 45 |
| 16 | 81 926 | 94 324 | 05 676 | 87 601 | 44 |
| 17 | 81 940 | 94 350 | 05 650 | 87 590 | 43 |
| 18 | 81 955 | 94 375 | 05 625 | 87 579 | 42 |
| 19 | 81 969 | 94 401 | 05 599 | 87 568 | 41 |
| 20 | 81 983 | 94 426 | 05 574 | 87 557 | 40 |
| 21 | 81 998 | 94 452 | 05 548 | 87 546 | 39 |
| 22 | 82 012 | 94 477 | 05 523 | 87 535 | 38 |
| 23 | 82 026 | 94 503 | 05 497 | 87 524 | 37 |
| 24 | 82 041 | 94 528 | 05 472 | 87 513 | 36 |
| 25 | 82 055 | 94 554 | 05 446 | 87 501 | 35 |
| 26 | 82 069 | 94 579 | 05 421 | 87 490 | 34 |
| 27 | 82 084 | 94 604 | 05 396 | 87 479 | 33 |
| 28 | 82 098 | 94 630 | 05 370 | 87 468 | 32 |
| 29 | 82 112 | 94 655 | 05 345 | 87 457 | 31 |
| 30 | 82 126 | 94 681 | 05 319 | 87 446 | 30 |
| 31 | 82 141 | 94 706 | 05 294 | 87 434 | 29 |
| 32 | 82 155 | 94 732 | 05 268 | 87 423 | 28 |
| 33 | 82 169 | 94 757 | 05 243 | 87 412 | 27 |
| 34 | 82 184 | 94 783 | 05 217 | 87 401 | 26 |
| 35 | 82 198 | 94 808 | 05 192 | 87 390 | 25 |
| 36 | 82 212 | 94 834 | 05 166 | 87 378 | 24 |
| 37 | 82 226 | 94 859 | 05 141 | 87 367 | 23 |
| 38 | 82 240 | 94 884 | 05 116 | 87 356 | 22 |
| 39 | 82 255 | 94 910 | 05 090 | 87 345 | 21 |
| 40 | 82 269 | 94 935 | 05 065 | 87 334 | 20 |
| 41 | 82 283 | 94 961 | 05 039 | 87 322 | 19 |
| 42 | 82 297 | 94 986 | 05 014 | 87 311 | 18 |
| 43 | 82 311 | 95 012 | 04 988 | 87 300 | 17 |
| 44 | 82 326 | 95 037 | 04 963 | 87 288 | 16 |
| 45 | 82 340 | 95 062 | 04 938 | 87 277 | 15 |
| 46 | 82 354 | 95 088 | 04 912 | 87 266 | 14 |
| 47 | 82 368 | 95 113 | 04 887 | 87 255 | 13 |
| 48 | 82 382 | 95 139 | 04 861 | 87 243 | 12 |
| 49 | 82 396 | 95 164 | 04 836 | 87 232 | 11 |
| 50 | 82 410 | 95 190 | 04 810 | 87 221 | 10 |
| 51 | 82 424 | 95 215 | 04 785 | 87 209 | 9 |
| 52 | 82 439 | 95 240 | 04 760 | 87 198 | 8 |
| 53 | 82 453 | 95 266 | 04 734 | 87 187 | 7 |
| 54 | 82 467 | 95 291 | 04 709 | 87 175 | 6 |
| 55 | 82 481 | 95 317 | 04 683 | 87 164 | 5 |
| 56 | 82 495 | 95 342 | 04 658 | 87 153 | 4 |
| 57 | 82 509 | 95 368 | 04 632 | 87 141 | 3 |
| 58 | 82 523 | 95 393 | 04 607 | 87 130 | 2 |
| 59 | 82 537 | 95 418 | 04 582 | 87 119 | 1 |
| 60 | 82 551 | 95 444 | 04 556 | 87 107 | 0 |
| ' | log cos 0 | log cot 10 | log tan 0 | log sin 0 | ' |

| ' | log sin 0 | log tan 0 | log cot 10 | log cos 0 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 82 551 | 95 444 | 04 556 | 87 107 | 60 |
| 1 | 82 565 | 95 469 | 04 531 | 87 096 | 59 |
| 2 | 82 579 | 95 495 | 04 505 | 87 085 | 58 |
| 3 | 82 593 | 95 520 | 04 480 | 87 073 | 57 |
| 4 | 82 607 | 95 545 | 04 455 | 87 062 | 56 |
| 5 | 82 621 | 95 571 | 04 429 | 87 050 | 55 |
| 6 | 82 635 | 95 596 | 04 404 | 87 039 | 54 |
| 7 | 82 649 | 95 622 | 04 378 | 87 028 | 53 |
| 8 | 82 663 | 95 647 | 04 353 | 87 016 | 52 |
| 9 | 82 677 | 95 672 | 04 328 | 87 005 | 51 |
| 10 | 82 691 | 95 698 | 04 302 | 86 993 | 50 |
| 11 | 82 705 | 95 723 | 04 277 | 86 982 | 49 |
| 12 | 82 719 | 95 748 | 04 252 | 86 970 | 48 |
| 13 | 82 733 | 95 774 | 04 226 | 86 959 | 47 |
| 14 | 82 747 | 95 799 | 04 201 | 86 947 | 46 |
| 15 | 82 761 | 95 825 | 04 175 | 86 936 | 45 |
| 16 | 82 775 | 95 850 | 04 150 | 86 924 | 44 |
| 17 | 82 788 | 95 875 | 04 125 | 86 913 | 43 |
| 18 | 82 802 | 95 901 | 04 099 | 86 902 | 42 |
| 19 | 82 816 | 95 926 | 04 074 | 86 890 | 41 |
| 20 | 82 830 | 95 952 | 04 048 | 86 879 | 40 |
| 21 | 82 844 | 95 977 | 04 023 | 86 867 | 39 |
| 22 | 82 858 | 96 002 | 03 998 | 86 855 | 38 |
| 23 | 82 872 | 96 028 | 03 972 | 86 844 | 37 |
| 24 | 82 885 | 96 053 | 03 947 | 86 832 | 36 |
| 25 | 82 899 | 96 078 | 03 922 | 86 821 | 35 |
| 26 | 82 913 | 96 104 | 03 896 | 86 809 | 34 |
| 27 | 82 927 | 96 129 | 03 871 | 86 798 | 33 |
| 28 | 82 941 | 96 155 | 03 845 | 86 786 | 32 |
| 29 | 82 955 | 96 180 | 03 820 | 86 775 | 31 |
| 30 | 82 968 | 96 205 | 03 795 | 86 763 | 30 |
| 31 | 82 982 | 96 231 | 03 769 | 86 752 | 29 |
| 32 | 82 996 | 96 256 | 03 744 | 86 740 | 28 |
| 33 | 83 010 | 96 281 | 03 719 | 86 728 | 27 |
| 34 | 83 023 | 96 307 | 03 693 | 86 717 | 26 |
| 35 | 83 037 | 96 332 | 03 668 | 86 705 | 25 |
| 36 | 83 051 | 96 357 | 03 643 | 86 694 | 24 |
| 37 | 83 065 | 96 383 | 03 617 | 86 682 | 23 |
| 38 | 83 078 | 96 408 | 03 592 | 86 670 | 22 |
| 39 | 83 092 | 96 433 | 03 567 | 86 659 | 21 |
| 40 | 83 106 | 96 459 | 03 541 | 86 647 | 20 |
| 41 | 83 120 | 96 484 | 03 516 | 86 635 | 19 |
| 42 | 83 133 | 96 510 | 03 490 | 86 624 | 18 |
| 43 | 83 147 | 96 535 | 03 465 | 86 612 | 17 |
| 44 | 83 161 | 96 560 | 03 440 | 86 600 | 16 |
| 45 | 83 174 | 96 586 | 03 414 | 86 589 | 15 |
| 46 | 83 188 | 96 611 | 03 389 | 86 577 | 14 |
| 47 | 83 202 | 96 636 | 03 364 | 86 565 | 13 |
| 48 | 83 215 | 96 662 | 03 338 | 86 554 | 12 |
| 49 | 83 229 | 96 687 | 03 313 | 86 542 | 11 |
| 50 | 83 242 | 96 712 | 03 288 | 86 530 | 10 |
| 51 | 83 256 | 96 738 | 03 262 | 86 518 | 9 |
| 52 | 83 270 | 96 763 | 03 237 | 86 507 | 8 |
| 53 | 83 283 | 96 788 | 03 212 | 86 495 | 7 |
| 54 | 83 297 | 96 814 | 03 186 | 86 483 | 6 |
| 55 | 83 310 | 96 839 | 03 161 | 86 472 | 5 |
| 56 | 83 324 | 96 864 | 03 136 | 86 460 | 4 |
| 57 | 83 338 | 96 890 | 03 110 | 86 448 | 3 |
| 58 | 83 351 | 96 915 | 03 085 | 86 436 | 2 |
| 59 | 83 365 | 96 940 | 03 060 | 86 425 | 1 |
| 60 | 83 378 | 96 966 | 03 034 | 86 413 | 0 |
| ' | log cos 0 | log cot 10 | log tan 0 | log sin 0 | ' |

43°

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 83 378 | 96 966 | 03 034 | 86 413 | 60 |
| 1 | 83 392 | 96 991 | 03 009 | 86 401 | 59 |
| 2 | 83 405 | 97 016 | 02 984 | 86 389 | 58 |
| 3 | 83 419 | 97 042 | 02 958 | 86 377 | 57 |
| 4 | 83 432 | 97 067 | 02 933 | 86 366 | 56 |
| 5 | 83 446 | 97 092 | 02 908 | 86 354 | 55 |
| 6 | 83 459 | 97 118 | 02 882 | 86 342 | 54 |
| 7 | 83 473 | 97 143 | 02 857 | 86 330 | 53 |
| 8 | 83 486 | 97 168 | 02 832 | 86 318 | 52 |
| 9 | 83 500 | 97 193 | 02 807 | 86 306 | 51 |
| 10 | 83 513 | 97 219 | 02 781 | 86 295 | 50 |
| 11 | 83 527 | 97 244 | 02 756 | 86 283 | 49 |
| 12 | 83 540 | 97 269 | 02 731 | 86 271 | 48 |
| 13 | 83 554 | 97 295 | 02 705 | 86 259 | 47 |
| 14 | 83 567 | 97 320 | 02 680 | 86 247 | 46 |
| 15 | 83 581 | 97 345 | 02 655 | 86 235 | 45 |
| 16 | 83 594 | 97 371 | 02 629 | 86 223 | 44 |
| 17 | 83 608 | 97 396 | 02 604 | 86 211 | 43 |
| 18 | 83 621 | 97 421 | 02 579 | 86 200 | 42 |
| 19 | 83 634 | 97 447 | 02 553 | 86 188 | 41 |
| 20 | 83 648 | 97 472 | 02 528 | 86 176 | 40 |
| 21 | 83 661 | 97 497 | 02 503 | 86 164 | 39 |
| 22 | 83 674 | 97 523 | 02 477 | 86 152 | 38 |
| 23 | 83 688 | 97 548 | 02 452 | 86 140 | 37 |
| 24 | 83 701 | 97 573 | 02 427 | 86 128 | 36 |
| 25 | 83 715 | 97 598 | 02 402 | 86 116 | 35 |
| 26 | 83 728 | 97 624 | 02 376 | 86 104 | 34 |
| 27 | 83 741 | 97 649 | 02 351 | 86 092 | 33 |
| 28 | 83 755 | 97 674 | 02 326 | 86 080 | 32 |
| 29 | 83 768 | 97 700 | 02 300 | 86 068 | 31 |
| 30 | 83 781 | 97 725 | 02 275 | 86 056 | 30 |
| 31 | 83 795 | 97 750 | 02 250 | 86 044 | 29 |
| 32 | 83 808 | 97 776 | 02 224 | 86 032 | 28 |
| 33 | 83 821 | 97 801 | 02 199 | 86 020 | 27 |
| 34 | 83 834 | 97 826 | 02 174 | 86 008 | 26 |
| 35 | 83 848 | 97 851 | 02 149 | 85 996 | 25 |
| 36 | 83 861 | 97 877 | 02 123 | 85 984 | 24 |
| 37 | 83 874 | 97 902 | 02 098 | 85 972 | 23 |
| 38 | 83 887 | 97 927 | 02 073 | 85 960 | 22 |
| 39 | 83 901 | 97 953 | 02 047 | 85 948 | 21 |
| 40 | 83 914 | 97 978 | 02 022 | 85 936 | 20 |
| 41 | 83 927 | 98 003 | 01 997 | 85 924 | 19 |
| 42 | 83 940 | 98 029 | 01 971 | 85 912 | 18 |
| 43 | 83 954 | 98 054 | 01 946 | 85 900 | 17 |
| 44 | 83 967 | 98 079 | 01 921 | 85 888 | 16 |
| 45 | 83 980 | 98 104 | 01 896 | 85 876 | 15 |
| 46 | 83 993 | 98 130 | 01 870 | 85 864 | 14 |
| 47 | 84 006 | 98 155 | 01 845 | 85 851 | 13 |
| 48 | 84 020 | 98 180 | 01 820 | 85 839 | 12 |
| 49 | 84 033 | 98 206 | 01 794 | 85 827 | 11 |
| 50 | 84 046 | 98 231 | 01 769 | 85 815 | 10 |
| 51 | 84 059 | 98 256 | 01 744 | 85 803 | 9 |
| 52 | 84 072 | 98 281 | 01 719 | 85 791 | 8 |
| 53 | 84 085 | 98 307 | 01 693 | 85 779 | 7 |
| 54 | 84 098 | 98 332 | 01 668 | 85 766 | 6 |
| 55 | 84 112 | 98 357 | 01 643 | 85 754 | 5 |
| 56 | 84 125 | 98 383 | 01 617 | 85 742 | 4 |
| 57 | 84 138 | 98 408 | 01 592 | 85 730 | 3 |
| 58 | 84 151 | 98 433 | 01 567 | 85 718 | 2 |
| 59 | 84 164 | 98 458 | 01 542 | 85 706 | 1 |
| 60 | 84 177 | 98 484 | 01 516 | 85 693 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

46°

44°

49

| ' | log sin 9 | log tan 9 | log cot 10 | log cos 9 | ' |
|----|--------------|---------------|---------------|--------------|----|
| 0 | 84 177 | 98 484 | 01 516 | 85 693 | 60 |
| 1 | 84 190 | 98 509 | 01 491 | 85 681 | 59 |
| 2 | 84 203 | 98 534 | 01 466 | 85 669 | 58 |
| 3 | 84 216 | 98 560 | 01 440 | 85 657 | 57 |
| 4 | 84 229 | 98 585 | 01 415 | 85 645 | 56 |
| 5 | 84 242 | 98 610 | 01 390 | 85 632 | 55 |
| 6 | 84 255 | 98 635 | 01 365 | 85 620 | 54 |
| 7 | 84 269 | 98 661 | 01 339 | 85 608 | 53 |
| 8 | 84 282 | 98 686 | 01 314 | 85 596 | 52 |
| 9 | 84 295 | 98 711 | 01 289 | 85 583 | 51 |
| 10 | 84 308 | 98 737 | 01 263 | 85 571 | 50 |
| 11 | 84 321 | 98 762 | 01 238 | 85 559 | 49 |
| 12 | 84 334 | 98 787 | 01 213 | 85 547 | 48 |
| 13 | 84 347 | 98 812 | 01 188 | 85 534 | 47 |
| 14 | 84 360 | 98 838 | 01 162 | 85 522 | 46 |
| 15 | 84 373 | 98 863 | 01 137 | 85 510 | 45 |
| 16 | 84 385 | 98 888 | 01 112 | 85 497 | 44 |
| 17 | 84 398 | 98 913 | 01 087 | 85 485 | 43 |
| 18 | 84 411 | 98 939 | 01 061 | 85 473 | 42 |
| 19 | 84 424 | 98 964 | 01 036 | 85 460 | 41 |
| 20 | 84 437 | 98 989 | 01 011 | 85 448 | 40 |
| 21 | 84 450 | 99 015 | 00 985 | 85 436 | 39 |
| 22 | 84 463 | 99 040 | 00 960 | 85 423 | 38 |
| 23 | 84 476 | 99 065 | 00 935 | 85 411 | 37 |
| 24 | 84 489 | 99 090 | 00 910 | 85 399 | 36 |
| 25 | 84 502 | 99 116 | 00 884 | 85 386 | 35 |
| 26 | 84 515 | 99 141 | 00 859 | 85 374 | 34 |
| 27 | 84 528 | 99 166 | 00 834 | 85 361 | 33 |
| 28 | 84 540 | 99 191 | 00 809 | 85 349 | 32 |
| 29 | 84 553 | 99 217 | 00 783 | 85 337 | 31 |
| 30 | 84 566 | 99 242 | 00 758 | 85 324 | 30 |
| 31 | 84 579 | 99 267 | 00 733 | 85 312 | 29 |
| 32 | 84 592 | 99 293 | 00 707 | 85 299 | 28 |
| 33 | 84 605 | 99 318 | 00 682 | 85 287 | 27 |
| 34 | 84 618 | 99 343 | 00 657 | 85 274 | 26 |
| 35 | 84 630 | 99 368 | 00 632 | 85 262 | 25 |
| 36 | 84 643 | 99 394 | 00 606 | 85 250 | 24 |
| 37 | 84 656 | 99 419 | 00 581 | 85 237 | 23 |
| 38 | 84 669 | 99 444 | 00 556 | 85 225 | 22 |
| 39 | 84 682 | 99 469 | 00 531 | 85 212 | 21 |
| 40 | 84 694 | 99 495 | 00 505 | 85 200 | 20 |
| 41 | 84 707 | 99 520 | 00 480 | 85 187 | 19 |
| 42 | 84 720 | 99 545 | 00 455 | 85 175 | 18 |
| 43 | 84 733 | 99 570 | 00 430 | 85 162 | 17 |
| 44 | 84 745 | 99 596 | 00 404 | 85 150 | 16 |
| 45 | 84 758 | 99 621 | 00 379 | 85 137 | 15 |
| 46 | 84 771 | 99 646 | 00 354 | 85 125 | 14 |
| 47 | 84 784 | 99 672 | 00 328 | 85 112 | 13 |
| 48 | 84 796 | 99 697 | 00 303 | 85 100 | 12 |
| 49 | 84 809 | 99 722 | 00 278 | 85 087 | 11 |
| 50 | 84 822 | 99 747 | 00 253 | 85 074 | 10 |
| 51 | 84 835 | 99 773 | 00 227 | 85 062 | 9 |
| 52 | 84 847 | 99 798 | 00 202 | 85 049 | 8 |
| 53 | 84 860 | 99 823 | 00 177 | 85 037 | 7 |
| 54 | 84 873 | 99 848 | 00 152 | 85 024 | 6 |
| 55 | 84 885 | 99 874 | 00 126 | 85 012 | 5 |
| 56 | 84 898 | 99 899 | 00 101 | 84 999 | 4 |
| 57 | 84 911 | 99 924 | 00 076 | 84 986 | 3 |
| 58 | 84 923 | 99 949 | 00 051 | 84 974 | 2 |
| 59 | 84 936 | 99 97 | 00 025 | 84 961 | 1 |
| 60 | 84 949 | 00 000 | 00 000 | 84 949 | 0 |
| ' | log cos 9 | log cot 10 | log tan 9 | log sin 9 | ' |

45°

TABLE IV.

FOR DETERMINING WITH GREATER ACCURACY THAN CAN BE DONE BY
MEANS OF TABLE III.:

1. $\log \sin$, $\log \tan$, and $\log \cot$, when the angle is between 0° and 2° ;
2. $\log \cos$, $\log \tan$, and $\log \cot$, when the angle is between 88° and 90° ;
3. The value of the angle when the logarithm of the function does not lie between the limits 8.54684 and 11.45816.

FORMULAS FOR THE USE OF THE NUMBERS S AND T.

I. When the angle α is between 0° and 2° :

$$\begin{aligned}\log \sin \alpha &= \log a'' + S, \\ \log \tan \alpha &= \log a'' + T, \\ \log \cot \alpha &= \text{colog } \tan \alpha.\end{aligned}$$

$$\begin{aligned}\log a'' &= \log \sin \alpha - S, \\ &= \log \tan \alpha - T, \\ &= \text{colog } \cot \alpha - T.\end{aligned}$$

II. When the angle α is between 88° and 90° :

$$\begin{aligned}\log \cos \alpha &= \log (90^\circ - \alpha)'' + S, \\ \log \cot \alpha &= \log (90^\circ - \alpha)'' + T, \\ \log \tan \alpha &= \text{colog } \cot \alpha.\end{aligned}$$

$$\begin{aligned}\log (90^\circ - \alpha)' &= \log \cos \alpha - S, \\ &= \log \cot \alpha - T, \\ &= \text{colog } \tan \alpha - T, \\ \text{and } \alpha &= 90^\circ - (90^\circ - \alpha).\end{aligned}$$

VALUES OF S AND T.

| α'' | S | $\log \sin \alpha$ | α'' | T | $\log \tan \alpha$ | α | T | $\log \tan \alpha$ |
|------------|----------|--------------------|------------|----------|--------------------|----------|----------|--------------------|
| 0 | | — | 0 | | — | 5 146 | | 8.39 713 |
| 2 409 | 4.68 557 | 8.06 740 | 200 | 4.68 557 | 6.90 660 | 5 424 | 4.68 567 | 8.41 999 |
| 3 417 | 4.68 556 | 8.21 920 | 1 726 | 4.68 558 | 7.92 263 | 5 689 | 4.68 568 | 8.44 072 |
| 3 823 | 4.68 555 | 8.26 795 | 2 432 | 4.68 559 | 8.07 156 | 5 941 | 4.68 569 | 8.45 955 |
| 4 190 | 4.68 555 | 8.30 776 | 2 976 | 4.68 560 | 8.15 924 | 6 184 | 4.68 570 | 8.47 697 |
| 4 840 | 4.68 554 | 8.37 038 | 3 434 | 4.68 561 | 8.22 142 | 6 417 | 4.68 571 | 8.49 305 |
| 5 414 | 4.68 553 | 8.41 904 | 3 838 | 4.68 562 | 8.26 973 | 6 642 | 4.68 572 | 8.50 802 |
| 5 932 | 4.68 552 | 8.45 872 | 4 204 | 4.68 563 | 8.30 930 | 6 859 | 4.68 573 | 8.52 200 |
| 6 408 | 4.68 551 | 8.49 223 | 4 540 | 4.68 564 | 8.34 270 | 7 070 | 4.68 574 | 8.53 516 |
| 6 633 | 4.68 550 | 8.50 721 | 4 699 | 4.68 565 | 8.35 766 | 7 173 | 4.68 575 | 8.54 145 |
| 6 851 | 4.68 550 | 8.52 125 | 4 853 | 4.68 565 | 8.37 167 | 7 274 | 4.68 575 | 8.54 753 |
| 7 267 | 4.68 549 | 8.54 684 | 5 146 | 4.68 566 | 8.39 713 | | | |
| α'' | S | $\log \sin \alpha$ | α'' | T | $\log \tan \alpha$ | α | T | $\log \tan \alpha$ |

TABLE IV.

This table (page 50) must be used when great accuracy is desired in working with angles between 0° and 2° , or between 88° and 90° .

The values of S and T are such that when the angle a is expressed in seconds,

$$S = \log \sin a - \log a'',$$

$$T = \log \tan a - \log a''.$$

Hence, follow the formulas given on the page containing the table.

The values of S and T are printed with the characteristic 10 too large, and in using them -10 must always be annexed.

Find $\log \sin 0^\circ 58' 17''$.

$$0^\circ 58' 17'' = 3497.''$$

$$\log 3497 = 3.54370$$

$$S = 4.68555 - 10$$

$$\log \sin 0^\circ 58' 17'' = 8.22925 - 10$$

Find $\log \cos 88^\circ 26' 41.2''$.

$$90^\circ - 88^\circ 26' 41.2'' = 1^\circ 33' 18.8''$$

$$= 5598.8.''$$

$$\log 5598.8 = 3.74809$$

$$S = 4.68552 - 10$$

$$\log \cos 88^\circ 26' 41.2'' = 8.43361 - 10$$

Find $\log \tan 0^\circ 52' 47.5''$.

$$0^\circ 52' 47.5'' = 3167.5.''$$

$$\log 3167.5 = 3.50072$$

$$T = 4.68561 - 10$$

$$\log \tan 0^\circ 52' 47.5'' = 8.18633 - 10$$

Find $\log \tan 89^\circ 54' 37.362''$.

$$90^\circ - 89^\circ 54' 37.362'' = 322.638''.$$

$$\log 322.638 = 2.50871$$

$$T = 5.68558 - 10$$

$$\log \cot 89^\circ 54' 37.362'' = 7.19429 - 10$$

$$\log \tan 89^\circ 54' 37.362'' = 2.80571.$$

Find the angle, if $\log \sin = 6.72306 - 10$.

$$6.72306 - 10$$

$$S = 4.68557 - 10$$

$$\text{Subtract, } 2.03749 = \log 109.015.$$

$$109.015'' = 0^\circ 1' 49.015''.$$

Find the angle for which $\log \cot = 1.67604$.

$$\text{colog } \cot = 8.32396 - 10$$

$$T = 4.68564 - 10$$

$$\text{Subtract, } 3.63832 = \log 4348.3.$$

$$4348.3'' = 1^\circ 12' 28.3''.$$

Find the angle for which $\log \tan = 1.55407$.

$$\text{colog } \tan = 8.44593 - 10$$

$$T = 4.68569 - 10$$

$$\text{Subtract, } 3.76024 = \log 5757.6.$$

$$5757.6'' = 1^\circ 35' 57.6'',$$

$$\text{and } 90^\circ - 1^\circ 35' 57.6'' = 88^\circ 24' 2.4'' = \text{angle required.}$$

TABLE V.

SHOWING LENGTHS IN NAUTICAL MILES AND STATUTE MILES OF DEGREES
OF LATITUDE AND LONGITUDE IN DIFFERENT LATITUDES.

| DEGREE OF THE PARALLEL. | | | DEGREE OF THE MERIDIAN. | | |
|-----------------------------|--------------------|-------------------|---------------------------------|--------------------|-------------------|
| Latitude of Parallel. | Nautical Miles. | Statute Miles. | Latitude of Middle Point. | Nautical Miles. | Statute Miles. |
| 20° | 56.404 | 65.018 | 20° | 59.664 | 68.777 |
| 21° | 56.039 | 64.598 | | | |
| 22° | 55.657 | 64.158 | | | |
| 23° | 55.258 | 63.698 | | | |
| 24° | 54.843 | 63.219 | | | |
| 25° | 54.411 | 62.721 | 25° | 59.706 | 68.825 |
| 26° | 53.962 | 62.204 | | | |
| 27° | 53.497 | 61.668 | | | |
| 28° | 53.016 | 61.113 | | | |
| 29° | 52.518 | 60.540 | | | |
| 30° | 52.005 | 59.948 | 30° | 59.749 | 68.875 |
| 31° | 51.476 | 59.338 | | | |
| 32° | 50.931 | 58.709 | | | |
| 33° | 50.370 | 58.063 | | | |
| 34° | 49.794 | 57.399 | | | |
| 35° | 49.203 | 56.718 | 35° | 59.796 | 68.929 |
| 36° | 48.597 | 56.019 | | | |
| 37° | 47.976 | 55.304 | | | |
| 38° | 47.341 | 54.571 | | | |
| 39° | 46.960 | 53.822 | | | |
| 40° | 46.026 | 53.056 | 40° | 59.847 | 68.987 |
| 41° | 45.348 | 52.274 | | | |
| 42° | 44.654 | 51.476 | | | |
| 43° | 43.949 | 50.662 | | | |
| 44° | 43.230 | 49.833 | | | |
| 45° | 42.497 | 48.988 | 45° | 59.899 | 69.048 |
| 46° | 41.752 | 48.128 | | | |
| 47° | 40.993 | 47.254 | | | |
| 48° | 40.222 | 46.365 | | | |
| 49° | 39.439 | 45.462 | | | |
| 50° | 38.643 | 44.545 | 50° | 59.951 | 69.108 |

TABLE VI.

MISCELLANEOUS FORMULÆ, AND EQUIVALENTS OF METRES, CHAINS, AND FEET.

| | | | |
|--|-------------------------|---|-------------------------|
| $\pi \dots = 3.14159265$ | Logarithm. 0.4971499 | $\sqrt{\pi} \dots = 1.77245385$ | Logarithm. 0.2485749 |
| $\frac{1}{\pi} \dots = 0.31830989$ | 9.5028501 — 10 | $\frac{1}{\sqrt{\pi}} \dots = 0.56418958$ | 9.7514251 — 10 |
| $\pi^2 \dots = 9.86960440$ | 0.9942997 | $\sqrt[3]{\pi} \dots = 1.46459189$ | 0.1657166 |
| $\frac{1}{\pi^2} \dots = 0.10132118$ | 9.0057006 — 10 | | |
| Circumference of circle, diameter being unity, } Area of circle, radius being unity } = $\pi = 3.14159265$ Surface of sphere, diameter being unity } | | | 0.4971499 |
| Area of a circle, diameter being unity = $\frac{\pi}{4} = 0.7853982$ | | | 9.8950899 — 10 |
| Volume of sphere, diameter being unity = $\frac{\pi}{6} = 0.52359878$ | | | 9.7189986 — 10 |
| Volume of sphere, radius being unity = $\frac{4\pi}{3} = 4.1887902$ | | | 0.6220886 |
| Arc whose length is equal to the radius : | | | |
| Expressed in degrees = $\frac{180}{\pi} = 57.2957795^\circ$ | | | 1.7581226 |
| Expressed in minutes = $\frac{10800}{\pi} = 3437.74677'$ | | | 3.5362739 |
| Expressed in seconds = $\frac{648000}{\pi} = 206264.806$ | | | 5.3144251 |
| If radius is unity : | | | |
| Length of arc for one degree = $\frac{\pi}{180} = 0.0174533$ | | | 8.2418774 — 10 |
| Length of arc for one minute = $\frac{\pi}{10800} = 0.0002909$ | | | 6.4637261 — 10 |
| Length of arc for one second = $\frac{\pi}{648000} = 0.00000485$ | | | 4.68557487 — 10 |
| Sine of one second = 0.00000485 | | | 4.68557487 — 10 |
| Base of Hyperbolic or Napier's System of Logarithms . = 2.7182818 | | | 0.4342945 |
| Modulus of Common or Briggs' System of Logarithms . = 0.4342945 | | | 9.6377843 — 10 |
| Equatorial radius of the earth in feet = 20923600 | | | |
| Polar radius of the earth in feet = 20853657 | | | |
| Length of degree of latitude at the equator, in feet . = 362748.33 | | | |
| Length of degree of latitude at 45°, in feet = 364571.77 | | | |

| | FEET. | METRES. | CHAINS. | METRES. | FEET. |
|--|-------|---------|---------|---------|---------|
| | 1 | 0.3048 | 0.0151 | 1 | 3.2809 |
| | 2 | 0.6096 | 0.0303 | 2 | 6.5617 |
| | 3 | 0.9144 | 0.0455 | 3 | 9.8426 |
| | 4 | 1.2192 | 0.0606 | 4 | 13.1235 |
| | 5 | 1.5240 | 0.0758 | 5 | 16.4044 |
| | 6 | 1.8288 | 0.0909 | 6 | 19.6852 |
| | 7 | 2.1336 | 0.1061 | 7 | 22.9661 |
| | 8 | 2.4384 | 0.1212 | 8 | 26.2470 |
| | 9 | 2.7432 | 0.1364 | 9 | 29.5278 |
| | 10 | 3.0480 | 0.1515 | 10 | 32.8087 |

TABLE VII.—TRAVERSE TABLE.

This table gives the latitude and departure to three places of decimals for distances from 1 to 10, corresponding to bearings from 0° to 90° , at intervals of $15'$.

If the bearing does not exceed 45° , it is found in the *left-hand* column, and the designations of the columns under "Distance" are taken from the *top* of the page; but if the bearing exceeds 45° , it is found in the *right-hand* column, and the designations of the columns under "Distance" are taken from the *bottom* of the page.

The method of using the table will be made plain by the following examples :

1. Let it be required to find the latitude and departure of a line running N. $35^\circ 15'$ E. 6 chains.

On page 60, left-hand column, look for $35^\circ 15'$; opposite this bearing, in the vertical column headed "Distance 6," are found 4.900 and 3.463, under the headings "Latitude" and "Departure" respectively. Hence latitude, or northing, = 4.900 chains, and departure, or easting, = 3.463 chains.

2. Let it be required to find the latitude and departure of a line running S. 87° W. 2 chains.

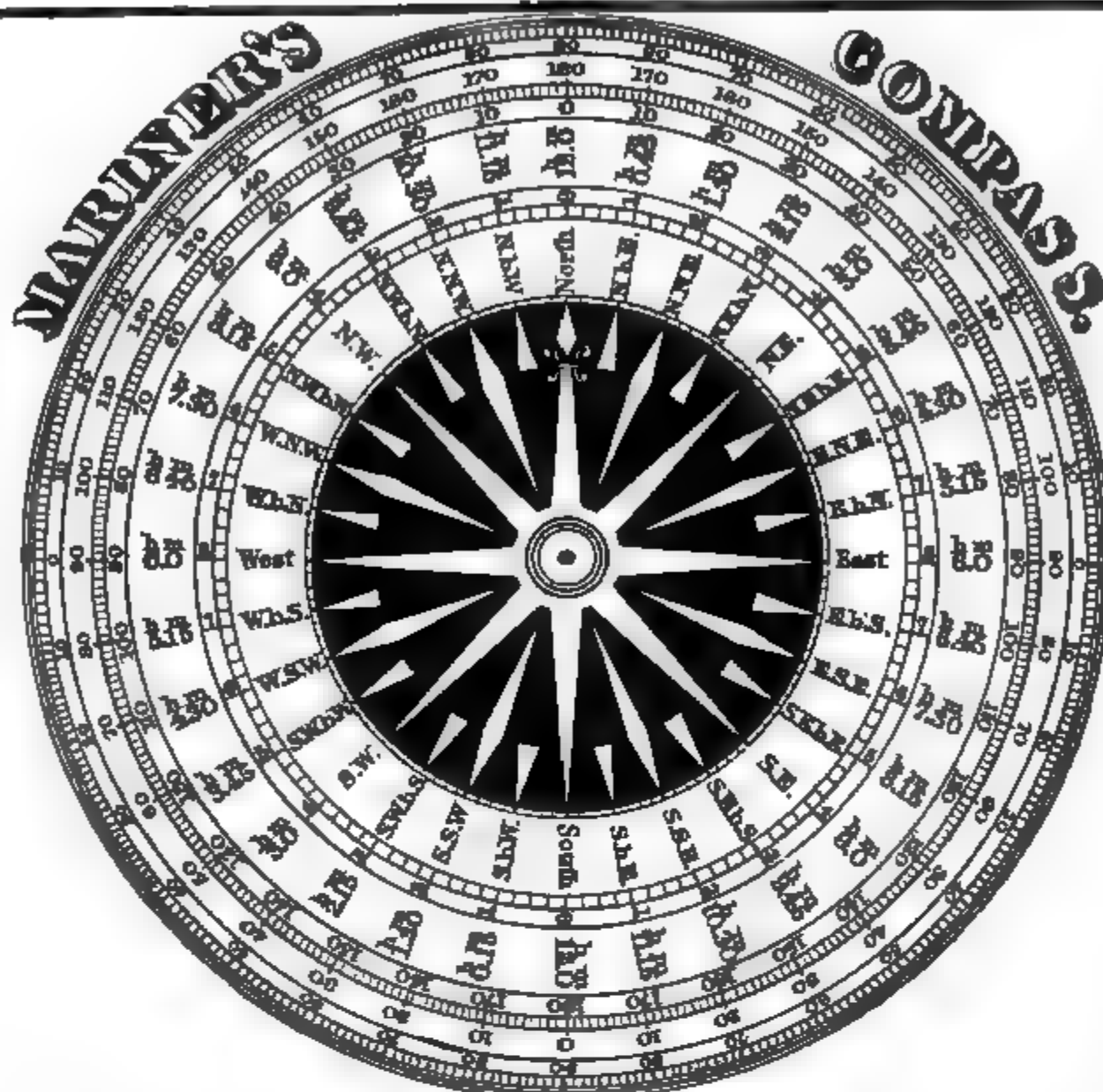
As the bearing exceeds 45° , we look in the right-hand column on page 56, and opposite 87° , in the column marked "Distance 2," we find (taking the designations of the columns from the bottom of the page) latitude — 0.105 chains, and departure = 1.997 chains. Hence latitude, or southing, = 0.105 chains, and departure, or westing, = 1.997 chains.

3. Let it be required to find the latitude and departure of a line running N. $15^\circ 45'$ W. 27.36 chains.

In this case, we find the required number for each figure of the distance separately, arranging the work as in the following table. In practice, only the last columns under "Latitude" and "Departure" are written.

| Distance. | Latitude. | Departure. |
|---------------------|---------------------------|--------------------------|
| 20 = 2×10 | $1.926 \times 10 = 19.26$ | $0.543 \times 10 = 5.43$ |
| 7 | 6.737 | 1.90 |
| 0.3 = $3 \div 10$ | $2.887 \div 10 = 0.289$ | $0.814 \div 10 = 0.081$ |
| 0.06 = $6 \div 100$ | $5.775 \div 100 = 0.058$ | $1.628 \div 100 = 0.016$ |
| 27.36 | 26.334 | 7.427 |

Hence latitude = 26.334 chains, and departure = 7.427 chains.



A TABLE OF THE ANGLES

Which every Point and Quarter Point of the Compass makes with the Meridian.

| North. | | Points. | ° ' " | Points. | South. | |
|------------|------------|---------|----------|---------|------------|------------|
| N. by E. | N. by W. | 0-1/4 | 2 48 45 | 0-1/4 | S. by E. | S. by W. |
| | | 0-1/2 | 5 37 30 | 0-1/2 | | |
| | | 0-3/4 | 8 26 15 | 0-3/4 | | |
| | | 1 | 11 15 0 | 1 | | |
| N.N.E. | N.N.W. | 1-1/4 | 14 3 45 | 1-1/4 | S.S.E. | S.S.W. |
| | | 1-1/2 | 16 52 30 | 1-1/2 | | |
| | | 1-3/4 | 19 41 15 | 1-3/4 | | |
| | | 2 | 22 30 0 | 2 | | |
| N.E. by N. | N.W. by N. | 2-1/4 | 25 18 45 | 2-1/4 | S.E. by S. | S.W. by S. |
| | | 2-1/2 | 28 7 30 | 2-1/2 | | |
| | | 2-3/4 | 30 56 15 | 2-3/4 | | |
| | | 3 | 33 45 0 | 3 | | |
| N.E. | N.W. | 3-1/4 | 36 33 45 | 3-1/4 | S.E. | S.W. |
| | | 3-1/2 | 39 22 30 | 3-1/2 | | |
| | | 3-3/4 | 42 11 15 | 3-3/4 | | |
| | | 4 | 45 0 0 | 4 | | |
| N.E. by E. | N.W. by W. | 4-1/4 | 47 48 45 | 4-1/4 | S.E. by E. | S.W. by W. |
| | | 4-1/2 | 50 37 30 | 4-1/2 | | |
| | | 4-3/4 | 53 26 15 | 4-3/4 | | |
| | | 5 | 56 15 0 | 5 | | |
| E.N.E. | W.N.W. | 5-1/4 | 59 3 45 | 5-1/4 | E.S.E. | W.S.W. |
| | | 5-1/2 | 61 42 30 | 5-1/2 | | |
| | | 5-3/4 | 64 41 15 | 5-3/4 | | |
| | | 6 | 67 30 0 | 6 | | |
| E. by N. | W. by N. | 6-1/4 | 70 18 45 | 6-1/4 | E. by S. | W. by S. |
| | | 6-1/2 | 73 7 30 | 6-1/2 | | |
| | | 6-3/4 | 75 56 15 | 6-3/4 | | |
| | | 7 | 78 45 0 | 7 | | |
| East. | West. | 7-1/4 | 81 33 45 | 7-1/4 | East. | West. |
| | | 7-1/2 | 84 22 30 | 7-1/2 | | |
| | | 7-3/4 | 87 11 15 | 7-3/4 | | |
| | | 8 | 90 0 0 | 8 | | |

TABLE VII—TRAVERSE TABLE.

| Bearing. | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. |
|----------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|----------|
| ° / | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / |
| 0 15 | 1.000 | 0.004 | 2.000 | 0.009 | 3.000 | 0.013 | 4.000 | 0.017 | 5.000 | 0.022 | 89 45 |
| 30 | 1.000 | 0.009 | 2.000 | 0.017 | 3.000 | 0.026 | 4.000 | 0.035 | 5.000 | 0.044 | 30 |
| 45 | 1.000 | 0.013 | 2.000 | 0.026 | 3.000 | 0.039 | 4.000 | 0.052 | 5.000 | 0.065 | 15 |
| 1 0 | 1.000 | 0.017 | 2.000 | 0.035 | 3.000 | 0.052 | 3.999 | 0.070 | 4.999 | 0.087 | 89 0 |
| 15 | 1.000 | 0.022 | 2.000 | 0.044 | 2.999 | 0.065 | 3.999 | 0.087 | 4.999 | 0.109 | 45 |
| 30 | 1.000 | 0.026 | 1.999 | 0.052 | 2.999 | 0.079 | 3.999 | 0.105 | 4.998 | 0.131 | 30 |
| 45 | 1.000 | 0.031 | 1.999 | 0.061 | 2.999 | 0.092 | 3.998 | 0.122 | 4.998 | 0.153 | 15 |
| 2 0 | 0.999 | 0.035 | 1.999 | 0.070 | 2.998 | 0.105 | 3.998 | 0.140 | 4.997 | 0.174 | 88 0 |
| 15 | 0.999 | 0.039 | 1.998 | 0.079 | 2.998 | 0.118 | 3.997 | 0.157 | 4.996 | 0.196 | 45 |
| 30 | 0.999 | 0.044 | 1.998 | 0.087 | 2.997 | 0.131 | 3.996 | 0.174 | 4.995 | 0.218 | 30 |
| 45 | 0.999 | 0.048 | 1.998 | 0.096 | 2.997 | 0.144 | 3.995 | 0.192 | 4.994 | 0.240 | 15 |
| 3 0 | 0.999 | 0.052 | 1.997 | 0.105 | 2.996 | 0.157 | 3.995 | 0.209 | 4.993 | 0.262 | 87 0 |
| 15 | 0.998 | 0.057 | 1.997 | 0.113 | 2.995 | 0.170 | 3.994 | 0.227 | 4.992 | 0.283 | 45 |
| 30 | 0.998 | 0.061 | 1.996 | 0.122 | 2.994 | 0.183 | 3.993 | 0.244 | 4.991 | 0.305 | 30 |
| 45 | 0.998 | 0.065 | 1.996 | 0.131 | 2.994 | 0.196 | 3.991 | 0.262 | 4.989 | 0.327 | 15 |
| 4 0 | 0.998 | 0.070 | 1.995 | 0.140 | 2.993 | 0.209 | 3.990 | 0.279 | 4.988 | 0.349 | 86 0 |
| 15 | 0.997 | 0.074 | 1.995 | 0.148 | 2.992 | 0.222 | 3.989 | 0.296 | 4.986 | 0.371 | 45 |
| 30 | 0.997 | 0.078 | 1.994 | 0.157 | 2.991 | 0.235 | 3.988 | 0.314 | 4.985 | 0.392 | 30 |
| 45 | 0.997 | 0.083 | 1.993 | 0.166 | 2.990 | 0.248 | 3.986 | 0.331 | 4.983 | 0.414 | 15 |
| 5 0 | 0.996 | 0.087 | 1.992 | 0.174 | 2.989 | 0.261 | 3.985 | 0.349 | 4.981 | 0.436 | 85 0 |
| 15 | 0.996 | 0.092 | 1.992 | 0.183 | 2.987 | 0.275 | 3.983 | 0.366 | 4.979 | 0.458 | 45 |
| 30 | 0.995 | 0.096 | 1.991 | 0.192 | 2.986 | 0.288 | 3.982 | 0.383 | 4.977 | 0.479 | 30 |
| 45 | 0.995 | 0.100 | 1.990 | 0.200 | 2.985 | 0.301 | 3.980 | 0.401 | 4.975 | 0.501 | 15 |
| 6 0 | 0.995 | 0.105 | 1.989 | 0.209 | 2.984 | 0.314 | 3.978 | 0.418 | 4.973 | 0.523 | 84 0 |
| 15 | 0.994 | 0.109 | 1.988 | 0.218 | 2.982 | 0.327 | 3.976 | 0.435 | 4.970 | 0.544 | 45 |
| 30 | 0.994 | 0.113 | 1.987 | 0.226 | 2.981 | 0.340 | 3.974 | 0.453 | 4.968 | 0.566 | 30 |
| 45 | 0.993 | 0.118 | 1.986 | 0.235 | 2.979 | 0.353 | 3.972 | 0.470 | 4.965 | 0.588 | 15 |
| 7 0 | 0.993 | 0.122 | 1.985 | 0.244 | 2.978 | 0.366 | 3.970 | 0.487 | 4.963 | 0.609 | 83 0 |
| 15 | 0.992 | 0.126 | 1.984 | 0.252 | 2.976 | 0.379 | 3.968 | 0.505 | 4.960 | 0.631 | 45 |
| 30 | 0.991 | 0.131 | 1.983 | 0.261 | 2.974 | 0.392 | 3.966 | 0.522 | 4.957 | 0.653 | 30 |
| 45 | 0.991 | 0.135 | 1.982 | 0.270 | 2.973 | 0.405 | 3.963 | 0.539 | 4.954 | 0.674 | 15 |
| 8 0 | 0.990 | 0.139 | 1.981 | 0.278 | 2.971 | 0.418 | 3.961 | 0.557 | 4.951 | 0.696 | 82 0 |
| 15 | 0.990 | 0.143 | 1.979 | 0.287 | 2.969 | 0.430 | 3.959 | 0.574 | 4.948 | 0.717 | 45 |
| 30 | 0.989 | 0.148 | 1.978 | 0.296 | 2.967 | 0.443 | 3.956 | 0.591 | 4.945 | 0.739 | 30 |
| 45 | 0.988 | 0.152 | 1.977 | 0.304 | 2.965 | 0.456 | 3.953 | 0.608 | 4.942 | 0.761 | 15 |
| 9 0 | 0.988 | 0.156 | 1.975 | 0.313 | 2.963 | 0.469 | 3.951 | 0.626 | 4.938 | 0.782 | 81 0 |
| 15 | 0.987 | 0.161 | 1.974 | 0.321 | 2.961 | 0.482 | 3.948 | 0.643 | 4.935 | 0.804 | 45 |
| 30 | 0.986 | 0.165 | 1.973 | 0.330 | 2.959 | 0.495 | 3.945 | 0.660 | 4.931 | 0.825 | 30 |
| 45 | 0.986 | 0.169 | 1.971 | 0.339 | 2.957 | 0.508 | 3.942 | 0.677 | 4.928 | 0.847 | 15 |
| 10 0 | 0.985 | 0.174 | 1.970 | 0.347 | 2.954 | 0.521 | 3.939 | 0.695 | 4.924 | 0.868 | 80 0 |
| 15 | 0.984 | 0.178 | 1.968 | 0.356 | 2.952 | 0.534 | 3.936 | 0.712 | 4.920 | 0.890 | 45 |
| 30 | 0.983 | 0.182 | 1.967 | 0.364 | 2.950 | 0.547 | 3.933 | 0.729 | 4.916 | 0.911 | 30 |
| 45 | 0.982 | 0.187 | 1.965 | 0.373 | 2.947 | 0.560 | 3.930 | 0.746 | 4.912 | 0.933 | 15 |
| 11 0 | 0.982 | 0.191 | 1.963 | 0.382 | 2.945 | 0.572 | 3.927 | 0.763 | 4.908 | 0.954 | 79 0 |
| 15 | 0.981 | 0.195 | 1.962 | 0.390 | 2.942 | 0.585 | 3.923 | 0.780 | 4.904 | 0.975 | 45 |
| 30 | 0.980 | 0.199 | 1.960 | 0.399 | 2.940 | 0.598 | 3.920 | 0.797 | 4.900 | 0.997 | 30 |
| 45 | 0.979 | 0.204 | 1.958 | 0.407 | 2.937 | 0.611 | 3.916 | 0.815 | 4.895 | 1.018 | 15 |
| 12 0 | 0.978 | 0.208 | 1.956 | 0.416 | 2.934 | 0.624 | 3.913 | 0.832 | 4.891 | 1.040 | 78 0 |
| 15 | 0.977 | 0.212 | 1.954 | 0.424 | 2.932 | 0.637 | 3.909 | 0.849 | 4.886 | 1.061 | 45 |
| 30 | 0.976 | 0.216 | 1.953 | 0.433 | 2.929 | 0.649 | 3.905 | 0.866 | 4.881 | 1.082 | 30 |
| 45 | 0.975 | 0.221 | 1.951 | 0.441 | 2.926 | 0.662 | 3.901 | 0.883 | 4.877 | 1.103 | 15 |
| 13 0 | 0.974 | 0.225 | 1.949 | 0.450 | 2.923 | 0.675 | 3.897 | 0.900 | 4.872 | 1.125 | 77 0 |
| 15 | 0.973 | 0.229 | 1.947 | 0.458 | 2.920 | 0.688 | 3.894 | 0.917 | 4.867 | 1.146 | 45 |
| 30 | 0.972 | 0.233 | 1.945 | 0.467 | 2.917 | 0.700 | 3.889 | 0.934 | 4.862 | 1.167 | 30 |
| 45 | 0.971 | 0.238 | 1.943 | 0.475 | 2.914 | 0.713 | 3.885 | 0.951 | 4.857 | 1.188 | 15 |
| 14 0 | 0.970 | 0.242 | 1.941 | 0.484 | 2.911 | 0.726 | 3.881 | 0.968 | 4.851 | 1.210 | 76 0 |
| 15 | 0.969 | 0.246 | 1.938 | 0.492 | 2.908 | 0.738 | 3.877 | 0.985 | 4.846 | 1.231 | 45 |
| 30 | 0.968 | 0.250 | 1.936 | 0.501 | 2.904 | 0.751 | 3.873 | 1.002 | 4.841 | 1.252 | 30 |
| 45 | 0.967 | 0.255 | 1.934 | 0.509 | 2.901 | 0.764 | 3.868 | 1.018 | 4.835 | 1.273 | 15 |
| 15 0 | 0.966 | 0.259 | 1.932 | 0.518 | 2.898 | 0.776 | 3.864 | 1.035 | 4.830 | 1.294 | 75 0 |
| ° / | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / |
| Bearing. | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. |

75°—90°

| Bearing. | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. |
|----------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|--------------|-------|----------|
| ° / | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / |
| 0 15 | 6.000 | 0.026 | 7.000 | 0.031 | 8.000 | 0.035 | 9.000 | 0.039 | 10.000 | 0.044 | 89 45 |
| 30 | 6.000 | 0.052 | 7.000 | 0.061 | 8.000 | 0.070 | 9.000 | 0.079 | 10.000 | 0.087 | 30 |
| 45 | 5.999 | 0.079 | 6.999 | 0.092 | 7.999 | 0.105 | 8.999 | 0.118 | 9.999 | 0.131 | 15 |
| 1 0 | 5.999 | 0.105 | 6.999 | 0.122 | 7.999 | 0.140 | 8.999 | 0.157 | 9.999 | 0.175 | 89 0 |
| 15 | 5.999 | 0.131 | 6.998 | 0.153 | 7.998 | 0.175 | 8.998 | 0.196 | 9.998 | 0.218 | 45 |
| 30 | 5.998 | 0.157 | 6.998 | 0.183 | 7.997 | 0.209 | 8.997 | 0.236 | 9.997 | 0.262 | 30 |
| 45 | 5.997 | 0.183 | 6.997 | 0.214 | 7.996 | 0.244 | 8.996 | 0.275 | 9.995 | 0.305 | 15 |
| 2 0 | 5.996 | 0.209 | 6.996 | 0.244 | 7.995 | 0.279 | 8.995 | 0.314 | 9.994 | 0.349 | 88 0 |
| 15 | 5.995 | 0.236 | 6.995 | 0.275 | 7.994 | 0.314 | 8.993 | 0.353 | 9.992 | 0.393 | 45 |
| 30 | 5.994 | 0.262 | 6.993 | 0.305 | 7.992 | 0.349 | 8.991 | 0.393 | 9.991 | 0.436 | 30 |
| 45 | 5.993 | 0.288 | 6.992 | 0.336 | 7.991 | 0.384 | 8.990 | 0.432 | 9.989 | 0.480 | 15 |
| 3 0 | 5.992 | 0.314 | 6.990 | 0.366 | 7.989 | 0.419 | 8.988 | 0.471 | 9.986 | 0.523 | 87 0 |
| 15 | 5.990 | 0.340 | 6.989 | 0.397 | 7.987 | 0.454 | 8.986 | 0.510 | 9.984 | 0.567 | 45 |
| 30 | 5.989 | 0.366 | 6.987 | 0.427 | 7.985 | 0.488 | 8.983 | 0.549 | 9.981 | 0.611 | 30 |
| 45 | 5.987 | 0.392 | 6.985 | 0.458 | 7.983 | 0.523 | 8.981 | 0.589 | 9.979 | 0.654 | 15 |
| 4 0 | 5.985 | 0.419 | 6.983 | 0.488 | 7.981 | 0.558 | 8.978 | 0.628 | 9.976 | 0.698 | 86 0 |
| 15 | 5.984 | 0.445 | 6.981 | 0.519 | 7.978 | 0.593 | 8.975 | 0.667 | 9.973 | 0.741 | 45 |
| 30 | 5.982 | 0.471 | 6.978 | 0.549 | 7.975 | 0.628 | 8.972 | 0.706 | 9.969 | 0.785 | 30 |
| 45 | 5.979 | 0.497 | 6.976 | 0.580 | 7.973 | 0.662 | 8.969 | 0.745 | 9.966 | 0.828 | 15 |
| 5 0 | 5.977 | 0.523 | 6.973 | 0.610 | 7.970 | 0.697 | 8.966 | 0.784 | 9.962 | 0.872 | 85 0 |
| 15 | 5.975 | 0.549 | 6.971 | 0.641 | 7.966 | 0.732 | 8.962 | 0.824 | 9.958 | 0.915 | 45 |
| 30 | 5.972 | 0.575 | 6.968 | 0.671 | 7.963 | 0.767 | 8.959 | 0.863 | 9.954 | 0.959 | 30 |
| 45 | 5.970 | 0.601 | 6.965 | 0.701 | 7.960 | 0.802 | 8.955 | 0.902 | 9.950 | 1.002 | 15 |
| 6 0 | 5.967 | 0.627 | 6.962 | 0.732 | 7.956 | 0.836 | 8.951 | 0.941 | 9.945 | 1.045 | 84 0 |
| 15 | 5.964 | 0.653 | 6.958 | 0.762 | 7.952 | 0.871 | 8.947 | 0.980 | 9.941 | 1.089 | 45 |
| 30 | 5.961 | 0.679 | 6.955 | 0.792 | 7.949 | 0.906 | 8.942 | 1.019 | 9.936 | 1.132 | 30 |
| 45 | 5.958 | 0.705 | 6.951 | 0.823 | 7.945 | 0.940 | 8.938 | 1.058 | 9.931 | 1.175 | 15 |
| 7 0 | 5.955 | 0.731 | 6.948 | 0.853 | 7.940 | 0.975 | 8.933 | 1.097 | 9.926 | 1.219 | 83 0 |
| 15 | 5.952 | 0.757 | 6.944 | 0.883 | 7.936 | 1.010 | 8.928 | 1.136 | 9.920 | 1.262 | 45 |
| 30 | 5.949 | 0.783 | 6.940 | 0.914 | 7.932 | 1.044 | 8.923 | 1.175 | 9.914 | 1.305 | 30 |
| 45 | 5.945 | 0.809 | 6.936 | 0.944 | 7.927 | 1.079 | 8.918 | 1.214 | 9.909 | 1.349 | 15 |
| 8 0 | 5.942 | 0.835 | 6.932 | 0.974 | 7.922 | 1.113 | 8.912 | 1.253 | 9.903 | 1.392 | 82 0 |
| 15 | 5.938 | 0.861 | 6.928 | 1.004 | 7.917 | 1.148 | 8.907 | 1.291 | 9.897 | 1.435 | 45 |
| 30 | 5.934 | 0.887 | 6.923 | 1.035 | 7.912 | 1.182 | 8.901 | 1.330 | 9.890 | 1.478 | 30 |
| 45 | 5.930 | 0.913 | 6.919 | 1.065 | 7.907 | 1.217 | 8.895 | 1.369 | 9.884 | 1.521 | 15 |
| 9 0 | 5.926 | 0.939 | 6.914 | 1.095 | 7.902 | 1.251 | 8.889 | 1.408 | 9.877 | 1.564 | 81 0 |
| 15 | 5.922 | 0.964 | 6.909 | 1.125 | 7.896 | 1.286 | 8.883 | 1.447 | 9.870 | 1.607 | 45 |
| 30 | 5.918 | 0.990 | 6.904 | 1.155 | 7.890 | 1.320 | 8.877 | 1.485 | 9.863 | 1.651 | 30 |
| 45 | 5.913 | 1.016 | 6.899 | 1.185 | 7.884 | 1.355 | 8.870 | 1.524 | 9.856 | 1.694 | 15 |
| 10 0 | 5.909 | 1.042 | 6.894 | 1.216 | 7.878 | 1.389 | 8.863 | 1.563 | 9.848 | 1.737 | 80 0 |
| 15 | 5.904 | 1.068 | 6.888 | 1.246 | 7.872 | 1.424 | 8.856 | 1.601 | 9.840 | 1.779 | 45 |
| 30 | 5.900 | 1.093 | 6.883 | 1.276 | 7.866 | 1.458 | 8.849 | 1.640 | 9.833 | 1.822 | 30 |
| 45 | 5.895 | 1.119 | 6.877 | 1.306 | 7.860 | 1.492 | 8.842 | 1.679 | 9.825 | 1.865 | 15 |
| 11 0 | 5.890 | 1.145 | 6.871 | 1.336 | 7.853 | 1.526 | 8.835 | 1.717 | 9.816 | 1.908 | 79 0 |
| 15 | 5.885 | 1.171 | 6.866 | 1.366 | 7.846 | 1.561 | 8.827 | 1.756 | 9.808 | 1.951 | 45 |
| 30 | 5.880 | 1.196 | 6.859 | 1.396 | 7.839 | 1.595 | 8.819 | 1.794 | 9.799 | 1.994 | 30 |
| 45 | 5.874 | 1.222 | 6.853 | 1.425 | 7.832 | 1.629 | 8.811 | 1.833 | 9.791 | 2.036 | 15 |
| 12 0 | 5.869 | 1.247 | 6.847 | 1.455 | 7.825 | 1.663 | 8.803 | 1.871 | 9.782 | 2.079 | 78 0 |
| 15 | 5.863 | 1.273 | 6.841 | 1.485 | 7.818 | 1.697 | 8.795 | 1.910 | 9.772 | 2.122 | 45 |
| 30 | 5.858 | 1.299 | 6.834 | 1.515 | 7.810 | 1.732 | 8.787 | 1.948 | 9.763 | 2.164 | 30 |
| 45 | 5.852 | 1.324 | 6.827 | 1.545 | 7.803 | 1.766 | 8.778 | 1.986 | 9.753 | 2.207 | 15 |
| 13 0 | 5.846 | 1.350 | 6.821 | 1.575 | 7.795 | 1.800 | 8.769 | 2.025 | 9.744 | 2.250 | 77 0 |
| 15 | 5.840 | 1.375 | 6.814 | 1.604 | 7.787 | 1.834 | 8.760 | 2.063 | 9.734 | 2.292 | 45 |
| 30 | 5.834 | 1.401 | 6.807 | 1.634 | 7.779 | 1.868 | 8.751 | 2.101 | 9.724 | 2.335 | 30 |
| 45 | 5.828 | 1.426 | 6.799 | 1.664 | 7.771 | 1.902 | 8.742 | 2.139 | 9.713 | 2.377 | 15 |
| 14 0 | 5.822 | 1.452 | 6.792 | 1.693 | 7.762 | 1.935 | 8.733 | 2.177 | 9.703 | 2.419 | 76 0 |
| 15 | 5.815 | 1.477 | 6.785 | 1.723 | 7.754 | 1.969 | 8.723 | 2.215 | 9.692 | 2.462 | 45 |
| 30 | 5.809 | 1.502 | 6.777 | 1.753 | 7.745 | 2.003 | 8.713 | 2.253 | 9.682 | 2.504 | 30 |
| 45 | 5.802 | 1.528 | 6.769 | 1.782 | 7.736 | 2.037 | 8.703 | 2.291 | 9.671 | 2.546 | 15 |
| 15 0 | 5.796 | 1.553 | 6.761 | 1.812 | 7.727 | 2.071 | 8.693 | 2.329 | 9.659 | 2.588 | 75 0 |
| ° / | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / |
| Bearing. | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. |

| Bearing. | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. |
|----------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|----------|
| ° / | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / |
| 15 15 | 0.965 | 0.263 | 1.930 | 0.526 | 2.894 | 0.789 | 3.859 | 1.052 | 4.824 | 1.315 | 74 45 |
| 30 | 0.964 | 0.267 | 1.927 | 0.534 | 2.891 | 0.802 | 3.855 | 1.069 | 4.818 | 1.336 | 30 |
| 45 | 0.962 | 0.271 | 1.925 | 0.543 | 2.887 | 0.814 | 3.850 | 1.086 | 4.812 | 1.357 | 15 |
| 16 0 | 0.961 | 0.276 | 1.923 | 0.551 | 2.884 | 0.827 | 3.845 | 1.103 | 4.806 | 1.378 | 74 0 |
| 15 | 0.960 | 0.280 | 1.920 | 0.560 | 2.880 | 0.839 | 3.840 | 1.119 | 4.800 | 1.399 | 45 |
| 30 | 0.959 | 0.284 | 1.918 | 0.568 | 2.876 | 0.852 | 3.835 | 1.136 | 4.794 | 1.420 | 30 |
| 45 | 0.958 | 0.288 | 1.915 | 0.576 | 2.873 | 0.865 | 3.830 | 1.153 | 4.788 | 1.441 | 15 |
| 17 0 | 0.956 | 0.292 | 1.913 | 0.585 | 2.869 | 0.877 | 3.825 | 1.169 | 4.782 | 1.462 | 73 0 |
| 15 | 0.955 | 0.297 | 1.910 | 0.593 | 2.865 | 0.890 | 3.820 | 1.186 | 4.775 | 1.483 | 45 |
| 30 | 0.954 | 0.301 | 1.907 | 0.601 | 2.861 | 0.902 | 3.815 | 1.203 | 4.769 | 1.504 | 30 |
| 45 | 0.952 | 0.305 | 1.905 | 0.610 | 2.857 | 0.915 | 3.810 | 1.220 | 4.762 | 1.524 | 15 |
| 18 0 | 0.951 | 0.309 | 1.902 | 0.618 | 2.853 | 0.927 | 3.804 | 1.236 | 4.755 | 1.545 | 72 0 |
| 15 | 0.950 | 0.313 | 1.899 | 0.626 | 2.849 | 0.939 | 3.799 | 1.253 | 4.748 | 1.566 | 45 |
| 30 | 0.948 | 0.317 | 1.897 | 0.635 | 2.845 | 0.952 | 3.793 | 1.269 | 4.742 | 1.587 | 30 |
| 45 | 0.947 | 0.321 | 1.894 | 0.643 | 2.841 | 0.964 | 3.788 | 1.286 | 4.735 | 1.607 | 15 |
| 19 0 | 0.946 | 0.326 | 1.891 | 0.651 | 2.837 | 0.977 | 3.782 | 1.302 | 4.728 | 1.628 | 71 0 |
| 15 | 0.944 | 0.330 | 1.888 | 0.659 | 2.832 | 0.989 | 3.776 | 1.319 | 4.720 | 1.648 | 45 |
| 30 | 0.943 | 0.334 | 1.885 | 0.668 | 2.828 | 1.001 | 3.771 | 1.335 | 4.713 | 1.669 | 30 |
| 45 | 0.941 | 0.338 | 1.882 | 0.676 | 2.824 | 1.014 | 3.765 | 1.352 | 4.706 | 1.690 | 15 |
| 20 0 | 0.940 | 0.342 | 1.879 | 0.684 | 2.819 | 1.026 | 3.759 | 1.368 | 4.698 | 1.710 | 70 0 |
| 15 | 0.938 | 0.346 | 1.876 | 0.692 | 2.815 | 1.038 | 3.753 | 1.384 | 4.691 | 1.731 | 45 |
| 30 | 0.937 | 0.350 | 1.873 | 0.700 | 2.810 | 1.051 | 3.747 | 1.401 | 4.683 | 1.751 | 30 |
| 45 | 0.935 | 0.354 | 1.870 | 0.709 | 2.805 | 1.063 | 3.741 | 1.417 | 4.676 | 1.771 | 15 |
| 21 0 | 0.934 | 0.358 | 1.867 | 0.717 | 2.801 | 1.075 | 3.734 | 1.433 | 4.668 | 1.792 | 69 0 |
| 15 | 0.932 | 0.362 | 1.864 | 0.725 | 2.796 | 1.087 | 3.728 | 1.450 | 4.660 | 1.812 | 45 |
| 30 | 0.930 | 0.367 | 1.861 | 0.733 | 2.791 | 1.100 | 3.722 | 1.466 | 4.652 | 1.833 | 30 |
| 45 | 0.929 | 0.371 | 1.858 | 0.741 | 2.786 | 1.112 | 3.715 | 1.482 | 4.644 | 1.853 | 15 |
| 22 0 | 0.927 | 0.375 | 1.854 | 0.749 | 2.782 | 1.124 | 3.709 | 1.498 | 4.636 | 1.873 | 68 0 |
| 15 | 0.926 | 0.379 | 1.851 | 0.757 | 2.777 | 1.136 | 3.702 | 1.515 | 4.628 | 1.893 | 45 |
| 30 | 0.924 | 0.383 | 1.848 | 0.765 | 2.772 | 1.148 | 3.696 | 1.531 | 4.619 | 1.913 | 30 |
| 45 | 0.922 | 0.387 | 1.844 | 0.773 | 2.767 | 1.160 | 3.689 | 1.547 | 4.611 | 1.934 | 15 |
| 23 0 | 0.921 | 0.391 | 1.841 | 0.781 | 2.762 | 1.172 | 3.682 | 1.563 | 4.603 | 1.954 | 67 0 |
| 15 | 0.919 | 0.395 | 1.838 | 0.789 | 2.756 | 1.184 | 3.675 | 1.579 | 4.594 | 1.974 | 45 |
| 30 | 0.917 | 0.399 | 1.834 | 0.797 | 2.751 | 1.196 | 3.668 | 1.595 | 4.585 | 1.994 | 30 |
| 45 | 0.915 | 0.403 | 1.831 | 0.805 | 2.746 | 1.208 | 3.661 | 1.611 | 4.577 | 2.014 | 15 |
| 24 0 | 0.914 | 0.407 | 1.827 | 0.813 | 2.741 | 1.220 | 3.654 | 1.627 | 4.568 | 2.034 | 66 0 |
| 15 | 0.912 | 0.411 | 1.824 | 0.821 | 2.735 | 1.232 | 3.647 | 1.643 | 4.559 | 2.054 | 45 |
| 30 | 0.910 | 0.415 | 1.820 | 0.829 | 2.730 | 1.244 | 3.640 | 1.659 | 4.550 | 2.073 | 30 |
| 45 | 0.908 | 0.419 | 1.816 | 0.837 | 2.724 | 1.256 | 3.633 | 1.675 | 4.541 | 2.093 | 15 |
| 25 0 | 0.906 | 0.423 | 1.813 | 0.845 | 2.719 | 1.268 | 3.625 | 1.690 | 4.532 | 2.113 | 65 0 |
| 15 | 0.904 | 0.427 | 1.809 | 0.853 | 2.713 | 1.280 | 3.618 | 1.706 | 4.522 | 2.133 | 45 |
| 30 | 0.903 | 0.431 | 1.805 | 0.861 | 2.708 | 1.292 | 3.610 | 1.722 | 4.513 | 2.153 | 30 |
| 45 | 0.901 | 0.434 | 1.801 | 0.869 | 2.702 | 1.303 | 3.603 | 1.738 | 4.503 | 2.172 | 15 |
| 26 0 | 0.899 | 0.438 | 1.798 | 0.877 | 2.696 | 1.315 | 3.595 | 1.753 | 4.494 | 2.192 | 64 0 |
| 15 | 0.897 | 0.442 | 1.794 | 0.885 | 2.691 | 1.327 | 3.587 | 1.769 | 4.484 | 2.211 | 45 |
| 30 | 0.895 | 0.446 | 1.790 | 0.892 | 2.685 | 1.339 | 3.580 | 1.785 | 4.475 | 2.231 | 30 |
| 45 | 0.893 | 0.450 | 1.786 | 0.900 | 2.679 | 1.350 | 3.572 | 1.800 | 4.465 | 2.250 | 15 |
| 27 0 | 0.891 | 0.454 | 1.782 | 0.908 | 2.673 | 1.362 | 3.564 | 1.816 | 4.455 | 2.270 | 63 0 |
| 15 | 0.889 | 0.458 | 1.778 | 0.916 | 2.667 | 1.374 | 3.556 | 1.831 | 4.445 | 2.289 | 45 |
| 30 | 0.887 | 0.462 | 1.774 | 0.923 | 2.661 | 1.385 | 3.548 | 1.847 | 4.435 | 2.309 | 30 |
| 45 | 0.885 | 0.466 | 1.770 | 0.931 | 2.655 | 1.397 | 3.540 | 1.862 | 4.425 | 2.328 | 15 |
| 28 0 | 0.883 | 0.469 | 1.766 | 0.939 | 2.649 | 1.408 | 3.532 | 1.878 | 4.415 | 2.347 | 62 0 |
| 15 | 0.881 | 0.473 | 1.762 | 0.947 | 2.643 | 1.420 | 3.524 | 1.893 | 4.404 | 2.367 | 45 |
| 30 | 0.879 | 0.477 | 1.758 | 0.954 | 2.636 | 1.431 | 3.515 | 1.909 | 4.394 | 2.386 | 30 |
| 45 | 0.877 | 0.481 | 1.753 | 0.962 | 2.630 | 1.443 | 3.507 | 1.924 | 4.384 | 2.405 | 15 |
| 29 0 | 0.875 | 0.485 | 1.749 | 0.970 | 2.624 | 1.454 | 3.498 | 1.939 | 4.373 | 2.424 | 61 0 |
| 15 | 0.872 | 0.489 | 1.745 | 0.977 | 2.617 | 1.466 | 3.490 | 1.954 | 4.362 | 2.443 | 45 |
| 30 | 0.870 | 0.492 | 1.741 | 0.985 | 2.611 | 1.477 | 3.481 | 1.970 | 4.352 | 2.462 | 30 |
| 45 | 0.868 | 0.496 | 1.736 | 0.992 | 2.605 | 1.489 | 3.473 | 1.985 | 4.341 | 2.481 | 15 |
| 30 0 | 0.866 | 0.500 | 1.732 | 1.000 | 2.598 | 1.500 | 3.464 | 2.000 | 4.330 | 2.500 | 60 0 |
| ° / | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / |
| Bearing. | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. |

| Bearing. | | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. | |
|----------|----|-------------|-------|-------------|-------|-------------|-------|-------------|-------|--------------|-------|----------|----|
| ° / | | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / | |
| 15 | 15 | 5.789 | 1.578 | 6.754 | 1.841 | 7.718 | 2.104 | 8.683 | 2.367 | 9.648 | 2.630 | 74 | 45 |
| | 30 | 5.782 | 1.603 | 6.745 | 1.871 | 7.709 | 2.138 | 8.673 | 2.405 | 9.636 | 2.672 | | 30 |
| | 45 | 5.775 | 1.629 | 6.737 | 1.900 | 7.700 | 2.172 | 8.662 | 2.443 | 9.625 | 2.714 | | 15 |
| 16 | 0 | 5.768 | 1.654 | 6.729 | 1.929 | 7.690 | 2.205 | 8.651 | 2.481 | 9.613 | 2.756 | 74 | 0 |
| | 15 | 5.760 | 1.679 | 6.720 | 1.959 | 7.680 | 2.239 | 8.640 | 2.518 | 9.601 | 2.798 | | 45 |
| | 30 | 5.753 | 1.704 | 6.712 | 1.988 | 7.671 | 2.272 | 8.629 | 2.556 | 9.588 | 2.840 | | 30 |
| | 45 | 5.745 | 1.729 | 6.703 | 2.017 | 7.661 | 2.306 | 8.618 | 2.594 | 9.576 | 2.882 | | 15 |
| 17 | 0 | 5.738 | 1.754 | 6.694 | 2.047 | 7.650 | 2.339 | 8.607 | 2.631 | 9.563 | 2.924 | 73 | 0 |
| | 15 | 5.730 | 1.779 | 6.685 | 2.076 | 7.640 | 2.372 | 8.595 | 2.669 | 9.550 | 2.965 | | 45 |
| | 30 | 5.722 | 1.804 | 6.676 | 2.105 | 7.630 | 2.406 | 8.583 | 2.706 | 9.537 | 3.007 | | 30 |
| | 45 | 5.714 | 1.829 | 6.667 | 2.134 | 7.619 | 2.439 | 8.572 | 2.744 | 9.524 | 3.049 | | 15 |
| 18 | 0 | 5.706 | 1.854 | 6.657 | 2.163 | 7.608 | 2.472 | 8.560 | 2.781 | 9.511 | 3.090 | 72 | 0 |
| | 15 | 5.698 | 1.879 | 6.648 | 2.192 | 7.598 | 2.505 | 8.547 | 2.818 | 9.497 | 3.132 | | 45 |
| | 30 | 5.690 | 1.904 | 6.638 | 2.221 | 7.587 | 2.538 | 8.535 | 2.856 | 9.483 | 3.173 | | 30 |
| | 45 | 5.682 | 1.929 | 6.629 | 2.250 | 7.575 | 2.572 | 8.522 | 2.893 | 9.469 | 3.214 | | 15 |
| 19 | 0 | 5.673 | 1.953 | 6.619 | 2.279 | 7.564 | 2.605 | 8.510 | 2.930 | 9.455 | 3.256 | 71 | 0 |
| | 15 | 5.665 | 1.978 | 6.609 | 2.308 | 7.553 | 2.638 | 8.497 | 2.967 | 9.441 | 3.297 | | 45 |
| | 30 | 5.656 | 2.003 | 6.598 | 2.337 | 7.541 | 2.670 | 8.484 | 3.004 | 9.426 | 3.338 | | 30 |
| | 45 | 5.647 | 2.028 | 6.588 | 2.365 | 7.529 | 2.703 | 8.471 | 3.041 | 9.412 | 3.379 | | 15 |
| 20 | 0 | 5.638 | 2.052 | 6.578 | 2.394 | 7.518 | 2.736 | 8.457 | 3.078 | 9.397 | 3.420 | 70 | 0 |
| | 15 | 5.629 | 2.077 | 6.567 | 2.423 | 7.506 | 2.769 | 8.444 | 3.115 | 9.382 | 3.461 | | 45 |
| | 30 | 5.620 | 2.101 | 6.557 | 2.451 | 7.493 | 2.802 | 8.430 | 3.152 | 9.367 | 3.502 | | 30 |
| | 45 | 5.611 | 2.126 | 6.546 | 2.480 | 7.481 | 2.834 | 8.416 | 3.189 | 9.351 | 3.543 | | 15 |
| 21 | 0 | 5.601 | 2.150 | 6.535 | 2.509 | 7.469 | 2.867 | 8.402 | 3.225 | 9.336 | 3.584 | 69 | 0 |
| | 15 | 5.592 | 2.175 | 6.524 | 2.537 | 7.456 | 2.900 | 8.388 | 3.262 | 9.320 | 3.624 | | 45 |
| | 30 | 5.582 | 2.199 | 6.513 | 2.566 | 7.443 | 2.932 | 8.374 | 3.299 | 9.304 | 3.665 | | 30 |
| | 45 | 5.573 | 2.223 | 6.502 | 2.594 | 7.430 | 2.964 | 8.359 | 3.335 | 9.288 | 3.706 | | 15 |
| 22 | 0 | 5.563 | 2.248 | 6.490 | 2.622 | 7.417 | 2.997 | 8.345 | 3.371 | 9.272 | 3.746 | 68 | 0 |
| | 15 | 5.553 | 2.272 | 6.479 | 2.651 | 7.404 | 3.029 | 8.330 | 3.408 | 9.255 | 3.787 | | 45 |
| | 30 | 5.543 | 2.296 | 6.467 | 2.679 | 7.391 | 3.061 | 8.315 | 3.444 | 9.239 | 3.827 | | 30 |
| | 45 | 5.533 | 2.320 | 6.455 | 2.707 | 7.378 | 3.094 | 8.300 | 3.480 | 9.222 | 3.867 | | 15 |
| 23 | 0 | 5.523 | 2.344 | 6.444 | 2.735 | 7.364 | 3.126 | 8.285 | 3.517 | 9.205 | 3.907 | 67 | 0 |
| | 15 | 5.513 | 2.368 | 6.432 | 2.763 | 7.350 | 3.158 | 8.269 | 3.553 | 9.188 | 3.947 | | 45 |
| | 30 | 5.502 | 2.392 | 6.419 | 2.791 | 7.336 | 3.190 | 8.254 | 3.589 | 9.171 | 3.988 | | 30 |
| | 45 | 5.492 | 2.416 | 6.407 | 2.819 | 7.322 | 3.222 | 8.238 | 3.625 | 9.153 | 4.028 | | 15 |
| 24 | 0 | 5.481 | 2.440 | 6.395 | 2.847 | 7.308 | 3.254 | 8.222 | 3.661 | 9.136 | 4.067 | 66 | 0 |
| | 15 | 5.471 | 2.464 | 6.382 | 2.875 | 7.294 | 3.286 | 8.206 | 3.696 | 9.118 | 4.107 | | 45 |
| | 30 | 5.460 | 2.488 | 6.370 | 2.903 | 7.280 | 3.318 | 8.190 | 3.732 | 9.100 | 4.147 | | 30 |
| | 45 | 5.449 | 2.512 | 6.357 | 2.931 | 7.265 | 3.349 | 8.173 | 3.768 | 9.081 | 4.187 | | 15 |
| 25 | 0 | 5.438 | 2.536 | 6.344 | 2.958 | 7.250 | 3.381 | 8.157 | 3.804 | 9.063 | 4.226 | 65 | 0 |
| | 15 | 5.427 | 2.559 | 6.331 | 2.986 | 7.236 | 3.413 | 8.140 | 3.839 | 9.045 | 4.266 | | 45 |
| | 30 | 5.416 | 2.583 | 6.318 | 3.014 | 7.221 | 3.444 | 8.123 | 3.875 | 9.026 | 4.305 | | 30 |
| | 45 | 5.404 | 2.607 | 6.305 | 3.041 | 7.206 | 3.476 | 8.106 | 3.910 | 9.007 | 4.345 | | 15 |
| 26 | 0 | 5.393 | 2.630 | 6.292 | 3.069 | 7.190 | 3.507 | 8.089 | 3.945 | 8.988 | 4.384 | 64 | 0 |
| | 15 | 5.381 | 2.654 | 6.278 | 3.096 | 7.175 | 3.538 | 8.072 | 3.981 | 8.969 | 4.423 | | 45 |
| | 30 | 5.370 | 2.677 | 6.265 | 3.123 | 7.160 | 3.570 | 8.054 | 4.016 | 8.949 | 4.462 | | 30 |
| | 45 | 5.358 | 2.701 | 6.251 | 3.151 | 7.144 | 3.601 | 8.037 | 4.051 | 8.930 | 4.501 | | 15 |
| 27 | 0 | 5.346 | 2.724 | 6.237 | 3.178 | 7.128 | 3.632 | 8.019 | 4.086 | 8.910 | 4.540 | 63 | 0 |
| | 15 | 5.334 | 2.747 | 6.223 | 3.205 | 7.112 | 3.663 | 8.001 | 4.121 | 8.890 | 4.579 | | 45 |
| | 30 | 5.322 | 2.770 | 6.209 | 3.232 | 7.096 | 3.694 | 7.983 | 4.156 | 8.870 | 4.618 | | 30 |
| | 45 | 5.310 | 2.794 | 6.195 | 3.259 | 7.080 | 3.725 | 7.965 | 4.190 | 8.850 | 4.656 | | 15 |
| 28 | 0 | 5.298 | 2.817 | 6.181 | 3.286 | 7.064 | 3.756 | 7.947 | 4.225 | 8.829 | 4.695 | 62 | 0 |
| | 15 | 5.285 | 2.840 | 6.166 | 3.313 | 7.047 | 3.787 | 7.928 | 4.260 | 8.809 | 4.733 | | 45 |
| | 30 | 5.273 | 2.863 | 6.152 | 3.340 | 7.031 | 3.817 | 7.909 | 4.294 | 8.788 | 4.772 | | 30 |
| | 45 | 5.260 | 2.886 | 6.137 | 3.367 | 7.014 | 3.848 | 7.891 | 4.329 | 8.767 | 4.810 | | 15 |
| 29 | 0 | 5.248 | 2.909 | 6.122 | 3.394 | 6.997 | 3.878 | 7.872 | 4.363 | 8.746 | 4.848 | 61 | 0 |
| | 15 | 5.235 | 2.932 | 6.107 | 3.420 | 6.980 | 3.909 | 7.852 | 4.398 | 8.725 | 4.886 | | 45 |
| | 30 | 5.222 | 2.955 | 6.093 | 3.447 | 6.963 | 3.939 | 7.833 | 4.432 | 8.704 | 4.924 | | 30 |
| | 45 | 5.209 | 2.977 | 6.077 | 3.474 | 6.946 | 3.970 | 7.814 | 4.466 | 8.682 | 4.962 | | 15 |
| 30 | 0 | 5.196 | 3.000 | 6.062 | 3.500 | 6.928 | 4.000 | 7.794 | 4.500 | 8.660 | 5.000 | 60 | 0 |
| ° / | | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / | |
| Bearing. | | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. | |

| Bearing. | | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. | | |
|----------|----|-------------|-------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|----------|----|---|
| ° / | | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / | | |
| 80 | 15 | 0.864 | 0.504 | 1.728 | 1.008 | 2.592 | 1.511 | 3.455 | 2.015 | 4.319 | 2.519 | 50 | 45 | |
| | 30 | 0.862 | 0.508 | 1.723 | 1.015 | 2.585 | 1.523 | 3.447 | 2.030 | 4.308 | 2.538 | | 30 | |
| | 45 | 0.859 | 0.511 | 1.719 | 1.023 | 2.578 | 1.534 | 3.438 | 2.045 | 4.297 | 2.556 | | 15 | |
| | 81 | 0 | 0.857 | 0.515 | 1.714 | 1.030 | 2.572 | 1.545 | 3.429 | 2.060 | 4.286 | 2.575 | 50 | 0 |
| | 15 | 0.855 | 0.519 | 1.710 | 1.038 | 2.565 | 1.556 | 3.420 | 2.075 | 4.275 | 2.594 | | 45 | |
| 81 | 30 | 0.853 | 0.522 | 1.705 | 1.045 | 2.558 | 1.567 | 3.411 | 2.090 | 4.263 | 2.612 | | 30 | |
| | 45 | 0.850 | 0.526 | 1.701 | 1.052 | 2.551 | 1.579 | 3.401 | 2.105 | 4.252 | 2.631 | | 15 | |
| | 82 | 0 | 0.848 | 0.530 | 1.696 | 1.060 | 2.544 | 1.590 | 3.392 | 2.120 | 4.240 | 2.650 | 50 | 0 |
| | 15 | 0.846 | 0.534 | 1.691 | 1.067 | 2.537 | 1.601 | 3.383 | 2.134 | 4.229 | 2.668 | | 45 | |
| | 30 | 0.843 | 0.537 | 1.687 | 1.075 | 2.530 | 1.612 | 3.374 | 2.149 | 4.217 | 2.686 | | 30 | |
| 82 | 45 | 0.841 | 0.541 | 1.682 | 1.082 | 2.523 | 1.623 | 3.364 | 2.164 | 4.205 | 2.705 | | 15 | |
| | 83 | 0 | 0.839 | 0.545 | 1.677 | 1.089 | 2.516 | 1.634 | 3.355 | 2.179 | 4.193 | 2.723 | 50 | 0 |
| | 15 | 0.836 | 0.548 | 1.673 | 1.097 | 2.509 | 1.645 | 3.345 | 2.193 | 4.181 | 2.741 | | 45 | |
| | 30 | 0.834 | 0.552 | 1.668 | 1.104 | 2.502 | 1.656 | 3.336 | 2.208 | 4.169 | 2.760 | | 30 | |
| | 45 | 0.831 | 0.556 | 1.663 | 1.111 | 2.494 | 1.667 | 3.326 | 2.222 | 4.157 | 2.778 | | 15 | |
| 84 | 0 | 0.829 | 0.559 | 1.658 | 1.118 | 2.487 | 1.678 | 3.316 | 2.237 | 4.145 | 2.796 | 50 | 0 | |
| | 15 | 0.827 | 0.563 | 1.653 | 1.126 | 2.480 | 1.688 | 3.306 | 2.251 | 4.133 | 2.814 | | 45 | |
| | 30 | 0.824 | 0.566 | 1.648 | 1.133 | 2.472 | 1.699 | 3.297 | 2.266 | 4.121 | 2.832 | | 30 | |
| | 45 | 0.822 | 0.570 | 1.643 | 1.140 | 2.465 | 1.710 | 3.287 | 2.280 | 4.108 | 2.850 | | 15 | |
| | 85 | 0 | 0.819 | 0.574 | 1.638 | 1.147 | 2.457 | 1.721 | 3.277 | 2.294 | 4.096 | 2.868 | 50 | 0 |
| 85 | 15 | 0.817 | 0.577 | 1.633 | 1.154 | 2.450 | 1.731 | 3.267 | 2.309 | 4.083 | 2.886 | | 45 | |
| | 30 | 0.814 | 0.581 | 1.628 | 1.161 | 2.442 | 1.742 | 3.257 | 2.323 | 4.071 | 2.904 | | 30 | |
| | 45 | 0.812 | 0.584 | 1.623 | 1.168 | 2.435 | 1.753 | 3.246 | 2.337 | 4.058 | 2.921 | | 15 | |
| | 86 | 0 | 0.809 | 0.588 | 1.618 | 1.176 | 2.427 | 1.763 | 3.236 | 2.351 | 4.045 | 2.939 | 50 | 0 |
| | 15 | 0.806 | 0.591 | 1.613 | 1.183 | 2.419 | 1.774 | 3.226 | 2.365 | 4.032 | 2.957 | | 45 | |
| 86 | 30 | 0.804 | 0.595 | 1.608 | 1.190 | 2.412 | 1.784 | 3.215 | 2.379 | 4.019 | 2.974 | | 30 | |
| | 45 | 0.801 | 0.598 | 1.603 | 1.197 | 2.404 | 1.795 | 3.205 | 2.393 | 4.006 | 2.992 | | 15 | |
| | 87 | 0 | 0.799 | 0.602 | 1.597 | 1.204 | 2.396 | 1.805 | 3.195 | 2.407 | 3.993 | 3.009 | 50 | 0 |
| | 15 | 0.796 | 0.605 | 1.592 | 1.211 | 2.388 | 1.816 | 3.184 | 2.421 | 3.980 | 3.026 | | 45 | |
| | 30 | 0.793 | 0.609 | 1.587 | 1.218 | 2.380 | 1.826 | 3.173 | 2.435 | 3.967 | 3.044 | | 30 | |
| 87 | 45 | 0.791 | 0.612 | 1.581 | 1.224 | 2.372 | 1.837 | 3.163 | 2.449 | 3.953 | 3.061 | | 15 | |
| | 88 | 0 | 0.788 | 0.616 | 1.576 | 1.231 | 2.364 | 1.847 | 3.152 | 2.463 | 3.940 | 3.078 | 50 | 0 |
| | 15 | 0.785 | 0.619 | 1.571 | 1.238 | 2.356 | 1.857 | 3.141 | 2.476 | 3.927 | 3.095 | | 45 | |
| | 30 | 0.783 | 0.623 | 1.565 | 1.245 | 2.348 | 1.868 | 3.130 | 2.490 | 3.913 | 3.113 | | 30 | |
| | 45 | 0.780 | 0.626 | 1.560 | 1.252 | 2.340 | 1.878 | 3.120 | 2.504 | 3.899 | 3.130 | | 15 | |
| 88 | 0 | 0.777 | 0.629 | 1.554 | 1.259 | 2.331 | 1.888 | 3.109 | 2.517 | 3.886 | 3.147 | 50 | 0 | |
| | 15 | 0.774 | 0.633 | 1.549 | 1.265 | 2.323 | 1.898 | 3.098 | 2.531 | 3.872 | 3.164 | | 45 | |
| | 30 | 0.772 | 0.636 | 1.543 | 1.272 | 2.315 | 1.908 | 3.086 | 2.544 | 3.858 | 3.180 | | 30 | |
| | 45 | 0.769 | 0.639 | 1.538 | 1.279 | 2.307 | 1.918 | 3.075 | 2.558 | 3.844 | 3.197 | | 15 | |
| | 40 | 0 | 0.766 | 0.643 | 1.532 | 1.286 | 2.298 | 1.928 | 3.064 | 2.571 | 3.830 | 3.214 | 50 | 0 |
| 40 | 15 | 0.763 | 0.646 | 1.526 | 1.292 | 2.290 | 1.938 | 3.053 | 2.584 | 3.816 | 3.231 | | 45 | |
| | 30 | 0.760 | 0.649 | 1.521 | 1.299 | 2.281 | 1.948 | 3.042 | 2.598 | 3.802 | 3.247 | | 30 | |
| | 45 | 0.758 | 0.653 | 1.515 | 1.306 | 2.273 | 1.958 | 3.030 | 2.611 | 3.788 | 3.264 | | 15 | |
| | 41 | 0 | 0.755 | 0.656 | 1.509 | 1.312 | 2.264 | 1.968 | 3.019 | 2.624 | 3.774 | 3.280 | 40 | 0 |
| | 15 | 0.752 | 0.659 | 1.504 | 1.319 | 2.256 | 1.978 | 3.007 | 2.637 | 3.759 | 3.297 | | 45 | |
| 41 | 30 | 0.749 | 0.663 | 1.498 | 1.325 | 2.247 | 1.988 | 2.996 | 2.650 | 3.745 | 3.313 | | 30 | |
| | 45 | 0.746 | 0.666 | 1.492 | 1.332 | 2.238 | 1.998 | 2.984 | 2.664 | 3.730 | 3.329 | | 15 | |
| | 42 | 0 | 0.743 | 0.669 | 1.486 | 1.338 | 2.229 | 2.007 | 2.973 | 2.677 | 3.716 | 3.346 | 40 | 0 |
| | 15 | 0.740 | 0.672 | 1.480 | 1.345 | 2.221 | 2.017 | 2.961 | 2.689 | 3.701 | 3.362 | | 45 | |
| | 30 | 0.737 | 0.676 | 1.475 | 1.351 | 2.212 | 2.027 | 2.949 | 2.702 | 3.686 | 3.378 | | 30 | |
| 42 | 45 | 0.734 | 0.679 | 1.469 | 1.358 | 2.203 | 2.036 | 2.937 | 2.715 | 3.672 | 3.394 | | 15 | |
| | 43 | 0 | 0.731 | 0.682 | 1.463 | 1.364 | 2.194 | 2.046 | 2.925 | 2.728 | 3.657 | 3.410 | 40 | 0 |
| | 15 | 0.728 | 0.685 | 1.457 | 1.370 | 2.185 | 2.056 | 2.913 | 2.741 | 3.642 | 3.426 | | 45 | |
| | 30 | 0.725 | 0.688 | 1.451 | 1.377 | 2.176 | 2.065 | 2.901 | 2.753 | 3.627 | 3.442 | | 30 | |
| | 45 | 0.722 | 0.692 | 1.445 | 1.383 | 2.167 | 2.075 | 2.889 | 2.766 | 3.612 | 3.458 | | 15 | |
| 43 | 0 | 0.719 | 0.695 | 1.439 | 1.389 | 2.158 | 2.084 | 2.877 | 2.779 | 3.597 | 3.473 | 40 | 0 | |
| | 15 | 0.716 | 0.698 | 1.433 | 1.396 | 2.149 | 2.093 | 2.865 | 2.791 | 3.582 | 3.489 | | 45 | |
| | 30 | 0.713 | 0.701 | 1.427 | 1.402 | 2.140 | 2.103 | 2.853 | 2.804 | 3.566 | 3.505 | | 30 | |
| | 45 | 0.710 | 0.704 | 1.420 | 1.408 | 2.131 | 2.112 | 2.841 | 2.816 | 3.551 | 3.520 | | 15 | |
| | 44 | 0 | 0.707 | 0.707 | 1.414 | 1.414 | 2.121 | 2.121 | 2.828 | 2.828 | 3.536 | 3.536 | 40 | 0 |
| ° / | | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / | | |
| Bearing. | | Distance 1. | | Distance 2. | | Distance 3. | | Distance 4. | | Distance 5. | | Bearing. | | |

| Bearing. | | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. |
|----------|----|-------------|-------|-------------|-------|-------------|-------|-------------|-------|--------------|-------|----------|
| ° / | | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | ° / |
| 80 | 15 | 5.183 | 3.023 | 6.047 | 3.526 | 6.911 | 4.030 | 7.775 | 4.534 | 8.638 | 5.038 | 50 45 |
| | 30 | 5.170 | 3.045 | 6.031 | 3.553 | 6.893 | 4.060 | 7.755 | 4.568 | 8.616 | 5.075 | 30 |
| | 45 | 5.156 | 3.068 | 6.016 | 3.579 | 6.875 | 4.090 | 7.735 | 4.602 | 8.594 | 5.113 | 15 |
| 81 | 0 | 5.143 | 3.090 | 6.000 | 3.605 | 6.857 | 4.120 | 7.715 | 4.635 | 8.572 | 5.150 | 50 0 |
| | 15 | 5.129 | 3.113 | 5.984 | 3.631 | 6.839 | 4.150 | 7.694 | 4.669 | 8.549 | 5.188 | 45 |
| | 30 | 5.116 | 3.135 | 5.968 | 3.657 | 6.821 | 4.180 | 7.674 | 4.702 | 8.526 | 5.225 | 30 |
| | 45 | 5.102 | 3.157 | 5.952 | 3.683 | 6.803 | 4.210 | 7.653 | 4.736 | 8.504 | 5.262 | 15 |
| 82 | 0 | 5.088 | 3.180 | 5.936 | 3.709 | 6.784 | 4.239 | 7.632 | 4.769 | 8.481 | 5.299 | 58 0 |
| | 15 | 5.074 | 3.202 | 5.920 | 3.735 | 6.766 | 4.269 | 7.612 | 4.802 | 8.457 | 5.336 | 45 |
| | 30 | 5.060 | 3.224 | 5.904 | 3.761 | 6.747 | 4.298 | 7.591 | 4.836 | 8.434 | 5.373 | 30 |
| | 45 | 5.046 | 3.246 | 5.887 | 3.787 | 6.728 | 4.328 | 7.569 | 4.869 | 8.410 | 5.410 | 15 |
| 83 | 0 | 5.032 | 3.268 | 5.871 | 3.812 | 6.709 | 4.357 | 7.548 | 4.902 | 8.387 | 5.446 | 57 0 |
| | 15 | 5.018 | 3.290 | 5.854 | 3.838 | 6.690 | 4.386 | 7.527 | 4.935 | 8.363 | 5.483 | 45 |
| | 30 | 5.003 | 3.312 | 5.837 | 3.864 | 6.671 | 4.416 | 7.505 | 4.967 | 8.339 | 5.519 | 30 |
| | 45 | 4.989 | 3.333 | 5.820 | 3.889 | 6.652 | 4.445 | 7.483 | 5.000 | 8.315 | 5.556 | 15 |
| 84 | 0 | 4.974 | 3.355 | 5.803 | 3.914 | 6.632 | 4.474 | 7.461 | 5.033 | 8.290 | 5.592 | 56 0 |
| | 15 | 4.960 | 3.377 | 5.786 | 3.940 | 6.613 | 4.502 | 7.439 | 5.065 | 8.266 | 5.628 | 45 |
| | 30 | 4.945 | 3.398 | 5.769 | 3.965 | 6.593 | 4.531 | 7.417 | 5.098 | 8.241 | 5.664 | 30 |
| | 45 | 4.930 | 3.420 | 5.752 | 3.990 | 6.573 | 4.560 | 7.395 | 5.130 | 8.217 | 5.700 | 15 |
| 85 | 0 | 4.915 | 3.441 | 5.734 | 4.015 | 6.553 | 4.589 | 7.372 | 5.162 | 8.192 | 5.736 | 55 0 |
| | 15 | 4.900 | 3.463 | 5.716 | 4.040 | 6.533 | 4.617 | 7.350 | 5.194 | 8.166 | 5.772 | 45 |
| | 30 | 4.885 | 3.484 | 5.699 | 4.065 | 6.513 | 4.646 | 7.327 | 5.226 | 8.141 | 5.807 | 30 |
| | 45 | 4.869 | 3.505 | 5.681 | 4.090 | 6.493 | 4.674 | 7.304 | 5.258 | 8.116 | 5.843 | 15 |
| 86 | 0 | 4.854 | 3.527 | 5.663 | 4.115 | 6.472 | 4.702 | 7.281 | 5.290 | 8.090 | 5.878 | 54 0 |
| | 15 | 4.839 | 3.548 | 5.645 | 4.139 | 6.452 | 4.730 | 7.258 | 5.322 | 8.064 | 5.913 | 45 |
| | 30 | 4.823 | 3.569 | 5.627 | 4.164 | 6.431 | 4.759 | 7.235 | 5.353 | 8.039 | 5.948 | 30 |
| | 45 | 4.808 | 3.590 | 5.609 | 4.188 | 6.410 | 4.787 | 7.211 | 5.385 | 8.013 | 5.983 | 15 |
| 87 | 0 | 4.792 | 3.611 | 5.590 | 4.213 | 6.389 | 4.815 | 7.188 | 5.416 | 7.986 | 6.018 | 53 0 |
| | 15 | 4.776 | 3.632 | 5.572 | 4.237 | 6.368 | 4.842 | 7.164 | 5.448 | 7.960 | 6.053 | 45 |
| | 30 | 4.760 | 3.653 | 5.554 | 4.261 | 6.347 | 4.870 | 7.140 | 5.479 | 7.934 | 6.088 | 30 |
| | 45 | 4.744 | 3.673 | 5.535 | 4.286 | 6.326 | 4.898 | 7.116 | 5.510 | 7.907 | 6.122 | 15 |
| 88 | 0 | 4.728 | 3.694 | 5.516 | 4.310 | 6.304 | 4.925 | 7.092 | 5.541 | 7.880 | 6.157 | 52 0 |
| | 15 | 4.712 | 3.715 | 5.497 | 4.334 | 6.283 | 4.953 | 7.068 | 5.572 | 7.853 | 6.191 | 45 |
| | 30 | 4.696 | 3.735 | 5.478 | 4.358 | 6.261 | 4.980 | 7.043 | 5.603 | 7.826 | 6.225 | 30 |
| | 45 | 4.679 | 3.756 | 5.459 | 4.381 | 6.239 | 5.007 | 7.019 | 5.633 | 7.799 | 6.259 | 15 |
| 89 | 0 | 4.663 | 3.776 | 5.440 | 4.405 | 6.217 | 5.035 | 6.994 | 5.664 | 7.772 | 6.293 | 51 0 |
| | 15 | 4.646 | 3.796 | 5.421 | 4.429 | 6.195 | 5.062 | 6.970 | 5.694 | 7.744 | 6.327 | 45 |
| | 30 | 4.630 | 3.816 | 5.401 | 4.453 | 6.173 | 5.089 | 6.945 | 5.725 | 7.716 | 6.361 | 30 |
| | 45 | 4.613 | 3.837 | 5.382 | 4.476 | 6.151 | 5.116 | 6.920 | 5.755 | 7.688 | 6.394 | 15 |
| 40 | 0 | 4.596 | 3.857 | 5.362 | 4.500 | 6.128 | 5.142 | 6.894 | 5.785 | 7.660 | 6.428 | 50 0 |
| | 15 | 4.579 | 3.877 | 5.343 | 4.523 | 6.106 | 5.169 | 6.869 | 5.815 | 7.632 | 6.461 | 45 |
| | 30 | 4.562 | 3.897 | 5.323 | 4.546 | 6.083 | 5.196 | 6.844 | 5.845 | 7.604 | 6.495 | 30 |
| | 45 | 4.545 | 3.917 | 5.303 | 4.569 | 6.061 | 5.222 | 6.818 | 5.875 | 7.576 | 6.528 | 15 |
| 41 | 0 | 4.528 | 3.936 | 5.283 | 4.592 | 6.038 | 5.248 | 6.792 | 5.905 | 7.547 | 6.561 | 49 0 |
| | 15 | 4.511 | 3.956 | 5.263 | 4.615 | 6.015 | 5.275 | 6.767 | 5.934 | 7.518 | 6.594 | 45 |
| | 30 | 4.494 | 3.976 | 5.243 | 4.638 | 5.992 | 5.301 | 6.741 | 5.964 | 7.490 | 6.626 | 30 |
| | 45 | 4.476 | 3.995 | 5.222 | 4.661 | 5.968 | 5.327 | 6.715 | 5.993 | 7.461 | 6.659 | 15 |
| 42 | 0 | 4.459 | 4.015 | 5.202 | 4.684 | 5.945 | 5.353 | 6.688 | 6.022 | 7.431 | 6.691 | 48 0 |
| | 15 | 4.441 | 4.034 | 5.182 | 4.707 | 5.922 | 5.379 | 6.662 | 6.051 | 7.402 | 6.724 | 45 |
| | 30 | 4.424 | 4.054 | 5.161 | 4.729 | 5.898 | 5.405 | 6.635 | 6.080 | 7.373 | 6.756 | 30 |
| | 45 | 4.406 | 4.073 | 5.140 | 4.752 | 5.875 | 5.430 | 6.609 | 6.109 | 7.343 | 6.788 | 15 |
| 43 | 0 | 4.388 | 4.092 | 5.119 | 4.774 | 5.851 | 5.456 | 6.582 | 6.138 | 7.314 | 6.820 | 47 0 |
| | 15 | 4.370 | 4.111 | 5.099 | 4.796 | 5.827 | 5.481 | 6.555 | 6.167 | 7.284 | 6.852 | 45 |
| | 30 | 4.352 | 4.130 | 5.078 | 4.818 | 5.803 | 5.507 | 6.528 | 6.195 | 7.254 | 6.884 | 30 |
| | 45 | 4.334 | 4.149 | 5.057 | 4.841 | 5.779 | 5.532 | 6.501 | 6.224 | 7.224 | 6.915 | 15 |
| 44 | 0 | 4.316 | 4.168 | 5.035 | 4.863 | 5.755 | 5.557 | 6.474 | 6.252 | 7.193 | 6.947 | 46 0 |
| | 15 | 4.298 | 4.187 | 5.014 | 4.885 | 5.730 | 5.582 | 6.447 | 6.280 | 7.163 | 6.978 | 45 |
| | 30 | 4.280 | 4.206 | 4.993 | 4.906 | 5.706 | 5.607 | 6.419 | 6.308 | 7.133 | 7.009 | 30 |
| | 45 | 4.261 | 4.224 | 4.971 | 4.928 | 5.681 | 5.632 | 6.392 | 6.336 | 7.102 | 7.040 | 15 |
| 45 | 0 | 4.243 | 4.243 | 4.950 | 4.950 | 5.657 | 5.657 | 6.364 | 6.364 | 7.071 | 7.071 | 45 0 |
| ° / | | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | ° / |
| Bearing. | | Distance 6. | | Distance 7. | | Distance 8. | | Distance 9. | | Distance 10. | | Bearing. |

62 TABLE VIII.—NATURAL SINES AND COSINES.

| / | 0° | | 1° | | 2° | | 3° | | 4° | | / |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 0000 | 1000 | 0175 | 9998 | 0349 | 9994 | 0523 | 9986 | 0698 | 9976 | 60 |
| 1 | 0003 | 1000 | 0177 | 9998 | 0352 | 9994 | 0526 | 9986 | 0700 | 9975 | 59 |
| 2 | 0006 | 1000 | 0180 | 9998 | 0355 | 9994 | 0529 | 9986 | 0703 | 9975 | 58 |
| 3 | 0009 | 1000 | 0183 | 9998 | 0358 | 9994 | 0532 | 9986 | 0706 | 9975 | 57 |
| 4 | 0012 | 1000 | 0186 | 9998 | 0361 | 9993 | 0535 | 9986 | 0709 | 9975 | 56 |
| 5 | 0015 | 1000 | 0189 | 9998 | 0364 | 9993 | 0538 | 9986 | 0712 | 9975 | 55 |
| 6 | 0017 | 1000 | 0192 | 9998 | 0366 | 9993 | 0541 | 9985 | 0715 | 9974 | 54 |
| 7 | 0020 | 1000 | 0195 | 9998 | 0369 | 9993 | 0544 | 9985 | 0718 | 9974 | 53 |
| 8 | 0023 | 1000 | 0198 | 9998 | 0372 | 9993 | 0547 | 9985 | 0721 | 9974 | 52 |
| 9 | 0026 | 1000 | 0201 | 9998 | 0375 | 9993 | 0550 | 9985 | 0724 | 9974 | 51 |
| 10 | 0029 | 1000 | 0204 | 9998 | 0378 | 9993 | 0552 | 9985 | 0727 | 9974 | 50 |
| 11 | 0032 | 1000 | 0207 | 9998 | 0381 | 9993 | 0555 | 9985 | 0729 | 9973 | 49 |
| 12 | 0035 | 1000 | 0209 | 9998 | 0384 | 9993 | 0558 | 9984 | 0732 | 9973 | 48 |
| 13 | 0038 | 1000 | 0212 | 9998 | 0387 | 9993 | 0561 | 9984 | 0735 | 9973 | 47 |
| 14 | 0041 | 1000 | 0215 | 9998 | 0390 | 9992 | 0564 | 9984 | 0738 | 9973 | 46 |
| 15 | 0044 | 1000 | 0218 | 9998 | 0393 | 9992 | 0567 | 9984 | 0741 | 9973 | 45 |
| 16 | 0047 | 1000 | 0221 | 9998 | 0396 | 9992 | 0570 | 9984 | 0744 | 9972 | 44 |
| 17 | 0049 | 1000 | 0224 | 9997 | 0398 | 9992 | 0573 | 9984 | 0747 | 9972 | 43 |
| 18 | 0052 | 1000 | 0227 | 9997 | 0401 | 9992 | 0576 | 9983 | 0750 | 9972 | 42 |
| 19 | 0055 | 1000 | 0230 | 9997 | 0404 | 9992 | 0579 | 9983 | 0753 | 9972 | 41 |
| 20 | 0058 | 1000 | 0233 | 9997 | 0407 | 9992 | 0581 | 9983 | 0756 | 9971 | 40 |
| 21 | 0061 | 1000 | 0236 | 9997 | 0410 | 9992 | 0584 | 9983 | 0758 | 9971 | 39 |
| 22 | 0064 | 1000 | 0239 | 9997 | 0413 | 9991 | 0587 | 9983 | 0761 | 9971 | 38 |
| 23 | 0067 | 1000 | 0241 | 9997 | 0416 | 9991 | 0590 | 9983 | 0764 | 9971 | 37 |
| 24 | 0070 | 1000 | 0244 | 9997 | 0419 | 9991 | 0593 | 9982 | 0767 | 9971 | 36 |
| 25 | 0073 | 1000 | 0247 | 9997 | 0422 | 9991 | 0596 | 9982 | 0770 | 9970 | 35 |
| 26 | 0076 | 1000 | 0250 | 9997 | 0425 | 9991 | 0599 | 9982 | 0773 | 9970 | 34 |
| 27 | 0079 | 1000 | 0253 | 9997 | 0427 | 9991 | 0602 | 9982 | 0776 | 9970 | 33 |
| 28 | 0081 | 1000 | 0256 | 9997 | 0430 | 9991 | 0605 | 9982 | 0779 | 9970 | 32 |
| 29 | 0084 | 1000 | 0259 | 9997 | 0433 | 9991 | 0608 | 9982 | 0782 | 9969 | 31 |
| 30 | 0087 | 1000 | 0262 | 9997 | 0436 | 9990 | 0610 | 9981 | 0785 | 9969 | 30 |
| 31 | 0090 | 1000 | 0265 | 9996 | 0439 | 9990 | 0613 | 9981 | 0787 | 9969 | 29 |
| 32 | 0093 | 1000 | 0268 | 9996 | 0442 | 9990 | 0616 | 9981 | 0790 | 9969 | 28 |
| 33 | 0096 | 1000 | 0270 | 9996 | 0445 | 9990 | 0619 | 9981 | 0793 | 9968 | 27 |
| 34 | 0099 | 1000 | 0273 | 9996 | 0448 | 9990 | 0622 | 9981 | 0796 | 9968 | 26 |
| 35 | 0102 | 9999 | 0276 | 9996 | 0451 | 9990 | 0625 | 9980 | 0799 | 9968 | 25 |
| 36 | 0105 | 9999 | 0279 | 9996 | 0454 | 9990 | 0628 | 9980 | 0802 | 9968 | 24 |
| 37 | 0108 | 9999 | 0282 | 9996 | 0457 | 9990 | 0631 | 9980 | 0805 | 9968 | 23 |
| 38 | 0111 | 9999 | 0285 | 9996 | 0459 | 9989 | 0634 | 9980 | 0808 | 9967 | 22 |
| 39 | 0113 | 9999 | 0288 | 9996 | 0462 | 9989 | 0637 | 9980 | 0811 | 9967 | 21 |
| 40 | 0116 | 9999 | 0291 | 9996 | 0465 | 9989 | 0640 | 9980 | 0814 | 9967 | 20 |
| 41 | 0119 | 9999 | 0294 | 9996 | 0468 | 9989 | 0642 | 9979 | 0816 | 9967 | 19 |
| 42 | 0122 | 9999 | 0297 | 9996 | 0471 | 9989 | 0645 | 9979 | 0819 | 9966 | 18 |
| 43 | 0125 | 9999 | 0300 | 9996 | 0474 | 9989 | 0648 | 9979 | 0822 | 9966 | 17 |
| 44 | 0128 | 9999 | 0302 | 9995 | 0477 | 9989 | 0651 | 9979 | 0825 | 9966 | 16 |
| 45 | 0131 | 9999 | 0305 | 9995 | 0480 | 9988 | 0654 | 9979 | 0828 | 9966 | 15 |
| 46 | 0134 | 9999 | 0308 | 9995 | 0483 | 9988 | 0657 | 9978 | 0831 | 9965 | 14 |
| 47 | 0137 | 9999 | 0311 | 9995 | 0486 | 9988 | 0660 | 9978 | 0834 | 9965 | 13 |
| 48 | 0140 | 9999 | 0314 | 9995 | 0488 | 9988 | 0663 | 9978 | 0837 | 9965 | 12 |
| 49 | 0143 | 9999 | 0317 | 9995 | 0491 | 9988 | 0666 | 9978 | 0840 | 9965 | 11 |
| 50 | 0145 | 9999 | 0320 | 9995 | 0494 | 9988 | 0669 | 9978 | 0843 | 9964 | 10 |
| 51 | 0148 | 9999 | 0323 | 9995 | 0497 | 9988 | 0671 | 9977 | 0845 | 9964 | 9 |
| 52 | 0151 | 9999 | 0326 | 9995 | 0500 | 9987 | 0674 | 9977 | 0848 | 9964 | 8 |
| 53 | 0154 | 9999 | 0329 | 9995 | 0503 | 9987 | 0677 | 9977 | 0851 | 9964 | 7 |
| 54 | 0157 | 9999 | 0332 | 9995 | 0506 | 9987 | 0680 | 9977 | 0854 | 9963 | 6 |
| 55 | 0160 | 9999 | 0334 | 9994 | 0509 | 9987 | 0683 | 9977 | 0857 | 9963 | 5 |
| 56 | 0163 | 9999 | 0337 | 9994 | 0512 | 9987 | 0686 | 9976 | 0860 | 9963 | 4 |
| 57 | 0166 | 9999 | 0340 | 9994 | 0515 | 9987 | 0689 | 9976 | 0863 | 9963 | 3 |
| 58 | 0169 | 9999 | 0343 | 9994 | 0518 | 9987 | 0692 | 9976 | 0866 | 9962 | 2 |
| 59 | 0172 | 9999 | 0346 | 9994 | 0520 | 9986 | 0695 | 9976 | 0869 | 9962 | 1 |
| 60 | 0175 | 9999 | 0349 | 9994 | 0523 | 9986 | 0698 | 9976 | 0872 | 9962 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| / | 89° | | 88° | | 87° | | 86° | | 85° | | / |

| ° | 5° | | 6° | | 7° | | 8° | | 9° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 0872 | 9962 | 1045 | 9945 | 1219 | 9925 | 1392 | 9903 | 1564 | 9877 | 60 |
| 1 | 0874 | 9962 | 1048 | 9945 | 1222 | 9925 | 1395 | 9902 | 1567 | 9876 | 59 |
| 2 | 0877 | 9461 | 1051 | 9945 | 1224 | 9925 | 1397 | 9902 | 1570 | 9876 | 58 |
| 3 | 0880 | 9961 | 1054 | 9944 | 1227 | 9924 | 1400 | 9901 | 1573 | 9876 | 57 |
| 4 | 0883 | 9961 | 1057 | 9944 | 1230 | 9924 | 1403 | 9901 | 1576 | 9875 | 56 |
| 5 | 0886 | 9961 | 1060 | 9944 | 1233 | 9924 | 1406 | 9901 | 1579 | 9875 | 55 |
| 6 | 0889 | 9960 | 1063 | 9943 | 1236 | 9923 | 1409 | 9900 | 1582 | 9874 | 54 |
| 7 | 0892 | 9960 | 1066 | 9943 | 1239 | 9923 | 1412 | 9900 | 1584 | 9874 | 53 |
| 8 | 0895 | 9960 | 1068 | 9943 | 1241 | 9923 | 1415 | 9899 | 1587 | 9873 | 52 |
| 9 | 0898 | 9960 | 1071 | 9942 | 1245 | 9922 | 1418 | 9899 | 1590 | 9873 | 51 |
| 10 | 0901 | 9959 | 1074 | 9942 | 1248 | 9922 | 1421 | 9899 | 1593 | 9872 | 50 |
| 11 | 0903 | 9959 | 1077 | 9942 | 1250 | 9922 | 1423 | 9898 | 1596 | 9872 | 49 |
| 12 | 0906 | 9959 | 1080 | 9942 | 1253 | 9921 | 1426 | 9898 | 1599 | 9871 | 48 |
| 13 | 0909 | 9959 | 1083 | 9941 | 1256 | 9921 | 1429 | 9897 | 1602 | 9871 | 47 |
| 14 | 0912 | 9958 | 1086 | 9941 | 1259 | 9920 | 1432 | 9897 | 1605 | 9870 | 46 |
| 15 | 0915 | 9958 | 1089 | 9941 | 1262 | 9920 | 1435 | 9897 | 1607 | 9870 | 45 |
| 16 | 0918 | 9958 | 1092 | 9940 | 1265 | 9920 | 1438 | 9896 | 1610 | 9869 | 44 |
| 17 | 0921 | 9958 | 1094 | 9940 | 1268 | 9919 | 1441 | 9896 | 1613 | 9869 | 43 |
| 18 | 0924 | 9957 | 1097 | 9940 | 1271 | 9919 | 1444 | 9895 | 1616 | 9869 | 42 |
| 19 | 0927 | 9957 | 1100 | 9939 | 1274 | 9919 | 1446 | 9895 | 1619 | 9868 | 41 |
| 20 | 0929 | 9957 | 1103 | 9939 | 1276 | 9918 | 1449 | 9894 | 1622 | 9868 | 40 |
| 21 | 0932 | 9956 | 1106 | 9939 | 1279 | 9918 | 1452 | 9894 | 1625 | 9867 | 39 |
| 22 | 0935 | 9956 | 1109 | 9938 | 1282 | 9917 | 1455 | 9894 | 1628 | 9867 | 38 |
| 23 | 0938 | 9956 | 1112 | 9938 | 1285 | 9917 | 1458 | 9893 | 1630 | 9866 | 37 |
| 24 | 0941 | 9956 | 1115 | 9938 | 1288 | 9917 | 1461 | 9893 | 1633 | 9866 | 36 |
| 25 | 0944 | 9955 | 1118 | 9937 | 1291 | 9916 | 1464 | 9892 | 1636 | 9865 | 35 |
| 26 | 0947 | 9955 | 1120 | 9937 | 1294 | 9916 | 1467 | 9892 | 1639 | 9865 | 34 |
| 27 | 0950 | 9955 | 1123 | 9937 | 1297 | 9916 | 1469 | 9891 | 1642 | 9864 | 33 |
| 28 | 0953 | 9955 | 1126 | 9936 | 1299 | 9915 | 1472 | 9891 | 1645 | 9864 | 32 |
| 29 | 0956 | 9954 | 1129 | 9936 | 1302 | 9915 | 1475 | 9891 | 1648 | 9863 | 31 |
| 30 | 0958 | 9954 | 1132 | 9936 | 1305 | 9914 | 1478 | 9890 | 1650 | 9863 | 30 |
| 31 | 0961 | 9954 | 1135 | 9935 | 1308 | 9914 | 1481 | 9890 | 1653 | 9862 | 29 |
| 32 | 9964 | 9953 | 1138 | 9935 | 1311 | 9914 | 1484 | 9889 | 1656 | 9862 | 28 |
| 33 | 0967 | 9953 | 1141 | 9935 | 1314 | 9913 | 1487 | 9889 | 1659 | 9861 | 27 |
| 34 | 0970 | 9953 | 1144 | 9934 | 1317 | 9913 | 1490 | 9888 | 1662 | 9861 | 26 |
| 35 | 0973 | 9553 | 1146 | 9934 | 1320 | 9913 | 1492 | 9888 | 1665 | 9860 | 25 |
| 36 | 0976 | 9952 | 1149 | 9934 | 1323 | 9912 | 1495 | 9888 | 1668 | 9860 | 24 |
| 37 | 0979 | 9952 | 1152 | 9933 | 1325 | 9912 | 1498 | 9887 | 1671 | 9859 | 23 |
| 38 | 0982 | 9952 | 1155 | 9933 | 1328 | 9911 | 1501 | 9887 | 1673 | 9859 | 22 |
| 39 | 0985 | 9951 | 1158 | 9933 | 1331 | 9911 | 1504 | 9886 | 1676 | 9859 | 21 |
| 40 | 0987 | 9951 | 1161 | 9932 | 1334 | 9911 | 1507 | 9886 | 1679 | 9858 | 20 |
| 41 | 0990 | 9951 | 1164 | 9932 | 1337 | 9910 | 1510 | 9885 | 1682 | 9858 | 19 |
| 42 | 0993 | 9951 | 1167 | 9932 | 1340 | 9910 | 1513 | 9885 | 1685 | 9857 | 18 |
| 43 | 0996 | 9950 | 1170 | 9931 | 1343 | 9909 | 1515 | 9884 | 1688 | 9857 | 17 |
| 44 | 0999 | 9950 | 1172 | 9931 | 1346 | 9909 | 1518 | 9884 | 1691 | 9856 | 16 |
| 45 | 1002 | 9950 | 1175 | 9931 | 1349 | 9909 | 1521 | 9884 | 1693 | 9856 | 15 |
| 46 | 1005 | 9949 | 1178 | 9930 | 1351 | 9908 | 1524 | 9883 | 1696 | 9855 | 14 |
| 47 | 1008 | 9949 | 1181 | 9930 | 1354 | 9908 | 1527 | 9883 | 1699 | 9855 | 13 |
| 48 | 1011 | 9949 | 1184 | 9930 | 1357 | 9907 | 1530 | 9882 | 1702 | 9854 | 12 |
| 49 | 1013 | 9949 | 1187 | 9929 | 1360 | 9907 | 1533 | 9882 | 1705 | 9854 | 11 |
| 50 | 1016 | 9948 | 1190 | 9929 | 1363 | 9907 | 1536 | 9881 | 1708 | 9853 | 10 |
| 51 | 1019 | 9948 | 1193 | 9929 | 1366 | 9906 | 1538 | 9881 | 1711 | 9853 | 9 |
| 52 | 1022 | 9948 | 1196 | 9928 | 1369 | 9906 | 1541 | 9880 | 1714 | 9852 | 8 |
| 53 | 1025 | 9947 | 1198 | 9928 | 1372 | 9905 | 1544 | 9880 | 1716 | 9852 | 7 |
| 54 | 1028 | 9947 | 1201 | 9928 | 1374 | 9905 | 1547 | 9880 | 1719 | 9851 | 6 |
| 55 | 1031 | 9947 | 1204 | 9927 | 1377 | 9905 | 1550 | 9879 | 1722 | 9851 | 5 |
| 56 | 1034 | 9946 | 1207 | 9927 | 1380 | 9904 | 1553 | 9879 | 1725 | 9850 | 4 |
| 57 | 1037 | 9946 | 1210 | 9927 | 1383 | 9904 | 1556 | 9878 | 1728 | 9850 | 3 |
| 58 | 1039 | 9946 | 1213 | 9926 | 1386 | 9903 | 1559 | 9878 | 1731 | 9849 | 2 |
| 59 | 1042 | 9946 | 1216 | 9926 | 1389 | 9903 | 1561 | 9877 | 1734 | 9849 | 1 |
| 60 | 1045 | 9945 | 1219 | 9925 | 1392 | 9903 | 1564 | 9877 | 1736 | 9848 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 84° | | 83° | | 82° | | 81° | | 80° | | ° |

| ° | 10° | | 11° | | 12° | | 13° | | 14° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 1736 | 9848 | 1908 | 9816 | 2079 | 9781 | 2250 | 9744 | 2419 | 9703 | 60 |
| 1 | 1739 | 9848 | 1911 | 9816 | 2082 | 9781 | 2252 | 9743 | 2422 | 9702 | 59 |
| 2 | 1742 | 9847 | 1914 | 9815 | 2085 | 9780 | 2255 | 9742 | 2425 | 9702 | 58 |
| 3 | 1745 | 9847 | 1917 | 9815 | 2088 | 9780 | 2258 | 9742 | 2428 | 9701 | 57 |
| 4 | 1748 | 9846 | 1920 | 9814 | 2090 | 9779 | 2261 | 9741 | 2431 | 9700 | 56 |
| 5 | 1751 | 9846 | 1922 | 9813 | 2093 | 9778 | 2264 | 9740 | 2433 | 9699 | 55 |
| 6 | 1754 | 9845 | 1925 | 9813 | 2096 | 9778 | 2267 | 9740 | 2436 | 9699 | 54 |
| 7 | 1757 | 9845 | 1928 | 9812 | 2099 | 9777 | 2269 | 9739 | 2439 | 9698 | 53 |
| 8 | 1759 | 9844 | 1931 | 9812 | 2102 | 9777 | 2272 | 9738 | 2442 | 9697 | 52 |
| 9 | 1762 | 9843 | 1934 | 9811 | 2105 | 9776 | 2275 | 9738 | 2445 | 9697 | 51 |
| 10 | 1765 | 9843 | 1937 | 9811 | 2108 | 9775 | 2278 | 9737 | 2447 | 9696 | 50 |
| 11 | 1768 | 9842 | 1939 | 9810 | 2110 | 9775 | 2281 | 9736 | 2450 | 9695 | 49 |
| 12 | 1771 | 9842 | 1942 | 9810 | 2113 | 9774 | 2284 | 9736 | 2453 | 9694 | 48 |
| 13 | 1774 | 9841 | 1945 | 9809 | 2116 | 9774 | 2286 | 9735 | 2456 | 9694 | 47 |
| 14 | 1777 | 9841 | 1948 | 9808 | 2119 | 9773 | 2289 | 9734 | 2459 | 9693 | 46 |
| 15 | 1779 | 9840 | 1951 | 9808 | 2122 | 9772 | 2292 | 9734 | 2462 | 9692 | 45 |
| 16 | 1782 | 9840 | 1954 | 9807 | 2125 | 9772 | 2295 | 9733 | 2464 | 9692 | 44 |
| 17 | 1785 | 9839 | 1957 | 9807 | 2127 | 9771 | 2298 | 9732 | 2467 | 9691 | 43 |
| 18 | 1788 | 9839 | 1959 | 9806 | 2130 | 9770 | 2300 | 9732 | 2470 | 9690 | 42 |
| 19 | 1791 | 9838 | 1962 | 9806 | 2133 | 9770 | 2303 | 9731 | 2473 | 9689 | 41 |
| 20 | 1794 | 9838 | 1965 | 9805 | 2136 | 9769 | 2306 | 9730 | 2476 | 9689 | 40 |
| 21 | 1797 | 9837 | 1968 | 9804 | 2139 | 9769 | 2309 | 9730 | 2478 | 9688 | 39 |
| 22 | 1799 | 9837 | 1971 | 9804 | 2142 | 9768 | 2312 | 9729 | 2481 | 9687 | 38 |
| 23 | 1802 | 9836 | 1974 | 9803 | 2145 | 9767 | 2315 | 9728 | 2484 | 9687 | 37 |
| 24 | 1805 | 9836 | 1977 | 9803 | 2147 | 9767 | 2317 | 9728 | 2487 | 9686 | 36 |
| 25 | 1808 | 9835 | 1979 | 9802 | 2150 | 9766 | 2320 | 9727 | 2490 | 9685 | 35 |
| 26 | 1811 | 9835 | 1982 | 9802 | 2153 | 9765 | 2323 | 9726 | 2493 | 9684 | 34 |
| 27 | 1814 | 9834 | 1985 | 9801 | 2156 | 9765 | 2326 | 9726 | 2495 | 9684 | 33 |
| 28 | 1817 | 9834 | 1988 | 9800 | 2159 | 9764 | 2329 | 9725 | 2498 | 9683 | 32 |
| 29 | 1819 | 9833 | 1991 | 9800 | 2162 | 9764 | 2332 | 9724 | 2501 | 9682 | 31 |
| 30 | 1822 | 9833 | 1994 | 9799 | 2164 | 9763 | 2334 | 9724 | 2504 | 9681 | 30 |
| 31 | 1825 | 9832 | 1997 | 9799 | 2167 | 9762 | 2337 | 9723 | 2507 | 9681 | 29 |
| 32 | 1828 | 9831 | 1999 | 9798 | 2170 | 9762 | 2340 | 9722 | 2509 | 9680 | 28 |
| 33 | 1831 | 9831 | 2002 | 9798 | 2173 | 9761 | 3343 | 9722 | 2512 | 9679 | 27 |
| 34 | 1834 | 9830 | 2005 | 9797 | 2176 | 9760 | 2346 | 9721 | 2515 | 9679 | 26 |
| 35 | 1837 | 9830 | 2008 | 9796 | 2179 | 9760 | 2349 | 9720 | 2518 | 9678 | 25 |
| 36 | 1840 | 9829 | 2011 | 9796 | 2181 | 9759 | 2351 | 9720 | 2521 | 9677 | 24 |
| 37 | 1842 | 9829 | 2014 | 9795 | 2184 | 9759 | 2354 | 9719 | 2524 | 9676 | 23 |
| 38 | 1845 | 9828 | 2016 | 9795 | 2187 | 9758 | 2357 | 9718 | 2526 | 9676 | 22 |
| 39 | 1848 | 9828 | 2019 | 9794 | 2190 | 9757 | 2360 | 9718 | 2529 | 9675 | 21 |
| 40 | 1851 | 9827 | 2022 | 9793 | 2193 | 9757 | 2363 | 9717 | 2532 | 9674 | 20 |
| 41 | 1854 | 9827 | 2025 | 9793 | 2196 | 9756 | 2366 | 9716 | 2535 | 9673 | 19 |
| 42 | 1857 | 9826 | 2028 | 9792 | 2198 | 9755 | 2368 | 9715 | 2538 | 9673 | 18 |
| 43 | 1860 | 9826 | 2031 | 9792 | 2201 | 9755 | 2371 | 9715 | 2540 | 9672 | 17 |
| 44 | 1862 | 9825 | 2034 | 9791 | 2204 | 9754 | 2374 | 9714 | 2543 | 9671 | 16 |
| 45 | 1865 | 9825 | 2036 | 9790 | 2207 | 9753 | 2377 | 9713 | 2546 | 9670 | 15 |
| 46 | 1868 | 9824 | 2039 | 9790 | 2210 | 9753 | 2380 | 9713 | 2549 | 9670 | 14 |
| 47 | 1871 | 9823 | 2042 | 9789 | 2213 | 9752 | 2383 | 9712 | 2552 | 9669 | 13 |
| 48 | 1474 | 9823 | 2045 | 9789 | 2215 | 9751 | 2385 | 9711 | 2554 | 9668 | 12 |
| 49 | 1877 | 9822 | 2048 | 9788 | 2218 | 9751 | 2388 | 9711 | 2557 | 9667 | 11 |
| 50 | 1880 | 9822 | 2051 | 9787 | 2221 | 9750 | 2391 | 9710 | 2560 | 9667 | 10 |
| 51 | 1882 | 9821 | 2054 | 9787 | 2224 | 9750 | 2394 | 9709 | 2563 | 9666 | 9 |
| 52 | 1885 | 9821 | 2056 | 9786 | 2227 | 9749 | 2397 | 9709 | 2566 | 9665 | 8 |
| 53 | 1888 | 9820 | 2059 | 9786 | 2230 | 9748 | 2399 | 9708 | 2569 | 9665 | 7 |
| 54 | 1891 | 9820 | 2062 | 9785 | 2233 | 9748 | 2402 | 9707 | 2571 | 9664 | 6 |
| 55 | 1894 | 9819 | 2065 | 9784 | 2235 | 9747 | 2405 | 9706 | 2574 | 9663 | 5 |
| 56 | 1897 | 9818 | 2068 | 9784 | 2238 | 9746 | 2408 | 9706 | 2577 | 9662 | 4 |
| 57 | 1900 | 9818 | 2071 | 9783 | 2241 | 9746 | 2411 | 9705 | 2580 | 9662 | 3 |
| 58 | 1902 | 9817 | 2073 | 9783 | 2244 | 9745 | 2414 | 9704 | 2583 | 9661 | 2 |
| 59 | 1905 | 9817 | 2076 | 9782 | 2247 | 9744 | 2416 | 9704 | 2585 | 9660 | 1 |
| 60 | 1908 | 9816 | 2079 | 9781 | 2250 | 9744 | 2419 | 9703 | 2588 | 9659 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 79° | | 78° | | 77° | | 76° | | 75° | | ° |

| ° | 15° | | 16° | | 17° | | 18° | | 19° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 2588 | 9659 | 2756 | 9613 | 2924 | 9563 | 3090 | 9511 | 3256 | 9455 | 60 |
| 1 | 2591 | 9659 | 2759 | 9612 | 2926 | 9562 | 3093 | 9510 | 3258 | 9454 | 59 |
| 2 | 2594 | 9658 | 2762 | 9611 | 2929 | 9561 | 3096 | 9509 | 3261 | 9453 | 58 |
| 3 | 2597 | 9657 | 2765 | 9610 | 2932 | 9560 | 3098 | 9508 | 3264 | 9452 | 57 |
| 4 | 2599 | 9656 | 2768 | 9609 | 2935 | 9560 | 3101 | 9507 | 3267 | 9451 | 56 |
| 5 | 2602 | 9655 | 2770 | 9609 | 2938 | 9559 | 3104 | 9506 | 3269 | 9450 | 55 |
| 6 | 2605 | 9655 | 2773 | 9608 | 2940 | 9558 | 3107 | 9505 | 3272 | 9449 | 54 |
| 7 | 2608 | 9654 | 2776 | 9607 | 2943 | 9557 | 3110 | 9504 | 3275 | 9449 | 53 |
| 8 | 2611 | 9653 | 2779 | 9606 | 2946 | 9556 | 3112 | 9503 | 3278 | 9448 | 52 |
| 9 | 2613 | 9652 | 2782 | 9605 | 2949 | 9555 | 3115 | 9502 | 3280 | 9447 | 51 |
| 10 | 2616 | 9652 | 2784 | 9605 | 2952 | 9555 | 3118 | 9502 | 3283 | 9446 | 50 |
| 11 | 2619 | 9651 | 2787 | 9604 | 2954 | 9554 | 3121 | 9501 | 3286 | 9445 | 49 |
| 12 | 2622 | 9650 | 2790 | 9603 | 2957 | 9553 | 3123 | 9500 | 3289 | 9444 | 48 |
| 13 | 2625 | 9649 | 2793 | 9602 | 2960 | 9552 | 3126 | 9499 | 3291 | 9443 | 47 |
| 14 | 2628 | 9649 | 2795 | 9601 | 2963 | 9551 | 3129 | 9498 | 3294 | 9442 | 46 |
| 15 | 2630 | 9648 | 2798 | 9600 | 2965 | 9550 | 3132 | 9497 | 3297 | 9441 | 45 |
| 16 | 2633 | 9647 | 2801 | 9600 | 2968 | 9549 | 3134 | 9496 | 3300 | 9440 | 44 |
| 17 | 2636 | 9646 | 2804 | 9599 | 2971 | 9548 | 3137 | 9495 | 3302 | 9439 | 43 |
| 18 | 2639 | 9646 | 2807 | 9598 | 2974 | 9548 | 3140 | 9494 | 3305 | 9438 | 42 |
| 19 | 2642 | 9645 | 2809 | 9597 | 2977 | 9547 | 3143 | 9493 | 3308 | 9437 | 41 |
| 20 | 2644 | 9644 | 2812 | 9596 | 2979 | 9546 | 3145 | 9492 | 3311 | 9436 | 40 |
| 21 | 2647 | 9643 | 2815 | 9596 | 2982 | 9545 | 3148 | 9492 | 3313 | 9435 | 39 |
| 22 | 2650 | 9642 | 2818 | 9595 | 2985 | 9544 | 3151 | 9491 | 3316 | 9434 | 38 |
| 23 | 2653 | 9642 | 2821 | 9594 | 2988 | 9543 | 3154 | 9490 | 3319 | 9433 | 37 |
| 24 | 2656 | 9641 | 2823 | 9593 | 2990 | 9542 | 3156 | 9489 | 3322 | 9432 | 36 |
| 25 | 2658 | 9640 | 2826 | 9592 | 2993 | 9542 | 3159 | 9488 | 3324 | 9431 | 35 |
| 26 | 2661 | 9639 | 2829 | 9591 | 2996 | 9541 | 3162 | 9487 | 3327 | 9430 | 34 |
| 27 | 2664 | 9639 | 2832 | 9591 | 2999 | 9540 | 3165 | 9486 | 3330 | 9429 | 33 |
| 28 | 2667 | 9638 | 2835 | 9590 | 3002 | 9539 | 3168 | 9485 | 3333 | 9428 | 32 |
| 29 | 2670 | 9637 | 2837 | 9589 | 3004 | 9538 | 3170 | 9484 | 3335 | 9427 | 31 |
| 30 | 2672 | 9636 | 2840 | 9588 | 3007 | 9537 | 3173 | 9483 | 3338 | 9426 | 30 |
| 31 | 2675 | 9636 | 2843 | 9587 | 3010 | 9536 | 3176 | 9482 | 3341 | 9425 | 29 |
| 32 | 2678 | 9635 | 2846 | 9587 | 3013 | 9535 | 3179 | 9481 | 3344 | 9424 | 28 |
| 33 | 2681 | 9634 | 2849 | 9586 | 3015 | 9535 | 3181 | 9480 | 3346 | 9423 | 27 |
| 34 | 2684 | 9633 | 2851 | 9585 | 3018 | 9534 | 3184 | 9480 | 3349 | 9423 | 26 |
| 35 | 2686 | 9632 | 2854 | 9584 | 3021 | 9533 | 3187 | 9479 | 3352 | 9422 | 25 |
| 36 | 2689 | 9632 | 2857 | 9583 | 3024 | 9532 | 3190 | 9478 | 3355 | 9421 | 24 |
| 37 | 2692 | 9631 | 2860 | 9582 | 3026 | 9531 | 3192 | 9477 | 3357 | 9420 | 23 |
| 38 | 2695 | 9630 | 2862 | 9582 | 3029 | 9530 | 3195 | 9476 | 3360 | 9419 | 22 |
| 39 | 2698 | 9629 | 2865 | 9581 | 3032 | 9529 | 3198 | 9475 | 3363 | 9418 | 21 |
| 40 | 2700 | 9628 | 2868 | 9580 | 3035 | 9528 | 3201 | 9474 | 3365 | 9417 | 20 |
| 41 | 2703 | 9628 | 2871 | 9579 | 3038 | 9527 | 3203 | 9473 | 3368 | 9416 | 19 |
| 42 | 2706 | 9627 | 2874 | 9578 | 3040 | 9527 | 3206 | 9472 | 3371 | 9415 | 18 |
| 43 | 2709 | 9626 | 2876 | 9577 | 3043 | 9526 | 3209 | 9471 | 3374 | 9414 | 17 |
| 44 | 2712 | 9625 | 2879 | 9577 | 3046 | 9525 | 3212 | 9470 | 3376 | 9413 | 16 |
| 45 | 2714 | 9625 | 2882 | 9576 | 3049 | 9524 | 3214 | 9469 | 3379 | 9412 | 15 |
| 46 | 2717 | 9624 | 2885 | 9575 | 3051 | 9523 | 3217 | 9468 | 3382 | 9411 | 14 |
| 47 | 2720 | 9623 | 2888 | 9574 | 3054 | 9522 | 3220 | 9467 | 3385 | 9410 | 13 |
| 48 | 2723 | 9622 | 2890 | 9573 | 3057 | 9521 | 3223 | 9466 | 3387 | 9409 | 12 |
| 49 | 2726 | 9621 | 2893 | 9572 | 3060 | 9520 | 3225 | 9466 | 3390 | 9408 | 11 |
| 50 | 2728 | 9621 | 2896 | 9572 | 3062 | 9520 | 3228 | 9465 | 3393 | 9407 | 10 |
| 51 | 2731 | 9620 | 2899 | 9571 | 3065 | 9519 | 3231 | 9464 | 3396 | 9406 | 9 |
| 52 | 2734 | 9619 | 2901 | 9570 | 3068 | 9518 | 3234 | 9463 | 3398 | 9405 | 8 |
| 53 | 2737 | 9618 | 2904 | 9569 | 3071 | 9517 | 3236 | 9462 | 3401 | 9404 | 7 |
| 54 | 2740 | 9617 | 2907 | 9568 | 3074 | 9516 | 3239 | 9461 | 3404 | 9403 | 6 |
| 55 | 2742 | 9617 | 2910 | 9567 | 3076 | 9515 | 3242 | 9460 | 3407 | 9402 | 5 |
| 56 | 2745 | 9616 | 2913 | 9566 | 3079 | 9514 | 3245 | 9459 | 3409 | 9401 | 4 |
| 57 | 2748 | 9615 | 2915 | 9566 | 3082 | 9513 | 3247 | 9458 | 3412 | 9400 | 3 |
| 58 | 2751 | 9614 | 2918 | 9565 | 3085 | 9512 | 3250 | 9457 | 3415 | 9399 | 2 |
| 59 | 2754 | 9613 | 2921 | 9564 | 3087 | 9511 | 3253 | 9456 | 3417 | 9398 | 1 |
| 60 | 2756 | 9613 | 2924 | 9563 | 3090 | 9511 | 3256 | 9455 | 3420 | 9397 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 74° | | 73° | | 72° | | 71° | | 70° | | ° |

| ° | 20° | | 21° | | 22° | | 23° | | 24° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 3420 | 9397 | 3584 | 9336 | 3746 | 9272 | 3907 | 9205 | 4067 | 9135 | 60 |
| 1 | 3423 | 9396 | 3586 | 9335 | 3749 | 9271 | 3910 | 9204 | 4070 | 9134 | 59 |
| 2 | 3426 | 9395 | 3589 | 9334 | 3751 | 9270 | 3913 | 9203 | 4073 | 9133 | 58 |
| 3 | 3428 | 9394 | 3592 | 9333 | 3754 | 9269 | 3915 | 9202 | 4075 | 9132 | 57 |
| 4 | 3431 | 9393 | 3595 | 9332 | 3757 | 9267 | 3918 | 9200 | 4078 | 9131 | 56 |
| 5 | 3434 | 9392 | 3597 | 9331 | 3760 | 9266 | 3921 | 9199 | 4081 | 9130 | 55 |
| 6 | 3437 | 9391 | 3600 | 9330 | 3762 | 9265 | 3923 | 9198 | 4083 | 9128 | 54 |
| 7 | 3439 | 9390 | 3603 | 9328 | 3765 | 9264 | 3926 | 9197 | 4086 | 9127 | 53 |
| 8 | 3442 | 9389 | 3605 | 9327 | 3768 | 9263 | 3929 | 9196 | 4089 | 9126 | 52 |
| 9 | 3445 | 9388 | 3608 | 9326 | 3770 | 9262 | 3931 | 9195 | 4091 | 9125 | 51 |
| 10 | 3448 | 9387 | 3611 | 9325 | 3773 | 9261 | 3934 | 9194 | 4094 | 9124 | 50 |
| 11 | 3450 | 9386 | 3614 | 9324 | 3776 | 9260 | 3937 | 9192 | 4097 | 9122 | 49 |
| 12 | 3453 | 9385 | 3616 | 9323 | 3778 | 9259 | 3939 | 9191 | 4099 | 9121 | 48 |
| 13 | 3456 | 9384 | 3619 | 9322 | 3781 | 9258 | 3942 | 9190 | 4102 | 9120 | 47 |
| 14 | 3458 | 9383 | 3622 | 9321 | 3784 | 9257 | 3945 | 9189 | 4105 | 9119 | 46 |
| 15 | 3461 | 9382 | 3624 | 9320 | 3786 | 9255 | 3947 | 9188 | 4107 | 9118 | 45 |
| 16 | 3464 | 9381 | 3627 | 9319 | 3789 | 9254 | 3950 | 9187 | 4110 | 9116 | 44 |
| 17 | 3467 | 9380 | 3630 | 9318 | 3792 | 9253 | 3953 | 9186 | 4112 | 9115 | 43 |
| 18 | 3469 | 9379 | 3633 | 9317 | 3795 | 9252 | 3955 | 9184 | 4115 | 9114 | 42 |
| 19 | 3472 | 9378 | 3635 | 9316 | 3797 | 9251 | 3958 | 9183 | 4118 | 9113 | 41 |
| 20 | 3475 | 9377 | 3638 | 9315 | 3800 | 9250 | 3961 | 9182 | 4120 | 9112 | 40 |
| 21 | 3478 | 9376 | 3641 | 9314 | 3803 | 9249 | 3963 | 9181 | 4123 | 9110 | 39 |
| 22 | 3480 | 9375 | 3643 | 9313 | 3805 | 9248 | 3966 | 9180 | 4126 | 9109 | 38 |
| 23 | 3483 | 9374 | 3646 | 9312 | 3808 | 9247 | 3969 | 9179 | 4128 | 9108 | 37 |
| 24 | 3486 | 9373 | 3649 | 9311 | 3811 | 9245 | 3971 | 9178 | 4131 | 9107 | 36 |
| 25 | 3488 | 9372 | 3651 | 9309 | 3813 | 9244 | 3974 | 9176 | 4134 | 9106 | 35 |
| 26 | 3491 | 9371 | 3654 | 9308 | 3816 | 9243 | 3977 | 9175 | 4136 | 9104 | 34 |
| 27 | 3494 | 9370 | 3657 | 9307 | 3819 | 9242 | 3979 | 9174 | 4139 | 9103 | 33 |
| 28 | 3497 | 9369 | 3660 | 9306 | 3821 | 9241 | 3982 | 9173 | 4142 | 9102 | 32 |
| 29 | 3499 | 9368 | 3662 | 9305 | 3824 | 9240 | 3985 | 9172 | 4144 | 9101 | 31 |
| 30 | 3502 | 9367 | 3665 | 9304 | 3827 | 9239 | 3987 | 9171 | 4147 | 9100 | 30 |
| 31 | 3505 | 9366 | 3668 | 9303 | 3830 | 9238 | 3990 | 9169 | 4150 | 9098 | 29 |
| 32 | 3508 | 9365 | 3670 | 9302 | 3832 | 9237 | 3993 | 9168 | 4152 | 9097 | 28 |
| 33 | 3510 | 9364 | 3673 | 9301 | 3835 | 9235 | 3995 | 9167 | 4155 | 9096 | 27 |
| 34 | 3513 | 9363 | 3676 | 9300 | 3838 | 9234 | 3998 | 9166 | 4158 | 9095 | 26 |
| 35 | 3516 | 9362 | 3679 | 9299 | 3840 | 9233 | 4001 | 9165 | 4160 | 9094 | 25 |
| 36 | 3518 | 9361 | 3681 | 9298 | 3843 | 9232 | 4003 | 9164 | 4163 | 9092 | 24 |
| 37 | 3521 | 9360 | 3684 | 9297 | 3846 | 9231 | 4006 | 9162 | 4165 | 9091 | 23 |
| 38 | 3524 | 9359 | 3687 | 9296 | 3848 | 9230 | 4009 | 9161 | 4168 | 9090 | 22 |
| 39 | 3527 | 9358 | 3689 | 9295 | 3851 | 9229 | 4011 | 9160 | 4171 | 9088 | 21 |
| 40 | 3529 | 9356 | 3692 | 9293 | 3854 | 9228 | 4014 | 9159 | 4173 | 9088 | 20 |
| 41 | 3532 | 9355 | 3695 | 9292 | 3856 | 9227 | 4017 | 9158 | 4176 | 9086 | 19 |
| 42 | 3535 | 9354 | 3697 | 9291 | 3859 | 9225 | 4019 | 9157 | 4179 | 9085 | 18 |
| 43 | 3537 | 9353 | 3700 | 9290 | 3862 | 9224 | 4022 | 9155 | 4181 | 9084 | 17 |
| 44 | 3540 | 9352 | 3703 | 9289 | 3864 | 9223 | 4025 | 9154 | 4184 | 9083 | 16 |
| 45 | 3543 | 9351 | 3706 | 9288 | 3867 | 9222 | 4027 | 9153 | 4187 | 9081 | 15 |
| 46 | 3546 | 9350 | 3708 | 9287 | 3870 | 9221 | 4030 | 9152 | 4189 | 9080 | 14 |
| 47 | 3548 | 9349 | 3711 | 9286 | 3872 | 9220 | 4033 | 9151 | 4192 | 9079 | 13 |
| 48 | 3551 | 9348 | 3714 | 9285 | 3875 | 9219 | 4035 | 9150 | 4195 | 9078 | 12 |
| 49 | 3554 | 9347 | 3716 | 9284 | 3878 | 9218 | 4038 | 9148 | 4197 | 9077 | 11 |
| 50 | 3557 | 9346 | 3719 | 9283 | 3881 | 9216 | 4041 | 9147 | 4200 | 9075 | 10 |
| 51 | 3559 | 9345 | 3722 | 9282 | 3883 | 9215 | 4043 | 9146 | 4202 | 9074 | 9 |
| 52 | 3562 | 9344 | 3724 | 9281 | 3886 | 9214 | 4046 | 9145 | 4205 | 9073 | 8 |
| 53 | 3565 | 9343 | 3727 | 9279 | 3889 | 9213 | 4049 | 9144 | 4208 | 9072 | 7 |
| 54 | 3567 | 9342 | 3730 | 9278 | 3891 | 9212 | 4051 | 9143 | 4210 | 9070 | 6 |
| 55 | 3570 | 9341 | 3733 | 9277 | 3894 | 9211 | 4054 | 9141 | 4213 | 9069 | 5 |
| 56 | 3573 | 9340 | 3735 | 9276 | 3897 | 9210 | 4057 | 9140 | 4216 | 9068 | 4 |
| 57 | 3576 | 9339 | 3738 | 9275 | 3899 | 9208 | 4059 | 9139 | 4218 | 9067 | 3 |
| 58 | 3578 | 9338 | 3741 | 9274 | 3902 | 9207 | 4062 | 9138 | 4221 | 9066 | 2 |
| 59 | 3581 | 9337 | 3743 | 9273 | 3905 | 9206 | 4065 | 9137 | 4224 | 9064 | 1 |
| 60 | 3584 | 9336 | 3746 | 9272 | 3907 | 9205 | 4067 | 9135 | 4226 | 9063 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 69° | | 68° | | 67° | | 66° | | 65° | | ° |

| / | 25° | | 26° | | 27° | | 28° | | 29° | | / |
|-----|------|------|------|------|------|------|------|------|------|------|-----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 4226 | 9063 | 4384 | 8988 | 4540 | 8910 | 4695 | 8829 | 4848 | 8746 | 60 |
| 1 | 4229 | 9062 | 4386 | 8987 | 4542 | 8909 | 4697 | 8828 | 4851 | 8745 | 59 |
| 2 | 4231 | 9061 | 4389 | 8985 | 4545 | 8907 | 4700 | 8827 | 4853 | 8743 | 58 |
| 3 | 4234 | 9059 | 4392 | 8984 | 4548 | 8906 | 4702 | 8825 | 4856 | 8742 | 57 |
| 4 | 4237 | 9058 | 4394 | 8983 | 4550 | 8905 | 4705 | 8824 | 4858 | 8741 | 56 |
| 5 | 4239 | 9057 | 4397 | 8982 | 4553 | 8903 | 4708 | 8823 | 4861 | 8739 | 55 |
| 6 | 4242 | 9056 | 4399 | 8980 | 4555 | 8902 | 4710 | 8821 | 4863 | 8738 | 54 |
| 7 | 4245 | 9054 | 4402 | 8979 | 4558 | 8901 | 4713 | 8820 | 4866 | 8736 | 53 |
| 8 | 4247 | 9053 | 4405 | 8978 | 4561 | 8899 | 4715 | 8819 | 4868 | 8735 | 52 |
| 9 | 4250 | 9052 | 4407 | 8976 | 4563 | 8898 | 4718 | 8817 | 4871 | 8733 | 51 |
| 10 | 4253 | 9051 | 4410 | 8975 | 4566 | 8897 | 4720 | 8816 | 4874 | 8732 | 50 |
| 11 | 4255 | 9050 | 4412 | 8974 | 4568 | 8895 | 4723 | 8814 | 4876 | 8731 | 49 |
| 12 | 4258 | 9048 | 4415 | 8973 | 4571 | 8894 | 4726 | 8813 | 4879 | 8729 | 48 |
| 13 | 4260 | 9047 | 4418 | 8971 | 4574 | 8893 | 4728 | 8812 | 4881 | 8728 | 47 |
| 14 | 4263 | 9046 | 4420 | 8970 | 4576 | 8892 | 4731 | 8810 | 4884 | 8726 | 46 |
| 15 | 4266 | 9045 | 4423 | 8969 | 4579 | 8890 | 4733 | 8809 | 4886 | 8725 | 45 |
| 16 | 4268 | 9043 | 4425 | 8967 | 4581 | 8889 | 4736 | 8808 | 4889 | 8724 | 44 |
| 17 | 4271 | 9042 | 4428 | 8966 | 4584 | 8888 | 4738 | 8806 | 4891 | 8722 | 43 |
| 18 | 4274 | 9041 | 4431 | 8965 | 4586 | 8886 | 4741 | 8805 | 4894 | 8721 | 42 |
| 19 | 4276 | 9040 | 4433 | 8964 | 4589 | 8885 | 4743 | 8803 | 4896 | 8719 | 41 |
| 20 | 4279 | 9038 | 4436 | 8962 | 4592 | 8884 | 4746 | 8802 | 4899 | 8718 | 40 |
| 21 | 4281 | 9037 | 4439 | 8961 | 4594 | 8882 | 4749 | 8801 | 4901 | 8716 | 39 |
| 22 | 4284 | 9036 | 4441 | 8960 | 4597 | 8881 | 4751 | 8799 | 4904 | 8715 | 38 |
| 23 | 4287 | 9035 | 4444 | 8958 | 4599 | 8879 | 4754 | 8798 | 4907 | 8714 | 37 |
| 24 | 4289 | 9033 | 4446 | 8957 | 4602 | 8878 | 4756 | 8796 | 4909 | 8712 | 36 |
| 25 | 4292 | 9032 | 4449 | 8956 | 4605 | 8877 | 4759 | 8795 | 4912 | 8711 | 35 |
| 26 | 4295 | 9031 | 4452 | 8955 | 4607 | 8875 | 4761 | 8794 | 4914 | 8709 | 34 |
| 27 | 4297 | 9030 | 4454 | 8953 | 4610 | 8874 | 4764 | 8792 | 4917 | 8708 | 33 |
| 28 | 4300 | 9028 | 4457 | 8952 | 4612 | 8873 | 4766 | 8791 | 4919 | 8706 | 32 |
| 29 | 4302 | 9027 | 4459 | 8951 | 4615 | 8871 | 4769 | 8790 | 4922 | 8705 | 31 |
| 30 | 4305 | 9026 | 4462 | 8949 | 4617 | 8870 | 4772 | 8788 | 4924 | 8704 | 30 |
| 31 | 4308 | 9025 | 4465 | 8948 | 4620 | 8869 | 4774 | 8787 | 4927 | 8702 | 29 |
| 32 | 4310 | 9023 | 4467 | 8947 | 4623 | 8867 | 4777 | 8785 | 4929 | 8701 | 28 |
| 33 | 4313 | 9022 | 4470 | 8945 | 4625 | 8866 | 4779 | 8784 | 4932 | 8699 | 27 |
| 34 | 4316 | 9021 | 4472 | 8944 | 4628 | 8865 | 4782 | 8783 | 4934 | 8698 | 26 |
| 35 | 4318 | 9020 | 4475 | 8943 | 4630 | 8863 | 4784 | 8781 | 4937 | 8696 | 25 |
| 36 | 4321 | 9018 | 4478 | 8942 | 4633 | 8862 | 4787 | 8780 | 4939 | 8695 | 24 |
| 37 | 4323 | 9017 | 4480 | 8940 | 4636 | 8861 | 4789 | 8778 | 4942 | 8694 | 23 |
| 38 | 4326 | 9016 | 4483 | 8939 | 4638 | 8859 | 4792 | 8777 | 4944 | 8692 | 22 |
| 39 | 4329 | 9015 | 4485 | 8938 | 4641 | 8858 | 4795 | 8776 | 4947 | 8691 | 21 |
| 40 | 4331 | 9013 | 4488 | 8936 | 4643 | 8857 | 4797 | 8774 | 4950 | 8689 | 20 |
| 41 | 4334 | 9012 | 4491 | 8935 | 4646 | 8855 | 4800 | 8773 | 4952 | 8688 | 19 |
| 42 | 4337 | 9011 | 4493 | 8934 | 4648 | 8854 | 4802 | 8771 | 4955 | 8686 | 18 |
| 43 | 4339 | 9010 | 4496 | 8932 | 4651 | 8853 | 4805 | 8770 | 4957 | 8685 | 17 |
| 44 | 4342 | 9008 | 4498 | 8931 | 4654 | 8851 | 4807 | 8769 | 4960 | 8683 | 16 |
| 45 | 4344 | 9007 | 4501 | 8930 | 4656 | 8850 | 4810 | 8767 | 4962 | 8682 | 15 |
| 46 | 4347 | 9006 | 4504 | 8928 | 4659 | 8849 | 4812 | 8766 | 4965 | 8681 | 14 |
| 47 | 4350 | 9004 | 4506 | 8927 | 4661 | 8847 | 4815 | 8764 | 4967 | 8679 | 13 |
| 48 | 4352 | 9003 | 4509 | 8926 | 4664 | 8846 | 4818 | 8763 | 4970 | 8678 | 12 |
| 49 | 4355 | 9002 | 4511 | 8925 | 4666 | 8844 | 4820 | 8762 | 4972 | 8676 | 11 |
| 50 | 4358 | 9001 | 4514 | 8923 | 4669 | 8843 | 4823 | 8760 | 4975 | 8675 | 10 |
| 51 | 4360 | 8999 | 4517 | 8922 | 4672 | 8842 | 4825 | 8759 | 4977 | 8673 | 9 |
| 52 | 4363 | 8998 | 4519 | 8921 | 4674 | 8840 | 4828 | 8757 | 4980 | 8672 | 8 |
| 53 | 4365 | 8997 | 4522 | 8919 | 4677 | 8839 | 4830 | 8756 | 4982 | 8670 | 7 |
| 54 | 4368 | 8996 | 4524 | 8918 | 4679 | 8838 | 4833 | 8755 | 4985 | 8669 | 6 |
| 55 | 4371 | 8994 | 4527 | 8917 | 4682 | 8836 | 4835 | 8753 | 4987 | 8668 | 5 |
| 56 | 4373 | 8993 | 4530 | 8915 | 4684 | 8835 | 4838 | 8752 | 4990 | 8666 | 4 |
| 57 | 4376 | 8992 | 4532 | 8914 | 4687 | 8834 | 4840 | 8750 | 4992 | 8665 | 3 |
| 58 | 4378 | 8990 | 4535 | 8913 | 4690 | 8832 | 4843 | 8749 | 4995 | 8663 | 2 |
| 59 | 4381 | 8989 | 4537 | 8911 | 4692 | 8831 | 4846 | 8748 | 4997 | 8662 | 1 |
| 60 | 4384 | 8988 | 4540 | 8910 | 4695 | 8829 | 4848 | 8746 | 5000 | 8660 | 0 |
| cos | | sin | cos | | sin | cos | | sin | cos | | sin |
| / | 64° | | 63° | | 62° | | 61° | | 60° | | / |

| | 50° | | 31° | | 32° | | 33° | | 34° | | |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 5000 | 8660 | 5150 | 8572 | 5299 | 8480 | 5446 | 8387 | 5592 | 8290 | 60 |
| 1 | 5003 | 8659 | 5153 | 8570 | 5302 | 8479 | 5449 | 8385 | 5594 | 8289 | 59 |
| 2 | 5005 | 8657 | 5155 | 8569 | 5304 | 8477 | 5451 | 8384 | 5597 | 8287 | 58 |
| 3 | 5008 | 8656 | 5158 | 8567 | 5307 | 8476 | 5454 | 8382 | 5599 | 8285 | 57 |
| 4 | 5010 | 8654 | 5160 | 8566 | 5309 | 8474 | 5456 | 8380 | 5602 | 8284 | 56 |
| 5 | 5013 | 8653 | 5163 | 8564 | 5312 | 8473 | 5459 | 8379 | 5604 | 8282 | 55 |
| 6 | 5015 | 8652 | 5165 | 8563 | 5314 | 8471 | 5461 | 8377 | 5606 | 8281 | 54 |
| 7 | 5018 | 8650 | 5168 | 8561 | 5316 | 8470 | 5463 | 8376 | 5609 | 8279 | 53 |
| 8 | 5020 | 8649 | 5170 | 8560 | 5319 | 8468 | 5466 | 8374 | 5611 | 8277 | 52 |
| 9 | 5023 | 8647 | 5173 | 8558 | 5321 | 8467 | 5468 | 8372 | 5614 | 8276 | 51 |
| 10 | 5025 | 8646 | 5175 | 8557 | 5324 | 8465 | 5471 | 8371 | 5616 | 8274 | 50 |
| 11 | 5028 | 8644 | 5178 | 8555 | 5326 | 8463 | 5473 | 8369 | 5618 | 8272 | 49 |
| 12 | 5030 | 8643 | 5180 | 8554 | 5329 | 8462 | 5476 | 8368 | 5621 | 8271 | 48 |
| 13 | 5033 | 8641 | 5183 | 8552 | 5331 | 8460 | 5478 | 8366 | 5623 | 8269 | 47 |
| 14 | 5035 | 8640 | 5185 | 8551 | 5334 | 8459 | 5480 | 8364 | 5626 | 8268 | 46 |
| 15 | 5038 | 8638 | 5188 | 8549 | 5336 | 8457 | 5483 | 8363 | 5628 | 8266 | 45 |
| 16 | 5040 | 8637 | 5190 | 8548 | 5339 | 8456 | 5485 | 8361 | 5630 | 8264 | 44 |
| 17 | 5043 | 8635 | 5193 | 8546 | 5341 | 8454 | 5488 | 8360 | 5633 | 8263 | 43 |
| 18 | 5045 | 8634 | 5195 | 8545 | 5344 | 8453 | 5490 | 8358 | 5635 | 8261 | 42 |
| 19 | 5048 | 8632 | 5198 | 8543 | 5346 | 8451 | 5493 | 8356 | 5638 | 8259 | 41 |
| 20 | 5050 | 8631 | 5200 | 8542 | 5348 | 8450 | 5495 | 8355 | 5640 | 8258 | 40 |
| 21 | 5053 | 8630 | 5203 | 8540 | 5351 | 8448 | 5498 | 8353 | 5642 | 8256 | 39 |
| 22 | 5055 | 8628 | 5205 | 8539 | 5353 | 8446 | 5500 | 8352 | 5645 | 8254 | 38 |
| 23 | 5058 | 8627 | 5208 | 8537 | 5356 | 8445 | 5502 | 8350 | 5647 | 8253 | 37 |
| 24 | 5060 | 8625 | 5210 | 8536 | 5358 | 8443 | 5505 | 8348 | 5650 | 8251 | 36 |
| 25 | 5063 | 8624 | 5213 | 8534 | 5361 | 8442 | 5507 | 8347 | 5652 | 8249 | 35 |
| 26 | 5065 | 8622 | 5215 | 8532 | 5363 | 8440 | 5510 | 8345 | 5654 | 8248 | 34 |
| 27 | 5068 | 8621 | 5218 | 8531 | 5366 | 8439 | 5512 | 8344 | 5657 | 8246 | 33 |
| 28 | 5070 | 8619 | 5220 | 8529 | 5368 | 8437 | 5515 | 8342 | 5659 | 8245 | 32 |
| 29 | 5073 | 8618 | 5223 | 8528 | 5371 | 8435 | 5517 | 8340 | 5662 | 8243 | 31 |
| 30 | 5075 | 8616 | 5225 | 8526 | 5373 | 8434 | 5519 | 8339 | 5664 | 8241 | 30 |
| 31 | 5078 | 8615 | 5227 | 8525 | 5375 | 8432 | 5522 | 8337 | 5666 | 8240 | 29 |
| 32 | 5080 | 8613 | 5230 | 8523 | 5378 | 8431 | 5524 | 8336 | 5669 | 8238 | 28 |
| 33 | 5083 | 8612 | 5232 | 8522 | 5380 | 8429 | 5527 | 8334 | 5671 | 8236 | 27 |
| 34 | 5085 | 8610 | 5235 | 8520 | 5383 | 8428 | 5529 | 8332 | 5674 | 8235 | 26 |
| 35 | 5088 | 8609 | 5237 | 8519 | 5385 | 8426 | 5531 | 8331 | 5676 | 8233 | 25 |
| 36 | 5090 | 8607 | 5240 | 8517 | 5388 | 8425 | 5534 | 8329 | 5678 | 8231 | 24 |
| 37 | 5093 | 8606 | 5242 | 8516 | 5390 | 8423 | 5536 | 8328 | 5681 | 8230 | 23 |
| 38 | 5095 | 8604 | 5245 | 8514 | 5393 | 8421 | 5539 | 8326 | 5683 | 8228 | 22 |
| 39 | 5098 | 8603 | 5247 | 8513 | 5395 | 8420 | 5541 | 8324 | 5686 | 8226 | 21 |
| 40 | 5100 | 8601 | 5250 | 8511 | 5398 | 8418 | 5544 | 8323 | 5688 | 8225 | 20 |
| 41 | 5103 | 8600 | 5252 | 8510 | 5400 | 8417 | 5546 | 8321 | 5690 | 8223 | 19 |
| 42 | 5105 | 8599 | 5255 | 8508 | 5402 | 8415 | 5548 | 8320 | 5693 | 8221 | 18 |
| 43 | 5108 | 8597 | 5257 | 8507 | 5405 | 8414 | 5551 | 8318 | 5695 | 8220 | 17 |
| 44 | 5110 | 8596 | 5260 | 8505 | 5407 | 8412 | 5553 | 8316 | 5698 | 8218 | 16 |
| 45 | 5113 | 8594 | 5262 | 8504 | 5410 | 8410 | 5556 | 8315 | 5700 | 8216 | 15 |
| 46 | 5115 | 8593 | 5265 | 8502 | 5412 | 8409 | 5558 | 8313 | 5702 | 8215 | 14 |
| 47 | 5118 | 8591 | 5267 | 8500 | 5415 | 8407 | 5561 | 8311 | 5705 | 8213 | 13 |
| 48 | 5120 | 8590 | 5270 | 8499 | 5417 | 8406 | 5563 | 8310 | 5707 | 8211 | 12 |
| 49 | 5123 | 8588 | 5272 | 8497 | 5420 | 8404 | 5565 | 8308 | 5710 | 8210 | 11 |
| 50 | 5125 | 8587 | 5275 | 8496 | 5422 | 8403 | 5568 | 8307 | 5712 | 8208 | 10 |
| 51 | 5128 | 8585 | 5277 | 8494 | 5424 | 8401 | 5570 | 8305 | 5714 | 8207 | 9 |
| 52 | 5130 | 8584 | 5279 | 8493 | 5427 | 8399 | 5573 | 8303 | 5717 | 8205 | 8 |
| 53 | 5133 | 8582 | 5282 | 8491 | 5429 | 8398 | 5575 | 8302 | 5719 | 8203 | 7 |
| 54 | 5135 | 8581 | 5284 | 8490 | 5432 | 8396 | 5577 | 8300 | 5721 | 8202 | 6 |
| 55 | 5138 | 8579 | 5287 | 8488 | 5434 | 8395 | 5580 | 8299 | 5724 | 8200 | 5 |
| 56 | 5140 | 8578 | 5289 | 8487 | 5437 | 8393 | 5582 | 8297 | 5726 | 8198 | 4 |
| 57 | 5143 | 8576 | 5292 | 8485 | 5439 | 8391 | 5585 | 8295 | 5729 | 8197 | 3 |
| 58 | 5145 | 8575 | 5294 | 8484 | 5442 | 8390 | 5587 | 8294 | 5731 | 8195 | 2 |
| 59 | 5148 | 8573 | 5297 | 8482 | 5444 | 8388 | 5590 | 8292 | 5733 | 8193 | 1 |
| 60 | 5150 | 8572 | 5299 | 8480 | 5446 | 8387 | 5592 | 8290 | 5736 | 8192 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | 50° | | 58° | | 57° | | 56° | | 55° | | |

| ° | 35° | | 36° | | 37° | | 38° | | 39° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 5736 | 8192 | 5878 | 8090 | 6018 | 7986 | 6157 | 7880 | 6293 | 7771 | 60 |
| 1 | 5738 | 8190 | 5880 | 8088 | 6020 | 7985 | 6159 | 7878 | 6295 | 7770 | 59 |
| 2 | 5741 | 8188 | 5883 | 8087 | 6023 | 7983 | 6161 | 7877 | 6298 | 7768 | 58 |
| 3 | 5743 | 8187 | 5885 | 8085 | 6025 | 7981 | 6163 | 7875 | 6300 | 7766 | 57 |
| 4 | 5745 | 8185 | 5887 | 8083 | 6027 | 7979 | 6166 | 7873 | 6302 | 7764 | 56 |
| 5 | 5748 | 8183 | 5890 | 8082 | 6030 | 7978 | 6168 | 7871 | 6305 | 7762 | 55 |
| 6 | 5750 | 8181 | 5892 | 8080 | 6032 | 7976 | 6170 | 7869 | 6307 | 7760 | 54 |
| 7 | 5752 | 8180 | 5894 | 8078 | 6034 | 7974 | 6173 | 7868 | 6309 | 7759 | 53 |
| 8 | 5755 | 8178 | 5897 | 8076 | 6037 | 7972 | 6175 | 7866 | 6311 | 7757 | 52 |
| 9 | 5757 | 8176 | 5899 | 8075 | 6039 | 7971 | 6177 | 7864 | 6314 | 7755 | 51 |
| 10 | 5760 | 8175 | 5901 | 8073 | 6041 | 7969 | 6180 | 7862 | 6316 | 7753 | 50 |
| 11 | 5762 | 8173 | 5904 | 8071 | 6044 | 7967 | 6182 | 7860 | 6318 | 7751 | 49 |
| 12 | 5764 | 8171 | 5906 | 8070 | 6046 | 7965 | 6184 | 7859 | 6320 | 7749 | 48 |
| 13 | 5767 | 8170 | 5908 | 8068 | 6048 | 7964 | 6186 | 7857 | 6323 | 7748 | 47 |
| 14 | 5769 | 8168 | 5911 | 8066 | 6051 | 7962 | 6189 | 7855 | 6325 | 7746 | 46 |
| 15 | 5771 | 8166 | 5913 | 8064 | 6053 | 7960 | 6191 | 7853 | 6327 | 7744 | 45 |
| 16 | 5774 | 8165 | 5915 | 8063 | 6055 | 7958 | 6193 | 7851 | 6329 | 7742 | 44 |
| 17 | 5776 | 8163 | 5918 | 8061 | 6058 | 7956 | 6196 | 7850 | 6332 | 7740 | 43 |
| 18 | 5779 | 8161 | 5920 | 8059 | 6060 | 7955 | 6198 | 7848 | 6334 | 7738 | 42 |
| 19 | 5781 | 8160 | 5922 | 8058 | 6062 | 7953 | 6200 | 7846 | 6336 | 7737 | 41 |
| 20 | 5783 | 8158 | 5925 | 8056 | 6065 | 7951 | 6202 | 7844 | 6338 | 7735 | 40 |
| 21 | 5786 | 8156 | 5927 | 8054 | 6067 | 7950 | 6205 | 7842 | 6341 | 7733 | 39 |
| 22 | 5788 | 8155 | 5930 | 8052 | 6069 | 7948 | 6207 | 7841 | 6343 | 7731 | 38 |
| 23 | 5790 | 8153 | 5932 | 8051 | 6071 | 7946 | 6209 | 7839 | 6345 | 7729 | 37 |
| 24 | 5793 | 8151 | 5934 | 8049 | 6074 | 7944 | 6211 | 7837 | 6347 | 7727 | 36 |
| 25 | 5795 | 8150 | 5937 | 8047 | 6076 | 7942 | 6214 | 7835 | 6350 | 7725 | 35 |
| 26 | 5798 | 8148 | 5939 | 8045 | 6078 | 7941 | 6216 | 7833 | 6352 | 7724 | 34 |
| 27 | 5800 | 8146 | 5941 | 8044 | 6081 | 7939 | 6218 | 7832 | 6354 | 7722 | 33 |
| 28 | 5802 | 8145 | 5944 | 8042 | 6083 | 7937 | 6221 | 7830 | 6356 | 7720 | 32 |
| 29 | 5805 | 8143 | 5946 | 8040 | 6085 | 7935 | 6223 | 7828 | 6359 | 7718 | 31 |
| 30 | 5807 | 8141 | 5948 | 8039 | 6088 | 7934 | 6225 | 7826 | 6361 | 7716 | 30 |
| 31 | 5809 | 8139 | 5951 | 8037 | 6090 | 7932 | 6227 | 7824 | 6363 | 7714 | 29 |
| 32 | 5812 | 8138 | 5953 | 8035 | 6092 | 7930 | 6230 | 7822 | 6365 | 7713 | 28 |
| 33 | 5814 | 8136 | 5955 | 8033 | 6095 | 7928 | 6232 | 7821 | 6368 | 7711 | 27 |
| 34 | 5816 | 8134 | 5958 | 8032 | 6097 | 7926 | 6234 | 7819 | 6370 | 7709 | 26 |
| 35 | 5819 | 8133 | 5960 | 8030 | 6099 | 7925 | 6237 | 7817 | 6372 | 7707 | 25 |
| 36 | 5821 | 8131 | 5962 | 8028 | 6101 | 7923 | 6239 | 7815 | 6374 | 7705 | 24 |
| 37 | 5824 | 8129 | 5965 | 8026 | 6104 | 7921 | 6241 | 7813 | 6376 | 7703 | 23 |
| 38 | 5826 | 8128 | 5967 | 8025 | 6106 | 7919 | 6243 | 7812 | 6379 | 7701 | 22 |
| 39 | 5828 | 8126 | 5969 | 8023 | 6108 | 7918 | 6246 | 7810 | 6381 | 7700 | 21 |
| 40 | 5831 | 8124 | 5972 | 8021 | 6111 | 7916 | 6248 | 7808 | 6383 | 7698 | 20 |
| 41 | 5833 | 8123 | 5974 | 8020 | 6113 | 7914 | 6250 | 7806 | 6385 | 7696 | 19 |
| 42 | 5835 | 8121 | 5976 | 8018 | 6115 | 7912 | 6252 | 7804 | 6388 | 7694 | 18 |
| 43 | 5838 | 8119 | 5979 | 8016 | 6118 | 7910 | 6255 | 7802 | 6390 | 7692 | 17 |
| 44 | 5840 | 8117 | 5981 | 8014 | 6120 | 7909 | 6257 | 7801 | 6392 | 7690 | 16 |
| 45 | 5842 | 8116 | 5983 | 8013 | 6122 | 7907 | 6259 | 7799 | 6394 | 7688 | 15 |
| 46 | 5845 | 8114 | 5986 | 8011 | 6124 | 7905 | 6262 | 7797 | 6397 | 7687 | 14 |
| 47 | 5847 | 8112 | 5988 | 8009 | 6127 | 7903 | 6264 | 7795 | 6399 | 7685 | 13 |
| 48 | 5850 | 8111 | 5990 | 8007 | 6129 | 7902 | 6266 | 7793 | 6401 | 7683 | 12 |
| 49 | 5852 | 8109 | 5993 | 8006 | 6131 | 7900 | 6268 | 7792 | 6403 | 7681 | 11 |
| 50 | 5854 | 8107 | 5995 | 8004 | 6134 | 7898 | 6271 | 7790 | 6406 | 7679 | 10 |
| 51 | 5857 | 8106 | 5997 | 8002 | 6136 | 7896 | 6273 | 7788 | 6408 | 7677 | 9 |
| 52 | 5859 | 8104 | 6000 | 8000 | 6138 | 7894 | 6275 | 7786 | 6410 | 7675 | 8 |
| 53 | 5861 | 8102 | 6002 | 7999 | 6141 | 7893 | 6277 | 7784 | 6412 | 7674 | 7 |
| 54 | 5864 | 8100 | 6004 | 7997 | 6143 | 7891 | 6280 | 7782 | 6414 | 7672 | 6 |
| 55 | 5866 | 8099 | 6007 | 7995 | 6145 | 7889 | 6282 | 7781 | 6417 | 7670 | 5 |
| 56 | 5868 | 8097 | 6009 | 7993 | 6147 | 7887 | 6284 | 7779 | 6419 | 7668 | 4 |
| 57 | 5871 | 8095 | 6011 | 7992 | 6150 | 7885 | 6286 | 7777 | 6421 | 7666 | 3 |
| 58 | 5873 | 8094 | 6014 | 7990 | 6152 | 7884 | 6289 | 7775 | 6423 | 7664 | 2 |
| 59 | 5875 | 8092 | 6016 | 7988 | 6154 | 7882 | 6291 | 7773 | 6426 | 7662 | 1 |
| 60 | 5878 | 8090 | 6018 | 7986 | 6157 | 7880 | 6293 | 7771 | 6428 | 7660 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 54° | | 53° | | 52° | | 51° | | 50° | | ° |

| ° | 40° | | 41° | | 42° | | 43° | | 44° | | ° |
|----|------|------|------|------|------|------|------|------|------|------|----|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0 | 6428 | 7660 | 6561 | 7547 | 6691 | 7431 | 6820 | 7314 | 6947 | 7193 | 60 |
| 1 | 6430 | 7659 | 6563 | 7545 | 6693 | 7430 | 6822 | 7312 | 6949 | 7191 | 59 |
| 2 | 6432 | 7657 | 6565 | 7543 | 6696 | 7428 | 6824 | 7310 | 6951 | 7189 | 58 |
| 3 | 6435 | 7655 | 6567 | 7541 | 6698 | 7426 | 6826 | 7308 | 6953 | 7187 | 57 |
| 4 | 6437 | 7653 | 6569 | 7539 | 6700 | 7424 | 6828 | 7306 | 6955 | 7185 | 56 |
| 5 | 6439 | 7651 | 6572 | 7538 | 6702 | 7422 | 6831 | 7304 | 6957 | 7183 | 55 |
| 6 | 6441 | 7649 | 6574 | 7536 | 6704 | 7420 | 6833 | 7302 | 6959 | 7181 | 54 |
| 7 | 6443 | 7647 | 6576 | 7534 | 6706 | 7418 | 6835 | 7300 | 6961 | 7179 | 53 |
| 8 | 6446 | 7645 | 6578 | 7532 | 6709 | 7416 | 6837 | 7298 | 6963 | 7177 | 52 |
| 9 | 6448 | 7644 | 6580 | 7530 | 6711 | 7414 | 6839 | 7296 | 6965 | 7175 | 51 |
| 10 | 6450 | 7642 | 6583 | 7528 | 6713 | 7412 | 6841 | 7294 | 6967 | 7173 | 50 |
| 11 | 6452 | 7640 | 6585 | 7526 | 6715 | 7410 | 6843 | 7292 | 6970 | 7171 | 49 |
| 12 | 6455 | 7638 | 6587 | 7524 | 6717 | 7408 | 6845 | 7290 | 6972 | 7169 | 48 |
| 13 | 6457 | 7636 | 6589 | 7522 | 6719 | 7406 | 6848 | 7288 | 6974 | 7167 | 47 |
| 14 | 6459 | 7634 | 6591 | 7520 | 6722 | 7404 | 6850 | 7286 | 6976 | 7165 | 46 |
| 15 | 6461 | 7632 | 6593 | 7518 | 6724 | 7402 | 6852 | 7284 | 6978 | 7163 | 45 |
| 16 | 6463 | 7630 | 6596 | 7516 | 6726 | 7400 | 6854 | 7282 | 6980 | 7161 | 44 |
| 17 | 6466 | 7629 | 6598 | 7515 | 6728 | 7398 | 6856 | 7280 | 6982 | 7159 | 43 |
| 18 | 6468 | 7627 | 6600 | 7513 | 6730 | 7396 | 6858 | 7278 | 6984 | 7157 | 42 |
| 19 | 6470 | 7625 | 6602 | 7511 | 6732 | 7394 | 6860 | 7276 | 6986 | 7155 | 41 |
| 20 | 6472 | 7623 | 6604 | 7509 | 6734 | 7392 | 6862 | 7274 | 6988 | 7153 | 40 |
| 21 | 6475 | 7621 | 6607 | 7507 | 6737 | 7390 | 6865 | 7272 | 6990 | 7151 | 39 |
| 22 | 6477 | 7619 | 6609 | 7505 | 6739 | 7388 | 6867 | 7270 | 6992 | 7149 | 38 |
| 23 | 6479 | 7617 | 6611 | 7503 | 6741 | 7387 | 6869 | 7268 | 6995 | 7147 | 37 |
| 24 | 6481 | 7615 | 6613 | 7501 | 6743 | 7385 | 6871 | 7266 | 6997 | 7145 | 36 |
| 25 | 6483 | 7613 | 6615 | 7499 | 6745 | 7383 | 6873 | 7264 | 6999 | 7143 | 35 |
| 26 | 6486 | 7612 | 6617 | 7497 | 6747 | 7381 | 6875 | 7262 | 7001 | 7141 | 34 |
| 27 | 6488 | 7610 | 6620 | 7495 | 6749 | 7379 | 6877 | 7260 | 7003 | 7139 | 33 |
| 28 | 6490 | 7608 | 6622 | 7493 | 6752 | 7377 | 6879 | 7258 | 7005 | 7137 | 32 |
| 29 | 6492 | 7606 | 6624 | 7491 | 6754 | 7375 | 6881 | 7256 | 7007 | 7135 | 31 |
| 30 | 6494 | 7604 | 6626 | 7490 | 6756 | 7373 | 6884 | 7254 | 7009 | 7133 | 30 |
| 31 | 6497 | 7602 | 6628 | 7488 | 6758 | 7371 | 6886 | 7252 | 7011 | 7130 | 29 |
| 32 | 6499 | 7600 | 6631 | 7486 | 6760 | 7369 | 6888 | 7250 | 7013 | 7128 | 28 |
| 33 | 6501 | 7598 | 6633 | 7484 | 6762 | 7367 | 6890 | 7248 | 7015 | 7126 | 27 |
| 34 | 6503 | 7596 | 6635 | 7482 | 6764 | 7365 | 6892 | 7246 | 7017 | 7124 | 26 |
| 35 | 6506 | 7595 | 6637 | 7480 | 6767 | 7363 | 6894 | 7244 | 7019 | 7122 | 25 |
| 36 | 6508 | 7593 | 6639 | 7478 | 6769 | 7361 | 6896 | 7242 | 7022 | 7120 | 24 |
| 37 | 6510 | 7591 | 6641 | 7476 | 6771 | 7359 | 6898 | 7240 | 7024 | 7118 | 23 |
| 38 | 6512 | 7589 | 6644 | 7474 | 6773 | 7357 | 6900 | 7238 | 7026 | 7116 | 22 |
| 39 | 6514 | 7587 | 6646 | 7472 | 6775 | 7355 | 6903 | 7236 | 7028 | 7114 | 21 |
| 40 | 6517 | 7585 | 6648 | 7470 | 6777 | 7353 | 6905 | 7234 | 7030 | 7112 | 20 |
| 41 | 6519 | 7583 | 6650 | 7468 | 6779 | 7351 | 6907 | 7232 | 7032 | 7110 | 19 |
| 42 | 6521 | 7581 | 6652 | 7466 | 6782 | 7349 | 6909 | 7230 | 7034 | 7108 | 18 |
| 43 | 6523 | 7579 | 6654 | 7464 | 6784 | 7347 | 6911 | 7228 | 7036 | 7106 | 17 |
| 44 | 6525 | 7578 | 6657 | 7463 | 6786 | 7345 | 6913 | 7226 | 7038 | 7104 | 16 |
| 45 | 6528 | 7576 | 6659 | 7461 | 6788 | 7343 | 6915 | 7224 | 7040 | 7102 | 15 |
| 46 | 6530 | 7574 | 6661 | 7459 | 6790 | 7341 | 6917 | 7222 | 7042 | 7100 | 14 |
| 47 | 6532 | 7572 | 6663 | 7457 | 6792 | 7339 | 6919 | 7220 | 7044 | 7098 | 13 |
| 48 | 6534 | 7570 | 6665 | 7455 | 6794 | 7337 | 6921 | 7218 | 7046 | 7096 | 12 |
| 49 | 6536 | 7568 | 6667 | 7453 | 6797 | 7335 | 6924 | 7216 | 7048 | 7094 | 11 |
| 50 | 6539 | 7566 | 6670 | 7451 | 6799 | 7333 | 6926 | 7214 | 7050 | 7092 | 10 |
| 51 | 6541 | 7564 | 6672 | 7449 | 6801 | 7331 | 6928 | 7212 | 7053 | 7090 | 9 |
| 52 | 6543 | 7562 | 6674 | 7447 | 6803 | 7329 | 6930 | 7210 | 7055 | 7088 | 8 |
| 53 | 6545 | 7560 | 6676 | 7445 | 6805 | 7327 | 6932 | 7208 | 7057 | 7085 | 7 |
| 54 | 6547 | 7559 | 6678 | 7443 | 6807 | 7325 | 6934 | 7206 | 7059 | 7083 | 6 |
| 55 | 6550 | 7557 | 6680 | 7441 | 6809 | 7323 | 6936 | 7203 | 7061 | 7081 | 5 |
| 56 | 6552 | 7555 | 6683 | 7439 | 6811 | 7321 | 6938 | 7201 | 7063 | 7079 | 4 |
| 57 | 6554 | 7553 | 6685 | 7437 | 6814 | 7319 | 6940 | 7199 | 7065 | 7077 | 3 |
| 58 | 6556 | 7551 | 6687 | 7435 | 6816 | 7318 | 6942 | 7197 | 7067 | 7075 | 2 |
| 59 | 6558 | 7549 | 6689 | 7433 | 6818 | 7316 | 6944 | 7195 | 7069 | 7073 | 1 |
| 60 | 6561 | 7547 | 6691 | 7431 | 6820 | 7314 | 6947 | 7193 | 7071 | 7071 | 0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| ° | 40° | | 41° | | 42° | | 43° | | 44° | | ° |

TABLE IX.—NATURAL TANGENTS AND COTANGENTS. 71

| ° | 0° | | 1° | | 2° | | 3° | | 4° | | ° |
|----|------|----------|------|---------|------|---------|------|---------|------|---------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 0000 | Infinite | 0175 | 57.2900 | 0349 | 28.6363 | 0524 | 19.0811 | 0699 | 14.3007 | 60 |
| 1 | 0003 | 3437.75 | 0177 | 56.3506 | 0352 | 28.3994 | 0527 | 18.9755 | 0702 | 14.2411 | 59 |
| 2 | 0006 | 1718.87 | 0180 | 55.4415 | 0355 | 28.1664 | 0530 | 18.8711 | 0705 | 14.1821 | 58 |
| 3 | 0009 | 1145.92 | 0183 | 54.5613 | 0358 | 27.9372 | 0533 | 18.7678 | 0708 | 14.1235 | 57 |
| 4 | 0012 | 859.436 | 0186 | 53.7086 | 0361 | 27.7117 | 0536 | 18.6656 | 0711 | 14.0655 | 56 |
| 5 | 0015 | 687.549 | 0189 | 52.8821 | 0364 | 27.4899 | 0539 | 18.5645 | 0714 | 14.0079 | 55 |
| 6 | 0017 | 572.957 | 0192 | 52.0807 | 0367 | 27.2715 | 0542 | 18.4645 | 0717 | 13.9507 | 54 |
| 7 | 0020 | 491.106 | 0195 | 51.3032 | 0370 | 27.0566 | 0544 | 18.3655 | 0720 | 13.8940 | 53 |
| 8 | 0023 | 429.718 | 0198 | 50.5485 | 0373 | 26.8450 | 0547 | 18.2677 | 0723 | 13.8378 | 52 |
| 9 | 0026 | 381.971 | 0201 | 49.8157 | 0375 | 26.6367 | 0550 | 18.1708 | 0726 | 13.7821 | 51 |
| 10 | 0029 | 343.774 | 0204 | 49.1039 | 0378 | 26.4316 | 0553 | 18.0750 | 0729 | 13.7267 | 50 |
| 11 | 0032 | 312.521 | 0207 | 48.4121 | 0381 | 26.2296 | 0556 | 17.9802 | 0731 | 13.6719 | 49 |
| 12 | 0035 | 286.478 | 0209 | 47.7395 | 0384 | 26.0307 | 0559 | 17.8863 | 0734 | 13.6174 | 48 |
| 13 | 0038 | 264.441 | 0212 | 47.0853 | 0387 | 25.8348 | 0562 | 17.7934 | 0737 | 13.5634 | 47 |
| 14 | 0041 | 245.552 | 0215 | 46.4489 | 0390 | 25.6418 | 0565 | 17.7015 | 0740 | 13.5098 | 46 |
| 15 | 0044 | 229.182 | 0218 | 45.8294 | 0393 | 25.4517 | 0568 | 17.6106 | 0743 | 13.4566 | 45 |
| 16 | 0047 | 214.858 | 0221 | 45.2261 | 0396 | 25.2644 | 0571 | 17.5205 | 0746 | 13.4039 | 44 |
| 17 | 0049 | 202.219 | 0224 | 44.6386 | 0399 | 25.0798 | 0574 | 17.4314 | 0749 | 13.3515 | 43 |
| 18 | 0052 | 190.984 | 0227 | 44.0661 | 0402 | 24.8978 | 0577 | 17.3432 | 0752 | 13.2996 | 42 |
| 19 | 0055 | 180.932 | 0230 | 43.5081 | 0405 | 24.7185 | 0580 | 17.2558 | 0755 | 13.2480 | 41 |
| 20 | 0058 | 171.885 | 0233 | 42.9641 | 0407 | 24.5418 | 0582 | 17.1693 | 0758 | 13.1969 | 40 |
| 21 | 0061 | 163.700 | 0236 | 42.4335 | 0410 | 24.3675 | 0585 | 17.0837 | 0761 | 13.1461 | 39 |
| 22 | 0064 | 156.259 | 0239 | 41.9158 | 0413 | 24.1957 | 0588 | 16.9990 | 0764 | 13.0958 | 38 |
| 23 | 0067 | 149.465 | 0241 | 41.4106 | 0416 | 24.0263 | 0591 | 16.9150 | 0767 | 13.0458 | 37 |
| 24 | 0070 | 143.237 | 0244 | 40.9174 | 0419 | 23.8593 | 0594 | 16.8319 | 0769 | 12.9962 | 36 |
| 25 | 0073 | 137.507 | 0247 | 40.4358 | 0422 | 23.6945 | 0597 | 16.7496 | 0772 | 12.9469 | 35 |
| 26 | 0076 | 132.219 | 0250 | 39.9655 | 0425 | 23.5321 | 0600 | 16.6681 | 0775 | 12.8981 | 34 |
| 27 | 0079 | 127.321 | 0253 | 39.5059 | 0428 | 23.3718 | 0603 | 16.5874 | 0778 | 12.8496 | 33 |
| 28 | 0081 | 122.774 | 0256 | 39.0568 | 0431 | 23.2137 | 0606 | 16.5075 | 0781 | 12.8014 | 32 |
| 29 | 0084 | 118.540 | 0259 | 38.6177 | 0434 | 23.0577 | 0609 | 16.4283 | 0784 | 12.7536 | 31 |
| 30 | 0087 | 114.589 | 0262 | 38.1885 | 0437 | 22.9038 | 0612 | 16.3499 | 0787 | 12.7062 | 30 |
| 31 | 0090 | 110.892 | 0265 | 37.7686 | 0440 | 22.7519 | 0615 | 16.2722 | 0790 | 12.6591 | 29 |
| 32 | 0093 | 107.426 | 0268 | 37.3579 | 0442 | 22.6020 | 0617 | 16.1952 | 0793 | 12.6124 | 28 |
| 33 | 0096 | 104.171 | 0271 | 36.9560 | 0445 | 22.4541 | 0620 | 16.1190 | 0796 | 12.5660 | 27 |
| 34 | 0099 | 101.107 | 0274 | 36.5627 | 0448 | 22.3081 | 0623 | 16.0435 | 0799 | 12.5199 | 26 |
| 35 | 0102 | 98.2179 | 0276 | 36.1776 | 0451 | 22.1640 | 0626 | 15.9687 | 0802 | 12.4742 | 25 |
| 36 | 0105 | 95.4895 | 0279 | 35.8006 | 0454 | 22.0217 | 0629 | 15.8945 | 0805 | 12.4288 | 24 |
| 37 | 0108 | 92.9085 | 0282 | 35.4313 | 0457 | 21.8813 | 0632 | 15.8211 | 0808 | 12.3838 | 23 |
| 38 | 0111 | 90.4633 | 0285 | 35.0695 | 0460 | 21.7426 | 0635 | 15.7483 | 0810 | 12.3390 | 22 |
| 39 | 0113 | 88.1436 | 0288 | 34.7151 | 0463 | 21.6056 | 0638 | 15.6762 | 0813 | 12.2946 | 21 |
| 40 | 0116 | 85.9398 | 0291 | 34.3678 | 0466 | 21.4704 | 0641 | 15.6048 | 0816 | 12.2505 | 20 |
| 41 | 0119 | 83.8435 | 0294 | 34.0273 | 0469 | 21.3369 | 0644 | 15.5340 | 0819 | 12.2067 | 19 |
| 42 | 0122 | 81.8470 | 0297 | 33.6935 | 0472 | 21.2049 | 0647 | 15.4638 | 0822 | 12.1632 | 18 |
| 43 | 0125 | 79.9434 | 0300 | 33.3662 | 0475 | 21.0747 | 0650 | 15.3943 | 0825 | 12.1201 | 17 |
| 44 | 0128 | 78.1263 | 0303 | 33.0452 | 0477 | 20.9460 | 0653 | 15.3254 | 0828 | 12.0772 | 16 |
| 45 | 0131 | 76.3900 | 0306 | 32.7303 | 0480 | 20.8188 | 0655 | 15.2571 | 0831 | 12.0346 | 15 |
| 46 | 0134 | 74.7292 | 0308 | 32.4213 | 0483 | 20.6932 | 0658 | 15.1893 | 0834 | 11.9923 | 14 |
| 47 | 0137 | 73.1390 | 0311 | 32.1181 | 0486 | 20.5691 | 0661 | 15.1222 | 0837 | 11.9504 | 13 |
| 48 | 0140 | 71.6151 | 0314 | 31.8205 | 0489 | 20.4465 | 0664 | 15.0557 | 0840 | 11.9087 | 12 |
| 49 | 0143 | 70.1533 | 0317 | 31.5284 | 0492 | 20.3253 | 0667 | 14.9898 | 0843 | 11.8673 | 11 |
| 50 | 0146 | 68.7501 | 0320 | 31.2416 | 0495 | 20.2056 | 0670 | 14.9244 | 0846 | 11.8262 | 10 |
| 51 | 0148 | 67.4019 | 0323 | 30.9599 | 0498 | 20.0872 | 0673 | 14.8596 | 0849 | 11.7853 | 9 |
| 52 | 0151 | 66.1055 | 0326 | 30.6833 | 0501 | 19.9702 | 0676 | 14.7954 | 0851 | 11.7448 | 8 |
| 53 | 0154 | 64.8580 | 0329 | 30.4116 | 0504 | 19.8546 | 0679 | 14.7317 | 0854 | 11.7045 | 7 |
| 54 | 0157 | 63.6567 | 0332 | 30.1446 | 0507 | 19.7403 | 0682 | 14.6685 | 0857 | 11.6645 | 6 |
| 55 | 0160 | 62.4992 | 0335 | 29.8823 | 0509 | 19.6273 | 0685 | 14.6059 | 0860 | 11.6248 | 5 |
| 56 | 0163 | 61.3829 | 0338 | 29.6245 | 0512 | 19.5156 | 0688 | 14.5438 | 0863 | 11.5853 | 4 |
| 57 | 0166 | 60.3058 | 0340 | 29.3711 | 0515 | 19.4051 | 0690 | 14.4823 | 0866 | 11.5461 | 3 |
| 58 | 0169 | 59.2659 | 0343 | 29.1220 | 0518 | 19.2959 | 0693 | 14.4212 | 0869 | 11.5072 | 2 |
| 59 | 0172 | 58.2612 | 0346 | 28.8771 | 0521 | 19.1879 | 0696 | 14.3607 | 0872 | 11.4685 | 1 |
| 60 | 0175 | 57.2900 | 0349 | 28.6363 | 0524 | 19.0811 | 0699 | 14.3007 | 0875 | 11.4301 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| ° | 89° | | 88° | | 87° | | 86° | | 85° | | ° |

| ° | 5° | | 6° | | 7° | | 8° | | 9° | | ° |
|----|------|---------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 0875 | 11.4301 | 1051 | 9.5144 | 1228 | 8.1443 | 1405 | 7.1154 | 1584 | 6.3138 | 60 |
| 1 | 0878 | 11.3919 | 1054 | 9.4878 | 1231 | 8.1248 | 1408 | 7.1004 | 1587 | 6.3019 | 59 |
| 2 | 0881 | 11.3540 | 1057 | 9.4614 | 1234 | 8.1054 | 1411 | 7.0855 | 1590 | 6.2901 | 58 |
| 3 | 0884 | 11.3163 | 1060 | 9.4352 | 1237 | 8.0860 | 1414 | 7.0706 | 1593 | 6.2783 | 57 |
| 4 | 0887 | 11.2789 | 1063 | 9.4090 | 1240 | 8.0667 | 1417 | 7.0558 | 1596 | 6.2666 | 56 |
| 5 | 0890 | 11.2417 | 1066 | 9.3831 | 1243 | 8.0476 | 1420 | 7.0410 | 1599 | 6.2549 | 55 |
| 6 | 0892 | 11.2048 | 1069 | 9.3572 | 1246 | 8.0285 | 1423 | 7.0264 | 1602 | 6.2432 | 54 |
| 7 | 0895 | 11.1681 | 1072 | 9.3315 | 1249 | 8.0095 | 1426 | 7.0117 | 1605 | 6.2316 | 53 |
| 8 | 0898 | 11.1316 | 1075 | 9.3060 | 1251 | 7.9906 | 1429 | 6.9972 | 1608 | 6.2200 | 52 |
| 9 | 0901 | 11.0954 | 1078 | 9.2806 | 1254 | 7.9718 | 1432 | 6.9827 | 1611 | 6.2085 | 51 |
| 10 | 0904 | 11.0594 | 1080 | 9.2553 | 1257 | 7.9530 | 1435 | 6.9682 | 1614 | 6.1970 | 50 |
| 11 | 0907 | 11.0237 | 1083 | 9.2302 | 1260 | 7.9344 | 1438 | 6.9538 | 1617 | 6.1856 | 49 |
| 12 | 0910 | 10.9882 | 1086 | 9.2052 | 1263 | 7.9158 | 1441 | 6.9395 | 1620 | 6.1742 | 48 |
| 13 | 0913 | 10.9529 | 1089 | 9.1803 | 1266 | 7.8973 | 1444 | 6.9252 | 1623 | 6.1628 | 47 |
| 14 | 0916 | 10.9178 | 1092 | 9.1555 | 1269 | 7.8789 | 1447 | 6.9110 | 1626 | 6.1515 | 46 |
| 15 | 0919 | 10.8829 | 1095 | 9.1309 | 1272 | 7.8606 | 1450 | 6.8969 | 1629 | 6.1402 | 45 |
| 16 | 0922 | 10.8483 | 1098 | 9.1065 | 1275 | 7.8424 | 1453 | 6.8828 | 1632 | 6.1290 | 44 |
| 17 | 0925 | 10.8139 | 1101 | 9.0821 | 1278 | 7.8243 | 1456 | 6.8687 | 1635 | 6.1178 | 43 |
| 18 | 0928 | 10.7797 | 1104 | 9.0579 | 1281 | 7.8062 | 1459 | 6.8548 | 1638 | 6.1066 | 42 |
| 19 | 0931 | 10.7457 | 1107 | 9.0338 | 1284 | 7.7883 | 1462 | 6.8408 | 1641 | 6.0955 | 41 |
| 20 | 0934 | 10.7119 | 1110 | 9.0098 | 1287 | 7.7704 | 1465 | 6.8269 | 1644 | 6.0844 | 40 |
| 21 | 0936 | 10.6783 | 1113 | 8.9860 | 1290 | 7.7525 | 1468 | 6.8131 | 1647 | 6.0734 | 39 |
| 22 | 0939 | 10.6450 | 1116 | 8.9623 | 1293 | 7.7348 | 1471 | 6.7994 | 1650 | 6.0624 | 38 |
| 23 | 0942 | 10.6118 | 1119 | 8.9387 | 1296 | 7.7171 | 1474 | 6.7856 | 1653 | 6.0514 | 37 |
| 24 | 0945 | 10.5789 | 1122 | 8.9152 | 1299 | 7.6996 | 1477 | 6.7720 | 1655 | 6.0405 | 36 |
| 25 | 0948 | 10.5462 | 1125 | 8.8919 | 1302 | 7.6821 | 1480 | 6.7584 | 1658 | 6.0296 | 35 |
| 26 | 0951 | 10.5136 | 1128 | 8.8686 | 1305 | 7.6647 | 1483 | 6.7448 | 1661 | 6.0188 | 34 |
| 27 | 0954 | 10.4813 | 1131 | 8.8455 | 1308 | 7.6473 | 1486 | 6.7313 | 1664 | 6.0080 | 33 |
| 28 | 0957 | 10.4491 | 1134 | 8.8225 | 1311 | 7.6301 | 1489 | 6.7179 | 1667 | 5.9972 | 32 |
| 29 | 0960 | 10.4172 | 1136 | 8.7996 | 1314 | 7.6129 | 1492 | 6.7045 | 1670 | 5.9865 | 31 |
| 30 | 0963 | 10.3854 | 1139 | 8.7769 | 1317 | 7.5958 | 1495 | 6.6912 | 1673 | 5.9758 | 30 |
| 31 | 0966 | 10.3538 | 1142 | 8.7542 | 1319 | 7.5787 | 1497 | 6.6779 | 1676 | 5.9651 | 29 |
| 32 | 0969 | 10.3224 | 1145 | 8.7317 | 1322 | 7.5618 | 1500 | 6.6646 | 1679 | 5.9545 | 28 |
| 33 | 0972 | 10.2913 | 1148 | 8.7093 | 1325 | 7.5449 | 1503 | 6.6514 | 1682 | 5.9439 | 27 |
| 34 | 0975 | 10.2602 | 1151 | 8.6870 | 1328 | 7.5281 | 1506 | 6.6383 | 1685 | 5.9333 | 26 |
| 35 | 0978 | 10.2294 | 1154 | 8.6648 | 1331 | 7.5113 | 1509 | 6.6252 | 1688 | 5.9228 | 25 |
| 36 | 0981 | 10.1988 | 1157 | 8.6427 | 1334 | 7.4947 | 1512 | 6.6122 | 1691 | 5.9124 | 24 |
| 37 | 0983 | 10.1683 | 1160 | 8.6208 | 1337 | 7.4781 | 1515 | 6.5992 | 1694 | 5.9019 | 23 |
| 38 | 0986 | 10.1381 | 1163 | 8.5989 | 1340 | 7.4615 | 1518 | 6.5863 | 1697 | 5.8915 | 22 |
| 39 | 0989 | 10.1080 | 1166 | 8.5772 | 1343 | 7.4451 | 1521 | 6.5734 | 1700 | 5.8811 | 21 |
| 40 | 0992 | 10.0780 | 1169 | 8.5555 | 1346 | 7.4287 | 1524 | 6.5606 | 1703 | 5.8708 | 20 |
| 41 | 0995 | 10.0483 | 1172 | 8.5340 | 1349 | 7.4124 | 1527 | 6.5478 | 1706 | 5.8605 | 19 |
| 42 | 0998 | 10.0187 | 1175 | 8.5126 | 1352 | 7.3962 | 1530 | 6.5350 | 1709 | 5.8502 | 18 |
| 43 | 1001 | 9.9893 | 1178 | 8.4913 | 1355 | 7.3800 | 1533 | 6.5223 | 1712 | 5.8400 | 17 |
| 44 | 1004 | 9.9601 | 1181 | 8.4701 | 1358 | 7.3639 | 1536 | 6.5097 | 1715 | 5.8298 | 16 |
| 45 | 1007 | 9.9310 | 1184 | 8.4490 | 1361 | 7.3479 | 1539 | 6.4971 | 1718 | 5.8197 | 15 |
| 46 | 1010 | 9.9021 | 1187 | 8.4280 | 1364 | 7.3319 | 1542 | 6.4846 | 1721 | 5.8095 | 14 |
| 47 | 1013 | 9.8734 | 1189 | 8.4071 | 1367 | 7.3160 | 1545 | 6.4721 | 1724 | 5.7994 | 13 |
| 48 | 1016 | 9.8448 | 1192 | 8.3863 | 1370 | 7.3002 | 1548 | 6.4596 | 1727 | 5.7894 | 12 |
| 49 | 1019 | 9.8164 | 1195 | 8.3656 | 1373 | 7.2844 | 1551 | 6.4472 | 1730 | 5.7794 | 11 |
| 50 | 1022 | 9.7882 | 1198 | 8.3450 | 1376 | 7.2687 | 1554 | 6.4348 | 1733 | 5.7694 | 10 |
| 51 | 1025 | 9.7601 | 1201 | 8.3245 | 1379 | 7.2531 | 1557 | 6.4225 | 1736 | 5.7594 | 9 |
| 52 | 1028 | 9.7322 | 1204 | 8.3041 | 1382 | 7.2375 | 1560 | 6.4103 | 1739 | 5.7495 | 8 |
| 53 | 1030 | 9.7044 | 1207 | 8.2838 | 1385 | 7.2220 | 1563 | 6.3980 | 1742 | 5.7396 | 7 |
| 54 | 1033 | 9.6768 | 1210 | 8.2636 | 1388 | 7.2066 | 1566 | 6.3859 | 1745 | 5.7297 | 6 |
| 55 | 1036 | 9.6499 | 1213 | 8.2434 | 1391 | 7.1912 | 1569 | 6.3737 | 1748 | 5.7199 | 5 |
| 56 | 1039 | 9.6220 | 1216 | 8.2234 | 1394 | 7.1759 | 1572 | 6.3617 | 1751 | 5.7101 | 4 |
| 57 | 1042 | 9.5949 | 1219 | 8.2035 | 1397 | 7.1607 | 1575 | 6.3496 | 1754 | 5.7004 | 3 |
| 58 | 1045 | 9.5679 | 1222 | 8.1837 | 1399 | 7.1455 | 1578 | 6.3376 | 1757 | 5.6906 | 2 |
| 59 | 1048 | 9.5411 | 1225 | 8.1640 | 1402 | 7.1304 | 1581 | 6.3257 | 1760 | 5.6809 | 1 |
| 60 | 1051 | 9.5144 | 1228 | 8.1443 | 1405 | 7.1154 | 1584 | 6.3138 | 1763 | 5.6713 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| | 84° | | 83° | | 82° | | 81° | | 80° | | |

| ° | 10° | | 11° | | 12° | | 13° | | 14° | | ° |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 1763 | 5.6713 | 1944 | 5.1446 | 2126 | 4.7046 | 2309 | 4.3315 | 2493 | 4.0108 | 60 |
| 1 | 1766 | 5.6617 | 1947 | 5.1366 | 2129 | 4.6979 | 2312 | 4.3257 | 2496 | 4.0058 | 59 |
| 2 | 1769 | 5.6521 | 1950 | 5.1286 | 2132 | 4.6912 | 2315 | 4.3200 | 2499 | 4.0009 | 58 |
| 3 | 1772 | 5.6425 | 1953 | 5.1207 | 2135 | 4.6845 | 2318 | 4.3143 | 2503 | 3.9959 | 57 |
| 4 | 1775 | 5.6330 | 1956 | 5.1128 | 2138 | 4.6779 | 2321 | 4.3086 | 2506 | 3.9910 | 56 |
| 5 | 1778 | 5.6234 | 1959 | 5.1049 | 2141 | 4.6712 | 2324 | 4.3029 | 2509 | 3.9861 | 55 |
| 6 | 1781 | 5.6140 | 1962 | 5.0970 | 2144 | 4.6646 | 2327 | 4.2972 | 2512 | 3.9812 | 54 |
| 7 | 1784 | 5.6045 | 1965 | 5.0892 | 2147 | 4.6580 | 2330 | 4.2916 | 2515 | 3.9763 | 53 |
| 8 | 1787 | 5.5951 | 1968 | 5.0814 | 2150 | 4.6514 | 2333 | 4.2859 | 2518 | 3.9714 | 52 |
| 9 | 1790 | 5.5857 | 1971 | 5.0736 | 2153 | 4.6448 | 2336 | 4.2803 | 2521 | 3.9665 | 51 |
| 10 | 1793 | 5.5764 | 1974 | 5.0658 | 2156 | 4.6382 | 2339 | 4.2747 | 2524 | 3.9617 | 50 |
| 11 | 1796 | 5.5671 | 1977 | 5.0581 | 2159 | 4.6317 | 2342 | 4.2691 | 2527 | 3.9568 | 49 |
| 12 | 1799 | 5.5578 | 1980 | 5.0504 | 2162 | 4.6252 | 2345 | 4.2635 | 2530 | 3.9520 | 48 |
| 13 | 1802 | 5.5485 | 1983 | 5.0427 | 2165 | 4.6187 | 2349 | 4.2580 | 2533 | 3.9471 | 47 |
| 14 | 1805 | 5.5393 | 1986 | 5.0350 | 2168 | 4.6122 | 2352 | 4.2524 | 2537 | 3.9423 | 46 |
| 15 | 1808 | 5.5301 | 1989 | 5.0273 | 2171 | 4.6057 | 2355 | 4.2468 | 2540 | 3.9375 | 45 |
| 16 | 1811 | 5.5209 | 1992 | 5.0197 | 2174 | 4.5993 | 2358 | 4.2413 | 2543 | 3.9327 | 44 |
| 17 | 1814 | 5.5118 | 1995 | 5.0121 | 2177 | 4.5928 | 2361 | 4.2358 | 2546 | 3.9279 | 43 |
| 18 | 1817 | 5.5026 | 1998 | 5.0045 | 2180 | 4.5864 | 2364 | 4.2303 | 2549 | 3.9232 | 42 |
| 19 | 1820 | 5.4936 | 2001 | 4.9969 | 2183 | 4.5800 | 2367 | 4.2248 | 2552 | 3.9184 | 41 |
| 20 | 1823 | 5.4845 | 2004 | 4.9894 | 2186 | 4.5736 | 2370 | 4.2193 | 2555 | 3.9136 | 40 |
| 21 | 1826 | 5.4755 | 2007 | 4.9819 | 2189 | 4.5673 | 2373 | 4.2139 | 2558 | 3.9089 | 39 |
| 22 | 1829 | 5.4665 | 2010 | 4.9744 | 2193 | 4.5609 | 2376 | 4.2084 | 2561 | 3.9042 | 38 |
| 23 | 1832 | 5.4575 | 2013 | 4.9669 | 2196 | 4.5546 | 2379 | 4.2030 | 2564 | 3.8995 | 37 |
| 24 | 1835 | 5.4486 | 2016 | 4.9594 | 2199 | 4.5483 | 2382 | 4.1976 | 2568 | 3.8947 | 36 |
| 25 | 1838 | 5.4397 | 2019 | 4.9520 | 2202 | 4.5420 | 2385 | 4.1922 | 2571 | 3.8900 | 35 |
| 26 | 1841 | 5.4308 | 2022 | 4.9446 | 2205 | 4.5357 | 2388 | 4.1868 | 2574 | 3.8854 | 34 |
| 27 | 1844 | 5.4219 | 2025 | 4.9372 | 2208 | 4.5294 | 2392 | 4.1814 | 2577 | 3.8807 | 33 |
| 28 | 1847 | 5.4131 | 2028 | 4.9298 | 2211 | 4.5232 | 2395 | 4.1760 | 2580 | 3.8760 | 32 |
| 29 | 1850 | 5.4043 | 2031 | 4.9225 | 2214 | 4.5169 | 2398 | 4.1706 | 2583 | 3.8714 | 31 |
| 30 | 1853 | 5.3955 | 2035 | 4.9152 | 2217 | 4.5107 | 2401 | 4.1653 | 2586 | 3.8667 | 30 |
| 31 | 1856 | 5.3868 | 2038 | 4.9078 | 2220 | 4.5045 | 2404 | 4.1600 | 2589 | 3.8621 | 29 |
| 32 | 1859 | 5.3781 | 2041 | 4.9006 | 2223 | 4.4983 | 2407 | 4.1547 | 2592 | 3.8575 | 28 |
| 33 | 1862 | 5.3694 | 2044 | 4.8933 | 2226 | 4.4922 | 2410 | 4.1493 | 2595 | 3.8528 | 27 |
| 34 | 1865 | 5.3607 | 2047 | 4.8860 | 2229 | 4.4860 | 2413 | 4.1441 | 2599 | 3.8482 | 26 |
| 35 | 1868 | 5.3521 | 2050 | 4.8788 | 2232 | 4.4799 | 2416 | 4.1388 | 2602 | 3.8436 | 25 |
| 36 | 1871 | 5.3435 | 2053 | 4.8716 | 2235 | 4.4737 | 2419 | 4.1335 | 2605 | 3.8391 | 24 |
| 37 | 1874 | 5.3349 | 2056 | 4.8644 | 2238 | 4.4676 | 2422 | 4.1282 | 2608 | 3.8345 | 23 |
| 38 | 1877 | 5.3263 | 2059 | 4.8573 | 2241 | 4.4615 | 2425 | 4.1230 | 2611 | 3.8299 | 22 |
| 39 | 1880 | 5.3178 | 2062 | 4.8501 | 2244 | 4.4555 | 2428 | 4.1178 | 2614 | 3.8254 | 21 |
| 40 | 1883 | 5.3093 | 2065 | 4.8430 | 2247 | 4.4494 | 2432 | 4.1126 | 2617 | 3.8208 | 20 |
| 41 | 1887 | 5.3008 | 2068 | 4.8359 | 2251 | 4.4434 | 2435 | 4.1074 | 2620 | 3.8163 | 19 |
| 42 | 1890 | 5.2924 | 2071 | 4.8288 | 2254 | 4.4374 | 2438 | 4.1022 | 2623 | 3.8118 | 18 |
| 43 | 1893 | 5.2839 | 2074 | 4.8218 | 2257 | 4.4313 | 2441 | 4.0970 | 2627 | 3.8073 | 17 |
| 44 | 1896 | 5.2755 | 2077 | 4.8147 | 2260 | 4.4253 | 2444 | 4.0918 | 2630 | 3.8028 | 16 |
| 45 | 1899 | 5.2672 | 2080 | 4.8077 | 2263 | 4.4194 | 2447 | 4.0867 | 2633 | 3.7983 | 15 |
| 46 | 1902 | 5.2588 | 2083 | 4.8007 | 2266 | 4.4134 | 2450 | 4.0815 | 2636 | 3.7938 | 14 |
| 47 | 1905 | 5.2505 | 2086 | 4.7937 | 2269 | 4.4075 | 2453 | 4.0764 | 2639 | 3.7893 | 13 |
| 48 | 1908 | 5.2422 | 2089 | 4.7867 | 2272 | 4.4015 | 2456 | 4.0713 | 2642 | 3.7848 | 12 |
| 49 | 1911 | 5.2339 | 2092 | 4.7798 | 2275 | 4.3956 | 2459 | 4.0662 | 2645 | 3.7804 | 11 |
| 50 | 1914 | 5.2257 | 2095 | 4.7729 | 2278 | 4.3897 | 2462 | 4.0611 | 2648 | 3.7760 | 10 |
| 51 | 1917 | 5.2174 | 2098 | 4.7659 | 2281 | 4.3838 | 2465 | 4.0560 | 2651 | 3.7715 | 9 |
| 52 | 1920 | 5.2092 | 2101 | 4.7591 | 2284 | 4.3779 | 2469 | 4.0509 | 2655 | 3.7671 | 8 |
| 53 | 1923 | 5.2011 | 2104 | 4.7522 | 2287 | 4.3721 | 2472 | 4.0459 | 2658 | 3.7627 | 7 |
| 54 | 1926 | 5.1929 | 2107 | 4.7453 | 2290 | 4.3662 | 2475 | 4.0408 | 2661 | 3.7583 | 6 |
| 55 | 1929 | 5.1848 | 2110 | 4.7385 | 2293 | 4.3604 | 2478 | 4.0358 | 2664 | 3.7539 | 5 |
| 56 | 1932 | 5.1767 | 2113 | 4.7317 | 2296 | 4.3546 | 2481 | 4.0308 | 2667 | 3.7495 | 4 |
| 57 | 1935 | 5.1686 | 2116 | 4.7249 | 2299 | 4.3488 | 2484 | 4.0257 | 2670 | 3.7451 | 3 |
| 58 | 1938 | 5.1606 | 2119 | 4.7181 | 2303 | 4.3430 | 2487 | 4.0207 | 2673 | 3.7408 | 2 |
| 59 | 1941 | 5.1526 | 2123 | 4.7114 | 2306 | 4.3372 | 2490 | 4.0158 | 2676 | 3.7364 | 1 |
| 60 | 1944 | 5.1446 | 2126 | 4.7046 | 2309 | 4.3315 | 2493 | 4.0108 | 2679 | 3.7321 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| ° | 79° | | 78° | | 77° | | 76° | | 75° | | ° |

| ° | 20° | | 21° | | 22° | | 23° | | 24° | | ° |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 3640 | 2.7475 | 3839 | 2.6051 | 4040 | 2.4751 | 4245 | 2.3559 | 4452 | 2.2460 | 60 |
| 1 | 3643 | 2.7450 | 3842 | 2.6028 | 4044 | 2.4730 | 4248 | 2.3539 | 4456 | 2.2443 | 59 |
| 2 | 3646 | 2.7425 | 3845 | 2.6006 | 4047 | 2.4709 | 4252 | 2.3520 | 4459 | 2.2425 | 58 |
| 3 | 3650 | 2.7400 | 3849 | 2.5983 | 4050 | 2.4689 | 4255 | 2.3501 | 4463 | 2.2408 | 57 |
| 4 | 3653 | 2.7376 | 3852 | 2.5961 | 4054 | 2.4668 | 4258 | 2.3483 | 4466 | 2.2390 | 56 |
| 5 | 3656 | 2.7351 | 3855 | 2.5938 | 4057 | 2.4648 | 4262 | 2.3464 | 4470 | 2.2373 | 55 |
| 6 | 3659 | 2.7326 | 3859 | 2.5916 | 4061 | 2.4627 | 4265 | 2.3445 | 4473 | 2.2355 | 54 |
| 7 | 3663 | 2.7302 | 3862 | 2.5893 | 4064 | 2.4606 | 4269 | 2.3426 | 4477 | 2.2338 | 53 |
| 8 | 3666 | 2.7277 | 3865 | 2.5871 | 4067 | 2.4586 | 4272 | 2.3407 | 4480 | 2.2320 | 52 |
| 9 | 3669 | 2.7253 | 3869 | 2.5848 | 4071 | 2.4566 | 4276 | 2.3388 | 4484 | 2.2303 | 51 |
| 10 | 3673 | 2.7228 | 3872 | 2.5826 | 4074 | 2.4545 | 4279 | 2.3369 | 4487 | 2.2286 | 50 |
| 11 | 3676 | 2.7204 | 3875 | 2.5804 | 4078 | 2.4525 | 4283 | 2.3351 | 4491 | 2.2268 | 49 |
| 12 | 3679 | 2.7179 | 3879 | 2.5782 | 4081 | 2.4504 | 4286 | 2.3332 | 4494 | 2.2251 | 48 |
| 13 | 3683 | 2.7155 | 3882 | 2.5759 | 4084 | 2.4484 | 4289 | 2.3313 | 4498 | 2.2234 | 47 |
| 14 | 3686 | 2.7130 | 3885 | 2.5737 | 4088 | 2.4464 | 4293 | 2.3294 | 4501 | 2.2216 | 46 |
| 15 | 3689 | 2.7106 | 3889 | 2.5715 | 4091 | 2.4443 | 4296 | 2.3276 | 4505 | 2.2199 | 45 |
| 16 | 3693 | 2.7082 | 3892 | 2.5693 | 4095 | 2.4423 | 4300 | 2.3257 | 4508 | 2.2182 | 44 |
| 17 | 3696 | 2.7058 | 3895 | 2.5671 | 4098 | 2.4403 | 4303 | 2.3238 | 4512 | 2.2165 | 43 |
| 18 | 3699 | 2.7034 | 3899 | 2.5649 | 4101 | 2.4383 | 4307 | 2.3220 | 4515 | 2.2148 | 42 |
| 19 | 3702 | 2.7009 | 3902 | 2.5627 | 4105 | 2.4362 | 4310 | 2.3201 | 4519 | 2.2130 | 41 |
| 20 | 3706 | 2.6985 | 3906 | 2.5605 | 4108 | 2.4342 | 4314 | 2.3183 | 4522 | 2.2113 | 40 |
| 21 | 3709 | 2.6961 | 3909 | 2.5583 | 4111 | 2.4322 | 4317 | 2.3164 | 4526 | 2.2096 | 39 |
| 22 | 3712 | 2.6937 | 3912 | 2.5561 | 4115 | 2.4302 | 4320 | 2.3146 | 4529 | 2.2079 | 38 |
| 23 | 3716 | 2.6913 | 3916 | 2.5539 | 4118 | 2.4282 | 4324 | 2.3127 | 4533 | 2.2062 | 37 |
| 24 | 3719 | 2.6889 | 3919 | 2.5517 | 4122 | 2.4262 | 4327 | 2.3109 | 4536 | 2.2045 | 36 |
| 25 | 3722 | 2.6865 | 3922 | 2.5495 | 4125 | 2.4242 | 4331 | 2.3090 | 4540 | 2.2028 | 35 |
| 26 | 3726 | 2.6841 | 3926 | 2.5473 | 4129 | 2.4222 | 4334 | 2.3072 | 4543 | 2.2011 | 34 |
| 27 | 3729 | 2.6818 | 3929 | 2.5452 | 4132 | 2.4202 | 4338 | 2.3053 | 4547 | 2.1994 | 33 |
| 28 | 3732 | 2.6794 | 3932 | 2.5430 | 4135 | 2.4182 | 4341 | 2.3035 | 4550 | 2.1977 | 32 |
| 29 | 3736 | 2.6770 | 3936 | 2.5408 | 4139 | 2.4162 | 4345 | 2.3017 | 4554 | 2.1960 | 31 |
| 30 | 3739 | 2.6746 | 3939 | 2.5386 | 4142 | 2.4142 | 4348 | 2.2998 | 4557 | 2.1943 | 30 |
| 31 | 3742 | 2.6723 | 3942 | 2.5365 | 4146 | 2.4122 | 4352 | 2.2980 | 4561 | 2.1926 | 29 |
| 32 | 3745 | 2.6699 | 3946 | 2.5343 | 4149 | 2.4102 | 4355 | 2.2962 | 4564 | 2.1909 | 28 |
| 33 | 3749 | 2.6675 | 3949 | 2.5322 | 4152 | 2.4083 | 4359 | 2.2944 | 4568 | 2.1892 | 27 |
| 34 | 3752 | 2.6652 | 3953 | 2.5300 | 4156 | 2.4063 | 4362 | 2.2925 | 4571 | 2.1876 | 26 |
| 35 | 3755 | 2.6628 | 3956 | 2.5279 | 4159 | 2.4043 | 4365 | 2.2907 | 4575 | 2.1859 | 25 |
| 36 | 3759 | 2.6605 | 3959 | 2.5257 | 4163 | 2.4023 | 4369 | 2.2889 | 4578 | 2.1842 | 24 |
| 37 | 3762 | 2.6581 | 3963 | 2.5236 | 4166 | 2.4004 | 4372 | 2.2871 | 4582 | 2.1825 | 23 |
| 38 | 3765 | 2.6558 | 3966 | 2.5214 | 4169 | 2.3984 | 4376 | 2.2853 | 4585 | 2.1808 | 22 |
| 39 | 3769 | 2.6534 | 3969 | 2.5193 | 4173 | 2.3964 | 4379 | 2.2835 | 4589 | 2.1792 | 21 |
| 40 | 3772 | 2.6511 | 3973 | 2.5172 | 4176 | 2.3945 | 4383 | 2.2817 | 4592 | 2.1775 | 20 |
| 41 | 3775 | 2.6488 | 3976 | 2.5150 | 4180 | 2.3925 | 4386 | 2.2799 | 4596 | 2.1758 | 19 |
| 42 | 3779 | 2.6464 | 3979 | 2.5129 | 4183 | 2.3906 | 4390 | 2.2781 | 4599 | 2.1742 | 18 |
| 43 | 3782 | 2.6441 | 3983 | 2.5108 | 4187 | 2.3886 | 4393 | 2.2763 | 4603 | 2.1725 | 17 |
| 44 | 3785 | 2.6418 | 3986 | 2.5086 | 4190 | 2.3867 | 4397 | 2.2745 | 4607 | 2.1708 | 16 |
| 45 | 3789 | 2.6395 | 3990 | 2.5065 | 4193 | 2.3847 | 4400 | 2.2727 | 4610 | 2.1692 | 15 |
| 46 | 3792 | 2.6371 | 3993 | 2.5044 | 4197 | 2.3828 | 4404 | 2.2709 | 4614 | 2.1675 | 14 |
| 47 | 3795 | 2.6348 | 3996 | 2.5023 | 4200 | 2.3808 | 4407 | 2.2691 | 4617 | 2.1659 | 13 |
| 48 | 3799 | 2.6325 | 4000 | 2.5002 | 4204 | 2.3789 | 4411 | 2.2673 | 4621 | 2.1642 | 12 |
| 49 | 3802 | 2.6302 | 4003 | 2.4981 | 4207 | 2.3770 | 4414 | 2.2655 | 4624 | 2.1625 | 11 |
| 50 | 3805 | 2.6279 | 4006 | 2.4960 | 4210 | 2.3750 | 4417 | 2.2637 | 4628 | 2.1609 | 10 |
| 51 | 3809 | 2.6256 | 4010 | 2.4939 | 4214 | 2.3731 | 4421 | 2.2620 | 4631 | 2.1592 | 9 |
| 52 | 3812 | 2.6233 | 4013 | 2.4918 | 4217 | 2.3712 | 4424 | 2.2602 | 4635 | 2.1576 | 8 |
| 53 | 3815 | 2.6210 | 4017 | 2.4897 | 4221 | 2.3693 | 4428 | 2.2584 | 4638 | 2.1560 | 7 |
| 54 | 3819 | 2.6187 | 4020 | 2.4876 | 4224 | 2.3673 | 4431 | 2.2566 | 4642 | 2.1543 | 6 |
| 55 | 3822 | 2.6165 | 4023 | 2.4855 | 4228 | 2.3654 | 4435 | 2.2549 | 4645 | 2.1527 | 5 |
| 56 | 3825 | 2.6142 | 4027 | 2.4834 | 4231 | 2.3635 | 4438 | 2.2531 | 4649 | 2.1510 | 4 |
| 57 | 3829 | 2.6119 | 4030 | 2.4813 | 4234 | 2.3616 | 4442 | 2.2513 | 4652 | 2.1494 | 3 |
| 58 | 3832 | 2.6096 | 4033 | 2.4792 | 4238 | 2.3597 | 4445 | 2.2496 | 4656 | 2.1478 | 2 |
| 59 | 3835 | 2.6074 | 4037 | 2.4772 | 4241 | 2.3578 | 4449 | 2.2478 | 4660 | 2.1461 | 1 |
| 60 | 3839 | 2.6051 | 4040 | 2.4751 | 4245 | 2.3559 | 4452 | 2.2460 | 4663 | 2.1445 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| ° | 69° | | 68° | | 67° | | 66° | | 65° | | ° |

| ° | 25° | | 26° | | 27° | | 28° | | 29° | | ° |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 4663 | 2.1445 | 4877 | 2.0503 | 5095 | 1.9626 | 5317 | 1.8807 | 5543 | 1.8040 | 60 |
| 1 | 4667 | 2.1429 | 4881 | 2.0488 | 5099 | 1.9612 | 5321 | 1.8794 | 5547 | 1.8028 | 59 |
| 2 | 4670 | 2.1413 | 4885 | 2.0473 | 5103 | 1.9598 | 5325 | 1.8781 | 5551 | 1.8016 | 58 |
| 3 | 4674 | 2.1396 | 4888 | 2.0458 | 5106 | 1.9584 | 5328 | 1.8768 | 5555 | 1.8003 | 57 |
| 4 | 4677 | 2.1380 | 4892 | 2.0443 | 5110 | 1.9570 | 5332 | 1.8755 | 5558 | 1.7991 | 56 |
| 5 | 4681 | 2.1364 | 4895 | 2.0428 | 5114 | 1.9556 | 5336 | 1.8741 | 5562 | 1.7979 | 55 |
| 6 | 4684 | 2.1348 | 4899 | 2.0413 | 5117 | 1.9542 | 5340 | 1.8728 | 5566 | 1.7966 | 54 |
| 7 | 4688 | 2.1332 | 4903 | 2.0398 | 5121 | 1.9528 | 5343 | 1.8715 | 5570 | 1.7954 | 53 |
| 8 | 4691 | 2.1315 | 4906 | 2.0383 | 5125 | 1.9514 | 5347 | 1.8702 | 5574 | 1.7942 | 52 |
| 9 | 4695 | 2.1299 | 4910 | 2.0368 | 5128 | 1.9500 | 5351 | 1.8689 | 5577 | 1.7930 | 51 |
| 10 | 4699 | 2.1283 | 4913 | 2.0353 | 5132 | 1.9486 | 5354 | 1.8676 | 5581 | 1.7917 | 50 |
| 11 | 4702 | 2.1267 | 4917 | 2.0338 | 5136 | 1.9472 | 5358 | 1.8663 | 5585 | 1.7905 | 49 |
| 12 | 4706 | 2.1251 | 4921 | 2.0323 | 5139 | 1.9458 | 5362 | 1.8650 | 5589 | 1.7893 | 48 |
| 13 | 4709 | 2.1235 | 4924 | 2.0308 | 5143 | 1.9444 | 5366 | 1.8637 | 5593 | 1.7881 | 47 |
| 14 | 4713 | 2.1219 | 4928 | 2.0293 | 5147 | 1.9430 | 5369 | 1.8624 | 5596 | 1.7868 | 46 |
| 15 | 4716 | 2.1203 | 4931 | 2.0278 | 5150 | 1.9416 | 5373 | 1.8611 | 5600 | 1.7856 | 45 |
| 16 | 4720 | 2.1187 | 4935 | 2.0263 | 5154 | 1.9402 | 5377 | 1.8598 | 5604 | 1.7844 | 44 |
| 17 | 4723 | 2.1171 | 4939 | 2.0248 | 5158 | 1.9388 | 5381 | 1.8585 | 5608 | 1.7832 | 43 |
| 18 | 4727 | 2.1155 | 4942 | 2.0233 | 5161 | 1.9375 | 5384 | 1.8572 | 5612 | 1.7820 | 42 |
| 19 | 4731 | 2.1139 | 4946 | 2.0219 | 5165 | 1.9361 | 5388 | 1.8559 | 5616 | 1.7808 | 41 |
| 20 | 4734 | 2.1123 | 4950 | 2.0204 | 5169 | 1.9347 | 5392 | 1.8546 | 5619 | 1.7796 | 40 |
| 21 | 4738 | 2.1107 | 4953 | 2.0189 | 5172 | 1.9333 | 5396 | 1.8533 | 5623 | 1.7783 | 39 |
| 22 | 4741 | 2.1092 | 4957 | 2.0174 | 5176 | 1.9319 | 5399 | 1.8520 | 5627 | 1.7771 | 38 |
| 23 | 4745 | 2.1076 | 4960 | 2.0160 | 5180 | 1.9306 | 5403 | 1.8507 | 5631 | 1.7759 | 37 |
| 24 | 4748 | 2.1060 | 4964 | 2.0145 | 5184 | 1.9292 | 5407 | 1.8495 | 5635 | 1.7747 | 36 |
| 25 | 4752 | 2.1044 | 4968 | 2.0130 | 5187 | 1.9278 | 5411 | 1.8482 | 5639 | 1.7735 | 35 |
| 26 | 4755 | 2.1028 | 4971 | 2.0115 | 5191 | 1.9265 | 5415 | 1.8469 | 5642 | 1.7723 | 34 |
| 27 | 4759 | 2.1013 | 4975 | 2.0101 | 5195 | 1.9251 | 5418 | 1.8456 | 5646 | 1.7711 | 33 |
| 28 | 4763 | 2.0997 | 4979 | 2.0086 | 5198 | 1.9237 | 5422 | 1.8443 | 5650 | 1.7699 | 32 |
| 29 | 4766 | 2.0981 | 4982 | 2.0072 | 5202 | 1.9223 | 5426 | 1.8430 | 5654 | 1.7687 | 31 |
| 30 | 4770 | 2.0965 | 4986 | 2.0057 | 5206 | 1.9210 | 5430 | 1.8418 | 5658 | 1.7675 | 30 |
| 31 | 4773 | 2.0950 | 4989 | 2.0042 | 5209 | 1.9196 | 5433 | 1.8405 | 5662 | 1.7663 | 29 |
| 32 | 4777 | 2.0934 | 4993 | 2.0028 | 5213 | 1.9183 | 5437 | 1.8392 | 5665 | 1.7651 | 28 |
| 33 | 4780 | 2.0918 | 4997 | 2.0013 | 5217 | 1.9169 | 5441 | 1.8379 | 5669 | 1.7639 | 27 |
| 34 | 4784 | 2.0903 | 5000 | 1.9999 | 5220 | 1.9155 | 5445 | 1.8367 | 5673 | 1.7627 | 26 |
| 35 | 4788 | 2.0887 | 5004 | 1.9984 | 5224 | 1.9142 | 5448 | 1.8354 | 5677 | 1.7615 | 25 |
| 36 | 4791 | 2.0872 | 5008 | 1.9970 | 5228 | 1.9128 | 5452 | 1.8341 | 5681 | 1.7603 | 24 |
| 37 | 4795 | 2.0856 | 5011 | 1.9955 | 5232 | 1.9115 | 5456 | 1.8329 | 5685 | 1.7591 | 23 |
| 38 | 4798 | 2.0840 | 5015 | 1.9941 | 5235 | 1.9101 | 5460 | 1.8316 | 5688 | 1.7579 | 22 |
| 39 | 4802 | 2.0825 | 5019 | 1.9926 | 5239 | 1.9088 | 5464 | 1.8303 | 5692 | 1.7567 | 21 |
| 40 | 4806 | 2.0809 | 5022 | 1.9912 | 5243 | 1.9074 | 5467 | 1.8291 | 5696 | 1.7556 | 20 |
| 41 | 4809 | 2.0794 | 5026 | 1.9897 | 5246 | 1.9061 | 5471 | 1.8278 | 5700 | 1.7544 | 19 |
| 42 | 4813 | 2.0778 | 5029 | 1.9883 | 5250 | 1.9047 | 5475 | 1.8265 | 5704 | 1.7532 | 18 |
| 43 | 4816 | 2.0763 | 5033 | 1.9868 | 5254 | 1.9034 | 5479 | 1.8253 | 5708 | 1.7520 | 17 |
| 44 | 4820 | 2.0748 | 5037 | 1.9854 | 5258 | 1.9020 | 5482 | 1.8240 | 5712 | 1.7508 | 16 |
| 45 | 4823 | 2.0732 | 5040 | 1.9840 | 5261 | 1.9007 | 5486 | 1.8228 | 5715 | 1.7496 | 15 |
| 46 | 4827 | 2.0717 | 5044 | 1.9825 | 5265 | 1.8993 | 5490 | 1.8215 | 5719 | 1.7485 | 14 |
| 47 | 4831 | 2.0701 | 5048 | 1.9811 | 5269 | 1.8980 | 5494 | 1.8202 | 5723 | 1.7473 | 13 |
| 48 | 4834 | 2.0686 | 5051 | 1.9797 | 5272 | 1.8967 | 5498 | 1.8190 | 5727 | 1.7461 | 12 |
| 49 | 4838 | 2.0671 | 5055 | 1.9782 | 5276 | 1.8953 | 5501 | 1.8177 | 5731 | 1.7449 | 11 |
| 50 | 4841 | 2.0655 | 5059 | 1.9768 | 5280 | 1.8940 | 5505 | 1.8165 | 5735 | 1.7437 | 10 |
| 51 | 4845 | 2.0640 | 5062 | 1.9754 | 5284 | 1.8927 | 5509 | 1.8152 | 5739 | 1.7426 | 9 |
| 52 | 4849 | 2.0625 | 5066 | 1.9740 | 5287 | 1.8913 | 5513 | 1.8140 | 5743 | 1.7414 | 8 |
| 53 | 4852 | 2.0609 | 5070 | 1.9725 | 5291 | 1.8900 | 5517 | 1.8127 | 5746 | 1.7402 | 7 |
| 54 | 4856 | 2.0594 | 5073 | 1.9711 | 5295 | 1.8887 | 5520 | 1.8115 | 5750 | 1.7391 | 6 |
| 55 | 4859 | 2.0579 | 5077 | 1.9697 | 5298 | 1.8873 | 5524 | 1.8103 | 5754 | 1.7379 | 5 |
| 56 | 4863 | 2.0564 | 5081 | 1.9683 | 5302 | 1.8860 | 5528 | 1.8090 | 5758 | 1.7367 | 4 |
| 57 | 4867 | 2.0549 | 5084 | 1.9669 | 5306 | 1.8847 | 5532 | 1.8078 | 5762 | 1.7355 | 3 |
| 58 | 4870 | 2.0533 | 5088 | 1.9654 | 5310 | 1.8834 | 5535 | 1.8065 | 5766 | 1.7344 | 2 |
| 59 | 4874 | 2.0518 | 5092 | 1.9640 | 5313 | 1.8820 | 5539 | 1.8053 | 5770 | 1.7332 | 1 |
| 60 | 4877 | 2.0503 | 5095 | 1.9626 | 5317 | 1.8807 | 5543 | 1.8040 | 5774 | 1.7321 | 0 |
| ° | 64° | | 63° | | 62° | | 61° | | 60° | | ° |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |

| ° | 30° | | 31° | | 32° | | 33° | | 34° | | ° |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 5774 | 1.7321 | 6009 | 1.6643 | 6249 | 1.6003 | 6494 | 1.5399 | 6745 | 1.4826 | 60 |
| 1 | 5777 | 1.7309 | 6013 | 1.6632 | 6253 | 1.5993 | 6498 | 1.5389 | 6749 | 1.4816 | 59 |
| 2 | 5781 | 1.7297 | 6017 | 1.6621 | 6257 | 1.5983 | 6502 | 1.5379 | 6754 | 1.4807 | 58 |
| 3 | 5785 | 1.7286 | 6020 | 1.6610 | 6261 | 1.5972 | 6506 | 1.5369 | 6758 | 1.4798 | 57 |
| 4 | 5789 | 1.7274 | 6024 | 1.6599 | 6265 | 1.5962 | 6511 | 1.5359 | 6762 | 1.4788 | 56 |
| 5 | 5793 | 1.7262 | 6028 | 1.6588 | 6269 | 1.5952 | 6515 | 1.5350 | 6766 | 1.4779 | 55 |
| 6 | 5797 | 1.7251 | 6032 | 1.6577 | 6273 | 1.5941 | 6519 | 1.5340 | 6771 | 1.4770 | 54 |
| 7 | 5801 | 1.7239 | 6036 | 1.6566 | 6277 | 1.5931 | 6523 | 1.5330 | 6775 | 1.4761 | 53 |
| 8 | 5805 | 1.7228 | 6040 | 1.6555 | 6281 | 1.5921 | 6527 | 1.5320 | 6779 | 1.4751 | 52 |
| 9 | 5808 | 1.7216 | 6044 | 1.6545 | 6285 | 1.5911 | 6531 | 1.5311 | 6783 | 1.4742 | 51 |
| 10 | 5812 | 1.7205 | 6048 | 1.6534 | 6289 | 1.5900 | 6536 | 1.5301 | 6787 | 1.4733 | 50 |
| 11 | 5816 | 1.7193 | 6052 | 1.6523 | 6293 | 1.5890 | 6540 | 1.5291 | 6792 | 1.4724 | 49 |
| 12 | 5820 | 1.7182 | 6056 | 1.6512 | 6297 | 1.5880 | 6544 | 1.5282 | 6796 | 1.4715 | 48 |
| 13 | 5824 | 1.7170 | 6060 | 1.6501 | 6301 | 1.5869 | 6548 | 1.5272 | 6800 | 1.4705 | 47 |
| 14 | 5828 | 1.7159 | 6064 | 1.6490 | 6305 | 1.5859 | 6552 | 1.5262 | 6805 | 1.4696 | 46 |
| 15 | 5832 | 1.7147 | 6068 | 1.6479 | 6310 | 1.5849 | 6556 | 1.5253 | 6809 | 1.4687 | 45 |
| 16 | 5836 | 1.7136 | 6072 | 1.6469 | 6314 | 1.5839 | 6560 | 1.5243 | 6813 | 1.4678 | 44 |
| 17 | 5840 | 1.7124 | 6076 | 1.6458 | 6318 | 1.5829 | 6565 | 1.5233 | 6817 | 1.4669 | 43 |
| 18 | 5844 | 1.7113 | 6080 | 1.6447 | 6322 | 1.5818 | 6569 | 1.5224 | 6822 | 1.4659 | 42 |
| 19 | 5847 | 1.7102 | 6084 | 1.6436 | 6326 | 1.5808 | 6573 | 1.5214 | 6826 | 1.4650 | 41 |
| 20 | 5851 | 1.7090 | 6088 | 1.6426 | 6330 | 1.5798 | 6577 | 1.5204 | 6830 | 1.4641 | 40 |
| 21 | 5855 | 1.7079 | 6092 | 1.6415 | 6334 | 1.5788 | 6581 | 1.5195 | 6834 | 1.4632 | 39 |
| 22 | 5859 | 1.7067 | 6096 | 1.6404 | 6338 | 1.5778 | 6585 | 1.5185 | 6839 | 1.4623 | 38 |
| 23 | 5863 | 1.7056 | 6100 | 1.6393 | 6342 | 1.5768 | 6590 | 1.5175 | 6843 | 1.4614 | 37 |
| 24 | 5867 | 1.7045 | 6104 | 1.6383 | 6346 | 1.5757 | 6594 | 1.5166 | 6847 | 1.4605 | 36 |
| 25 | 5871 | 1.7033 | 6108 | 1.6372 | 6350 | 1.5747 | 6598 | 1.5156 | 6851 | 1.4596 | 35 |
| 26 | 5875 | 1.7022 | 6112 | 1.6361 | 6354 | 1.5737 | 6602 | 1.5147 | 6856 | 1.4586 | 34 |
| 27 | 5879 | 1.7011 | 6116 | 1.6351 | 6358 | 1.5727 | 6606 | 1.5137 | 6860 | 1.4577 | 33 |
| 28 | 5883 | 1.6999 | 6120 | 1.6340 | 6363 | 1.5717 | 6610 | 1.5127 | 6864 | 1.4568 | 32 |
| 29 | 5887 | 1.6988 | 6124 | 1.6329 | 6367 | 1.5707 | 6615 | 1.5118 | 6869 | 1.4559 | 31 |
| 30 | 5890 | 1.6977 | 6128 | 1.6319 | 6371 | 1.5697 | 6619 | 1.5108 | 6873 | 1.4550 | 30 |
| 31 | 5894 | 1.6965 | 6132 | 1.6308 | 6375 | 1.5687 | 6623 | 1.5099 | 6877 | 1.4541 | 29 |
| 32 | 5898 | 1.6954 | 6136 | 1.6297 | 6379 | 1.5677 | 6627 | 1.5089 | 6881 | 1.4532 | 28 |
| 33 | 5902 | 1.6943 | 6140 | 1.6287 | 6383 | 1.5667 | 6631 | 1.5080 | 6886 | 1.4523 | 27 |
| 34 | 5906 | 1.6932 | 6144 | 1.6276 | 6387 | 1.5657 | 6636 | 1.5070 | 6890 | 1.4514 | 26 |
| 35 | 5910 | 1.6920 | 6148 | 1.6265 | 6391 | 1.5647 | 6640 | 1.5061 | 6894 | 1.4505 | 25 |
| 36 | 5914 | 1.6909 | 6152 | 1.6255 | 6395 | 1.5637 | 6644 | 1.5051 | 6899 | 1.4496 | 24 |
| 37 | 5918 | 1.6898 | 6156 | 1.6244 | 6399 | 1.5627 | 6648 | 1.5042 | 6903 | 1.4487 | 23 |
| 38 | 5922 | 1.6887 | 6160 | 1.6234 | 6403 | 1.5617 | 6652 | 1.5032 | 6907 | 1.4478 | 22 |
| 39 | 5926 | 1.6875 | 6164 | 1.6223 | 6408 | 1.5607 | 6657 | 1.5023 | 6911 | 1.4469 | 21 |
| 40 | 5930 | 1.6864 | 6168 | 1.6212 | 6412 | 1.5597 | 6661 | 1.5013 | 6916 | 1.4460 | 20 |
| 41 | 5934 | 1.6853 | 6172 | 1.6202 | 6416 | 1.5587 | 6665 | 1.5004 | 6920 | 1.4451 | 19 |
| 42 | 5938 | 1.6842 | 6176 | 1.6191 | 6420 | 1.5577 | 6669 | 1.4994 | 6924 | 1.4442 | 18 |
| 43 | 5942 | 1.6831 | 6180 | 1.6181 | 6424 | 1.5567 | 6673 | 1.4985 | 6929 | 1.4433 | 17 |
| 44 | 5945 | 1.6820 | 6184 | 1.6170 | 6428 | 1.5557 | 6678 | 1.4975 | 6933 | 1.4424 | 16 |
| 45 | 5949 | 1.6808 | 6188 | 1.6160 | 6432 | 1.5547 | 6682 | 1.4966 | 6937 | 1.4415 | 15 |
| 46 | 5953 | 1.6797 | 6192 | 1.6149 | 6436 | 1.5537 | 6686 | 1.4957 | 6942 | 1.4406 | 14 |
| 47 | 5957 | 1.6786 | 6196 | 1.6139 | 6440 | 1.5527 | 6690 | 1.4947 | 6946 | 1.4397 | 13 |
| 48 | 5961 | 1.6775 | 6200 | 1.6128 | 6445 | 1.5517 | 6694 | 1.4938 | 6950 | 1.4388 | 12 |
| 49 | 5965 | 1.6764 | 6204 | 1.6118 | 6449 | 1.5507 | 6699 | 1.4928 | 6954 | 1.4379 | 11 |
| 50 | 5969 | 1.6753 | 6208 | 1.6107 | 6453 | 1.5497 | 6703 | 1.4919 | 6959 | 1.4370 | 10 |
| 51 | 5973 | 1.6742 | 6212 | 1.6097 | 6457 | 1.5487 | 6707 | 1.4910 | 6963 | 1.4361 | 9 |
| 52 | 5977 | 1.6731 | 6216 | 1.6087 | 6461 | 1.5477 | 6711 | 1.4900 | 6967 | 1.4352 | 8 |
| 53 | 5981 | 1.6720 | 6220 | 1.6076 | 6465 | 1.5468 | 6716 | 1.4891 | 6972 | 1.4344 | 7 |
| 54 | 5985 | 1.6709 | 6224 | 1.6066 | 6469 | 1.5458 | 6720 | 1.4882 | 6976 | 1.4335 | 6 |
| 55 | 5989 | 1.6698 | 6228 | 1.6055 | 6473 | 1.5448 | 6724 | 1.4872 | 6980 | 1.4326 | 5 |
| 56 | 5993 | 1.6687 | 6233 | 1.6045 | 6478 | 1.5438 | 6728 | 1.4863 | 6985 | 1.4317 | 4 |
| 57 | 5997 | 1.6676 | 6237 | 1.6034 | 6482 | 1.5428 | 6732 | 1.4854 | 6989 | 1.4308 | 3 |
| 58 | 6001 | 1.6665 | 6241 | 1.6024 | 6486 | 1.5418 | 6737 | 1.4844 | 6993 | 1.4299 | 2 |
| 59 | 6005 | 1.6654 | 6245 | 1.6014 | 6490 | 1.5408 | 6741 | 1.4835 | 6998 | 1.4290 | 1 |
| 60 | 6009 | 1.6643 | 6249 | 1.6003 | 6494 | 1.5399 | 6745 | 1.4826 | 7002 | 1.4281 | 0 |
| ° | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | ° |
| | 59° | | 58° | | 57° | | 56° | | 55° | | |

| ° | 35° | | 36° | | 37° | | 38° | | 39° | | ° |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 7002 | 1.4281 | 7265 | 1.3764 | 7536 | 1.3270 | 7813 | 1.2799 | 8098 | 1.2349 | 60 |
| 1 | 7006 | 1.4273 | 7270 | 1.3755 | 7540 | 1.3262 | 7818 | 1.2792 | 8103 | 1.2342 | 59 |
| 2 | 7011 | 1.4264 | 7274 | 1.3747 | 7545 | 1.3254 | 7822 | 1.2784 | 8107 | 1.2334 | 58 |
| 3 | 7015 | 1.4255 | 7279 | 1.3739 | 7549 | 1.3246 | 7827 | 1.2776 | 8112 | 1.2327 | 57 |
| 4 | 7019 | 1.4246 | 7283 | 1.3730 | 7554 | 1.3238 | 7832 | 1.2769 | 8117 | 1.2320 | 56 |
| 5 | 7024 | 1.4237 | 7288 | 1.3722 | 7558 | 1.3230 | 7836 | 1.2761 | 8122 | 1.2312 | 55 |
| 6 | 7028 | 1.4229 | 7292 | 1.3713 | 7563 | 1.3222 | 7841 | 1.2753 | 8127 | 1.2305 | 54 |
| 7 | 7032 | 1.4220 | 7297 | 1.3705 | 7568 | 1.3214 | 7846 | 1.2746 | 8132 | 1.2298 | 53 |
| 8 | 7037 | 1.4211 | 7301 | 1.3697 | 7572 | 1.3206 | 7850 | 1.2738 | 8136 | 1.2290 | 52 |
| 9 | 7041 | 1.4202 | 7306 | 1.3688 | 7577 | 1.3198 | 7855 | 1.2731 | 8141 | 1.2283 | 51 |
| 10 | 7046 | 1.4193 | 7310 | 1.3680 | 7581 | 1.3190 | 7860 | 1.2723 | 8146 | 1.2276 | 50 |
| 11 | 7050 | 1.4185 | 7314 | 1.3672 | 7586 | 1.3182 | 7865 | 1.2715 | 8151 | 1.2268 | 49 |
| 12 | 7054 | 1.4176 | 7319 | 1.3663 | 7590 | 1.3175 | 7869 | 1.2708 | 8156 | 1.2261 | 48 |
| 13 | 7059 | 1.4167 | 7323 | 1.3655 | 7595 | 1.3167 | 7874 | 1.2700 | 8161 | 1.2254 | 47 |
| 14 | 7063 | 1.4158 | 7328 | 1.3647 | 7600 | 1.3159 | 7879 | 1.2693 | 8165 | 1.2247 | 46 |
| 15 | 7067 | 1.4150 | 7332 | 1.3638 | 7604 | 1.3151 | 7883 | 1.2685 | 8170 | 1.2239 | 45 |
| 16 | 7072 | 1.4141 | 7337 | 1.3630 | 7609 | 1.3143 | 7888 | 1.2677 | 8175 | 1.2232 | 44 |
| 17 | 7076 | 1.4132 | 7341 | 1.3622 | 7613 | 1.3135 | 7893 | 1.2670 | 8180 | 1.2225 | 43 |
| 18 | 7080 | 1.4124 | 7346 | 1.3613 | 7618 | 1.3127 | 7898 | 1.2662 | 8185 | 1.2218 | 42 |
| 19 | 7085 | 1.4115 | 7350 | 1.3605 | 7623 | 1.3119 | 7902 | 1.2655 | 8190 | 1.2210 | 41 |
| 20 | 7089 | 1.4106 | 7355 | 1.3597 | 7627 | 1.3111 | 7907 | 1.2647 | 8195 | 1.2203 | 40 |
| 21 | 7094 | 1.4097 | 7359 | 1.3588 | 7632 | 1.3103 | 7912 | 1.2640 | 8199 | 1.2196 | 39 |
| 22 | 7098 | 1.4089 | 7364 | 1.3580 | 7636 | 1.3095 | 7916 | 1.2632 | 8204 | 1.2189 | 38 |
| 23 | 7102 | 1.4080 | 7368 | 1.3572 | 7641 | 1.3087 | 7921 | 1.2624 | 8209 | 1.2181 | 37 |
| 24 | 7107 | 1.4071 | 7373 | 1.3564 | 7646 | 1.3079 | 7926 | 1.2617 | 8214 | 1.2174 | 36 |
| 25 | 7111 | 1.4063 | 7377 | 1.3555 | 7650 | 1.3072 | 7931 | 1.2609 | 8219 | 1.2167 | 35 |
| 26 | 7115 | 1.4054 | 7382 | 1.3547 | 7655 | 1.3064 | 7935 | 1.2602 | 8224 | 1.2160 | 34 |
| 27 | 7120 | 1.4045 | 7386 | 1.3539 | 7659 | 1.3056 | 7940 | 1.2594 | 8229 | 1.2153 | 33 |
| 28 | 7124 | 1.4037 | 7391 | 1.3531 | 7664 | 1.3048 | 7945 | 1.2587 | 8234 | 1.2145 | 32 |
| 29 | 7129 | 1.4028 | 7395 | 1.3522 | 7669 | 1.3040 | 7950 | 1.2579 | 8238 | 1.2138 | 31 |
| 30 | 7133 | 1.4019 | 7400 | 1.3514 | 7673 | 1.3032 | 7954 | 1.2572 | 8243 | 1.2131 | 30 |
| 31 | 7137 | 1.4011 | 7404 | 1.3506 | 7678 | 1.3024 | 7959 | 1.2564 | 8248 | 1.2124 | 29 |
| 32 | 7142 | 1.4002 | 7409 | 1.3498 | 7683 | 1.3017 | 7964 | 1.2557 | 8253 | 1.2117 | 28 |
| 33 | 7146 | 1.3994 | 7413 | 1.3490 | 7687 | 1.3009 | 7969 | 1.2549 | 8258 | 1.2109 | 27 |
| 34 | 7151 | 1.3985 | 7418 | 1.3481 | 7692 | 1.3001 | 7973 | 1.2542 | 8263 | 1.2102 | 26 |
| 35 | 7155 | 1.3976 | 7422 | 1.3473 | 7696 | 1.2993 | 7978 | 1.2534 | 8268 | 1.2095 | 25 |
| 36 | 7159 | 1.3968 | 7427 | 1.3465 | 7701 | 1.2985 | 7983 | 1.2527 | 8273 | 1.2088 | 24 |
| 37 | 7164 | 1.3959 | 7431 | 1.3457 | 7706 | 1.2977 | 7988 | 1.2519 | 8278 | 1.2081 | 23 |
| 38 | 7168 | 1.3951 | 7436 | 1.3449 | 7710 | 1.2970 | 7992 | 1.2512 | 8283 | 1.2074 | 22 |
| 39 | 7173 | 1.3942 | 7440 | 1.3440 | 7715 | 1.2962 | 7997 | 1.2504 | 8287 | 1.2066 | 21 |
| 40 | 7177 | 1.3934 | 7445 | 1.3432 | 7720 | 1.2954 | 8002 | 1.2497 | 8292 | 1.2059 | 20 |
| 41 | 7181 | 1.3925 | 7449 | 1.3424 | 7724 | 1.2946 | 8007 | 1.2489 | 8297 | 1.2052 | 19 |
| 42 | 7186 | 1.3916 | 7454 | 1.3416 | 7729 | 1.2938 | 8012 | 1.2482 | 8302 | 1.2045 | 18 |
| 43 | 7190 | 1.3908 | 7458 | 1.3408 | 7734 | 1.2931 | 8016 | 1.2475 | 8307 | 1.2038 | 17 |
| 44 | 7195 | 1.3899 | 7463 | 1.3400 | 7738 | 1.2923 | 8021 | 1.2467 | 8312 | 1.2031 | 16 |
| 45 | 7199 | 1.3891 | 7467 | 1.3392 | 7743 | 1.2915 | 8026 | 1.2460 | 8317 | 1.2024 | 15 |
| 46 | 7203 | 1.3882 | 7472 | 1.3384 | 7747 | 1.2907 | 8031 | 1.2452 | 8322 | 1.2017 | 14 |
| 47 | 7208 | 1.3874 | 7476 | 1.3375 | 7752 | 1.2900 | 8035 | 1.2445 | 8327 | 1.2009 | 13 |
| 48 | 7212 | 1.3865 | 7481 | 1.3367 | 7757 | 1.2892 | 8040 | 1.2437 | 8332 | 1.2002 | 12 |
| 49 | 7217 | 1.3857 | 7485 | 1.3359 | 7761 | 1.2884 | 8045 | 1.2430 | 8337 | 1.1995 | 11 |
| 50 | 7221 | 1.3848 | 7490 | 1.3351 | 7766 | 1.2876 | 8050 | 1.2423 | 8342 | 1.1988 | 10 |
| 51 | 7226 | 1.3840 | 7495 | 1.3343 | 7771 | 1.2869 | 8055 | 1.2415 | 8346 | 1.1981 | 9 |
| 52 | 7230 | 1.3831 | 7499 | 1.3335 | 7775 | 1.2861 | 8059 | 1.2408 | 8351 | 1.1974 | 8 |
| 53 | 7234 | 1.3823 | 7504 | 1.3327 | 7780 | 1.2853 | 8064 | 1.2401 | 8356 | 1.1967 | 7 |
| 54 | 7239 | 1.3814 | 7508 | 1.3319 | 7785 | 1.2846 | 8069 | 1.2393 | 8361 | 1.1960 | 6 |
| 55 | 7243 | 1.3806 | 7513 | 1.3311 | 7789 | 1.2838 | 8074 | 1.2386 | 8366 | 1.1953 | 5 |
| 56 | 7248 | 1.3798 | 7517 | 1.3303 | 7794 | 1.2830 | 8079 | 1.2378 | 8371 | 1.1946 | 4 |
| 57 | 7252 | 1.3789 | 7522 | 1.3295 | 7799 | 1.2822 | 8083 | 1.2371 | 8376 | 1.1939 | 3 |
| 58 | 7257 | 1.3781 | 7526 | 1.3287 | 7803 | 1.2815 | 8088 | 1.2364 | 8381 | 1.1932 | 2 |
| 59 | 7261 | 1.3772 | 7531 | 1.3278 | 7808 | 1.2807 | 8093 | 1.2356 | 8386 | 1.1925 | 1 |
| 60 | 7265 | 1.3764 | 7536 | 1.3270 | 7813 | 1.2799 | 8098 | 1.2349 | 8391 | 1.1918 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| | 54° | | 53° | | 52° | | 51° | | 50° | | |

| | 40° | | 41° | | 42° | | 43° | | 44° | | |
|----|------|--------|------|--------|------|--------|------|--------|------|--------|----|
| | tan | cot | tan | cot | tan | cot | tan | cot | tan | cot | |
| 0 | 8391 | 1.1918 | 8693 | 1.1504 | 9004 | 1.1106 | 9325 | 1.0724 | 9657 | 1.0355 | 30 |
| 1 | 8396 | 1.1910 | 8698 | 1.1497 | 9009 | 1.1100 | 9331 | 1.0717 | 9663 | 1.0349 | 59 |
| 2 | 8401 | 1.1903 | 8703 | 1.1490 | 9015 | 1.1093 | 9336 | 1.0711 | 9668 | 1.0343 | 58 |
| 3 | 8406 | 1.1896 | 8708 | 1.1483 | 9020 | 1.1087 | 9341 | 1.0705 | 9674 | 1.0337 | 57 |
| 4 | 8411 | 1.1889 | 8713 | 1.1477 | 9025 | 1.1080 | 9347 | 1.0699 | 9679 | 1.0331 | 56 |
| 5 | 8416 | 1.1882 | 8718 | 1.1470 | 9030 | 1.1074 | 9352 | 1.0692 | 9685 | 1.0325 | 55 |
| 6 | 8421 | 1.1875 | 8724 | 1.1463 | 9036 | 1.1067 | 9358 | 1.0686 | 9691 | 1.0319 | 54 |
| 7 | 8426 | 1.1868 | 8729 | 1.1456 | 9041 | 1.1061 | 9363 | 1.0680 | 9696 | 1.0313 | 53 |
| 8 | 8431 | 1.1861 | 8734 | 1.1450 | 9046 | 1.1054 | 9369 | 1.0674 | 9702 | 1.0307 | 52 |
| 9 | 8436 | 1.1854 | 8739 | 1.1443 | 9052 | 1.1048 | 9374 | 1.0668 | 9708 | 1.0301 | 51 |
| 10 | 8441 | 1.1847 | 8744 | 1.1436 | 9057 | 1.1041 | 9380 | 1.0661 | 9713 | 1.0295 | 50 |
| 11 | 8446 | 1.1840 | 8749 | 1.1430 | 9062 | 1.1035 | 9385 | 1.0655 | 9719 | 1.0289 | 49 |
| 12 | 8451 | 1.1833 | 8754 | 1.1423 | 9067 | 1.1028 | 9391 | 1.0649 | 9725 | 1.0283 | 48 |
| 13 | 8456 | 1.1826 | 8759 | 1.1416 | 9073 | 1.1022 | 9396 | 1.0643 | 9730 | 1.0277 | 47 |
| 14 | 8461 | 1.1819 | 8765 | 1.1410 | 9078 | 1.1016 | 9402 | 1.0637 | 9736 | 1.0271 | 46 |
| 15 | 8466 | 1.1812 | 8770 | 1.1403 | 9083 | 1.1009 | 9407 | 1.0630 | 9742 | 1.0265 | 45 |
| 16 | 8471 | 1.1806 | 8775 | 1.1396 | 9089 | 1.1003 | 9413 | 1.0624 | 9747 | 1.0259 | 44 |
| 17 | 8476 | 1.1799 | 8780 | 1.1389 | 9094 | 1.0996 | 9418 | 1.0618 | 9753 | 1.0253 | 43 |
| 18 | 8481 | 1.1792 | 8785 | 1.1383 | 9099 | 1.0990 | 9424 | 1.0612 | 9759 | 1.0247 | 42 |
| 19 | 8486 | 1.1785 | 8790 | 1.1376 | 9105 | 1.0983 | 9429 | 1.0606 | 9764 | 1.0241 | 41 |
| 20 | 8491 | 1.1778 | 8796 | 1.1369 | 9110 | 1.0977 | 9435 | 1.0599 | 9770 | 1.0235 | 40 |
| 21 | 8496 | 1.1771 | 8801 | 1.1363 | 9115 | 1.0971 | 9440 | 1.0593 | 9776 | 1.0230 | 39 |
| 22 | 8501 | 1.1764 | 8806 | 1.1356 | 9121 | 1.0964 | 9446 | 1.0587 | 9781 | 1.0224 | 38 |
| 23 | 8506 | 1.1757 | 8811 | 1.1349 | 9126 | 1.0958 | 9451 | 1.0581 | 9787 | 1.0218 | 37 |
| 24 | 8511 | 1.1750 | 8816 | 1.1343 | 9131 | 1.0951 | 9457 | 1.0575 | 9793 | 1.0212 | 36 |
| 25 | 8516 | 1.1743 | 8821 | 1.1336 | 9137 | 1.0945 | 9462 | 1.0569 | 9798 | 1.0206 | 35 |
| 26 | 8521 | 1.1736 | 8827 | 1.1329 | 9142 | 1.0939 | 9468 | 1.0562 | 9804 | 1.0200 | 34 |
| 27 | 8526 | 1.1729 | 8832 | 1.1323 | 9147 | 1.0932 | 9473 | 1.0556 | 9810 | 1.0194 | 33 |
| 28 | 8531 | 1.1722 | 8837 | 1.1316 | 9153 | 1.0926 | 9479 | 1.0550 | 9816 | 1.0188 | 32 |
| 29 | 8536 | 1.1715 | 8842 | 1.1310 | 9158 | 1.0919 | 9484 | 1.0544 | 9821 | 1.0182 | 31 |
| 30 | 8541 | 1.1708 | 8847 | 1.1303 | 9163 | 1.0913 | 9490 | 1.0538 | 9827 | 1.0176 | 30 |
| 31 | 8546 | 1.1702 | 8852 | 1.1296 | 9169 | 1.0907 | 9495 | 1.0532 | 9833 | 1.0170 | 29 |
| 32 | 8551 | 1.1695 | 8858 | 1.1290 | 9174 | 1.0900 | 9501 | 1.0526 | 9838 | 1.0164 | 28 |
| 33 | 8556 | 1.1688 | 8863 | 1.1283 | 9179 | 1.0894 | 9506 | 1.0519 | 9844 | 1.0158 | 27 |
| 34 | 8561 | 1.1681 | 8868 | 1.1276 | 9185 | 1.0888 | 9512 | 1.0513 | 9850 | 1.0152 | 26 |
| 35 | 8566 | 1.1674 | 8873 | 1.1270 | 9190 | 1.0881 | 9517 | 1.0507 | 9856 | 1.0147 | 25 |
| 36 | 8571 | 1.1667 | 8878 | 1.1263 | 9195 | 1.0875 | 9523 | 1.0501 | 9861 | 1.0141 | 24 |
| 37 | 8576 | 1.1660 | 8884 | 1.1257 | 9201 | 1.0869 | 9528 | 1.0495 | 9867 | 1.0135 | 23 |
| 38 | 8581 | 1.1653 | 8889 | 1.1250 | 9206 | 1.0862 | 9534 | 1.0489 | 9873 | 1.0129 | 22 |
| 39 | 8586 | 1.1647 | 8894 | 1.1243 | 9212 | 1.0856 | 9540 | 1.0483 | 9879 | 1.0123 | 21 |
| 40 | 8591 | 1.1640 | 8899 | 1.1237 | 9217 | 1.0850 | 9545 | 1.0477 | 9884 | 1.0117 | 20 |
| 41 | 8596 | 1.1633 | 8904 | 1.1230 | 9222 | 1.0843 | 9551 | 1.0470 | 9890 | 1.0111 | 19 |
| 42 | 8601 | 1.1626 | 8910 | 1.1224 | 9228 | 1.0837 | 9556 | 1.0464 | 9896 | 1.0105 | 18 |
| 43 | 8606 | 1.1619 | 8915 | 1.1217 | 9233 | 1.0831 | 9562 | 1.0458 | 9902 | 1.0099 | 17 |
| 44 | 8611 | 1.1612 | 8920 | 1.1211 | 9239 | 1.0824 | 9567 | 1.0452 | 9907 | 1.0094 | 16 |
| 45 | 8617 | 1.1606 | 8925 | 1.1204 | 9244 | 1.0818 | 9573 | 1.0446 | 9913 | 1.0088 | 15 |
| 46 | 8622 | 1.1599 | 8931 | 1.1197 | 9249 | 1.0812 | 9578 | 1.0440 | 9919 | 1.0082 | 14 |
| 47 | 8627 | 1.1592 | 8936 | 1.1191 | 9255 | 1.0805 | 9584 | 1.0434 | 9925 | 1.0076 | 13 |
| 48 | 8632 | 1.1585 | 8941 | 1.1184 | 9260 | 1.0799 | 9590 | 1.0428 | 9930 | 1.0070 | 12 |
| 49 | 8637 | 1.1578 | 8946 | 1.1178 | 9266 | 1.0793 | 9595 | 1.0422 | 9936 | 1.0064 | 11 |
| 50 | 8642 | 1.1571 | 8952 | 1.1171 | 9271 | 1.0786 | 9601 | 1.0416 | 9942 | 1.0058 | 10 |
| 51 | 8647 | 1.1565 | 8957 | 1.1165 | 9276 | 1.0780 | 9606 | 1.0410 | 9948 | 1.0052 | 9 |
| 52 | 8652 | 1.1558 | 8962 | 1.1158 | 9282 | 1.0774 | 9612 | 1.0404 | 9954 | 1.0047 | 8 |
| 53 | 8657 | 1.1551 | 8967 | 1.1152 | 9287 | 1.0768 | 9618 | 1.0398 | 9959 | 1.0041 | 7 |
| 54 | 8662 | 1.1544 | 8972 | 1.1145 | 9293 | 1.0761 | 9623 | 1.0392 | 9965 | 1.0035 | 6 |
| 55 | 8667 | 1.1538 | 8978 | 1.1139 | 9298 | 1.0755 | 9629 | 1.0385 | 9971 | 1.0029 | 5 |
| 56 | 8672 | 1.1531 | 8983 | 1.1132 | 9303 | 1.0749 | 9634 | 1.0379 | 9977 | 1.0023 | 4 |
| 57 | 8678 | 1.1524 | 8988 | 1.1126 | 9309 | 1.0742 | 9640 | 1.0373 | 9983 | 1.0017 | 3 |
| 58 | 8683 | 1.1517 | 8994 | 1.1119 | 9314 | 1.0736 | 9646 | 1.0367 | 9988 | 1.0012 | 2 |
| 59 | 8688 | 1.1510 | 8999 | 1.1113 | 9320 | 1.0730 | 9651 | 1.0361 | 9994 | 1.0006 | 1 |
| 60 | 8693 | 1.1504 | 9004 | 1.1106 | 9325 | 1.0724 | 9657 | 1.0355 | 1000 | 1.0000 | 0 |
| | cot | tan | cot | tan | cot | tan | cot | tan | cot | tan | |
| | 49° | | 48° | | 47° | | 46° | | 45° | | |

INDEX.

| | PAGE |
|---|-------------------|
| Agonic line | 201 |
| Alignment in mines | 381-384 |
| obstacles to and problems on | 17, 120, 304, 370 |
| street..... | 360 |
| Angle of deflection | 108 |
| of depression; of elevation..... | 55 |
| horizontal; vertical..... | 55 |
| measurement between lines | 112 |
| measurement by repetition..... | 106 |
| measurement by series | 107 |
| traversing | 108 |
| problems | 113 |
| Areas, determination of | 35, 183, 187, 196 |
| Azimuth, determination by solar attachment..... | 94 |
| table of errors in | 95 |
| Balancing the survey | 161 |
| weights used | 165 |
| Bearings, compass | 102 |
| reverse | 103 |
| proof | 104 |
| changing | 116 |
| when the variation is known | 209 |
| to obtain true | 212 |
| Chain, engineer's | 4 |
| Gunter's; two-pole..... | 3 |
| required length of four-pole | 301 |
| Chaining | 4, 302, 334 |
| Circle, area of | 33, 43 |
| laying out | 228-231, 383, 384 |
| vertical | 388, 389 |
| Clinometer, the hanging | 390, 391 |
| Compass, surveyor's | 55 |
| adjustment..... | 56 |

| | PAGE |
|---|---------------------------------|
| Compass bearings..... | 102 |
| solar | 275 |
| principles and adjustments of solar | 280-285 |
| the hanging | 390 |
| the diurnal variation of | 200 |
| the secular variation of | 201, 208 |
| variation determined by old lines | 210 |
| Contour lines..... | 393 |
| Co-ordinates, application of rectangular..... | 45 |
| Corners, establishing | 304 |
| lot | 369 |
| quarter-section; witness | 307 |
| re-establishment of | 316 |
| Course, initial, reduced | 382 |
| special appliances..... | 390 |
| Cross-sectioning, angular | 394 |
| Curvature and refraction | 350, 351 |
| Curves, how laid out..... | 229, 230, 383, 384 |
| Declination, allowance for | 281 |
| formula expressing | 203 |
| line of no | 201 |
| of the needle | 199 |
| by Polaris | 214 |
| problems | 290, 291 |
| setting off..... | 285, 286 |
| Departure and latitude | 154 |
| correcting..... | 161 |
| Distances, measurement of inaccessible | 135-137 |
| Dividing land | 232 |
| Drainage | 377 |
| Ellipse, area of | 35 |
| laying out | 228-231 |
| Error in chaining | 7, 8 |
| caused by needle..... | 164 |
| the survey | 163 |
| Field notes, recording the | 22, 41, 142, 147, 318, 356, 385 |
| Formulas expressing magnetic declination..... | 203 |
| Grade lines | 357 |
| Grade line marking | 366 |
| Grades, determination of | 374 |
| Gradienter | 100, 329 |
| Graduation | 69 |

INDEX.

83

| | PAGE |
|--|------------------------|
| Heights, measurement of | 20, 129-131 |
| Hexagon, area of regular | 31 |
| Judicial functions of surveyors | 397 |
| Latitude and departure | 154 |
| correcting | 161 |
| difference of | 154 |
| finding | 85 |
| by circumpolar star | 92 |
| by the sun, with Saegmuller's attachment | 91 |
| setting off | 291 |
| Laying out land | 221 |
| Level, adjustment | 340-343 |
| wye | 337 |
| engraving of wye | 338, 339 |
| use of | 343, 387, 388 |
| Locke's hand | 333 |
| rod | 349 |
| Levelling-rods | 345 |
| Levelling defined | 349 |
| fore and back sights | 352, 353 |
| general method | 354, 355 |
| Line, agonic | 201 |
| azimuth of | 55 |
| bearing of | 55 |
| base | 308 |
| meridian | 54 |
| meridian distance of | 55 |
| obstacles in | 17, 122, 124, 126, 304 |
| inaccessibility of one end | 19, 126 |
| inaccessibility of both ends | 20, 127 |
| running a grade | 357 |
| Lines, offset and tie | 44-51 |
| parallel | 16 |
| perpendicular | 13 |
| ranging | 10 |
| marking | 303, 366 |
| preservation of points in | 361 |
| surveying section | 296 |
| surveying township | 298, 299, 301 |
| straightening boundary | 258 |
| variation determined by old | 210 |
| contour | 393 |

| | PAGE |
|--|-----------------------------|
| Local attraction | 103 |
| Longitude, difference of..... | 154 |
| Mapping, <i>see</i> Plotting. | |
| Meandering | 307 |
| Measurement, corrections..... | 335 |
| suggestions on | 336 |
| obstacles to, problems on..... | 18, 124 |
| Meridian, the principal..... | 309 |
| the magnetic | 54 |
| auxiliary | 310 |
| principal base and | 300 |
| establishing with transit..... | 218 |
| establishing with solar attachment..... | 93 |
| obtaining approximately | 219 |
| convergency of | 324 |
| inclination of..... | 322, 323 |
| deflection of range lines from..... | 324 |
| general rule for obtaining the double | 188 |
| Micrometer or stadia wires | 95, 96 |
| Needle, the magnetic..... | 54, 68 |
| adjustment of | 59 |
| declination of | 199 |
| remagnetization of..... | 60 |
| Objects to be noted in surveying public land | 316 |
| Obstacles | 17, 19, 120-127, 304, 370 |
| Octagon, area of regular | 31 |
| Omissions, supplying..... | 166 |
| Parallelogram, area of | 31, 38, 183 |
| laying out | 225, 226 |
| Parallels, standard..... | 300, 309 |
| Perpendiculars and parallels, problems on..... | 13, 15, 17, 120 |
| Pins, marking | 4, 302 |
| Plane-table surveying | 262 |
| Plane-table, engraving of | 263 |
| adjustments | 266 |
| surveying, various methods | 267-269 |
| use in topography | 303 |
| practical suggestions on | 273 |
| exercises | 274 |
| Plans..... | 372 |
| Plot, farm | 153 |
| Plotting | 26, 177, 180, 182, 385, 386 |

| | PAGE |
|--|---------------|
| Poles, or rods, for sighting..... | 4, 331 |
| of the earth | 55 |
| Polygon, area of..... | 31-40, 186 |
| laying out | 227, 228 |
| dividing up | 253-258 |
| Profiles | 363, 364, 373 |
| level notes for..... | 356, 365 |
| Public lands, survey of..... | 275 |
| origin of system of survey | 275, 295 |
| Record of levels for profile | 365 |
| Recording notes, <i>see</i> Field Notes. | |
| Rectangle, area of | 31, 37 |
| laying out | 224, 225 |
| Rectangular surveying | 297 |
| Refraction, allowance for..... | 287 |
| tables | 92, 288 |
| explanation of table of | 290 |
| problems in | 290, 291 |
| curvature and | 350 |
| Ring, area of..... | 37, 43 |
| Rods or poles | 4, 331 |
| levelling | 345, 348 |
| Scale, drawing to..... | 29 |
| to adopt | 30, 385 |
| unknown..... | 30 |
| Scales | 28 |
| Sections | 299 |
| subdividing | 314-316 |
| Segment, area of | 34, 43 |
| Sextant, area of | 34 |
| Shaft, transference of points down..... | 389 |
| Solar compass | 275 |
| engraving of | 277 |
| hour arc, polar axis | 279 |
| principles | 280 |
| adjustments..... | 283-285 |
| to use | 285 |
| running lines with | 292 |
| time for using | 295 |
| time of day by sun with | 294 |
| Solar attachment of the transit (Gurley's) | 79 |
| engraving of | 81 |

| | PAGE |
|--|----------|
| Solar attachment, adjustment of | 80 |
| running lines with | 87 |
| Saegmuller's | 88 |
| adjustment of Saegmuller's | 88, 89 |
| time and azimuth with | 94 |
| Squares, laying out | 223 |
| Stadia wires, or micrometer | 95, 96 |
| level line of sight perpendicular to | 97 |
| line of sight inclined to | 99 |
| Streets, direction and width | 358, 359 |
| alignment of | 360 |
| grade and profile | 362, 363 |
| preservation of points in | 361 |
| Table of areas of regular polygons | 33 |
| errors in azimuth | 95 |
| elongation and azimuth of Polaris | 216, 217 |
| inclination and convergency of meridians | 325 |
| refraction | 92 |
| secular variation | 206 |
| traverse | 156 |

TABLES AT END OF BOOK.

| | |
|---|-------------------|
| Tables of logarithms of numbers | 1-19 |
| approximate equations of time | 20 |
| logarithms of trigonometric functions | 21, 49 |
| for determining with greater accuracy | 50, 51 |
| lengths of degree of latitude and longitude | 52 |
| miscellaneous formulæ, and equivalents of metres, chains, and feet, | 53 |
| traverse | 54-61 |
| natural sines and cosines | 62-70 |
| natural tangents and cotangents | 71-79 |
| Tallying | 3 |
| Tape measures | 4, 332, 380 |
| Telescope, section of | 65, 338, 339 |
| adjusting level on | 78 |
| Testing a survey | 160 |
| Time by solar attachment | 94 |
| Topography | 362, 363, 391-393 |
| Township, surveying boundaries of | 298, 299, 301 |
| subdividing | 311-313 |

INDEX.

87

| | PAGE |
|---|-------------|
| Township plan..... | 296 |
| Transit, showing gradienter, vertical arc, etc..... | 329 |
| surveyor's (full circle) | 64 |
| adjustments..... | 71 |
| measurement of angles | 106 |
| attachments | 77 |
| adjustment of attachments | 78 |
| solar attachment (<i>see</i> Solar Attachment) | 79 |
| as first made | 396 |
| to take apart | 79 |
| Trapezium, area of | 39, 184 |
| dividing | 246-252 |
| Trapezoid, area of | 31, 38, 184 |
| dividing | 241-246 |
| Traverse table..... | 156 |
| Triangle, area of..... | 31, 35, 183 |
| dividing | 232-240 |
| laying out..... | 221-223 |
| Tripod..... | 71 |
| Tripods, extra | 381 |
| Vernier, the | 61 |
| the transit | 69 |
| determination of least count of..... | 61 |
| to read an instrument having | 62 |
| exercises on spacing..... | 62 |
| Vertical circle..... | 77 |
| to adjust | 77 |

89078558939



B89078558939A